# CSC 480: Artificial Intelligence

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### **Course Overview**

- Introduction
- Intelligent Agents
- Search
  - problem solving through search
  - uninformed search
  - informed search
- Games
  - games as search problems
- Knowledge and Reasoning
  - reasoning agents
  - propositional logicpredicate logic
  - knowledge-based systems

- Learning
  - learning from observation
  - neural networks
- Conclusions





# Chapter Overview Logic

- Motivation
- Objectives
- Propositional Logic
  - syntax
  - semantics
  - validity and inference
  - models
  - inference rules
  - complexity
  - limitations
  - Wumpus agents

- Predicate Logic
  - Principles
    - objects
    - relations
    - properties
  - Syntax
  - Semantics
  - Extensions and Variations
  - Usage
- Logic and the Wumpus World
  - reflex agent
  - change
- Important Concepts and Terms
- Chapter Summary





### **Motivation**

- formal methods to perform reasoning are required when dealing with knowledge
- propositional logic is a simple mechanism for basic reasoning tasks
  - it allows the description of the world via sentences
    - simple sentences can be combined into more complex ones
    - new sentences can be generated by inference rules applied to existing sentences
- predicate logic is more powerful, but also considerably more complex
  - it is very general, and can be used to model or emulate many other methods
  - although of high computational complexity, there is a subclass that can be treated by computers reasonably well





### **Objectives**

- know the important aspects of propositional and predicate logic
  - syntax, semantics, models, inference rules, complexity
- understand the limitations of propositional and predicate logic
- apply simple reasoning techniques to specific tasks
- learn about the basic principles of predicate logic
- apply predicate logic to the specification of knowledge-based systems and agents
- use inference rules to deduce new knowledge from existing knowledge bases





### **Logical Inference**

#### also referred to as deduction

- implements the entailment relation for sentences
  - operates at the semantic level
  - takes into account the meaning of sentences
- computers have difficulties reasoning at the semantic level
  - typically work at the syntactic level
  - derivation is used to approximate entailment
  - uses purely "mechanical" symbol manipulation without consideration of meaning
  - should be used with care since more constraints apply





### Validity and Satisfiability

#### validity

- a sentence is valid if it is true under all possible interpretations in all possible world states
  - independent of its intended or assigned meaning
  - independent of the state of affairs in the world under consideration
  - valid sentences are also called tautologies

### satisfiability

 a sentence is satisfiable if there is some interpretation in some world state (a model) such that the sentence is true

### relationship between satisfiability and validity

- a sentence is satisfiable iff ("if and only if") its negation is not valid
- a sentence is valid iff its negation is not satisfiable





# Computational Approaches to Inference

#### model checking based on truth tables

- generate all possible models and check them for validity or satisfiability
- exponential complexity, NP-complete
  - all combinations of truth values need to be considered

#### search

- use inference rules as successor functions for a search algorithm
- also exponential, but only worst-case
  - in practice, many problems have shorter proofs
  - only relevant propositions need to be considered





### **Propositional Logic**

- a relatively simple framework for reasoning
- can be extended for more expressiveness at the cost of computational overhead
- important aspects
  - syntax
  - semantics
  - validity and inference
  - \* models
  - inference rules
  - complexity





### **Syntax**

#### symbols

- logical constants True, False
- propositional symbols P, Q, ...
- logical connectives
  - conjunction A, disjunction V,
  - ❖ negation ¬,
  - implication ⇒, equivalence ⇔
  - there are other connectives
    - unary, binary, n-ary
- parentheses ( )

#### sentences

- constructed from simple sentences
- conjunction, disjunction, implication, equivalence, negation





# BNF Grammar Propositional Logic

Sentence → AtomicSentence | ComplexSentence

AtomicSentence  $\rightarrow$  True | False | P | Q | R | ...

