Machine Learning (911.236)

Exercise sheet **B**

PAC Learning

Exercise 1.   2 P.

Given real numbers $a_1 \leq b_1$ and $a_2 \leq b_2$, define the predictor

$$h_{a_1, b_1, a_2, b_2}(x_1, x_2) = \begin{cases} 1 & \text{if } a_1 \leq x_1 \leq b_1 \text{ and } a_2 \leq x_2 \leq b_2 \\ 0 & \text{else} \end{cases}$$

This defines a rectangle in $\mathbb{R}^2$ which labels all points as 1 if they are inside and 0 otherwise. Assume realizability and let A be an algorithm that returns the smallest rectangle which encloses all positive instances in the training set S. *Argue that A is an ERM algorithm.*

Exercise 2.   3 P.

Let $\mathcal{X} = \mathbb{R}^2$, $\mathcal{Y} = \{0, 1\}$ and consider hypotheses $h_r : \mathcal{X} \to \mathcal{Y}$ in $\mathcal{H}$ of the form

$$h_r(\boldsymbol{x}) = 1_{\|\boldsymbol{x}\| \leq r}(\boldsymbol{x}), \text{ with } r \in \mathbb{R}_+ \ .$$

In other words, our hypotheses are *concentric circles*. Show that this class is PAC-learnable (i.e., assume realizability) from training data of size

$$m \geq \left(\frac{1}{\epsilon}\right) \log\left(\frac{1}{\delta}\right) \ .$$