Øving 4

11.4

2.

$f(x)=x$ odde

$-\pi<x<\pi$

Minimum square error betyr at $A_0=a_0=0$, $A_n=a_n=0$, $B_n=b_n$

$b_n=\frac{2}{\pi}\int_0^{\pi}x\sin(nx)\,dx$

$=\frac{2}{\pi}\left[-\frac{x\cos(nx)}{n}\right]_0^{\pi}+\frac{2}{\pi}\int_0^{\pi}\frac{\cos(nx)}{n}\,dx$

$=\frac{2}{\pi}\left(\frac{-\pi\cos(n\pi)}{n}\right)+\frac{2}{\pi}\left[\frac{\sin(nx)}{n^2}\right]_0^{\pi}$

$=-\frac{2\cos(n\pi)}{n}$

$=-\frac{2(-1)^n}{n}$

$F(x)=\sum_{n=1}^{5}b_n\sin(nx)$

$=2\sin(x)-\sin(2x)+\frac{2}{3}\sin(3x)-\frac{1}{2}\sin(4x)+\frac{2}{5}\sin(5x)$

$E^*=\int_{-\pi}^{\pi}f^2\,dx-\pi\left[\sum_{n=1}^{5}b_n^2\right]$

$\Rightarrow E^*=\left[\frac{x^3}{3}\right]_{-\pi}^{\pi}-\pi\sum_{n=1}^{5}\left(-\frac{2(-1)^n}{n}\right)^2$

$=\frac{2\pi^3}{3}-\pi\sum_{n=1}^{5}\left(-\frac{2(-1)^n}{n}\right)^2$

$N=1$

$E^*=8.1$

$N=2$

$E^*=4.96$

$N=3$

$E^*=3.57$

$N=4$

$E^*=2.78$

$N=5$

$E^*=2.28$

Minimum for $N=5$

$\Rightarrow F(x)=2\sin(x)-\sin(2x)+\frac{2}{3}\sin(3x)-\frac{1}{2}\sin(4x)+\frac{2}{5}\sin(5x)$

3.

$f(x)=|x|$ like

$-\pi<x<\pi$

$\Rightarrow a_0=\frac{1}{\pi}\int_0^{\pi}|x|\,dx$

$=\frac{1}{\pi}\left(\frac{\pi^2}{2}\right)$

$=\frac{\pi}{2}$

$a_n=\frac{2}{\pi}\int_0^{\pi}|x|\cos(nx)\,dx$

$=\frac{2}{\pi}\underbrace{\left[\frac{x\sin(nx)}{n}\right]_0^{\pi}}_{0}-\frac{2}{\pi}\int_0^{\pi}\frac{\sin(nx)}{n}\,dx$

$=-\frac{2}{\pi}\left[-\frac{\cos(nx)}{n^2}\right]_0^{\pi}$

$=\frac{2}{\pi n^2}\left((-1)^n-1\right)$

$E^*=\int_{-\pi}^{\pi}|x|^2\,dx-\pi\left(2\frac{\pi}{2}+\sum_{n=1}^{N}\left(\frac{2((-1)^n-1)}{\pi n^2}\right)^2\right)$

$=\frac{2\pi^3}{3}-\pi\left(\pi+\sum_{n=1}^{N}\left(\frac{2((-1)^n-1)}{\pi n^2}\right)^2\right)$

$N=1$

$E^*=10.8$

$N=2$

$E^*=10.8$

$N=3$

$E^*=10.8$

$N=4$

$E^*=10.8$

$N=5$

$E^*=10.8$

Min for $N=1$

$\Rightarrow F(x)=\frac{2}{\pi}-\frac{4}{\pi}\cos(x)$

13.

$1+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}+\ldots=\frac{\pi^4}{96}$

$=1.014678032$

11.4

9.

$f(x)=x$

$-\pi<x<\pi$

$n=0$:

$c_0=0$

$n\neq 0$:

$c_n=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(x)e^{-inx}\,dx$

$=\frac{1}{2\pi}\int_{-\pi}^{\pi}xe^{-inx}\,dx$

$=\frac{1}{2\pi}\left[-\frac{xe^{-inx}}{in}\right]_{-\pi}^{\pi}+\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{e^{-inx}}{in}\,dx$

$=\frac{1}{2\pi}\left[\frac{ixe^{-inx}}{n}\right]_{-\pi}^{\pi}-\frac{i}{2\pi n}\int_{-\pi}^{\pi}e^{-inx}\,dx$