ComplexSentence → (Sentence)

Sentence → | Sentence Connective Sentence

| ¬ Sentence

Connective

$$\rightarrow \land |\lor| \Rightarrow |\Leftrightarrow$$

ambiguities are resolved through precedence ¬ ∧ ∨ ⇒ ⇔ or parentheses

e.g.  $\neg P \lor Q \land R \Rightarrow S$  is equivalent to  $((\neg P) \lor (Q \land R)) \Rightarrow S$ 





### **Semantics**

- interpretation of the propositional symbols and constants
  - symbols can stand for any arbitrary fact
    - sentences consisting of only a propositional symbols are satisfiable, but not valid
      - ◆ the value of the symbol can be True or False
      - must be explicitly stated in the model
  - the constants True and False have a fixed interpretation
    - True indicates that the world is as stated
    - False indicates that the world is not as stated
- specification of the logical connectives
  - frequently explicitly via truth tables





# Truth Tables for Common Connectives

Р	Q	¬P	P ∧ Q	P v Q	$P \Rightarrow Q$	P ⇔ Q
FALSE	FALSE	TRUE	FALSE	FALSE	TRUE	TRUE
FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE
TRUE	FALSE	FALSE	FALSE	TRUE	FALSE	FALSE
TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE





### Validity and Inference

- truth tables can be used to test sentences for validity
  - one row for each possible combination of truth values for the symbols in the sentence
  - the final value must be True for every sentence
  - a variation of the model checking approach
  - in general, not very practical for large sentences
    - can be very effective with customized improvements in specific domains, such as VLSI design





#### known facts about the Wumpus World

- there is a wumpus in [1,3] or in [2,2]
- there is no wumpus in [2,2]

### question (hypothesis)

is there a wumpus in [1,3]

#### \* task

prove or disprove the validity of the question

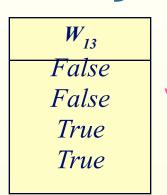
#### approach

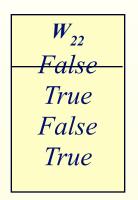
- construct a sentence that combines the above statements in an appropriate manner
  - so that it answers the questions
- construct a truth table that shows if the sentence is valid
  - incremental approach with truth tables for sub-sentences

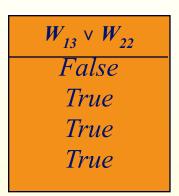




P	Q	$P \lor Q$
False -	False	False
False	True	True
True	False	True
True	True	True







#### Interpretation:

 $W_{13}$  Wumpus in [1,3]

 $W_{22}$  Wumpus in [2,2]

#### Facts:

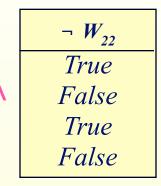
there is a wumpus in [1,3] or in [2,2]





P	Q	$P \wedge Q$
False	False	False
False	True	False
True	False	False
True	True	True

$W_{13} \vee W_{22}$	
False	
True	
True	
True	



#### Interpretation:

 $W_{13}$  Wumpus in [1,3]

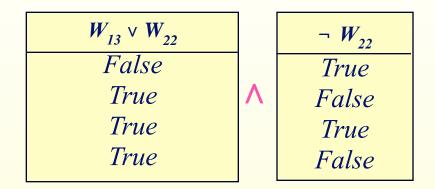
 $W_{22}$  Wumpus in [2,2]

#### Facts:

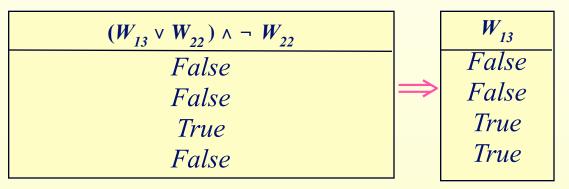
- there is a wumpus in [1,3] or in [2,2]
- there is no wumpus in [2,2]







P	Q	$P \Rightarrow Q$
False	False	True
False	True	True
True	False	False
True	True	True



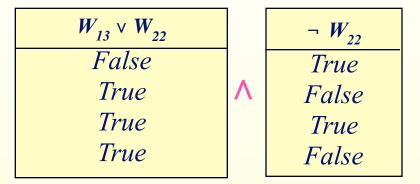
#### **Valid Sentence Definition:**

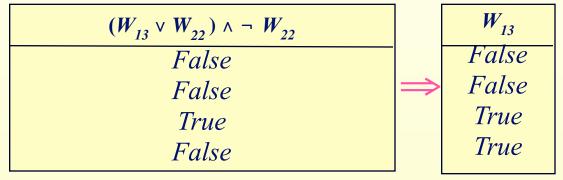
For all possible combinations, the value of the sentence must be true.

### **Question**:

Can we conclude that the wumpus is in [1,3]?







