Øving 9

13.7

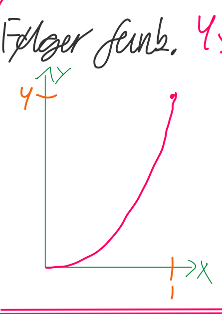
22.

$z=(2i)^{2i}$
$=e^{2i\,Ln(2i)}$

14.1

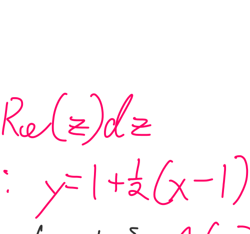
3.

$z(t)=t+4t^2 i,\ 0\leq t\leq 1$
$=x(t)+y(t)i$
$x(t)=t \Rightarrow x(0)=0,\ x(1)=1$
$y(t)=4t^2 \Rightarrow y(0)=0,\ y(1)=4$
Følger funk. $4x^2$



11.

Segment fra $(-1,2)$ til $(1,4)$
Stign.tall: $m=\frac{4-2}{1+1}=1$
Funk: $y=1(x-2)+2$
$=x$
$=t,\ t\in[-1,1]$
$\Rightarrow z(t)=t+it,\ t\in[-1,1]$



20.

$4(x-2)^2+5(y+1)^2=20$
Sirkel

22.

$\int_C Re(z)dz$
C: $y=1+\frac{1}{2}(x-1)^2$ fra $1+i$ til $3+3i$
Tester hvis $f(z)=Re(z)$ er anal.:
$z=u(x,y)+iv(x,y)$
$=x(t)+iy(t)$
$=x+i(1+\frac{1}{2}(x-1)^2)$
$=t+i(1+\frac{1}{2}(t-1)^2)$
$Re(z)=t$
$u_x=1$
$v_y=0$
Ikke anal. $\Rightarrow$ kan ikke bruke 1. metode
2. metode:
$\dot{z}(t)=1+i(t-1)$
$\int_C f(z)dz=\int_1^3 t(1+i(t-1))dt$
$=\int_1^3 t+it^2-it\,dt$
$=\left[\frac{1}{2}t^2+\frac{1}{3}it^3-\frac{1}{2}it^2\right]_1^3$
$=\frac{1}{2}9+\frac{1}{3}i27-\frac{1}{2}i9-\frac{1}{2}-\frac{1}{3}i+\frac{1}{2}i$
$=4+\frac{14}{3}i$

26.

$\int_C(z+z^{-1})dz$
C: Enhetssirkel mot klokke
$\Rightarrow z(t)=e^{it}$
Anal.?
$f(z)=z+z^{-1}$
$=e^{it}+e^{-it}$
$=\cos(t)+i\cancel{\sin(t)}+\cos(-t)+i\sin(-t)$
$=2\cos(t)$
$u=2\cos(t)=2\cos(x)$
$v=0$
$u_x=2\sin(x)$
$v_y=0$
Ikke anal.
2. metode
$\int_C(z+z^{-1})dz=\int_{2\pi}^{0}2\cos(t)dt$
$=-\int_0^{2\pi}2\cos(t)dt$
$=-2\int_0^{2\pi}\cos(t)dt$
$=-2[\sin(t)]_0^{2\pi}$
$=0$

29.

$\int_C Im(z^2)dz$

14.2

4.

13.

$f(z)=\frac{1}{z^4-1.2}$
$z=e^{it}$
Anal.?
$f(z)=\frac{1}{e^{4it}-1.2}$
$=$

22.

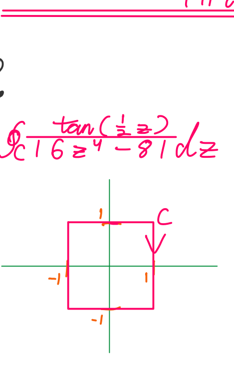
$\oint_C Re(z)dz$
C: $z(t)=\begin{cases}e^{it}, & t\in[0,\pi]\\ 0, & t\in(\pi,2\pi)\end{cases}$
Anal.?
$z(t)=e^{it},\ t\in[0,\pi]$
$=\cos(t)+i\sin(t)$
$=\cos(x)+i\sin(x)$
$Re(z)=\cos(x)$
$u=\cos(x)$
$v=0$
$u_x=-\sin(x)\neq v_y$
Ikke anal. $\Rightarrow$ Cauchy fungerer ikke
$\oint_C Re(z)dz=\int_0^{\pi}\cos(t)\cdot(-\sin(t)+i\cos(t))dt$
$=\int_0^{\pi}-\cos(t)\sin(t)dt+i\int_0^{\pi}\cos^2(t)dt$
$=\int_0^{\pi}-\frac{\sin(2t)}{2}dt+i\int_0^{\pi}\frac{1-\cos(2t)}{2}dt$
$=\underbrace{\left[\frac{\cos(2t)}{4}\right]_0^{\pi}}_{=0}+i\left(\underbrace{\left[\frac{1}{2}t\right]_0^{\pi}}_{=\frac{1}{2}\pi}-\underbrace{\left[\frac{\sin(2t)}{2}\right]_0^{\pi}}_{=\frac{1}{2}}\right)$
$=i\left(\frac{\pi}{2}-\frac{1}{2}\right)$

23.

$\oint_C\frac{2z-1}{z^2-z}dz$
Ikke anal. i $z=0$ og $z=1$ fordi nevner
Kan ikke bruke C.I. Thm.
$\oint_C\frac{2z-1}{z^2-z}dz=\oint_C\frac{2z-1}{z(z-1)}dz$
$=\oint_C\frac{A}{z}+\frac{B}{z-1}dz$
$=[A=1,\ B=1]$
$=\oint_C\frac{1}{z}+\frac{1}{z-1}dz$
$=\oint_C\frac{1}{z}dz+\oint_C\frac{1}{z-1}dz$
$=\oint_C z^{-1}dz+\oint_C(z-1)^{-1}dz$
$=2\pi i+2\pi i$
$=4\pi i$

28.

$\oint_C\frac{\tan(\frac{1}{2}z)}{16z^4-81}dz$



Ikke anal. i $z=\frac{3}{2},\ z=-\frac{3}{2},$