Project 2

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Problem 1

a)

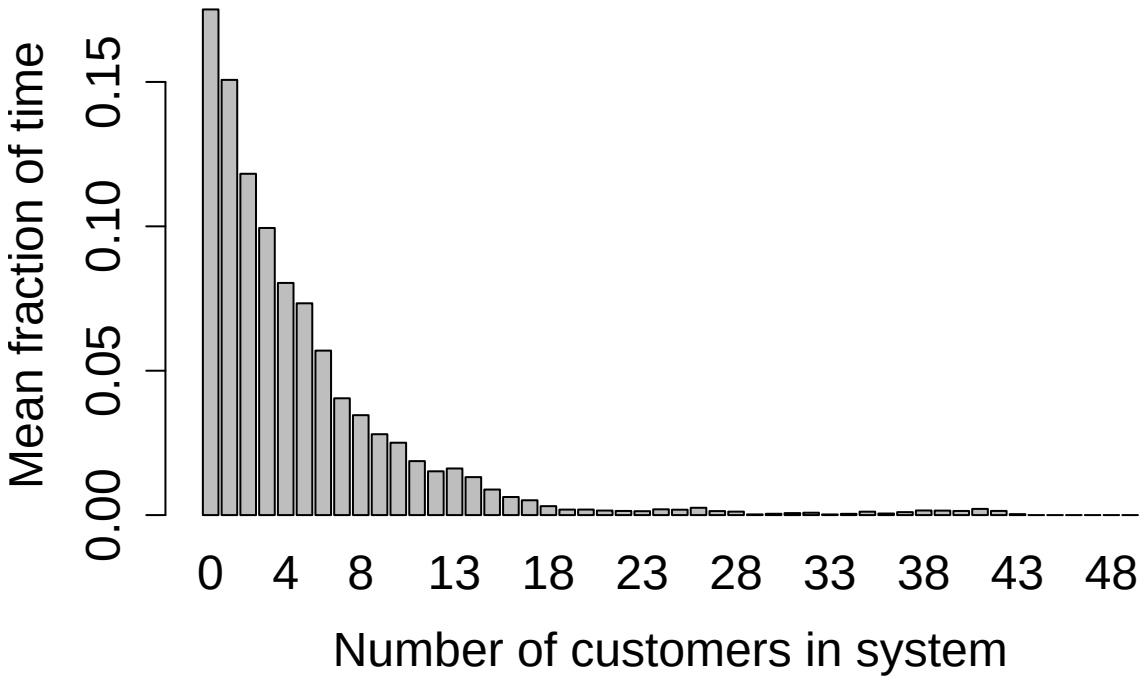
b)

```
set.seed(97)
lam = 5
mu = 6

# Initial conditions
x = c(0)
s = c(0)

# Simulate forward in time
B = 11908
for (i in 1:B) {
    cState = x[i]
    if (cState == 0) {
        s = c(s, tail(s, 1) + rexp(n = 1, rate = lam))
        x = c(x, 1)
    } else {
        s = c(s, tail(s, 1) + rexp(n = 1, rate = lam + mu))
        if (runif(1) < lam/(lam + mu)) {
            x = c(x, cState + 1)
        } else {
            x = c(x, cState - 1)
        }
    }
}
```

```
table(x)
```

```
## x
##    0    1    2    3    4    5    6    7    8    9   10   11   12   13   14   15
## 1042 1912 1565 1281 1111  959  757  576  445  368  326  252  207  211  174  122
##   16   17   18   19   20   21   22   23   24   25   26   27   28   29   30   31
##   99   75   44   25   24   23   17   14   17   22   25   24   14    4    5    7
##   32   33   34   35   36   37   38   39   40   41   42   43
##    8    8   10   12    8    9   18   22   24   23   15    5
```

```
numT = rep(0, 50)
hmm = 0
for (i in 1:(length(x) - 1)) {
    numT[x[i] + 1] = numT[x[i] + 1] + s[i + 1] - s[i]
    hmm = hmm + s[i + 1] - s[i]
}
barplot(numT/s[length(x)], names.arg = 0:(length(numT) - 1),
    xlab = "Number of customers in system", ylab = "Mean fraction of time",
    cex.axis = 1.5, cex.lab = 1.5, cex = 1.5)
```




```
# Expected number of requests in the system
print("Average number of requests in the system:")
```

```
## [1] "Average number of requests in the system:"
```

