Równania różniczkowe i różnicowe

Potencjał elektryczny – Problem nr. 5

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Sformułowanie silne:

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho}{\varepsilon_r}$$

$$\varphi'(0) + \varphi(0) = 5$$

$$\varphi(3) = 2$$

$$\rho = 1$$

$$\varepsilon_r = \begin{cases} 10 & \text{dla } x \in [0,1] \\ 5 & \text{dla } x \in (1,2] \\ 1 & \text{dla } x \in (2,3] \end{cases}$$

Gdzie φ to poszukiwana funkcja:

$$[0,3] \ni x \rightarrow \varphi(x) \in \mathbb{R}$$

Sformułowanie wariacyjne:

$$\varphi''(x) = -\frac{\rho}{\varepsilon_r}$$

$$\varphi''(x)v(x) = -\frac{\rho}{\varepsilon_r}v(x)$$

$$\int_0^3 \varphi''(x)v(x) = \int_0^3 -\frac{\rho}{\varepsilon_r}v(x)$$

$$[\varphi'(x)v(x)]_0^3 - \int_0^3 \varphi'(x)v(x) = -\int_0^3 \frac{\rho}{\varepsilon_r}v(x)$$

$$\varphi'(3)v(3) - \varphi'(0)v(0) - \int_0^3 \varphi'(x)v(x) = -\int_0^3 \frac{\rho}{\varepsilon_r}v(x)$$

$$-\varphi'(0)v(0) - \int_0^3 \varphi'(x)v(x) = -\int_0^3 \frac{\rho}{\varepsilon_r}v(x)$$

$$-\big(5 - \varphi(0)\big)v(0) - \int_0^3 \varphi'(x)v(x) = -\int_0^3 \frac{\rho}{\varepsilon_r}v(x)$$

$$-5v(0) + \varphi(0)v(0) - \int_0^3 \varphi'(x)v(x) = -\int_0^3 \frac{\rho}{\varepsilon_r}v(x)$$

$$\varphi(0)v(0) - \int_0^3 \varphi'(x)v(x) = 5v(0) - \int_0^3 \frac{\rho}{\varepsilon_r}v(x)$$

$$B(\varphi, v) = \varphi(0)v(0) - \int_0^3 \varphi'(x)v(x)$$

$$L(v) = 5v(0) - \int_0^3 \frac{\rho}{\varepsilon_r}v(x)$$

$$B(\varphi, v) = L(v)$$

$$\varphi = \tilde{\varphi} + w$$

$$\tilde{\varphi} = 2e_n$$

$$B(\tilde{\varphi} + w, v) = L(v)$$

$$B(\tilde{\varphi}, v) + B(w, v) = L(v)$$

$$B(w, v) = L(v) - B(\tilde{\varphi}, v)$$

$$B(w, v) = L(v) - B(2e_n, v)$$

$$B(w, v) = L(v) - 2B(e_n, v)$$

$$\tilde{L}(v) = L(v) - 2B(e_n, v)$$

$$B(w, v) = \tilde{L}(v)$$