University of Salzburg — Lecturer: Roland Kwitt

Machine Learning (911.236)

Exercise sheet **F**

Exercise 1. 5 P.

Construct an example showing that the $0-1$ loss function may suffer from **local minima**. In particular, take $\mathcal{X} = \mathbb{R}^2$ and construct a training sample

$$S \in (\mathcal{X} \times \{\pm 1\})^m$$

for which there exists a vector $\mathbf{w}$ and some $\epsilon > 0$ such that ...

- ... for any $\mathbf{w}'$ with

  $$\|\mathbf{w}' - \mathbf{w}\| \leq \epsilon$$

  we have

  $$L_S(\mathbf{w}) \leq L_S(\mathbf{w}')$$

but

- ... there exists some $\mathbf{w}^*$ such that

  $$L_S(\mathbf{w}^*) \leq L_S(\mathbf{w}) \quad .$$

This would show that $\mathbf{w}$ is **not** a global minimum of L_S.

Hint: Consider homogeneous halfspaces (parametrized by $\mathbf{w}$), i.e., $h_{\mathbf{w}}(\mathbf{x}) = \langle \mathbf{w}, \mathbf{x} \rangle$ and think about unit vectors and a very very small training set :) The Cauchy-Schwarz inequality might also be of great help!

Total #points: 5 P.