NTNU
Norwegian University of Science and Technology

Department of Mathematical Sciences

Examination paper for **MA2501 Numerical Methods**

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Examination date: 16th of May 2017

Examination time (from–to): 09:00–13:00

Permitted examination support material: Support material code C

- Approved basic calculator.
- The textbook: Cheney & Kincaid, Numerical Mathematics and Computing, 6th or 7th edition, including the list of errata.
- Rottmann, Matematisk formulae.
- Handout: Fixed point iterations.

Other information:
All answers should be justified and include enough details to make it clear which methods and/or results have been used.

All the (sub-)problems are worth 5 points each. The total value is 70 points.

Language: English

Number of pages: 2

Number of pages enclosed: 0

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Problem 1

a) Is the following matrix symmetric and positive definite?

$$\begin{pmatrix} 4.5 & -3.0 & -1.0 \\ -3.0 & 5.9 & 2.7 \\ -1.0 & 2.7 & 4.6 \end{pmatrix} \tag{1}$$

b) What quantity characterizes the expected accuracy one will get when one solves a linear equation system that contains a symmetrically positive definite coefficient matrix.

c) Given the following matrix A

$$\begin{pmatrix} 3 & 1 & 2 \\ 6 & 3 & 3 \\ -3 & 2 & -3 \end{pmatrix} \tag{2}$$

Find an LU factorization, i.e., find a lower triangular matrix L and an upper triangular matrix U such that A = LU.

Problem 2

a) The value of the function f(x) is given in the following points:

x_i	0.0	2.0	3.0
$f(x_i)$	1.0	17.0	31.0

Find the polynomial p(x) of the lowest possible degree that interpolates f(x) in these three points.

b) Find the polynomial q(x) to the lowest possible extent that satisfies the above interpolation conditions as well as the condition $f(1.0) = 7.0$.

c) Construct the Hermitic interpolation polynomial of degree 3 for the function $f(x) = x^5$, by using the points $x_0 = 0$, $x_1 = a$, and show that it is equal to $p_3(x) = 3a^2x^3 - 2a^3x^2$.

Problem 3

a) Construct the following quadrature formula

$$\int_{-1}^{1} f(x) \approx A_0 f(-1) + A_1 f(x_1) + A_2 f(1) \tag{3}$$

on the interval (-1,1); i.e., write up and solve four equations to determine x_1, A_0, A_1 and A_2. This is a so-called Lobatto quadrature formula.

b) Define the Gauss quadrature formula that has the same accuracy (i.e., integrates exact polynomials up to and including the order m) as for the Lobatto formula found in sub-question a) above. Verify the answer by integrating a polynomial of the degree $p = m$ (where you choose m as big as possible) with the two quadrature formulas.

c) Use the Lobatto and Gauss quadrature formulas found above to integrate numerically the function $f(x) = x^4$ on the interval $[-1, 1]$. Compare the calculated results with the exact solution and comments.

Problem 4 Given the following nonlinear system of equations:

$$x_1^2 + x_2^2 = 1 \tag{4}$$

$$x_1^3 - x_2 = 2 \tag{5}$$

This system has two sets of solutions, one in the domain $-1 \leq x_1, x_2 \leq 0$ and one in the domain $0 \leq x_1, x_2 \leq 1$.

a) Set up Newton's method for the nonlinear equation system.

b) Select a set of appropriate initial values for x_1 and x_2 and make two iterations of Newton's method.

c) Explain what happens if you select initial values that lie on the x_2 axis.

Problem 5

a) Explain the difference between explicit and implicit methods for solving initial value problems.

b) Explain when it is appropriate to choose implicit methods for solving initial value problems.