Mini Lecture: MCMC 24/2 Week 8.3

MCMC Coding Practice

Note (Conditional Dependency Graphs):

$$y_t \mid \tau_t \sim Bin(n_t, \pi(\tau_t))$$

for $\pi(\tau_t) = \frac{1}{1+e^{-\tau_t}}$

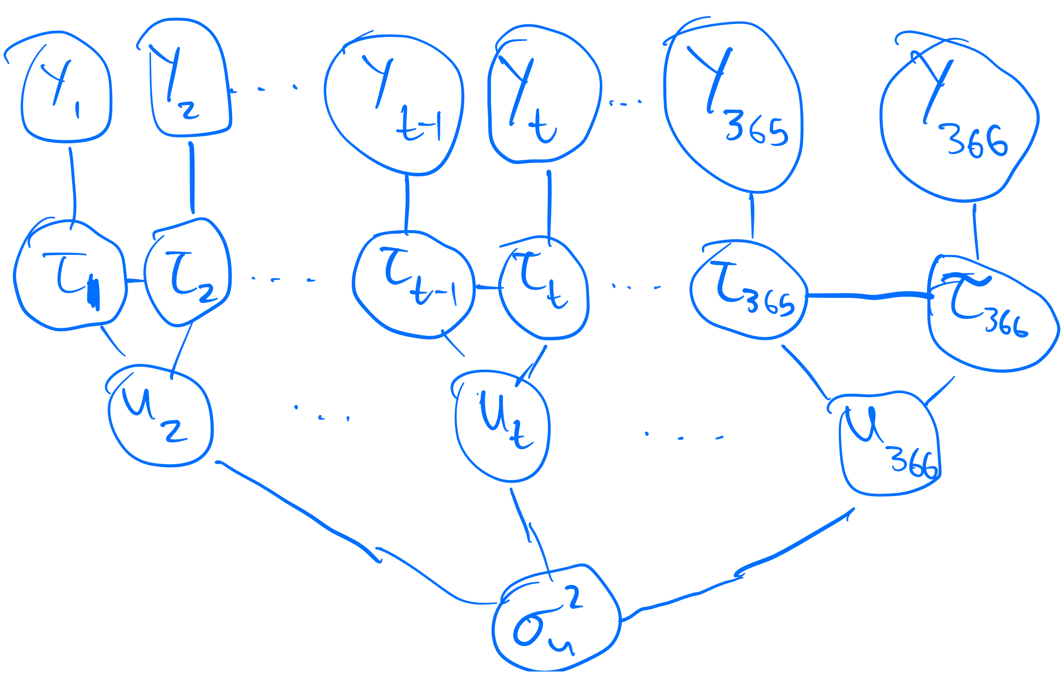
} data model

$$\tau_t = \tau_{t-1} + u_t, \quad u_t \mid \sigma_u^2 \overset{iid}{\sim} N(0, \sigma_u^2)$$