# $((W_{13} \lor W_{22}) \land \neg W_{22}) \Rightarrow W_{13}$ True True True True True

#### **Valid Sentence Definition:**

For all possible combinations, the value of the sentence must be true.



### Validity and Computers

- the computer may not have access to the real world, to check the truth value of sentences (facts)
  - humans often can do that, which greatly decreases the complexity of reasoning
  - humans also have experience in considering only important aspects, neglecting others
- if a conclusion can be drawn from premises, independent of their truth values, then the sentence is valid
  - usually too tedious for humans
  - may exclude potentially interesting sentences
    - where some, but not all interpretations are true





### Models

- if there is an interpretation for a sentence such that the sentence is true in a particular world, that world is called a model
  - refers to specific interpretations
- models can also be thought of as mathematical objects
  - these mathematical models can be viewed as equivalence classes for worlds that have the truth values indicated by the mapping under that interpretation
  - ◆ a model then is a mapping from proposition symbols to **True** or **False**





### **Models and Entailment**

• a sentence  $\alpha$  is entailed by a knowledge base KB if the models of the knowledge base KB are also models of the sentence  $\alpha$ 

KB 
$$\mid$$
=  $\alpha$ 

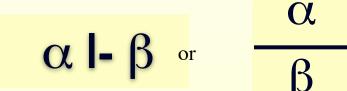
here: reasoning at the semantic level





### Inference and Derivation

- inference rules allow the construction of new sentences from existing sentences
  - lacktriangle notation: a sentence  $\beta$  can be derived from  $\alpha$



- an inference procedure generates new sentences on the basis of inference rules
- if all the new sentences are entailed, the inference procedure is called sound or truth-preserving

here: reasoning at the *syntactic* level





### Inference Rules

#### modus ponens

 from an implication and its premise one can infer the conclusion

#### and-elimination

 from a conjunct, one can infer any of the conjuncts

#### and-introduction

 from a list of sentences, one can infer their conjunction

#### or-introduction

 from a sentence, one can infer its disjunction with anything else

$$\alpha \Rightarrow \beta, \quad \alpha$$
 $\beta$ 

$$\frac{\alpha_1 \wedge \alpha_2 \wedge ... \wedge \alpha_n}{\alpha_i}$$

$$\begin{array}{c} \alpha_1, \, \alpha_2, \, \dots \, , \, \alpha_n \\ \hline \alpha_1 \, \wedge \, \alpha_2 \, \wedge \dots \, \wedge \, \alpha_n \end{array}$$

$$\alpha_1 \lor \alpha_2 \lor \dots \lor \alpha_n$$
 $\alpha_1 \lor \alpha_2 \lor \dots \lor \alpha_n$ 
WISSENSCHAFTEN



### Inference Rules

#### double-negation elimination

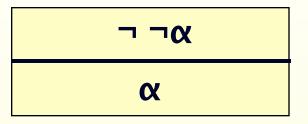
a double negations infers the positive sentence

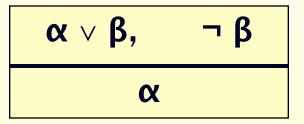
#### unit resolution

 if one of the disjuncts in a disjunction is false, then the other one must be true

#### resolution

- β cannot be true and false, so one of the other disjuncts must be true
- can also be restated as "implication is transitive"





$$\alpha \vee \beta$$
,  $\neg \beta \vee \gamma$ 

$$\alpha \vee \gamma$$

$$\neg \alpha \Rightarrow \beta, \beta \Rightarrow \gamma$$

$$\neg \alpha \Rightarrow \gamma$$

$$\neg \alpha \Rightarrow \gamma$$
HSCH
GEWANI
SCHAFT



### Complexity

- the truth-table method to inference is complete
  - enumerate the 2<sup>n</sup> rows of a table involving n symbols
  - computation time is exponential
- satisfiability for a set of sentences is NP-complete
  - so most likely there is no polynomial-time algorithm
  - in many practical cases, proofs can be found with moderate effort
- there is a class of sentences with polynomial inference procedures (Horn sentences or Horn clauses)
  - $P1 \land P2 \land ... \land Pn \Rightarrow Q$