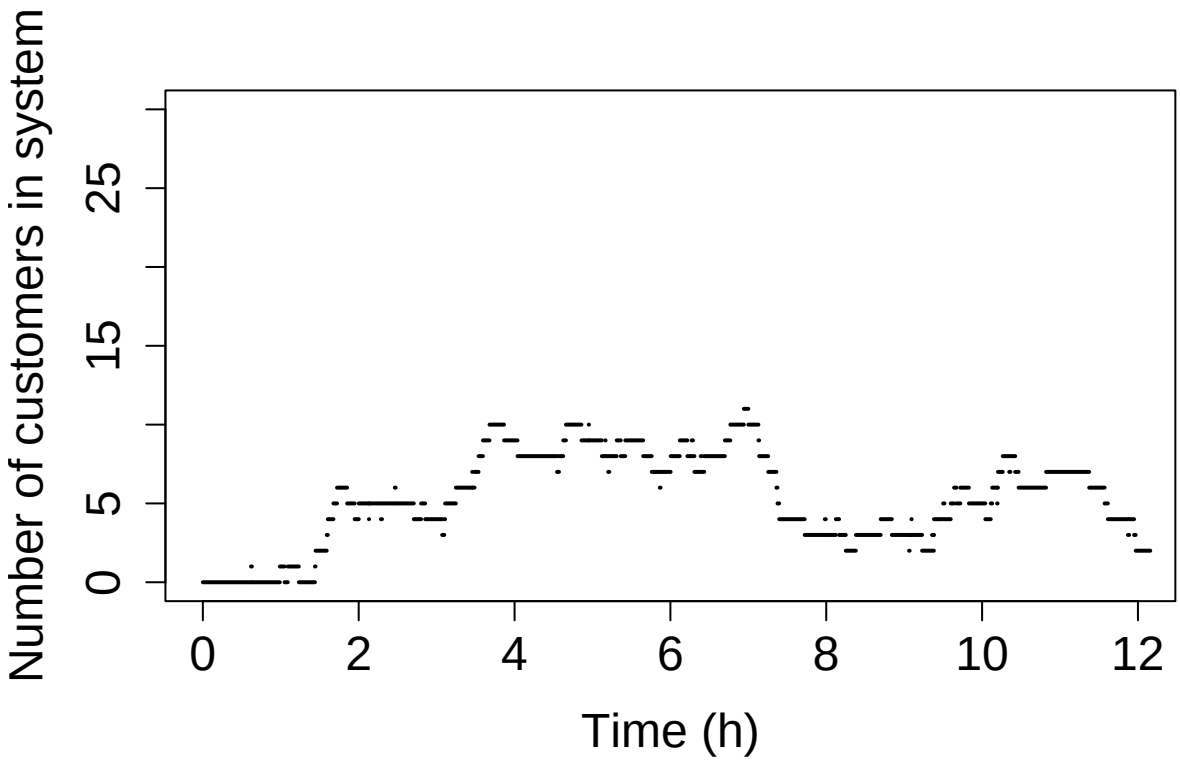
```
pi = numT/s[length(x) - 1]
L = sum((0:49) * pi)
print(L)
```

```
## [1] 4.867745
```

```
# Expected waiting time
cat("Exptected time spent in system:", L/lam, "hour(s)")
```

```
## Exptected time spent in system: 0.9735491 hour(s)
```

```
# 12 hours
plot(NULL, NULL, xlim = c(0, 12), ylim = c(0, 30), xlab = "Time (h)",
    ylab = "Number of customers in system", cex.axis = 1.5, cex.lab = 1.5)
for (i in 1:119) {
    lines(s[i:(i + 1)], rep(x[i], 2), lwd = 2)
}
```



c)

d)

e)

