Radiography

Computed Tomography (CT)[1 2]

[1] G. Dougherty. *Digital Image Processing for Medical Applications.* Cambridge University Press, 2009.

[2] D. Eppstein. *An Introduction to the Mathematics of Medical Imaging.* SIAM, 2008.

Computed Tomography

Introduction

Conventional X-Ray imaging produces *planar* images, i.e., projections of 3D objects onto 2D planes.

Tomographic imaging, e.g., computed tomography (CT), was developed to produce *transverse* images.

Working principle

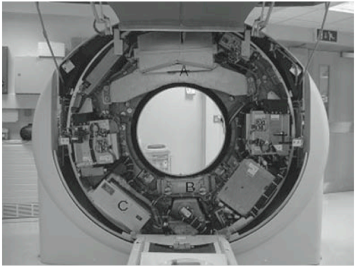
Scan a slice of tissue (with a narrow fan-shaped beam) from multiple angles $\rightarrow$ We obtain 1D projections of the object.

Tomography

In tomographic imaging, we "reconstruct" the transverse image from the 1D projections of the object.

Computed Tomography

Introduction



- Typically only one tube
- More filtering than projection radiography (previous lecture)
- Better approximation to monoenergetic source

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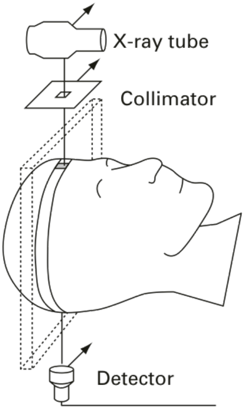
Working principle of a 1st generation CT scanner



Iterate:

1. Sweep beam linearly across the patient's head
2. Turn off tube & rotate by small angle ($\approx$1 degree), then goto step (1)

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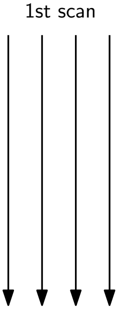
Working principle of a 1st generation CT scanner



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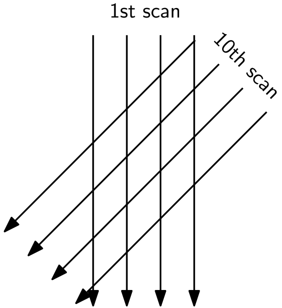
Working principle of a 1st generation CT scanner



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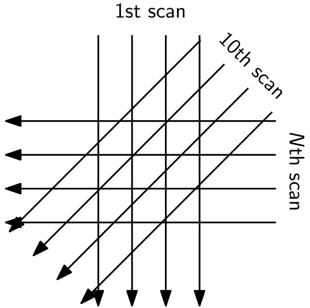
Working principle of a 1st generation CT scanner



Sampling interval & angle determine pixel size; collimator width determines slice thickness.

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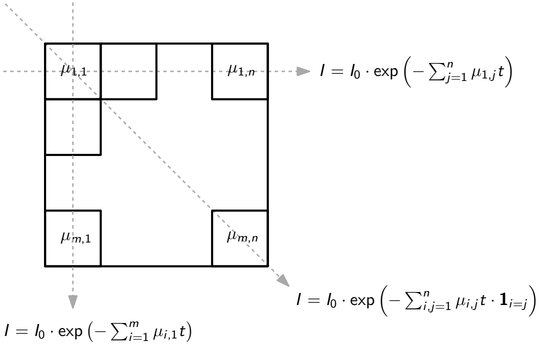
Working principle



Remember $I = I_0 \cdot e^{-\mu t}$. Hence, the ray-sum along a path is

$$-\log\left(\frac{I}{I_0}\right) = \sum \mu_{ij} t \quad \text{with } i, j \text{ appropriately} \quad .$$

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Working principle

We have seen that the measured X-ray intensity depends on the sum of attenuation coefficients.

X-Ray CT Image Reconstruction

Solve for

- the individual attenuation coefficients of each voxel

and

- assign a value (depending on those coefficients) to each pixel in the 2D-array that describes the transverse image.

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Working principle

Several correction factors needs to be implemented:

- X-Ray beam is (only approximately) mono-energetic and attenuation depends on energy.
- Effective X-Ray energy increases as it passes through patient – This effect is commonly known as “beam hardening”.

