University of Salzburg | Lecturer: Roland Kwitt

Machine Learning (911.236)

Exercise sheet **D**

Exercise 1. 4 P.

Given two hypothesis classes $\mathcal{F}$ and $\mathcal{G}$ of functions from domain $\mathcal{X}$ to label set $\{0, 1\}$. Show that

$$\mathrm{VC}(\mathcal{F} \cup \mathcal{G}) \leq \mathrm{VC}(\mathcal{F}) + \mathrm{VC}(\mathcal{G}) + 1$$

where $\mathrm{VC}(\cdot)$ denotes the VC dimension (as usual). A useful identity to use is

$$\binom{m}{i} = \binom{m}{m-i} \quad \text{for} \quad 0 \leq i \leq m \enspace .$$

Exercise 2. 4 P.

Consider the hypothesis class

$$\mathcal{B}_{\boldsymbol{\mu},r} = \{B_{\boldsymbol{\mu},r}, \boldsymbol{\mu} \in \mathbb{R}^d, r > 0\}$$

with

$$B_{\boldsymbol{\mu},r}(\mathbf{x}) = \begin{cases} 1, \text{if } \|\mathbf{x} - \boldsymbol{\mu}\| \leq r \\ 0, \text{otherwise} \end{cases}$$

The is the class of *closed balls* in $\mathbb{R}^d$ with center $\boldsymbol{\mu}$ and radius $r > 0$. Take the set

$$C = \{\mathbf{e}_1, \ldots, \mathbf{e}_d, \mathbf{0}\}$$

where $\mathbf{e}_i$ denotes the i-th unit vector and $\mathbf{0}$ is the "all zeros" vector. Show that this set of $d + 1$ points is shattered by $\mathcal{B}_{\boldsymbol{\mu},r}$. In other words, you have to show that for any $A \subseteq C$, the points in A can be labeled positively, while all points in $C \setminus A$ (i.e., C without A) are labeled as '0'. Of course, this shows that $\mathrm{VC}(B_{\boldsymbol{\mu},r}) \geq d + 1$. *Hint:* $\boldsymbol{\mu}$ could be chosen to be the sum over the unit vectors in A.

Exercise 3. 4 P.

Find the minimizer (wrt. α) of the function

$$g : \alpha \mapsto (1 - \epsilon)e^{-\alpha} + \epsilon e^{\alpha}$$

and provide an argument how this fits into the **AdaBoost** algorithm from the lecture.

Total #points: 12 P.