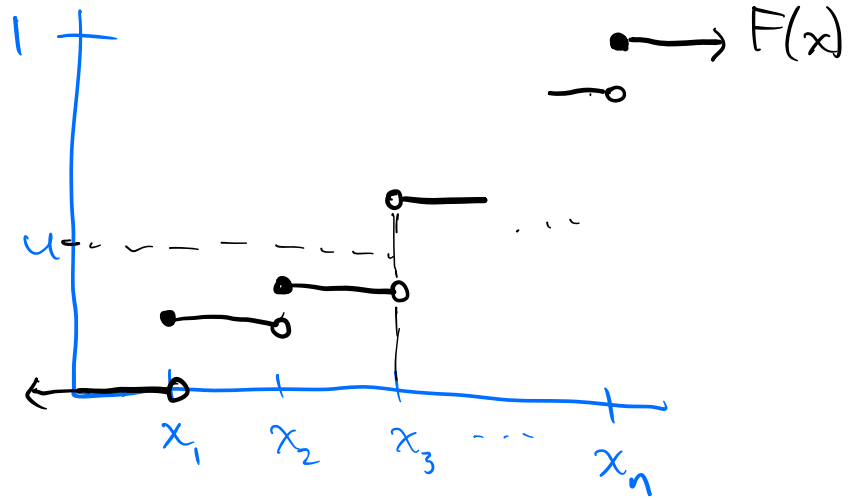
Lecture 6 24/1/23 Week 4.2

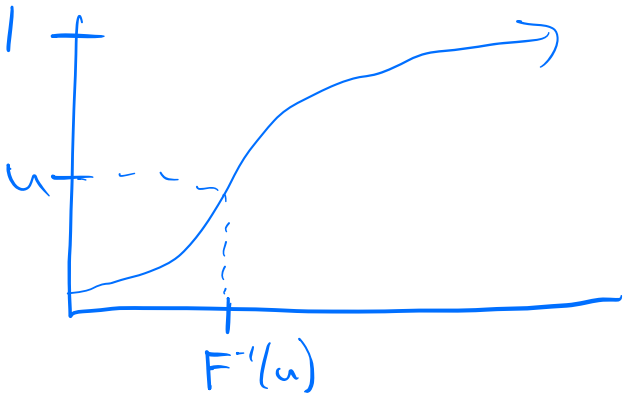
Module 1 Review

Discrete distn PIT

$$P(X = x_i) = p_i, \quad \text{for } i = 1, \ldots, n$$



Continuous PIT



Summary of sampling methods learned:

Sampling Method	When to use?	Needs Normalizing Constant?	Pros	Cons
Inversion Sampling	$(cdf)^{-1}$ available	yes	Direct (no accept/reject step)	$(cdf)^{-1}$ might not be available
Transformations of RVs (e.g. Box-Müller)	normal distns, location scale families, etc. (want $X \sim f$ to be 1-to-1 Fxn of simpler RV)	not necessarily	Direct	transformation might not exist
Rejection Sampling	Direct sampling unavailable, (can find g & c: but weighted resampling for c, adaptive rejection sampling for g)	no	Very generalizeable	- might be difficult to find c - reject some samples - How to choose g? – must have heavier tails than f – must be "similar to f" – should be easy to sample from - bad in high dimensions

Sampling Method	When to Use?	Needs Normalizing Constant?	Pros	Cons
Ratio of Uniforms	e.g. normalizing constant unknown, or sampling from C_f is easy.	no	If we can sample from C_f easily this is easy	- might need to use accept/reject step - often must calculate a_-, b_-, & b_+ easily e.g. differentiation, finding roots.
Mixtures	Fill	This	In	Yourself
Multivariate Normal				

Summary of Methods for Estimating Expectations

Integration Method	When to Use?	Needs Normalizing Constant	Pros	Cons
Analytical	Whenever possible!	yes	Exact, no computation required	Might be difficult to calculate
Monte Carlo	When an analytical solution unavailable, sampling from f possible	not necessarily	Unbiased, generalizable, simple	Inexact, can be imprecise, must draw from f
Importance Sampling	When an analytical solution is unavailable, if a choice of g can reduce the variance of our estimator	not necessarily (self-normalizing IS)	- Can reduce variance of estimate - no accept/reject step - standard IS is unbiased	- Inexact, - Self-normalizing IS is biased - could increase estimator variance if g is chosen poorly

Notes:

- For rejection and importance sampling, we need $f>0 \Rightarrow g>0$ (or $f \cdot h > 0 \Rightarrow g>0$)
- All of our sampling method so far are used to draw independant samples!
- In practice, all of these methods typically become more difficult in high dimensions.
- One method of integration we have not discussed but is important is numerical integration.