Eksamen

1. $f: \mathbb{R}^2 \to \mathbb{R}$

$f(x,y) = x^2 + y^2 - 2(x+y) + 2$

$\nabla f(x,y) = (2x-2, 2y-2) = (0,0)$

$\Rightarrow 2x-2=0$ og $2y-2=0$

$\Rightarrow x=1$ og $y=1$

$\Rightarrow$ Ekstremalpunkt i $(1,1)$

$f_{xx}(x,y)=2$

$f_{yy}(x,y)=2$

$f_{xy}(x,y)=0$

$f_{yx}(x,y)=0$

$f_{xx}(1,1)\cdot f_{yy}(1,1) - [f_{xy}(1,1)]^2 = 2\cdot 2 - 0^2$

$= 4$

Siden $4>0$ så betyr det at $(1,1)$ er lokalt minimum ved bruk av andrederiverttesten

$\{x^2+y^2=2\mathbb{R}^2\}, \mathbb{R}>0$

$x^2+y^2-2(x+y)+2 = x^2+y^2-2\mathbb{R}^2$

$-2(x+y)+2 = -2\mathbb{R}^2$

$x+y-1=\mathbb{R}^2$

$\Rightarrow$

2. $\frac{\sin(x^2)}{x^2+y^2}$

$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2)}{x^2+y^2}$

Langs $(x,0)$

$\lim_{(x,0)\to(0,0)} \frac{\sin(x^2)}{x^2} =$ divergerer

Langs $(0,y)$

$\lim_{(0,y)\to(0,0)} \frac{0}{y^2} = 0$

$\Rightarrow$ grensen eksisterer ikke

3. $P_0\left(0, \frac{1}{\sqrt{2}}, \frac{\pi}{4}\right)$

$x^2+y^2+\cos^2(z)=1$

$f(x,y,z)=x^2+y^2+\cos^2(z)-1=0$

$\frac{\partial f}{\partial z}\Big|_{P_0}=-2\cos(z)\sin(z)\Big|_{P_0}$

$=-2\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$

$=-1\neq 0$

Implisitte f. setn.

$\Rightarrow$ z kan skrives som $\phi(x,y)$

$\frac{\partial z}{\partial x}\Big|_{P_0}=-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}}\Bigg|_{P_0}$

$=-\frac{2x}{-2\cos(z)\sin(z)}\Big|_{P_0}$

$=0$

$\frac{\partial z}{\partial y}\Big|_{P_0}=-\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial z}}\Bigg|_{P_0}$

$=-\frac{2y}{-2\cos(z)\sin(z)}\Big|_{P_0}$

$=-\frac{\frac{2}{\sqrt{2}}}{-1}$

$=\frac{2}{\sqrt{2}}$

Tangentplan:

$z=z(x_0,y_0)+(x-x_0)\partial_x z(x_0,y_0)+(y-y_0)\partial_y z(x_0,y_0)$

$=z\left(0,\frac{1}{\sqrt{2}}\right)+0+\left(y-\frac{1}{\sqrt{2}}\right)\cdot\frac{2}{\sqrt{2}}$

$=z\left(0,\frac{1}{\sqrt{2}}\right)+\left(y-\frac{1}{\sqrt{2}}\right)\frac{2}{\sqrt{2}}$

4. $y=x^2$

$y=1+x^2$

$x=0$

$x=1$

$0\leq x\leq 1$

$x^2\leq y\leq 1+x^2$

$\int_0^1\int_{x^2}^{1+x^2}dy\,dx=\int_0^1 dx$

$=1$

$$\int_0 \int_{x^2} dy\,dx = \int_0 dx$$
$$= 1$$

$$(x,y) \mapsto (u,v) = (x^2, y)$$

$$x = \sqrt{u}$$

$$y = v$$

$$\begin{bmatrix} \partial_u x & \partial_v x \\ \partial_u y & \partial_v y \end{bmatrix} = \begin{bmatrix} \frac{1}{2\sqrt{u}} & 0 \\ 0 & 1 \end{bmatrix}$$ er Jakobimatrisen

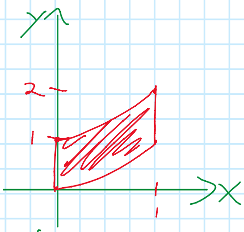
$$\begin{vmatrix} \frac{1}{2\sqrt{u}} & 0 \\ 0 & 1 \end{vmatrix} = \frac{1}{2\sqrt{u}}$$

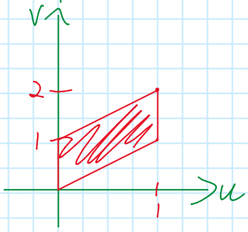
$x = \sqrt{u} \Rightarrow x^2 = u$ og ikke $-u$ fordi $\sqrt{u}^2 = u$

$$0 \leq x \leq 1 \Rightarrow 0 \leq u \leq 1$$

$$x^2 \leq y \leq 1 + x^2 \Rightarrow u \leq v \leq 1 + v$$

$$\Rightarrow \int_0^1 \int_{x^2}^{1+x^2} dy\,dx = \int_0^1 \int_u^{1+u} \frac{1}{2\sqrt{u}} dv\,du$$
$$= \int_0^1 \frac{1}{2\sqrt{u}} du$$
$$= \left[\sqrt{u}\right]_0^1$$
$$= 1$$





5. $\int_\delta F \cdot T\,ds$

$$\gamma : t \mapsto [t, t^2, (1-t)t],\ t \in [0,1]$$

$$F : \mathbb{R}^3 \mapsto \mathbb{R}^3$$

$$(x,y,z) \mapsto e^{xy\cos(z)}[y\cos(z), x\cos(z), -xy\sin(z)]$$

Viser at F er konservativt:

Ser at $F = \nabla\phi$ der $\phi = e^{xy\cos(z)}$

$\Rightarrow$ F er konservativt

$$\Rightarrow \int_\gamma F \cdot T\,ds = \phi(\gamma(1)) - \phi(\gamma(0))$$

$$= \phi(1,1,0) - \phi(0,0,0)$$

$$= e - 1$$

6. $F(x,y,z) = (y, -x, z)$

$x^2 + y^2 + z^2 = 81,\ z \geq 0$