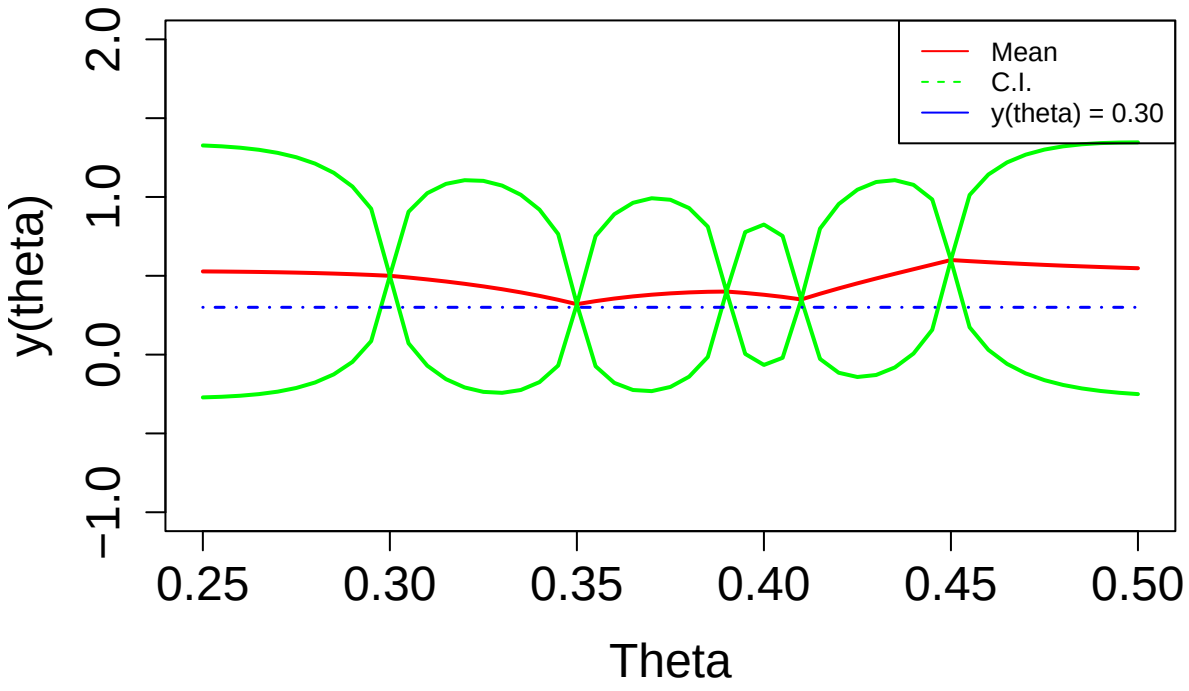
f)

g)

