University of Salzburg — Lecturer: Roland Kwitt

Machine Learning (911.236)

Exercise sheet **D**

Exercise 1. 5 P.

Let our domain be $\mathcal{X} = \mathbb{R}$ and consider a hypothesis class $\mathcal{H}$ with functions $h : \mathbb{R} \to \{-1, +1\}$ of the form

$$x \mapsto h(x) = \mathrm{sign}(\sin(\alpha x)), \quad \alpha \geq 0 \ .$$

We define

$$\mathrm{sign}(x) = \begin{cases} +1 & \text{if } x \geq 0 \\ -1 & \text{else} \ . \end{cases}$$

The exercise is to proof that $\mathrm{VC}(\mathcal{H}) = \infty$.

Strategy: To show the claim, it's enough to show that for any n, a set of points $\{x_1, \ldots, x_n\}$ can be shattered. We start with a set of points

$$(x_i, y_i), i = 1, \ldots, n \quad \text{and } y_i \in \{-1, +1\}$$

with

$$x_i = 2\pi \ 10^{-i}$$

We also set

$$\alpha = \frac{1}{2}\left(1 + \sum_{i=1}^{n} \frac{(1 - y_i)10^i}{2}\right)$$

Now, we are left to argue that we can generate any possible labeling, independent of the size n.

Exercise 2. 2 P.

Say we have $\mathcal{X} = \mathbb{R}$ and consider a hypothesis class $\mathcal{H}$ that consists of hypotheses $h : \mathbb{R} \to \{0, 1\}$ with

$$x \mapsto h(x) = \begin{cases} 1, \text{if } x \in [a, b] \cup [c, d] \\ 0, \text{else} \end{cases}$$

with $a < b$ and $c < d$. What is the VC dimension of $\mathcal{H}$, i.e., $\mathrm{VC}(\mathcal{H})$. Remember, to show $\mathrm{VC}(\mathcal{H}) = d$, first show $\mathrm{VC}(\mathcal{H}) >= d$ and then $\mathrm{VC}(\mathcal{H}) < d + 1$. This means we first find a set of size d that is shattered and then show that no set of size $d + 1$ is shattered. For this example, a visual argument suffices.