Øving 10

6.1

11. $\|x+y\|^2+\|x-y\|^2=\langle x+y,x+y\rangle+\langle x-y,x-y\rangle$

$$=\langle x,x+y\rangle+\langle y,x+y\rangle+\langle x,x-y\rangle-\langle y,x-y\rangle$$

$$=\langle x,x\rangle+\cancel{\langle x,y\rangle}+\cancel{\langle y,x\rangle}+\langle y,y\rangle+\langle x,x\rangle-\cancel{\langle x,y\rangle}-\cancel{\langle y,x\rangle}+\langle y,y\rangle$$

$$=2\langle x,x\rangle+2\langle y,y\rangle$$

$$=2\|x\|^2+2\|y\|^2$$

15.(a) $|\langle x,y\rangle|=\|x\|\cdot\|y\| \iff x=\alpha y$

$\alpha=\frac{\langle x,y\rangle}{\|y\|^2}$

$z=x-\alpha y$

$\langle z,y\rangle=\langle x-\alpha y,y\rangle$

$$=\langle x-\alpha y,y\rangle$$

$$=\langle x,y\rangle-\alpha\langle y,y\rangle$$

$$=\langle x,y\rangle-\frac{\langle x,y\rangle}{\|y\|^2}\cdot\|y\|^2$$

$$=\langle x,y\rangle-\langle x,y\rangle$$

$$=0$$

z,y ortogonal

$|\alpha|=\frac{|\langle x,y\rangle|}{\|y\|^2}$

$$=\frac{\|x\|\cdot\|y\|}{\|y\|^2}$$

$$=\frac{\|x\|}{\|y\|}$$

$\|x\|^2=\|\alpha y+z\|^2$

$$=\|\alpha y\|^2+\|z\|^2$$

$$=|\alpha|^2\|y\|^2+\|z\|^2$$

$$=\left(\frac{\|x\|}{\|y\|}\right)^2\|y\|^2+\|z\|^2$$

$$=\|x\|^2+\|z\|^2$$

$\Rightarrow z=0 \Rightarrow x=\alpha y$

(b) $\|x+y\|=\|x\|+\|y\|$

$$\geq 0 \iff x = ay$$

(b) $\|x+y\| = \|x\| + \|y\|$

$x = ay$

$\|x+y\| = \|ay+y\|$

$= \|y(a+1)\|$

$= \|y\| \, |a+1|$

$\|x\| + \|y\| = \|ay\| + \|y\|$

$= |a| \, \|y\| + \|y\|$

$= \|y\| (|a| + 1)$

$|a+1| = |a| + 1$

$\|x+y\| = \|x\| + \|y\|$ hvis $a \geq 0$

$\|x+y\| = \|x\| + \|y\|$

$\sqrt{\langle x+y, x+y \rangle} = \sqrt{\langle x,x \rangle} + \sqrt{\langle y,y \rangle}$

$\langle x,x \rangle + 2\langle x,y \rangle + \langle y,y \rangle = \langle x,x \rangle + 2\langle x,x \rangle \langle y,y \rangle + \langle y,y \rangle$

$2\langle x,y \rangle = 2\langle x,x \rangle \langle y,y \rangle$

$\langle x,y \rangle = \langle x,x \rangle \langle y,y \rangle$

$|\langle x,y \rangle| = \|x\| \cdot \|y\|$ når $x = ay,\ a \geq 0$

$\|\sum_{i=1}^{n} x_i\| = \sum_{i=1}^{n} \|x_i\|$ når $x_i = a_i x,\ a_i \geq 0,\ i \geq 2$

19.(a) $\|x \pm y\|^2 = \|x\| \pm 2\mathrm{Re}(\langle x,y \rangle) + \|y\|^2 \quad \forall x,y \in V$

$\|x+y\|^2 = \langle x+y, x+y \rangle$

$= \langle x, x+y \rangle + \langle y, x+y \rangle$

$= \langle x,x \rangle + \langle x,y \rangle + \langle y,x \rangle + \langle y,y \rangle$

$= \|x\|^2 + 2\mathrm{Re}(\langle x,y \rangle) + \|y\|^2$

$\|x-y\|^2 = \langle x-y, x-y \rangle$

$= \langle x, x-y \rangle - \langle y, x-y \rangle$

$= \langle x,x \rangle - \langle x,y \rangle - \langle x,y \rangle + \langle y,y \rangle$

$= \|x\|^2 - 2\mathrm{Re}(\langle x,y \rangle) + \|y\|^2$

$\Rightarrow \|x \pm y\|^2 = \|x\| \pm 2\mathrm{Re}(\langle x,y \rangle) + \|y\|^2$

$$= \|x\|^2 - 2Re(\langle x,y\rangle) + \|y\|^2$$

$$\Rightarrow \|x \pm y\|^2 = \|x\|^2 \pm 2Re(\langle x,y\rangle) + \|y\|^2$$

(b) $\big|\,\|x\| - \|y\|\,\big| \leq \|x-y\| \quad \forall x,y \in V$

Δ-ulikheten:

$$\|x+y\| \leq \|x\| + \|y\|$$

$$\|x\| = \|x-y+y\|$$
$$\leq \|x-y\| + \|y\|$$
$$\Rightarrow \|x\| - \|y\| \leq \|x-y\|$$

$$\|y\| = \|y-x+x\|$$
$$\leq \|y-x\| + \|x\|$$
$$\Rightarrow -\|y-x\| \leq \|x\| - \|y\|$$
$$-\|x-y\| \leq \|x\| - \|y\|$$

$$\Rightarrow -\|x-y\| \leq \|x\| - \|y\| \leq \|x-y\|$$

$$\Rightarrow \big|\,\|x\| - \|y\|\,\big| \leq \|x-y\|$$

6.2

2 (b) $V = \mathbb{R}^3$

$S = \{(1,1,1), (0,1,1), (0,0,1)\}$

$x = (1,0,1)$