### **Wumpus Logic**

- an agent can use propositional logic to reason about the Wumpus world
  - knowledge base contains
    - percepts
    - rules

$$\neg S_{1,1} \\ \neg S_{2,1} \\ S_{1,2}$$

$$\neg B_{1,1}$$

$$B_{2,1}$$

$$\neg B_{1,2}$$

$$\mathbf{R1:} \neg S_{1,1} \Longrightarrow \neg W_{1,1} \land \neg W_{1,2} \land \neg W_{2,1}$$

$$\mathbf{R2:} \neg S_{2,1} \Longrightarrow \neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{2,2} \wedge \neg W_{3,1}$$

$$\mathbf{R3:} \neg S_{1,2} \Longrightarrow \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,2} \wedge \neg W_{1,3}$$

R4: 
$$S_{1,2} \Longrightarrow W_{1,1} \vee W_{1,2} \vee W_{2,2} \vee W_{1,3}$$

. . .





### Finding the Wumpus

### two options

- construct truth table to show that W<sub>1,3</sub> is a valid sentence
  - rather tedious
- use inference rules
  - apply some inference rules to sentences already in the knowledge base





### **Action in the Wumpus World**

additional rules are required to determine actions for the agent

```
RM: A_{I,I} \wedge East_A \wedge W_{2,I} \Longrightarrow \neg Forward_A
RM + 1: ...
```

- the agent also needs to **ASK** the knowledge base what to do
- must ask specific questions
  - Can I go to the next square X<sub>i,i</sub>?
    - "next" is easy for humans, but must be enumerated here by giving a specific location
- general questions are not possible in propositional logic
  - Where should I go?





### **Propositional Wumpus Agent**

#### the size of the knowledge base even for a small wumpus world becomes immense

- explicit statements about the state of each square
- additional statements for actions, time
- easily reaches thousands of sentences for very small configurations

#### completely unmanageable for humans

- size
- lack of expressiveness, abstraction

### efficient methods exist for computers

- optimized variants of search algorithms
- sequential circuits
  - combinations of gates and registers
  - more efficient treatment of time
  - effectively a reflex agent with state
  - can be implemented in hardware





# **Exercise: Wumpus World in Propositional Logic**

- express important knowledge about the Wumpus world through sentences in propositional logic format
  - status of the environment
  - percepts of the agent in a specific situation
  - new insights obtained by reasoning
    - rules for the derivation of new sentences
    - new sentences
  - decisions made by the agent
  - actions performed by the agent
    - changes in the environment as a consequence of the actions
  - background
    - general properties of the Wumpus world
  - learning from experience
    - general properties of the Wumpus world





# From Propositional to Predicate Logic

- limitations of propositional logic in the Wumpus World
  - enumeration of statements
  - change
  - proposition as "representational device"
    - limited expressiveness
    - not very compatible with human reasoning
- usefulness of objects and relations between them
  - properties
  - internal structure
  - arbitrary relations
  - functions





# Knowledge Representation and Commitments

#### ontological commitment

- describes the basic entities that are used to describe the world
  - propositional logic
    - facts expressed through propositional symbols
  - first-order predicate logic
    - facts, objects (terms), relations (predicates)

### epistemological commitment

- describes how an agent expresses its believes about facts
  - true, false, unknown in binary logic
  - ♦ n+1 truth values in n-ary logic





# Formal Languages and Commitments

Language	Ontological Commitment	Epistemological Commitment	
Propositional Logic	facts	true, false, unknown	
First-order Logic	facts, objects, relations	true, false, unknown	
Temporal Logic	facts, objects, relations, times	true, false, unknown	
Probability Theory	facts	degree of belief $\in [0,1]$	
Fuzzy Logic	facts with degree of truth ∈ [0,1]	known interval value	

### **Predicate Logic**

#### new concepts

- complex objects and their properties
  - terms
- relations
  - predicates
  - quantifiers
- syntax
- semantics
- inference rules
- usage





### **Examples of Objects, Relations**

### "The smelly wumpus occupies square [1,3]"

- objects: wumpus, square<sub>1,3</sub>
- property: smelly
- relation: occupies

### "Two plus two equals four"

- objects: two, four
- relation: equals
- function: plus





### **Objects**

#### primarily distinguishable things in the real world

- e.g. people, cars, computers, programs, ...
- the set of objects determines the domain of a model

