University of Salzburg | Lecturer: Roland Kwitt

Medical Imaging – Proseminar (911.934)

Exercise sheet **A**

Exercise 1. 4 P.

Download the ZIP file linked below. It contains a (brain) MRI image (`a01.nii.gz`) with a (manual) segmentation (`a01-seg.nii.gz`). The segmentation image only contains integers specifying to which anatomical structure each voxel belongs to.

Download by clicking

Use either `Convert3D` or `ITKSnap`, see , to compute, for the image (`a01.nii.gz`),

1. the image size (in voxel),
2. the physical voxel size (in mm),
3. the image orientation (e.g., RAS, etc.),
4. the image origin, and
5. the range of the intensity values in the MRI image.

Exercise 2. 2 P.

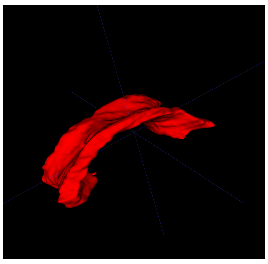
Use the MRI image from **Exercise 1** and convert the image orientation to *RPI* using `Convert3D`. Provide the `Convert3D` command.

Exercise 3. 4 P.

Use `Convert3D` to extract the middle transversal slice from the MRI image in **Exercise 1** and save it (1) as a PNG (visualize this) and (2) as a `.nii` file. What could be the problem with the PNG image here? Provide the `Convert3D` command!

Exercise 4. 5 P.

The ZIP file also contains a XML file `Hammers_mith_atlases_n30r95_label_indices_SPM12_20170315.xml` that lists the IDs of all anatomical brain structures present in `a01-seg.nii.gz`; Identify the ID of the *corpus callosum* and extract a binary volume (i.e., values in $\{0, 1\}$ with 1 for voxel belonging to the *corpus callosum* and 0 else). *Hint*: use the `-thresh` command line parameter of `Convert3D`). Visualize the corpus callosum in 3D (e.g., load the extracted corpus callosum as a *segmentation* in `ITKSnap` and update the 3D view) - this should look something like the image below.



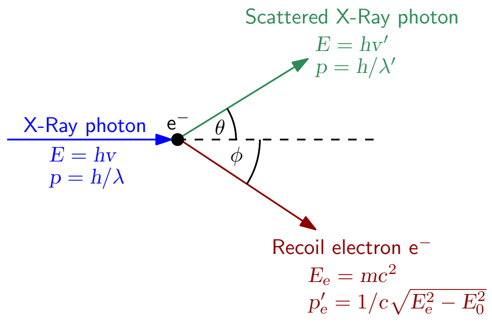
Finally, use Convert3D to compute the volume of this structure (in mm^3). Provide all Convert3D commands!

Exercise 5. 6 P.

This is a slightly longer exercise, but worth doing, as it gives detailed insight into where the Compton shift equation comes from. The latter is given by

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

where λ' is the wavelength of the scattered photon and λ is the wavelength of the incoming photon (λ_f and λ_i in lecture slides). Your task is to derive this equation, following the recipe below.



Background. For Compton scattering theory, we need relativistic mechanics, as (1) it involves the scattering of photons which are *massless* and (2) the energy transferred to the electron is comparable to its rest energy, E_0. In general, if we have an object moving at velocity v and rest mass m_0, we have the relativistic mass given by

$$m = \frac{m_0}{\sqrt{1 - (v/c)^2}}$$

The relativistic momentum (i.e., the product of mass and velocity) is $p = mv$ with p and v being vector quantities. From this, we can easily derive the following equality

$$(pc)^2 = E^2 - E_0^2 \tag{1}$$

which relates the magnitude of the relativistic momentum to its relativistic total energy E and its rest energy E_0 (for the total relativistic energy of the electron in Compton scattering, we would write E_e). From this, we obtain the famous *relativistic dispersion equation*

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

which expresses the relativistic total energy in terms of the rest mass, m_0 and the momentum p. Obviously, for a massless object (such as a photon), we have

$$E = pc \Rightarrow p = \frac{E}{c} = \frac{hv}{c} = \frac{h}{\lambda}$$

Recipe. Lets say we have the incoming photon at energy $E = hv$ and $p = h/\lambda$. The electron has rest energy $E_0 = m_0 c^2$ with $p_e = 0$ (right before the scattering happens, see figure). After scattering, the scattered photon (at angle θ) has $E' = hv'$ and $p' = h/\lambda'$. The recoil electron now has $E_e = mc^2$ and $p_e' = {}^1/_c\sqrt{E_e^2 - E_0}$ which we can easily derive from Eq. (1).

Start with the law of conservation of energy which dictates that

$$hv + E_0 = hv' + E_e$$

and then express p_e' in terms of the momentum of the incident photon, p, the momentum of the scattered photon, p', the rest energy of the electron, E_0, and the speed of light, c.

Once you have done that, use the law of conservation of momentum, to similarly express p_e. Essentially, the total momentum before the scattering event has to be the total momentum after the scattering event. Assume that the incident photon, with momentum p, is moving in the x direction (i.e., no y component, see figure) and the electron has no momentum (in x and y direction) before scattering. Hence, you should have

$$p = p' \cos\theta + p_e' \cos\phi \quad \text{and} \quad 0 = p' \sin\theta - p_e' \sin\phi$$

for the x and y component of the momentum. The trick is now to square both equations, add them, and simplify. This should yield another expression for p_e (or $p_e'^2$). Finally, equate the expressions for p_e' you got from conservation of energy and conservation of momentum, and simplify to get the Compton shift.

Total #points: 21 P.