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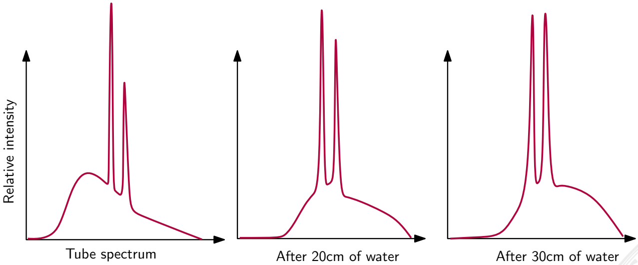
Working principle – Beam hardening

Beam hardening

The mean energy of an X-Ray beam increases as it passes through an object / patient.



Lower energy photons are attenuated more easily, higher energy photons are attenuated less easily $\rightarrow$ dark streaks & cupping.

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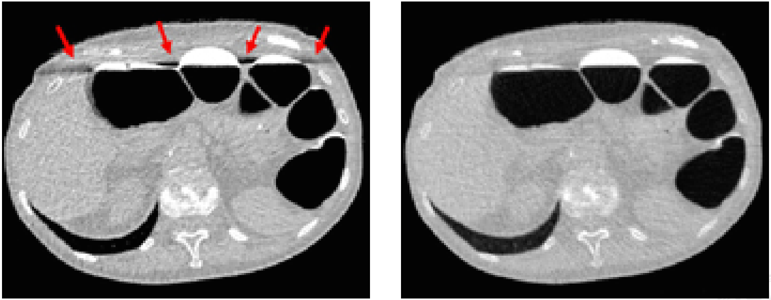
Working principle – Beam hardening

Beam hardening $\rightarrow$ streaking effects:



Left: Streaking effect; *Right*: Virtual CT image

(Courtesy of Janne Nappi, PhD.)

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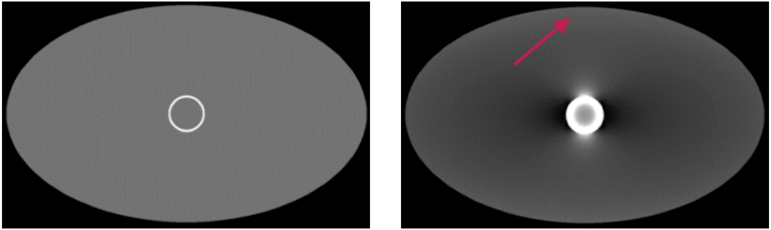
Working principle – Beam hardening

Beam hardening $\rightarrow$ “cupping” effects (simulated):

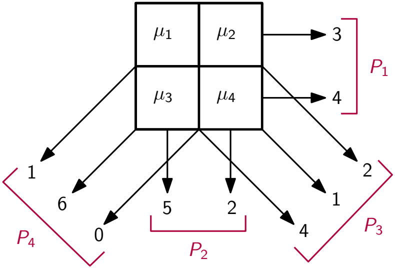


Beams passing through the center are “hardened” more than beams passing through edges.

We observe that

$$-\log\left(\frac{I}{I_0}\right)$$

is the line integral of linear attenuation coefficients at an effective energy. Next is an example of 4 voxel and their projections (P_i):



The question is “How do we obtain attenuation coefficients for each voxel”?

Various algorithms exist, e.g.,

- Backprojection
- Filtered backprojection
- Direct Fourier reconstruction

Backprojection (based on example from previous slide)

$$\xrightarrow{P_1} \begin{array}{c|c} 3 & 3 \\ \hline 4 & 4 \end{array} \xrightarrow{P_2} \begin{array}{c|c} 8 & 5 \\ \hline 9 & 6 \end{array} \xrightarrow{P_3} \begin{array}{c|c} 9 & 7 \\ \hline 13 & 7 \end{array} \xrightarrow{P_4} \begin{array}{c|c} 10 & 13 \\ \hline 19 & 7 \end{array}$$

No effort is taken to *distribute* the values over the voxel $\rightarrow$ suboptimal!