#### frequently includes abstract concepts

- colors, stories, light, money, love, ...
- in contrast to physical objects

#### properties

- describe specific aspects of objects
  - green, round, heavy, visible,
- can be used to distinguish between objects





### Relations

#### used to establish connections between objects

- unary relations refer to a single object
  - \* e.g. mother-of(John), brother-of(Jill), spouse-of(Joe)
  - often called functions
- binary relations relate two objects to each other
  - \* e.g. twins(John, Jill), married(Joe, Jane)
- ◆ n-ary relations relate n objects to each other
  - ❖ e.g. triplets(Jim, Tim, Wim), seven-dwarfs(D1, ..., D7)

#### relations can be defined by the designer or user

- ◆ neighbor, successor, next to, taller than, younger than, ...
- functions are a special type of relation
  - non-ambiguous: only one output for a given input
  - ◆ often distinguished from similar binary relations by appending -of
    - \* e.g. father(John, Jim) VS. father-of(John)
      - \*brother-of(John) is not a good example: ambiguous





### **Syntax**

#### based on sentences

- more complex than propositional logic
  - constants (propositional symbols), predicates, terms, quantifiers

#### constant symbols

```
A, B, C, Franz, Square<sub>1,3</sub>, ...
```

stand for unique objects (in a specific context)

#### predicate symbols

```
Adjacent-To, Younger-Than, ...
```

describes relations between objects

#### function symbols

```
Father-Of, Square-Position, ...
```

the given object is related to exactly one other object





### **Semantics**

#### relates sentences to models

in order to determine their truth values

#### provided by interpretations for the basic constructs

- usually suggested by meaningful names (intended interpretations)
- constants
  - the interpretation identifies the object in the real world
- predicate symbols
  - the interpretation specifies the particular relation in a model
  - may be explicitly defined through the set of tuples of objects that satisfy the relation
- function symbols
  - identifies the object referred to by a tuple of objects
  - may be defined implicitly through other functions, or explicitly through tables

#### interpretations for complex constructs

constructed from basic building blocks ("compositional semantics")



### **BNF Grammar Predicate Logic**

Sentence → AtomicSentence

| (Sentence Connective Sentence)

| Quantifier Variable, ... Sentence

| ¬ Sentence

AtomicSentence → Predicate(Term, ...) | Term = Term

Term → Function(Term, ...) | Constant | Variable

Connective  $\rightarrow \land |\lor| \Rightarrow |\Leftrightarrow$ 

Quantifier → ∀ | ∃

Constant  $\rightarrow A, B, C, X_1, X_2, Jim, Jack$ 

Variable  $\rightarrow$  a, b, c,  $x_1$ ,  $x_2$ , counter, position

Predicate → Adjacent-To, Younger-Than,

Function → Father-Of, Square-Position, Sqrt, Cosine

ambiguities are resolved through precedence or parentheses





### **Terms**

- logical expressions that specify objects
- constants and variables are terms
- more complex terms are constructed from function symbols and simpler terms, enclosed in parentheses
  - basically a complicated name of an object
- semantics is constructed from the basic components, and the definition of the functions involved
  - either through explicit descriptions (e.g. table), or via other functions





### **Atomic Sentences**

- state facts about objects and their relations
- specified through predicates and terms
  - the predicate identifies the relation, the terms identify the objects that have the relation
- an atomic sentence is true if the relation between the objects holds
  - this can be verified by looking it up in the set of tuples that define the relation





### **Examples Atomic Sentences**

```
Father(Jack, John)
Mother(Jill, John)
Sister(Jane, John)
Parents(Jack, Jill, John, Jane)
Married(Jack, Jill)
Married(Father-Of(John), Mother-Of(John))
Married(Father-Of(John), Mother-Of(Jane))
Married(Parents(Jack, Jill, John, Jane))
```





### **Complex Sentences**

- logical connectives can be used to build more complex sentences
- semantics is specified as in propositional logic





### **Examples Complex Sentences**

```
Father(Jack, John) ∧ Mother(Jill, John) ∧ Sister(Jane,
John)
¬ Sister(John, Jane)

Parents(Jack, Jill, John, Jane) ∧ Married(Jack, Jill)

Parents(Jack, Jill, John, Jane) ⇒ Married(Jack, Jill)

Older-Than(Jane, John) ∨ Older-Than(John, Jane)

Older(Father-Of(John), 30) ∨ Older (Mother-Of(John), 20)
```