$=\frac{i}{2\pi n}\left[xe^{-inx}\right]_{-\pi}^{\pi}-\frac{i}{2\pi n}\left[-\frac{e^{-inx}}{in}\right]_{-\pi}^{\pi}$

$=\frac{i}{2\pi n}\left(\pi e^{-in\pi}+\pi e^{in\pi}\right)-\frac{i^2}{2\pi n^2}\left(e^{-in\pi}-e^{in\pi}\right)$

$=\frac{in\pi e^{-in\pi}+in\pi e^{in\pi}+e^{-in\pi}-e^{in\pi}}{2\pi n}$

$\Rightarrow f(x)=\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}c_ne^{inx}$

11.7

1.

$$\int_0^{\infty}\frac{\cos(xw)+w\sin(xw)}{1+w^2}\,dw=\begin{cases}0, & x<0\\ \frac{\pi}{2}, & x=0\\ \pi e^{-x}, & x>0\end{cases}$$

$f(x)=\int_0^{\infty}A(w)\cos(wx)+B(w)\sin(wx)\,dw$

$A(w)=\frac{1}{\pi}\int_0^{\infty}\pi e^{-v}\cos(wv)\,dv$

$=\int_0^{\infty}e^{-v}\cos(wv)\,dv$

$=\int_0^{\infty}e^{-v}\cos(wv)\,dv$

$=\underbrace{\left[\frac{e^{-v}\sin(wv)}{w}\right]_0^{\infty}}_{0}+\int_0^{\infty}\frac{e^{-v}\sin(wv)}{w}\,dv$

$=\frac{1}{w^2}\left[-e^{-v}\cos(wv)\right]_0^{\infty}-\frac{1}{w^2}\underbrace{\int_0^{\infty}e^{-v}\cos(wv)\,dv}_{A(w)}$

$\Rightarrow A(w)+\frac{1}{w^2}A(w)=\frac{1}{w^2}\left(\lim_{x\to\infty}\left(-e^{-x}\cos(wx)\right)+e^0\cos(0)\right)$

$=\frac{1}{w^2}$

$\Rightarrow A(w)=\frac{1}{w^2+1}$

$B(w)=\frac{1}{\pi}\int_0^{\infty}\pi e^{-v}\sin(wv)\,dv$

$=[$

11.9

5.

$$f(x)=\begin{cases}e^x, & -a<x<a\\ 0, & \text{ellers}\end{cases}$$

$F[f](w)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-iwx}\,dx$

$=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a}e^xe^{-iwx}\,dx$

$=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a}e^{(1-iw)x}\,dx$

$=\frac{1}{\sqrt{2\pi}}\left[\frac{e^{(1-iw)x}}{(1-iw)}\right]_{-a}^{a}$

$=\frac{e^{(1-iw)a}-e^{-(1-iw)a}}{\sqrt{2\pi}(1-iw)}$

7. $f(x)=\begin{cases}x, & 0<x<a\\ 0, & \end{cases}$

$F[f](w)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-iwx}\,dx$

$=\frac{1}{\sqrt{2\pi}}\int_0^{a}xe^{-iwx}\,dx$

$=\frac{1}{\sqrt{2\pi}}\left(\left[-\frac{xe^{-iwx}}{iw}\right]_0^{a}+\frac{1}{iw}\int_0^{a}e^{-iwx}\,dx\right)$

$=\frac{1}{\sqrt{2\pi}}\left(-\frac{ae^{-iwa}}{iw}+\frac{1}{iw}\left[-\frac{e^{-iwx}}{iw}\right]_0^{a}\right)$

$=\frac{1}{\sqrt{2\pi}}\left(-\frac{ae^{-iwa}}{iw}+\frac{(-e^{-iwa}+1)}{(iw)^2}\right)$

$=\frac{1}{\sqrt{2\pi}}\left(\frac{-ae^{-iwa}\cdot iw-e^{-iwa}+1}{(iw)^2}\right)$

$=\frac{1}{\sqrt{2\pi}}\left(\frac{iwae^{-iwa}+e^{-iwa}-1}{w^2}\right)$

9. $f(x)=\begin{cases}|x|, & -1<x<1\\ 0, & \text{otherwise}\end{cases}$

$F[f](w)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-iwx}\,dx$

$=\frac{1}{\sqrt{2\pi}}\int_{-1}^{1}|x|e^{-iwx}\,dx$

$=\frac{2}{\sqrt{2\pi}}\int_0^{1}xe^{-iwx}\,dx$

$=\frac{2}{\sqrt{2\pi}}\left(\frac{iwe^{-iw}+e^{-iw}-1}{w^2}\right)$