University of Salzburg | Lecturer: Roland Kwitt

Machine Learning (911.236)

Exercise sheet **C**

Exercise 1. 5 P.

In the proof of the No-Free-Lunch (NFL) theorem, we started with a subset $C \subset \mathcal{X}$ of size $2m$ and studied what happens if our learning algorithm only sees training samples of size m. Now, assume that C is of size km with $k \geq 2$ and we only observe $|C|/k$ samples. Proof that the original lower bound of $1/4$ changes to $1/2 - 1/2k$. This means that there exists a distribution $\mathcal{D}$ over $\mathcal{X} \times \{0, 1\}$ such that (1) there exists a function $f : \mathcal{X} \to \{0, 1\}$ with $L_{\mathcal{D}}(f) = 0$ and (2) $\mathbb{E}_{S \sim \mathcal{D}^m}[L_{\mathcal{D}}(A(S))] \geq 1/2 - 1/2k$. Interpret this result.

Hint: You need to go through the proof of the NFL theorem and find the appropriate place where to inject these changes.

Exercise 2. 4 P.

Assume that someone gives you a coin and he/she tells you that the coin is *biased*. How many flips of the coin do you need to decide the direction of the bias? Consider the outcome of the coin flip as a sequence of independent Bernoulli random variables $X_1, \ldots, X_n$ with success probability p (say success means "heads" equals 1) and you estimate p with the empirical average $\hat{p}_n = 1/n \sum_i X_i$. If $\hat{p}_n \geq 1/2$ you decide a bias towards "heads", and "tails" otherwise. Show that if

$$n > \frac{1}{2\epsilon^2} \log\left(\frac{1}{\delta}\right) \ , \tag{1}$$

then you decide correctly with probability $1 - \delta$ for $\delta \in (0, 1)$ fixed. Remember that if $p = 1/2 - \epsilon, \epsilon > 0$, you make an error if $\hat{p}_n \geq 1/2$ (and vice versa).

Total #points: 9 P.