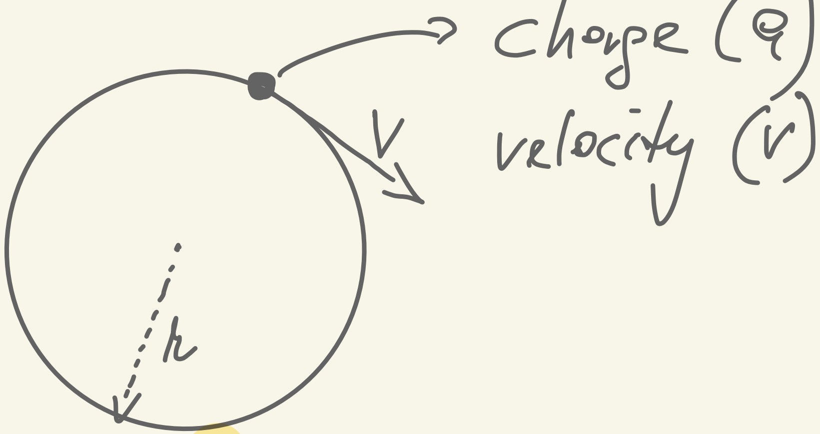
Atomic magnetic moments

In general, the magnetic moment of an atom is related to the overall angular momentum of its electrons.

(mag. moment .... dt. "magnetischer Moment" (vector quantity)
ang. momentum ....dt. „Bahndrehimpuls" (vec. quantity))



The ang. momentum (L) of this particle is

$$\boxed{L = m \cdot v \cdot r}$$

The abs. value of the magn. moment (m_m) is

$$m_m = I \cdot A$$

(current × area of enclosed region)

In our example

$$m_m = I \cdot \pi r^2$$

Current (I) is given by

$$I = \frac{q}{T}$$; T ... period, $\frac{1}{T}$ ... orbital frequency

With $T = \frac{2\pi r}{v}$ we get

$$I = \frac{q \cdot v}{2\pi r} \Rightarrow \boxed{m_m = \frac{1}{2} \cdot q \cdot v \cdot r}$$

with $L = m.v.r$, we get the classic relation

$$\boxed{m_m = \frac{q}{2m} \cdot L}$$

pos. or neg.

or in vec. notation

$$\vec{m_m} = q/_{2m} \cdot \vec{L}$$

In a quantum mech. treatment of atoms (where electrons are described through their wave function), three quantum numbers occur

$$n = 1, 2, 3, \ldots$$
$$l = 0, 1, \ldots n-1$$
$$m = -l, -l+1, \ldots +l$$

n ... main quant. number
l ... ang. momentum quantum number
m ... magnetic quantum number

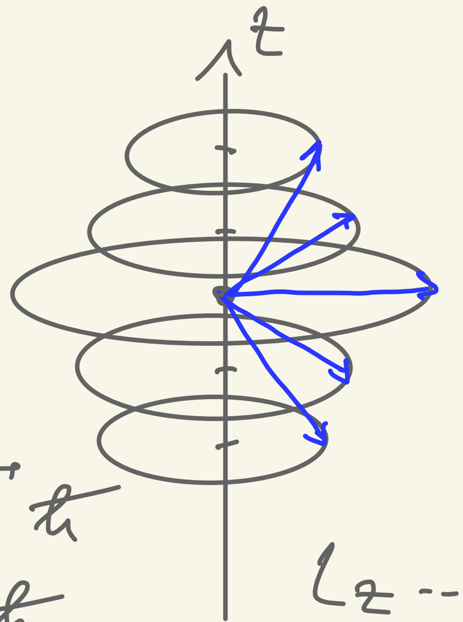
Ang. momentum of electron

$$L = \sqrt{l(l+1)} \cdot \hbar \quad \hbar \to \frac{h}{2\pi}$$

m specifies the component of the ang. momentum in a concrete direction. All directions are equal in atom but one can be selected through application of magn. field! (e.g., in z direction)

Then

$$L_z = m \cdot \hbar \quad (2l+1 \text{ values})$$



$l = 2$

$\sqrt{2 \cdot (2+1)}\,\hbar$

$= \sqrt{6}\,\hbar$

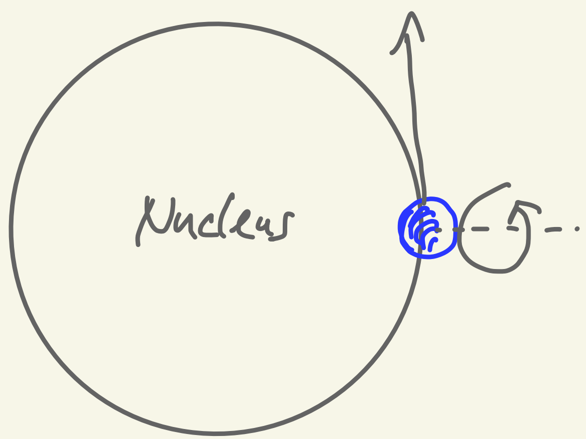
$L_z \ldots (2l+1) = 5$ values possible for z-component L_z

Pauli postulated that a <u>fourth</u> quant. number has to be assigned to an electron (taking one of <u>two</u> possible values).
Now known as spin quant. number.

→ fund. property of matter (like mass, charge magnetism)

For an electron in no electric or magn. field $s = \frac{1}{2}$.

<u>Just for better understanding:</u>



Bound electrons in atoms are always in a mag. field (gen. by movement in orbit). In that case spin can be parallel or anti-parallel to that field (e.g. in z direction).

$$S_z = m_s \cdot \hbar \quad \text{with} \quad m_s = \pm \frac{1}{2}$$

(in general $2s+1$ values)

Q: How large are the mag. moments due to orbital motion and spin? We know (calling mag. moments μ now)

$$\vec{\mu} = \frac{q}{2m_q} \cdot \vec{L}$$

with $q = -e$ and $m_q = m_e$ (electron)

$$\vec{\mu} = -\frac{e}{2m_e} \cdot \vec{L}$$

Applied to ang. mom. of electron in 1H:

$$\mu = -\frac{e}{2m_e} \cdot L$$

$$= -\frac{e}{2m_e} \cdot \sqrt{l \cdot (l+1)} \cdot \hbar$$

$$= \sqrt{(l+1) \cdot l} \cdot \mu_B$$

Bohr magneton

$$\mu_z = -\frac{e}{2m_e} \cdot m \hbar = -m \cdot \mu_B \qquad (z\text{-component})$$

<u>For the mag. moment due to spin:</u>

$$\mu = -\frac{e}{2m_e} \cdot \sqrt{s \cdot (s+1)} \hbar = \sqrt{s \cdot (s+1)} \mu_B$$

$$\mu_z = -\frac{e}{2m_e} \cdot m_s \cdot \hbar = \pm \frac{1}{2} \cdot \mu_B$$

(on slides, I used $\underline{\sigma}$ instead of $\underline{s}$).
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Overall, spin is a form of <u>ang. momentum</u>, but not due to rotation, but rather <u>intrinsic</u>.