Attention: Some sentences may look like tautologies, but only because we "automatically" assume the meaning of the name as the only interpretation (parasitic interpretation)



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### Quantifiers

- \* can be used to express properties of collections of objects
  - eliminates the need to explicitly enumerate all objects
- predicate logic uses two quantifiers
  - ♦ universal quantifier ∀
  - ◆ existential quantifier ∃





### **Universal Quantification**

- states that a predicate P holds for all objects x in the universe under discourse
   ∀x P(x)
- the sentence is true if and only if all the individual sentences where the variable x is replaced by the individual objects it can stand for are true





# **Example Universal Quantification**

 assume that x denotes the squares in the wumpus world

```
 \forall x \ \textit{Is-Empty}(x) \lor \textit{Contains-Agent}(x) \lor \textit{Contains-Wumpus}(x) \\ \text{is true if and only if all of the following sentences are true:} \\ \textit{Is-empty}(S_{11}) \lor \textit{Contains-Agent}(S_{11}) \lor \textit{Contains-Wumpus}(S_{11}) \\ \textit{Is-empty}(S_{12}) \lor \textit{Contains-Agent}(S_{12}) \lor \textit{Contains-Wumpus}(S_{12}) \\ \textit{Is-empty}(S_{13}) \lor \textit{Contains-Agent}(S_{13}) \lor \textit{Contains-Wumpus}(S_{13}) \\ \cdots \\ \textit{Is-empty}(S_{21}) \lor \textit{Contains-Agent}(S_{21}) \lor \textit{Contains-Wumpus}(S_{21}) \\ \cdots \\ \textit{Is-empty}(S_{44}) \lor \textit{Contains-Agent}(S_{44}) \lor \textit{Contains-Wumpus}(S_{44})
```

beware of the implicit (parasitic) interpretation fallacy!





# Usage of Universal Qualification

 universal quantification is frequently used to make statements like "All humans are mortal", "All cats are mammals", "All birds can fly",

• •

this can be expressed through sentences like

$$\forall x \; Human(x) \Rightarrow Mortal(x)$$

$$\forall x \ Cat(x) \Rightarrow Mammal(x)$$

$$\forall x \; Bird(x) \Rightarrow Can-Fly(x)$$

these sentences are equivalent to the explicit sentence about individuals

$$Human(John) \Rightarrow Mortal(John) \land$$

$$Human(Jane) \Rightarrow Mortal(Jane) \land$$

$$Human(Jill) \Rightarrow Mortal(Jill) \land$$





## **Existential Quantification**

- states that a predicate P holds for some objects in the universe  $\exists x \ P(x)$
- the sentence is true if and only if there is at least one true individual sentence where the variable x is replaced by the individual objects it can stand for





# **Example Existential Quantification**

assume that x denotes the squares in the wumpus world

```
is true if and only if at least one of the following sentences is true: Glitter(S_{11}) Glitter(S_{12}) Glitter(S_{13}) . . . Glitter(S_{21}) . . . Glitter(S_{44})
```





# Usage of Existential Qualification

- existential quantification is used to make statements like
  - "Some humans are computer scientists",
    "John has a sister who is a computer scientist"
    "Some birds can't fly", ...
- this can be expressed through sentences like
  - ∃ x Human(x) ∧ Computer-Scientist(x)
     ∃ x Sister(x, John) ∧ Computer-Scientist(x)
     ∃ x Bird(x) ∧ ¬ Can-Fly(x)
- these sentences are equivalent to the explicit sentence about individuals
  - → Human(John) ∧ ¬ Computer-Scientist(John) ∨
     Human(Jane) ∧ Computer-Scientist(Jane) ∨
     Human(Jill) ∧ ¬ Computer-Scientist(Jill) ∨





### **Multiple Quantifiers**

- more complex sentences can be formulated by using multiple variables and by nesting quantifiers
  - the order of quantification is important
  - variables must be introduced by quantifiers, and belong to the innermost quantifier that mention them
  - examples

```
\forall x, y \mid Parent(x,y) \Rightarrow Child(y,x)

\forall x \; Human(x) \; \exists \; y \; Mother(y,x)

\forall x \; Human(x) \; \exists \; y \; Loves(x, y)

\exists \; x \; Human(x) \; \forall \; y \; Loves(x, y)

\exists \; x \; Human(x) \; \forall \; y \; Loves(y,x)
```