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Projection & Backprojection

Problem: *We get high values!*

Countermeasures:

1. Remove the sum in any of the projections (here: 7).

$$\begin{array}{c|c} 10 & 13 \\ \hline 19 & 7 \end{array} \rightarrow \begin{array}{c|c} 3 & 6 \\ \hline 12 & 0 \end{array}$$

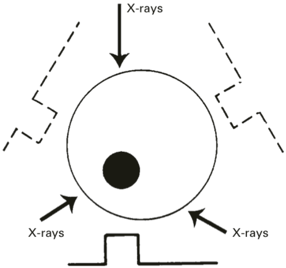
2. Normalize by the highest common factor:

$$\begin{array}{c|c} 3 & 6 \\ \hline 12 & 0 \end{array} \rightarrow \begin{array}{c|c} 1 & 2 \\ \hline 4 & 0 \end{array}$$

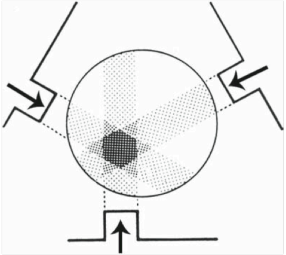
In our case, this gives the attenuation values that actually led to the projection results!

Computed Tomography

Working principle – Problems of "simple" backprojection



Projection



Backprojection

Star artifacts

The more images we have from different angles, the less prominent the object appears.

Why? Projections at an angle to the image grid intersect incomplete pixel.

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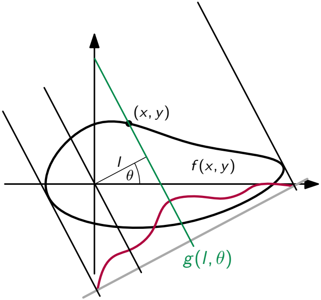
Projection & backprojection principles

The projection line through the point (x, y) can be specified as

$$x\cos(\theta) + y\sin(\theta) = l \quad \text{(“Hessesche Normalform”)}$$



$$g(l, \theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\delta(x\cos(\theta) + y\sin(\theta) - l)dxdy$$

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Projection & Backprojection

What is $g(\cdot,\cdot)$?

- if we fix l and $\theta \Rightarrow$ line integral of $f(x,y)$
- if we fix $\theta \Rightarrow$ projection of $f(x,y)$ at angle θ

In other words, $g(\cdot,\cdot)$ is the Radon transform of $f(x,y)$.

Image Generation

Projections are acquired for a selection of l and θ values and then CT image is *reconstructed* from these projections.

Backprojection (Variant 1)

- g is only measured at certain l
- coarse sampling $\rightarrow$ many points will not be assigned a value!
- Variant 1 is is what we have seen so far!

Backprojection (Variant 2) – Better Option!

For each angle θ, go through all sampling points in an image, find the corresponding l and take the $g(l, \theta)$ value.

$$b_\theta(x, y) = g(x\cos(\theta) + y\sin(\theta), \theta)$$

$$\begin{aligned} f_b(x, y) = \frac{1}{\pi}\int_0^\pi b_\theta(x, y)\mathrm{d}\theta &= \frac{1}{\pi}\int_0^\pi g(x\cos(\theta) + y\sin(\theta), \theta)\mathrm{d}\theta \\ &= \frac{1}{\pi}\int_0^\pi [g(l, \theta)]_{l = x\cos(\theta) + y\sin(\theta)}\mathrm{d}\theta \end{aligned}$$

Note: since $g(l, \theta)$ is only measured at certain l, we need to interpolate.

$f_b(x, y)$ is the backprojection summation image.

Computed Tomography

Radon Transform – An alternative view

A line in $\mathbb{R}^2$ is the set of points satisfying

$$ax + by = c$$

where $a^2 + b^2 \neq 0$. Equally, we can write

$$\frac{ax}{\sqrt{a^2+b^2}} + \frac{by}{\sqrt{a^2+b^2}} = \frac{c}{\sqrt{a^2+b^2}}.$$

Now,

$$\left(\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}\right) \in \mathbb{S}^1$$

where $\mathbb{S}^1$ is the unit circle in $\mathbb{R}^2$.

