

**Machine Learning (911.236)**

## Exercise sheet A

**Background (Probability, inequalities, ...)****Exercise 1.**

1 P.

Let  $X$  be a random variable that captures rolling a fair dice with 6 sides. Use Markov's inequality to bound  $\mathbb{P}[X \geq 4]$  and also compute the exact probability. Is the bound loose or tight?

**Exercise 2.**

2 P.

Assume we have a random variable  $X$  which takes on values  $> -90$  and we know  $\mathbb{E}[X] = -30$ . Bound  $\mathbb{P}[X \geq -20]$ .

Hint: How about defining a new (appropriate) random variable  $Y$  such that we can apply Markov's inequality?

**Exercise 3.**

2 P.

Consider the following problem with (binary) inputs  $x_i \in \{0, 1\}$ ,  $i = 1, \dots, 4$  and (binary) output  $y \in \{0, 1\}$ :

$x_1$	$x_2$	$x_3$	$x_4$	$y$
1	0	0	1	0
1	1	0	1	1
1	0	1	1	1
1	0	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Lets say our learning objective is to learn a function

$$y = f(x_1, x_2, x_3, x_4)$$

which maps our four boolean inputs to one boolean output  $y \in \{0, 1\}$ . To get a feeling for the (size of the) problem, we want to know how many such functions exist? When you have the solution, think about what happens for  $n$  inputs  $x_1, \dots, x_n$ ? What do you think is the big problem here? Plot the number of functions as a function of the number of inputs  $n$ .