

**Machine Learning (911.236)**

## Exercise sheet D

**Exercise 1.**

3 P.

Show that the condition

$$\mathbb{E}_{S \sim \mathcal{D}^m} [L_{\mathcal{D}}(A(S))] \geq \frac{1}{4}$$

we had in the lecture during our proof of the NFL theorem *suffices* for

$$\mathbb{P}_{S \sim \mathcal{D}^m} [L_{\mathcal{D}}(A(S)) \geq 1/8] \geq 1/7$$

to hold. Hint: use Lemma 1 below.

**Lemma 1** (see Lemma B.1 in Shalev-Schwartz & Ben-David's book). *Let  $Z$  be a random variable taking values in  $[0, 1]$  and assume that  $\mathbb{E}[Z] = \mu$ . Then, for any  $a \in (0, 1)$ ,*

$$\mathbb{P}[Z > 1 - a] \geq \frac{\mathbb{E}[Z] - (1 - a)}{a} ,$$

*which also implies that for every  $a \in (0, 1)$ , we have*

$$\mathbb{P}[Z > a] \geq \frac{\mathbb{E}[Z] - a}{1 - a} \geq \mathbb{E}[Z] - a .$$