Problem 2

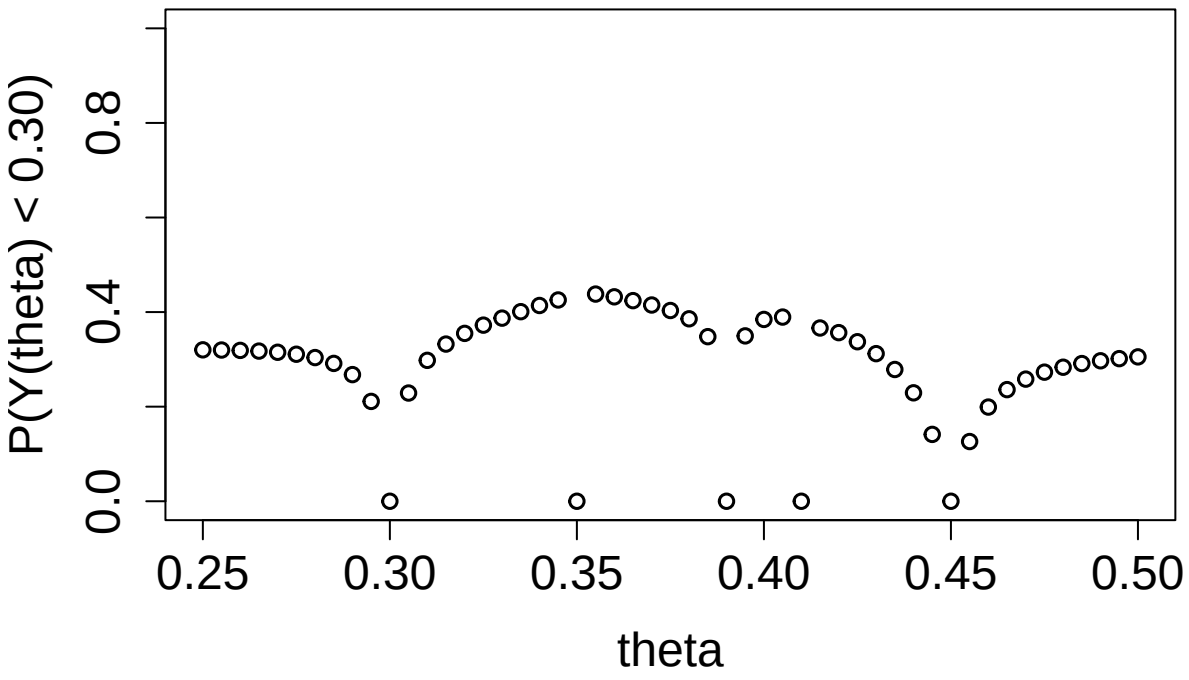
a)

```
set.seed(98)
xx <- seq(0.25, 0.5, 0.005)

mu = rep(0.5, length(xx))
Sig = matrix(0, nrow = length(xx), ncol = length(xx))
for (i in 1:length(xx)) {
    for (j in 1:length(xx)) {
        Sig[i, j] = 0.5^2 * (1 - 15 * abs(xx[i] - xx[j])) * exp(-15 *
            abs(xx[i] - xx[j]))
    }
}

SigC = Sig - Sig[, c(11, 21, 29, 33, 41), drop = FALSE] %*% solve(Sig[c(11,
    21, 29, 33, 41), c(11, 21, 29, 33, 41), drop = FALSE], Sig[c(11,
    21, 29, 33, 41), , drop = FALSE])

muC = mu + Sig[, c(11, 21, 29, 33, 41), drop = FALSE] %*% solve(Sig[c(11,
    21, 29, 33, 41), c(11, 21, 29, 33, 41), drop = FALSE], c(0.5,
    0.32, 0.4, 0.35, 0.6) - mu[c(11, 21, 29, 33, 41)])

nR = 100
yMat = matrix(0, nrow = nR, ncol = length(xx))
Lt = t(chol(SigC + diag(length(xx)) * 1e-08))
for (i in 1:nR) {
    yMat[i, ] = Lt %*% rnorm(length(xx)) + muC
}

# Plot
plot(NULL, NULL, xlim = c(xx[1], tail(xx, 1)), ylim = c(-1, 2),
    xlab = "Theta", ylab = "y(theta)", cex.axis = 1.5, cex.lab = 1.5,
    lwd = 1.5)
# for (i in 1:100) { lines(xx, yMat[i,], col = 1, lwd =
# 1.5) }

lines(xx, muC, col = "red", lwd = 2)
lines(xx, muC + 1.64 * sqrt(diag(SigC)), col = "green", lwd = 2)
lines(xx, muC - 1.64 * sqrt(diag(SigC)), col = "green", lwd = 2)

lines(c(0.25, 0.5), c(0.3, 0.3), col = "blue", lwd = 1.5, lty = 4)

legend(x = "topright", y = 0.8, legend = c("Mean", "C.I.", "y(theta) = 0.30"),
    col = c("red", "green", "blue"), lty = 1:2, cex = 0.8)
```



b)