### **Connections between ∀ and ∃**

- all statements made with one quantifier can be converted into equivalent statements with the other quantifier by using negation
  - ♦ ∀ is a conjunction over all objects under discourse
  - ◆ ∃ is a disjunction over all objects under discourse
  - ◆ De Morgan's rules apply to quantified sentences  $\forall x \neg P(x) \equiv \neg \exists x \ P(x)$   $\neg \forall x \ P(x) \equiv \exists x \ \neg P(x)$   $\forall x \ P(x) \equiv \exists x \ P(x)$
- strictly speaking, only one quantifier is necessary
  - using both is more convenient





### **Domains**

a section of the world we want to reason about

#### assertion

- a sentence added to the knowledge about the domain
- often uses the TELL construct

```
• e.g. TELL (KB-Fam, (Father(John) = Jim))
```

 sometimes ASSERT, RETRACT and MODIFY construct are used to make, withdraw and modify statements

#### axiom

- ◆ a statement with basic, factual, undisputed information about the domain
- often used as definitions to specify predicates in terms of already defined predicates

#### theorem

- statement entailed by the axioms
- it follows logically from the axioms





## **Example: Family Relationships**

- objects: people
- properties: gender, ...
  - expressed as unary predicates Male(x), Female(y)
- relations: parenthood, brotherhood, marriage
  - expressed through binary predicates Parent(x,y), Brother(x,y), ...
- functions: motherhood, fatherhood
  - Mother(x), Father(y)
  - because every person has exactly one mother and one father
  - \* there may also be a relation *Mother-of(x,y)*, *Father-of(x,y)*





## Family Relationships

 $\forall m,c \; Mother(c) = m \Leftrightarrow Female(m) \land Parent(m,c)$ 

 $\forall w, h \; Husband(h, w) \Leftrightarrow Male(h) \land Spouse(h, w)$ 

 $\forall x \; Male(x) \Leftrightarrow \neg Female(x)$ 

 $\forall g, c \ Grandparent(g,c) \Leftrightarrow \exists \ p \ Parent(g,p) \land Parent(p,c)$ 

 $\forall x,y \ Sibling(x,y) \Leftrightarrow \neg (x=y) \land \exists \ p \ Parent(p,x) \land Parent(p,y)$ 

. . .





# Logic and the Wumpus World

#### representation

suitability of logic to represent the critical aspects of the Wumpus World

#### reflex agent

specification of a reflex agent for the Wumpus World

#### change

dealing with aspects of the Wumpus World that change over time

#### model-based agent

specification using logic





# Reflex Agent in the Wumpus World

#### rules that directly connect percepts to actions

 $\forall s,b,g,u,c,t$   $Percept([s, b, Glitter, u,c], t) \Rightarrow Action(Grab(treasure(Glitter)), t)$ 

- ◆ *Grab(x)* is a predicate that indicates the agent is now holding x
- ◆ Action(a,t) indicates the agent is performing action a at time t
- treasure(Glitter) is a function that returns the object with the property Glitter
- requires many rules for different combinations of percepts at different times

#### can be simplified by intermediate predicates

∀ s,b,g,u,c,t	$Percept([Stench, b, g, u, c], t) \Rightarrow Stench(t)$
∀ s,b,g,u,c,t	$Percept([s, Breeze, g, u, c], t) \Rightarrow Breeze(t)$
∀ s,b,g,u,c,t	$Percept([s, b, Glitter, u, c], t) \Rightarrow AtGold(t)$
∀ s,b,g,u,c,t	$Percept([s, b, g, Bump, c], t) \Rightarrow Bump(t)$
∀ s,b,g,u,c,t	$Percept([s, b, g, u, Scream], t) \Rightarrow Scream(t)$
$\forall t$	$AtGold(t) \Rightarrow Action(Grab(treasure(Glitter)), t)$

. . .

- mainly abstraction over time
  - does not deal with duration (intervals)
- ◆ is it still a reflex agent?





