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Medical Imaging – Proseminar (911.934)

Exercise sheet **C**

For the following exercises, template code is provided in the Jupyter notebook `Exercise-Sheet-C-Notebook.ipynb`*. Please hand-in the updated version of this Jupyter notebook (with your solutions).*

Exercise 1. 2 P.

Consider *transverse relaxation* in MR imaging, i.e., the exponential decay (with T_2) of the the (x, y) component of the magnetization vector $\mathbf{M}(t)$. For the x-component (and also for the y-component), we know

$$M_x(t) = M_x(0)e^{-t/T_2}$$

where $M_x(0)$ denotes the initial value of the x-component at time $t = 0$. Assume $T_2 = 220$ ms and create a plot showing the time t (in ms) vs. $M_x(t)$ for $t \in [100, 300]$ ms. Calculate the exact value for $M_x(t)$ at $t = 233$ ms.

Exercise 2. 3 P.

For *transverse relaxation*, now consider *both* the x- and y-component and $T_2 = 220$ ms. Start with $\mathbf{M}(0) = [1, 0, 0]^\top$ and write $\mathbf{M}(t)$ as $\mathbf{A}(t)\mathbf{M}(0)$ where $\mathbf{A}$ is an appropriate 3×3 matrix. Obviously, $\mathbf{A}(t)$ needs to be such that this works for any other initial magnetization vector as well. Plot the (2-)norm $\|\mathbf{M}(t)\|$ over $t \in [100, 1000]$ ms for starting vector $\mathbf{M}(0) = [1, 0, 0]^\top$).

Exercise 3. 4 P.

For *longitudinal relaxation*, the situation is slightly different. For the z-component of $\mathbf{M}(t)$, i.e., $M_z(t)$, we have

$$M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T_1}$$

where M_0 is the magnetization at *equilibrium*, i.e., when $T \to \infty$. Remember that without any relaxation, $\mathbf{M}(t)$ precesses around the axis of the static magnetic field $\mathbf{B}_0$. Taking relaxation into account, this means that longitudinal relaxation reaches M_0 as $T \to \infty$ (and transversal magnetization shrinks to 0). **Combine** both *longitudinal* and *transversal* relaxation such that you can write

$$\mathbf{M}(t) = \mathbf{A}(t)\mathbf{M}(\mathbf{0}) + \mathbf{b}(t)$$

where $\mathbf{A}(t)$ and $\mathbf{b}(t)$ are set appropriately (with $M_0 = 1$ and $T_1 = 600$ ms, $T_2 = 120$ ms). Create a plot for $M_z(t)$ over $t \in [100, 2000]$ ms for starting vector $\mathbf{M}(0) = [1, 0, 0]^\top$.

Exercise 4. 5 P.

In this exercise, we simulate (1) the path of transversal relaxation and (2) how longitudinal magnetization approaches the equilibrium state. The code in the Jupyter notebook contains a function `zRot(phi)` which returns a rotation matrix $\mathbf{R}$ around the z-axis (and, by default, takes radians as input). Note that for the matrix $\mathbf{A}$ from *Exercise 3* and the rotation matrix $\mathbf{R}$, we have

$$\mathbf{AR} = \mathbf{RA} \ ,$$

i.e., they commute.

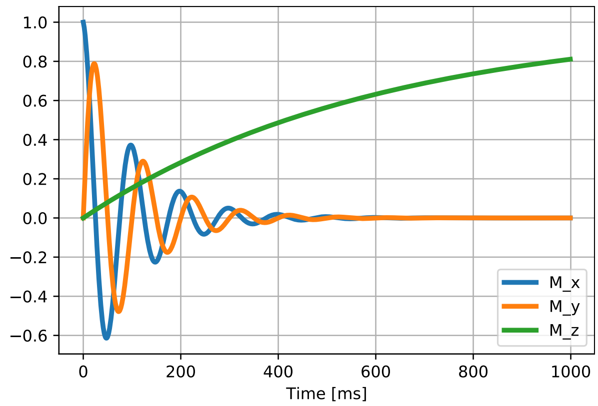
Write a function `freep(T,T1,T2,f)` which returns two matrices $\mathbf{A}$ and $\mathbf{B}$, such that we can write

$$\mathbf{M}(t) = \mathbf{A}(t)\mathbf{M}(0) + \mathbf{b}(t)$$

Note that `T` is the duration of the precession (here we look at the graph for 1000 ms), `T1`,`T2` are the longitudinal and transversal relaxation times (600 ms and 100 ms) and `f` is the frequency (10 Hz). Note that we can compute the rotation angle (around the z-axis) ϕ via (see notebook)

$$\phi = \frac{2\pi f T}{1000}$$

Use the `freep` function to start at $\mathbf{M}(0) = [1, 0, 0]^\top$ and plot $M_x(t)$, $M_y(t)$ as well as $M_z(t)$ over the time t (i.e., over the 1000 [ms]). It's important to note that you only need to use `freep` once: start with $\mathbf{M}(0)$ and use `T=1` to get $\mathbf{M}(t_1)$. Then use $\mathbf{M}(t_1)$ to get $\mathbf{M}(t_2)$, etc. *Below is an example of what this plot should look like, as a sanity check.*



1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0
200
400
600
800
1000
Time [ms]
M_x
M_y
M_z