Øving 6

12.6

11.

$A_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

$u_x(0,t) = 0 = u_x(L,t)$

$u(x,t) = f(x)$

$u_t = c^2 u_{xx}$

Skriver $u(x,t) = F(x)G(t)$

$\frac{G'(t)}{c^2 G(t)} = \frac{F''(x)}{F(x)} = -p^2$

$F''(x) + p^2 F(x) = 0$ (1)

$G'(t) + c^2p^2 G(t) = 0$ (2)

Løser (1), gir $F(x) = A\cos(px) + B\sin(px)$

Vet fra "boundary cond." $u(0,t) = F(0)G(t) = 0$

$u(L,t) = F(L)G(t) = 0$

12.

$L = \pi,\ c = 1$

$f(x) = x$

$A_0 = \frac{1}{L}\int_0^L f(x)dx$

$= \frac{1}{\pi}\int_0^{\pi} x\,dx$

$= \frac{1}{2\pi}\pi^2$

$= \frac{\pi}{2}$

$A_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

$= \frac{2}{\pi}\int_0^{\pi} x\sin(nx)dx$

$= [\text{Tidl. øvinger}]$

$= \frac{2}{\pi}\left(\left[-\frac{x\cos(nx)}{n}\right]_0^{\pi} + \left[\frac{\sin(nx)}{n^2}\right]_0^{\pi}\right)$

$= \frac{2}{\pi}\left(-\frac{\pi(-1)^n}{n}\right)$

$= -\frac{2(-1)^n}{n}$

$\underline{\underline{u(x,t) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\left(-\frac{2(-1)^n}{n}\right)\cos(nx)e^{-n^2t}}}$

14.

$L = \pi,\ c = 1$

$f(x) = \cos(2x)$

$A_0 = \frac{1}{\pi}\int_0^{\pi}\cos(2x)dx$

$= \frac{1}{\pi}\left[\frac{\sin(2x)}{2}\right]_0^{\pi}$

$= 0$

$A_n = \frac{2}{\pi}\int_0^{\pi}\cos(2x)\sin(nx)dx$

$= \frac{2}{\pi}\int_0^{\pi}\frac{\sin(nx+2x)+\sin(nx-2x)}{2}dx$

$= \frac{1}{\pi}\left(\int_0^{\pi}\sin(nx+2x)dx + \int_0^{\pi}\sin(nx-2x)dx\right)$

$= \frac{1}{\pi}\left(\left[-\frac{\cos(nx+2x)}{n+2}\right]_0^{\pi} + \left[-\frac{\cos(nx-2x)}{n-2}\right]_0^{\pi}\right)$

$= \frac{1}{\pi}\left(\frac{-(-1)^n+1}{n+2} + \frac{-(-1)^n+1}{n-2}\right)$

$\underline{\underline{u(x,t) = \sum_{n=1}^{\infty}\left(\frac{1}{\pi}\left(\frac{-(-1)^n+1}{n+2} + \frac{-(-1)^n+1}{n-2}\right)\right)\cos(nx)e^{-n^2t}}}$

16.

$H > 0$

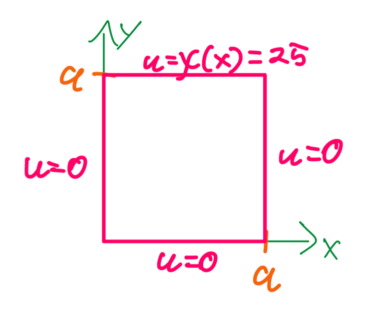
$u_t = c^2u_{xx} + H$

$L = \pi$

$u = v - \frac{Hx(x-\pi)}{2c^2}$

21.

$a = 24$



$A_n^* = \frac{2}{a\sinh(n\pi a/a)}\int_0^a f(x)\sin\left(\frac{n\pi x}{a}\right)dx$

$= \frac{1}{12\sinh(n\pi)}\int_0^{24} 25\sin\left(\frac{n\pi x}{24}\right)dx$

$= \frac{25}{12\sinh(n\pi)}\left[-\frac{24\cos(n\pi x/24)}{n\pi}\right]_0^{24}$

$= \frac{-50}{n\pi\sinh(n\pi)}(\cos(n\pi)-1)$

$= \frac{-50((-1)^n-1)}{n\pi\sinh(n\pi)}$

$= \begin{cases}\frac{100}{n\pi\sinh(n\pi)}, & n = \text{odde}\\ 0, & n = \text{like}\end{cases}$

$\underline{\underline{u(x,y) = \sum_{n\ \text{odde}}\frac{100}{n\pi\sinh(n\pi)}\sin\left(\frac{n\pi x}{24}\right)\sinh\left(\frac{n\pi y}{24}\right)}}$

12.7

1.

$2u_x + 3u_t = 0,\ u(x,0) = f(x)$

F-transf.

$2\mathcal{F}(u_x) + 3\mathcal{F}(u_t) = 0$

$2\hat{u}_x + 3\hat{u}_t = 0$

$u(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\hat{u}(w,t)e^{iwx}dw$

$u_x(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} iw\hat{u}(w,t)e^{iwx}dw$

$\hat{u}_t = iw\hat{u}$

$\hat{u}(w,0) = \hat{f}(w)$

$\hat{u}(w,t) = \hat{f}(w)e^{iwt}$

$\mathcal{F}^{-1}$-transf.:

$u(x,t) = \mathcal{F}^{-1}\left[\hat{f}(w)e^{iwt}\right]$

$= f(x+t)$

2.

$2tu_x + 3u_t = 0,\ u(x,0) = f(x)$

F-transf.

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