Consequently, we can parametrize a line by the unit vector $\mathbf{w} \in \mathbb{S}^1$ and $t \in \mathbb{R}$. The line $l_{\mathbf{w},t}$ is the set of points satisfying

$$\langle \mathbf{w}, (x, y) \rangle = t$$

We can also parametrize **w** as

$$\mathbf{w}(\theta) = (\cos(\theta), \sin(\theta)), \theta \in [0, 2\pi]$$

and we have

$$\cos(\theta)x + \sin(\theta)y = t$$

Also, the vector perpendicular to **w** is parametrized by

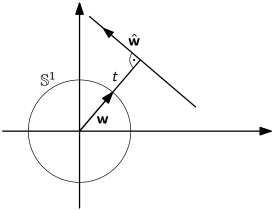
$$\widehat{\mathbf{w}}(\theta) = (-\sin(\theta), \cos(\theta)), \theta \in [0, 2\pi].$$

It holds that

$$\forall s \in \mathbb{R} : \langle \mathbf{w}, (t\mathbf{w} + s\widehat{\mathbf{w}}) \rangle = t$$

which allows us to parametrize $l_{\mathbf{w},t}$ as the set of points

$$l_{\mathbf{w},t} = \{t\mathbf{w} + s\widehat{\mathbf{w}} | s \in (-\infty, \infty)\}$$



$\widehat{\mathbf{w}}$ is chosen s.t. $\det(\mathbf{w}\widehat{\mathbf{w}}) = +1$.

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Radon Transform – An alternative view

Formally, the Radon transform is the integral transform

$$\mathcal{R}f(t,\mathbf{w}) = \int_{-\infty}^{\infty} f(t\mathbf{w} + s\widehat{\mathbf{w}})ds$$

mapping functions defined in $\mathbb{R}^2$ to functions defined on $\mathbb{R} \times \mathbb{S}^1$.

Remark. We assume that functions f are in the *natural domain*[3] of the Radon transform, i.e., f

1. is regular enough so that the restriction to $l_{\mathbf{w},t}$ is integrable.
2. goes to 0 rapidly so that the improper integrals converge.

[3] D. Eppstein. *An Introduction to the Mathematics of Medical Imaging*. SIAM, 2008.

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Radon Transform – An alternative view

Backprojection essentially *averages* the values of $\mathcal{R}f$ over the lines that pass through a point, i.e.,

$$f_b(x, y) = \frac{1}{2\pi} \int_0^{2\pi} \mathcal{R}f(\langle (x, y), \mathbf{w}(\theta) \rangle, \theta) d\theta$$

We have seen the discretization of this idea before in our examples.

Computed Tomography

Interpretation of CT image values – Hounsfield units (HU)

Instead of using the attenuation coefficients directly as gray values, Hounsfield units (HU) are used:

$$HU = 1000 \cdot \frac{\mu - \mu_{H_2O}}{\mu_{H_2O}}$$

(relative to the attenuation of water; truncated to the nearest integer)

Why do we want to use Hounsfield units? Minimize dependence on the energy of X-Ray beam; produces unitless values!

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Interpretation of CT image values – Hounsfield units (HU)

A brief overview of Hounsfield units:

Tissue type	HU
Bone	1000+
Hemorrhage	60-110
Liver	50-80
Muscle	44-69
Blood	42-58
Gray matter	32-44
White matter	24-36
Heart	24
Cerebrospinal fluid	0-22
Water	0
Fat	-20 to -100
Lung	-300
Air	-1000

(Note: Very dense bone has HU values of ≈ 3000, so the range is $\approx 4000 \rightarrow$ we will need 12 [bits])

Computed Tomography

Some sources of noise & artifacts

Main source of noise: Quantum noise[4]

(result of the statistical nature of X-ray emission)

Signal-to-noise Ratio (SNR) $\propto \sqrt{N}$, where N denotes the number of X-ray quanta / pixel.

$\rightarrow$ better SNR if we increase N, but also more radiation!

Countermeasures

- Larger voxels give better SNR (but reduced resolution)
- Smoothing during reconstruction (but resolution degrades)
- Better quantum efficiency of detectors

[4] each photon is a "quantum", i.e., a specific quantity of energy. Roughly speaking, due to the independent nature of photons, we get an uneven distribution of photons in an image area; this shows up as noise!