### **Limitations of Reflex Agents**

- the agent doesn't know its state
  - it doesn't know when to perform the climb action
    - it doesn't know if it has the gold, nor where the agent is
  - the agent may get into infinite loops
    - it will have to perform the same action for the same percepts





## Change in the Wumpus World

- in principle, the percept history contains all the relevant knowledge for the agent
  - by writing rules that can access past percepts, the agent can take into account previous information
  - this is sufficient for optimal action under given circumstances
  - may be very tedious, involving many rules
- it is usually better to keep a set of sentences about the current state of the world
  - must be updated for every percept and every action
  - can be considered a model of the world





### **Agent Movement**

- constructs that help the agent keep track of
  - its location
  - how it can move
- creates a simple map for the agent
  - ◆ current location of the agent
     At(Agent, [1,1], S₀)
     uses a Situation parameter S₀ to keep track of changes independent of specific time points
  - orientation of the agent  $Orientation(Agent, S_0)$
  - arrangement of locations, i.e. a map

$$\forall x, y \quad LocationToward([x,y],0) = [x+1,y]$$

$$\forall x, y \quad LocationToward([x,y],90) = [x, y+1]$$





### **Model-Based Agent**

#### knows about locations through its map

- can associate properties with the locations
- can be used to reason about safe places, the presence of gold, pits, the wumpus, etc.

```
\forall \ \textit{I,s} At(Agent,\textit{I,s}) \land Breeze(s) \Rightarrow Breezy(\textit{I})
...

\forall \ \textit{I_1, I_2,s} At(Wumpus,\textit{I_1,s}) \land Adjacent(\textit{I_1, I_2}) \Rightarrow Smelly(\textit{I_2})
...

\forall \ \textit{I_1, I_2, s} Smelly(\textit{I_1}) \Rightarrow (\exists \ \textit{I_2} \ At(Wumpus,\textit{I_2,s}) \land \neg (\textit{I_1} = \textit{I_2}) \land (Adjacent(\textit{I_1, I_2}))
...

\forall \ \textit{I_1, I_2, x, t} \ \neg At(Wumpus, x,t) \land \neg (\textit{I_1} = \textit{I_2}) \land \neg Pit(x)) \Leftrightarrow OK(x)
```

- such an agent will find the gold provided there is a safe sequence
- returning to the exit with the gold is difficult



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### **Goal-Based Agent**

- once the agent has the gold, it needs to return to the exit
  - $\diamond$  v s Holding(Gold, s)  $\Rightarrow$  GoalLocation([1,1],s)
- the agent can calculate a sequence of actions that will take it safely there
  - through inference
    - computationally rather expensive for larger worlds
    - difficult to distinguish good and bad solutions
  - through search
    - e.g. via the best-first search method
  - through planning
    - requires a special-purpose reasoning system





### **Utility-Based Agent**

- can distinguish between more and less desirable states
  - different goals, pits, ...
    - pots with different amounts of gold
    - find multiple solutions and distinguish between them
  - optimization of the route back to the exit
    - metrics to compare important properties of the route
  - performance measure for the agent
  - requires the ability to deal with natural numbers in the knowledge representation scheme
    - possible in predicate logic, but tedious





### **Important Concepts and Terms**

- agent
- and
- atomic sentence
- automated reasoning
- completeness
- conjunction
- constant
- disjunction
- domain
- existential quantifier
- fact
- false
- function
- implication
- inference mechanism
- inference rule
- interpretation
- knowledge representation
- logic
- model
- object
- or

- predicate
- predicate logic
- property
- proposition
- propositional logic
- propositional symbol
- quantifier
- query
- rational agent
- · reflex agent
- relation
- resolution
- satisfiable sentence
- semantics
- sentence
- soundness
- syntax
- term
- true
- universal quantifier
- valid sentence
- variable





### **Chapter Summary**

- logic can be used as the basis of formal knowledge representation and processing
  - syntax specifies the rules for constructing sentences
  - semantics establishes a relation between the sentences and their counterparts in the real world
  - simple sentences can be combined into more complex ones
  - new knowledge can be generated through inference rules from existing sentences
- propositional logic encodes knowledge about the world in simple sentences or formulae
- predicate logic is a formal language with constructs for the specifications of objects and their relations
  - models of reasonably complex worlds and agents can be constructed with predicate logic