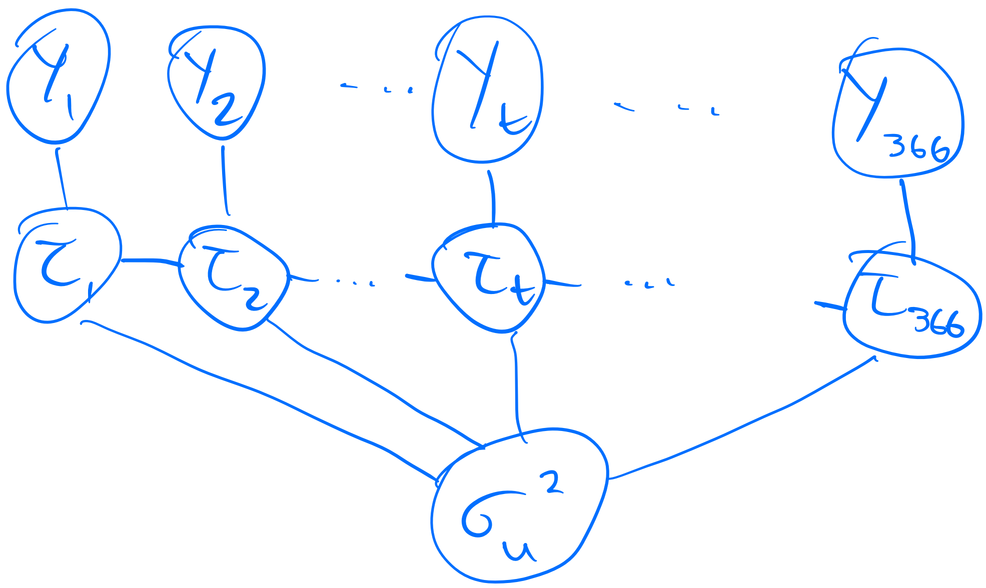
} latent model

$$\sigma_u^2 \sim \text{Inv-Gamma}(\alpha, \beta)$$

} hyperprior for σ_u^2

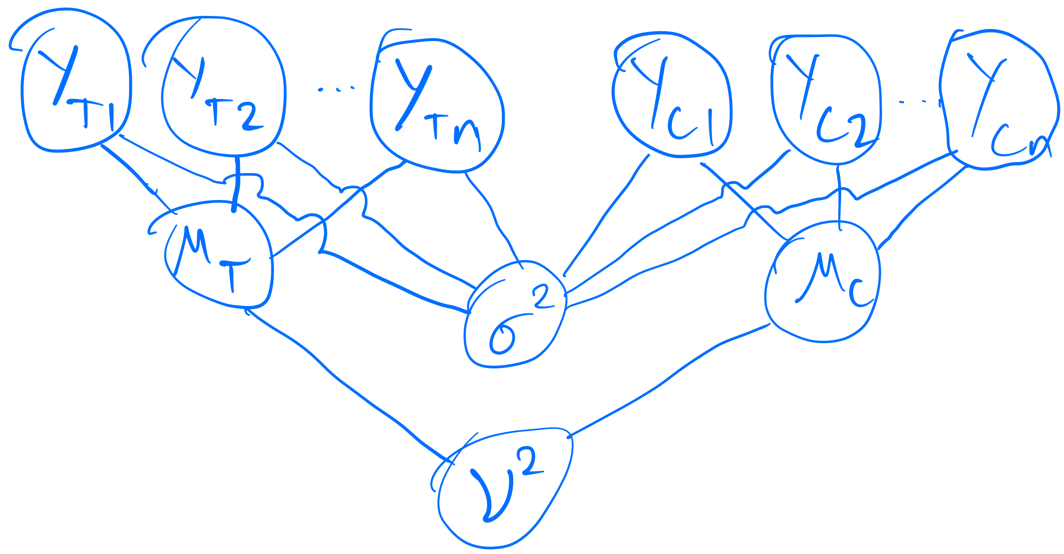


reparameterize:



This represents the model's conditional dependencies. For example, removing τ_1 and τ_2, we cannot reach Y_2 from Y_1 (and vice-versa). This means $Y_1 \perp Y_2 \mid \tau_1, \tau_2$. Note: we are only drawing connections that are explicitely stated in the Bayesian Hierarchical model given.

Worksheet Q 1:



$$Y_{T1} \mid \mu_T, \sigma^2 \sim N(\mu_T, \sigma^2)$$

$\Rightarrow$ We draw lines from μ_T & σ^2 to Y_{T1}

$$\mu_T, \mu_C \mid \nu^2 \overset{iid}{\sim} N(0, \nu^2)$$

$\Rightarrow$ we draw lines from ν^2 to μ_T, μ_C

Q 2:

$$p(\mu_T, \mu_C, \sigma^2, \nu^2 \mid y_T, y_C) = \cancel{p(y_T \mid \mu_T, \sigma^2)}$$

$$= p(y_T, y_C \mid \mu_T, \mu_C, \sigma^2, \nu^2) \times$$

$$\times\ p(\mu_T \mid \mu_C, \sigma^2, \nu^2) \times p(\mu_C \mid \sigma^2, \nu^2)$$

$$\times\, p(\sigma \mid \nu\;)\cdot p(\nu\;)$$

$$= p(y_T \mid \mu_T, \sigma^2) \times p(y_C \mid \mu_C, \sigma^2)$$

$$\times\, p(\mu_T \mid \nu^2) \times p(\mu_C \mid \nu^2) \times p(\sigma^2) \times p(\nu^2)$$