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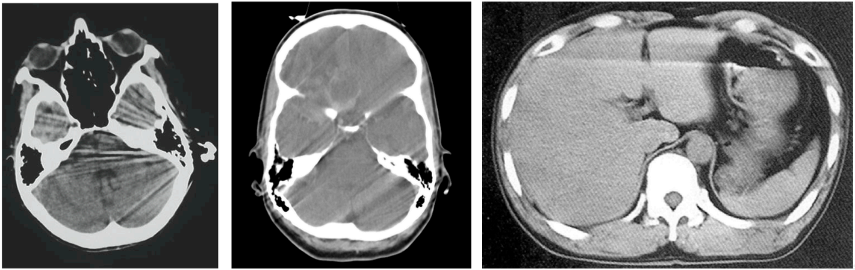
Some sources of noice & artifacts

Artifact from patient motion:



Characteristic streaks and "ghosting" (i.e., the image appears as if it is composed of superimposed images).

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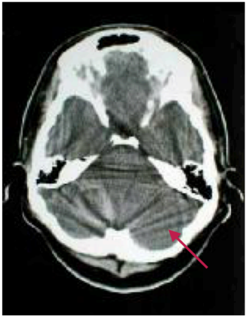
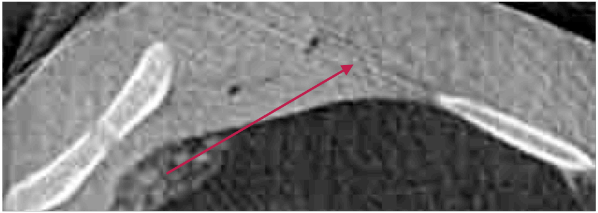
Some sources of noice & artifacts

Artifact from beam hardening:





Cause: already discussed (see previous slides)!

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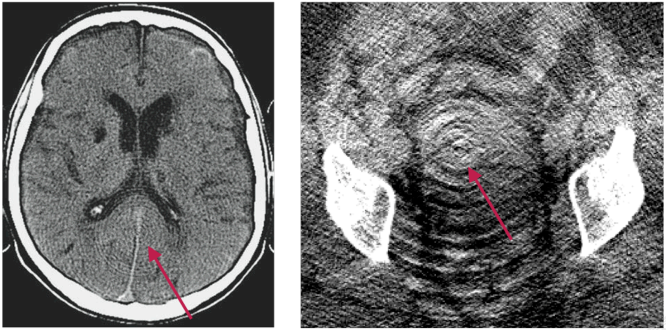
Some sources of noice & artifacts

Artifact from calibration issues:



Cause: poorly calibrated or defective detector elements; shows up as bright/dark ring(s) around the center of rotation.

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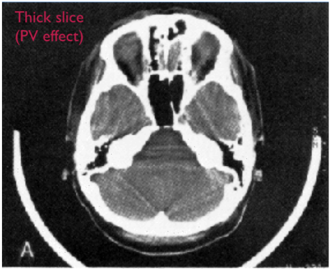
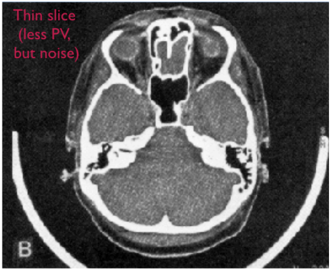
Some sources of noice & artifacts

Artifact from partial volume effects:





Cause: Object only *partially* overlaps a slice. For example, a highly attenuating material might appear with the density of soft tissue!

Computed Tomography

Advances in scanner technology

Early CT scanners acquired *a slice at a time*. Below is a sketch of the procedure (with stationary patient table):

1. Gantry spins 360° in one direction (one slice)
2. Gantry spins back (another slice)
3. Gantry stops and the patient table moves
4. Goto step (1)

Very time-consuming, hence long scan times!

What is used today?

Continuously rotating gantries $\rightarrow$ spiral or helical CT.

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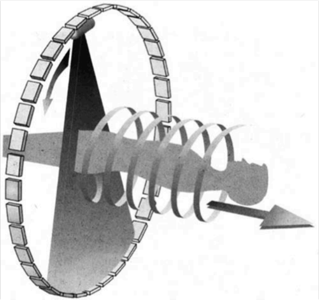
Advances in scanner technology



The data is acquired while the patient is moved through the scanner. The trajectory of X-Ray the beam traces out a helix.

Remark: This requires changes in the backprojection algorithm!

Computed Tomography

Applications of CT

CT is primarily used to acquire images of

- chest,
- lungs,
- abdomen,
- bones.

Good modality to diagnose pulmonary (i.e., lung) disease. *Why?*
Lungs are hard to diagnose using MRI or Ultrasound.