University of Salzburg | Lecturer: Roland Kwitt

Machine Learning (911.236)

Exercise sheet **B**

Exercise 1. 2 P.

Given an exponentially distributed random variable X, i.e., $X \sim \text{Exp}(\lambda)$, with rate $\lambda > 0$, upper bound $\mathbb{P}[X \geq a]$, with $a > 0$, using Markov's inequality.

Exercise 2. 4 P.

Given a random variable X and an *even* integer k, show that

$$\mathbb{P}\left[|X - \mathbb{E}[X]| > t\sqrt[k]{\mathbb{E}[(X - \mathbb{E}[X])^k]}\right] \leq \frac{1}{t^k}$$

assuming that $\mathbb{E}[(X - \mathbb{E}[X])^k] < \infty$.

Hint: Appropriately define random variable Y and use Markov's inequality.

Exercise 3. 4 P.

Say you have a discrete uniform random variable X on $\{\pm 1\}$, i.e., $X \sim \text{Unif}(\{\pm 1\})$. In other words, $\mathbb{P}[X = 1] = \mathbb{P}[X = -1] = 1/2$. Show that X is *sub-Gaussian* (see Definition 1 below).

Definition 1. *A real-valued random variable X is said to be σ-subgaussian if it has the property that there is some $\sigma > 0$ s.t. for every $\lambda \in \mathbb{R}$ one has*

$$\mathbb{E}[e^{tX}] \leq e^{\frac{\sigma^2\lambda^2}{2}}$$

Hint: There are multiple ways to show this. One is to first express $\mathbb{E}[e^{\lambda X}]$ (remembering that X is a random variable that follows a discrete uniform distribution on $\{\pm 1\}$), then do a series expansion of the exponential function, simplify (take a look at what cancels out), and use the series expansion of the exponential function again. A useful inequality will be $2^k k! \leq (2k)!$.

This is not just for fun, but we will use this type of random variables again later in the lecture.

Total #points: 10 P.