University of Salzburg — Lecturer: Roland Kwitt

Machine Learning (911.236)

Exercise sheet **A**

Exercise 1. 2 P.

Show that for any set A, the power set $\mathcal{P}(A)$ (or written as 2^A) is a σ-algebra on A. Remember that the power set is defined as the set of all subsets of A.

Exercise 2. 2 P.

Show that for any set A, $\{\emptyset, A\}$ is a σ-algebra.

Exercise 3. 2 P.

Show that if $(S_1, \mathcal{F}_1)$, $(S_2, \mathcal{F}_2)$ and $(S_3, \mathcal{F}_3)$ are measurable spaces and $f : S_1 \longrightarrow S_2$, $g : S_2 \longrightarrow S_3$ are measurable functions (with respect to the respective σ-algebras), then $g \circ f : S_1 \longrightarrow S_3, x \mapsto (g \circ f)(x) = g(f(x))$ is measurable.

Exercise 4. 2 P.

Say you have $S = \{a, b\}$ with σ-algebra $F = \mathcal{P}(S)$. Take a look at the following functions ($\mu_i, i = 1, \ldots, 4$) that assign to each element of F a value in $\mathbb{R} \cup \{\infty\}$:

- $\mu_1(\emptyset) = 0$, $\mu_1(\{a\}) = 5$, $\mu_1(\{b\}) = 6$ and $\mu_1(\{1, 2\}) = 11$
- $\mu_2(\emptyset) = 0$, $\mu_2(\{a\}) = 0$, $\mu_2(\{b\}) = 0$ and $\mu_2(\{1, 2\}) = 1$
- $\mu_3(\emptyset) = 0$, $\mu_3(\{a\}) = 0$, $\mu_3(\{b\}) = 1$ and $\mu_3(\{1, 2\}) = 1$
- $\mu_4(\emptyset) = 0$, $\mu_4(\{a\}) = 0$, $\mu_4(\{b\}) = \infty$ and $\mu_4(\{1, 2\}) = \infty$

Which of those μ_i is a *measure*, which is a *measure/probability measure* (or neither)? Provide an argument for each answer.

Exercise 5. 3 P.

Show that the intersection of two σ-algebras on set S is also a σ-algebra on S. E.g., take F_1 and F_2 σ-algebras over S and verify that (i) $\emptyset$ is in $F_1 \cap F_2$, (ii) the complement of any set in $F_1 \cap F_2$ is also in $F_1 \cap F_2$ and (iii) countable additivity holds.

Exercise 6. 3 P.

Suppose Jack is late to work on a given day with probability of *at most* 0.02. Bound the probability that this happens (i.e., Jack being late to work) *at least once* over a period of 20 days. *Do not make any independence assumptions.*

Total #points: 14 P.