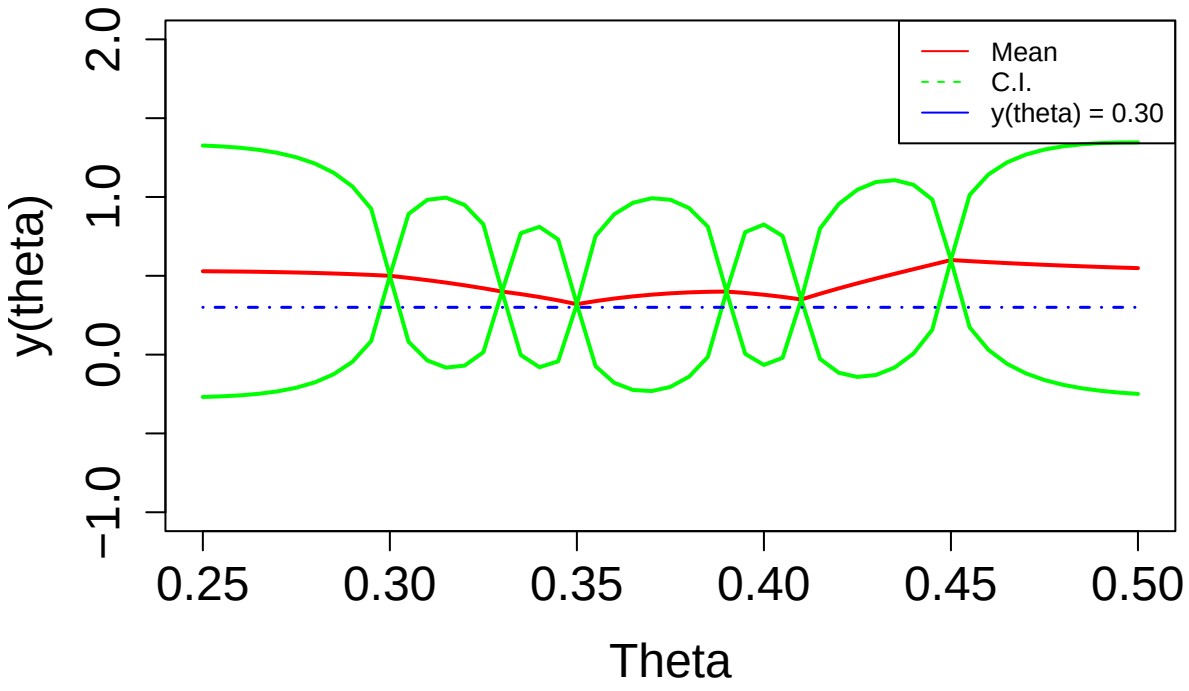
```
z0 <- rep(0, length(muC))
for (i in 1:length(muC)) {
    z0[i] <- (0.3 - muC[i])/(sqrt(diag(SigC))[i])
}

plot(NULL, NULL, xlim = c(xx[1], tail(xx, 1)), ylim = c(0, 1),
    xlab = "theta", ylab = "P(Y(theta) < 0.30)", cex.axis = 1.5,
    cex.lab = 1.5, lwd = 1.5)
for (i in 1:length(z0)) {
    points(xx[i], pnorm(z0[i]), col = 1, lwd = 1.5)
}
```



c)

