Kandidatnr: 10009. Problem 1

(i) $M \in \mathbb{R}^{n \times n}$

$I - M$ non-singular

$$\|(I-M)^{-1} - I\| = \max \frac{\|((I-M)^{-1} - I)x\|}{\|x\|}$$

$$= \max \frac{\|(I-M)^{-1}x - x\|}{\|x\|}$$

Δ-ie. $\leq \max$

(ii) $(1+\|M\|)^{-1} \leq \|(I-M)^{-1}\|$

Let $\|x_1\| = 1$ s.t. $\|A\| = \|Ax_1\|$

Then $x_2 = Ax_1 \cdot \frac{1}{\|A\|} = 1$

$$(1+\|M\|)^{-1} = \frac{1}{1+\|M\|}$$

$$= \frac{\|x_1\|}{1+\|M\|}$$

$$= \|(I-M)^{-1}(I-M) \cdot \frac{x_1}{1+\|M\|}\|$$

$$\leq \|(I-M)^{-1}x_2\|$$

$$\leq \|(I-M)^{-1}\|\|x_2\|$$

$$= \|(I-M)^{-1}\|$$