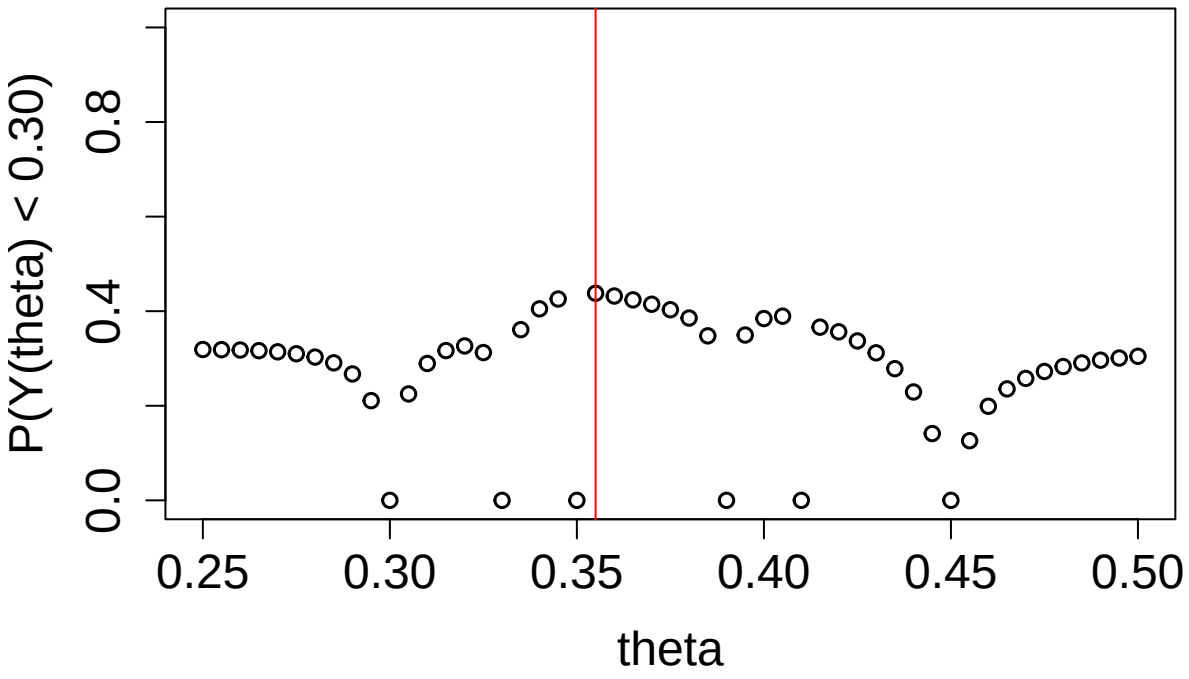
```
set.seed(98)
xx <- seq(0.25, 0.5, 0.005)

mu = rep(0.5, length(xx))
Sig = matrix(0, nrow = length(xx), ncol = length(xx))
for (i in 1:length(xx)) {
    for (j in 1:length(xx)) {
        Sig[i, j] = 0.5^2 * (1 - 15 * abs(xx[i] - xx[j])) * exp(-15 *
            abs(xx[i] - xx[j]))
    }
}

SigC = Sig - Sig[, c(11, 17, 21, 29, 33, 41), drop = FALSE] %*%
    solve(Sig[c(11, 17, 21, 29, 33, 41), c(11, 17, 21, 29, 33,
        41), drop = FALSE], Sig[c(11, 17, 21, 29, 33, 41), ,
        drop = FALSE])

muC = mu + Sig[, c(11, 17, 21, 29, 33, 41), drop = FALSE] %*%
    solve(Sig[c(11, 17, 21, 29, 33, 41), c(11, 17, 21, 29, 33,
        41), drop = FALSE], c(0.5, 0.4, 0.32, 0.4, 0.35, 0.6) -
        mu[c(11, 17, 21, 29, 33, 41)])
```

```r
nR = 100
yMat = matrix(0, nrow = nR, ncol = length(xx))
Lt = t(chol(SigC + diag(length(xx)) * 1e-08))
for (i in 1:nR) {
    yMat[i, ] = Lt %*% rnorm(length(xx)) + muC
}

# Plot
plot(NULL, NULL, xlim = c(xx[1], tail(xx, 1)), ylim = c(-1, 2),
    xlab = "Theta", ylab = "y(theta)", cex.axis = 1.5, cex.lab = 1.5,
    lwd = 1.5)
# for (i in 1:100) { lines(xx, yMat[i,], col = 1, lwd =
# 1.5) }

lines(xx, muC, col = "red", lwd = 2)
lines(xx, muC + 1.64 * sqrt(diag(SigC)), col = "green", lwd = 2)
lines(xx, muC - 1.64 * sqrt(diag(SigC)), col = "green", lwd = 2)

lines(c(0.25, 0.5), c(0.3, 0.3), col = "blue", lwd = 1.5, lty = 4)

legend(x = "topright", y = 0.92, legend = c("Mean", "C.I.", "y(theta) = 0.30"),
    col = c("red", "green", "blue"), lty = 1:2, cex = 0.8)
```




```r
z0 <- rep(0, length(muC))
for (i in 1:length(muC)) {
```

```
    z0[i] <- (0.3 - muC[i])/(sqrt(diag(SigC))[i])
}

plot(NULL, NULL, xlim = c(xx[1], tail(xx, 1)), ylim = c(0, 1),
    xlab = "theta", ylab = "P(Y(theta) < 0.30)", cex.axis = 1.5,
    cex.lab = 1.5, lwd = 1.5)

pnormc <- rep(0, length(z0))

for (i in 1:length(z0)) {
    pnormc[i] <- pnorm(z0[i])
    points(xx[i], pnorm(z0[i]), col = 1, lwd = 1.5)
}

abline(v = xx[which.max(pnormc)], col = "red")
```




```
cat("Largest prob. is", max(pnormc), "and this is given when theta =",
    xx[which.max(pnormc)])
```

```
## Largest prob. is 0.4378658 and this is given when theta = 0.355
```

For the biggest probability of getting $y(\theta) < 0.30$, they should use $\theta = 0.355$. This gives the larges probability.