Øving 8

13.4

2

$f(z)=iz\bar{z}$
$=0+i(x^2+y^2)$

$u_x=0$

$v_y=2y$

ikke analytisk

10.

$f(z)=\ln(|z|)+i\arg(z)$
$=\ln(r)+i\theta$
$=u+iv$

$\left.\begin{array}{l} u_r=\frac{1}{r} \\ v_\theta=1 \end{array}\right\} \Rightarrow u_r=\frac{1}{r}v_\theta$

$\left.\begin{array}{l} u_\theta=0 \\ v_r=0 \end{array}\right\} \Rightarrow v_r=-\frac{1}{r}u_\theta$

Analytisk

18.

$u=x^3-3xy^2$

$u_x=3x^2-3y^2$

$u_{xx}=6x$

$u_y=-6y$

$u_{yy}=-6$

$\Rightarrow u_{xx}+u_{yy}=0$ ✓

Funksjonen er harmonisk

30

(b)

(c)

13.5

20

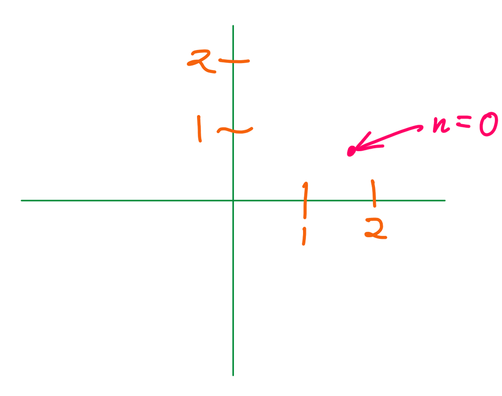
$e^z=4+3i$

$\Rightarrow e^x e^{iy}=4+3i$
$=re^{i\theta}$
$=\sqrt{4^2+3^2}\,e^{i\theta}$
$=5e^{i\theta}$

$e^x=5$

$\theta=\tan^{-1}\left(\frac{3}{4}\right)\approx 0.64 \Rightarrow y=0.64+2\pi n,\ n\in\mathbb{Z}$

$\Rightarrow z=\ln(5)+i(0.64+2\pi n),\ n\in\mathbb{Z}$



13.6

10.

$\sinh(3+4i)=\frac{1}{2}\left(e^{3+4i}-e^{-3-4i}\right)$
$=\frac{1}{2}\left(e^3\cos(4)+ie^3\sin(4)-\left(e^{-3}\cos(-4)+ie^{-3}\sin(-4)\right)\right)$
$=\frac{1}{2}\left(e^3\cos(4)+ie^3\sin(4)-e^{-3}\cos(4)+e^{-3}\sin(4)\right)$
$=\frac{1}{2}\left(\left(e^3-e^{-3}\right)\cos(4)+i\left(e^3+e^{-3}\right)\sin(4)\right)$
$=\frac{e^3-e^{-3}}{2}\cos(4)+i\frac{e^3+e^{-3}}{2}\sin(4)$
$=\sinh(3)\cos(4)+i\cosh(3)\sin(4)$

$\cosh(3+4i)=\frac{1}{2}\left(e^{3+4i}+e^{-3-4i}\right)$
$=\frac{1}{2}\left(e^3\cos(4)+ie^3\sin(4)+e^{-3}\cos(-4)+ie^{-3}\sin(-4)\right)$
$=\frac{1}{2}\left(\left(e^3+e^{-3}\right)\cos(4)+i\left(e^3-e^{-3}\right)\sin(4)\right)$
$=\cosh(3)\cos(4)+i\sinh(3)\sin(4)$

16.

$\sin(z)=100$

$\frac{1}{2i}\left(e^{iz}-e^{-iz}\right)=100 \quad \Big|\cdot 2ie^{iz}$

$\left(e^{iz}\right)^2-200ie^{iz}-1=0$

$e^{iz}=\frac{200i\pm\sqrt{(200i)^2-4}}{2}$

$=\begin{cases} i\left(100+\sqrt{10001}\right) \\ i\left(100-\sqrt{10001}\right) \end{cases}$

19.

$\sinh(z)=0$

$\frac{1}{2}\left(e^z-e^{-z}\right)=0$

$e^z-e^{-z}=e^x\cos(y)+ie^x\sin(y)-\left(e^{-x}\cos(-y)+ie^{-x}\sin(-y)\right)$
$=\left(e^x-e^{-x}\right)\cos(y)+i\left(e^x+e^{-x}\right)\sin(y)$
$=0$

$\Rightarrow \left(e^x-e^{-x}\right)\cos(y)=\left(e^x+e^{-x}\right)\sin(y)=0$

$\Rightarrow x=0$ og $y=\pi n,\ n\in\mathbb{Z}$

13.7

15.

$\ln\left(e^i\right)=i\pm 2n\pi i$ fra (4b)

17.

$\ln\left(i^2\right)=\ln(-1)$
$=\ln(|-1|)+i\tan^{-1}\left(\frac{0}{-1}\right)$
$=0+i\pi$
$=i\pi+2n\pi i,\ n\in\mathbb{Z}$

$2\ln(i)=2\left(\ln(|i|)+i\tan^{-1}\left(\frac{1}{0}\right)\right)$
$=2\left(0+i\frac{\pi}{2}\right)$
$=2\left(i\frac{\pi}{2}+2n\pi i\right),\ n\in\mathbb{Z}$
$=i\pi+4n\pi i$

$\Rightarrow \ln\left(i^2\right)\neq 2\ln(i)$

30.

(a) $\cos^{-1}(z)=w \Rightarrow z=\cos(w)=\frac{e^{iw}+e^{-iw}}{2}$

$z=\frac{e^{iw}+e^{-iw}}{2} \quad \Big|\cdot 2e^{iw}$

$\left(e^{iw}\right)^2-2ze^{iw}+1=0$

$e^{iw}=\frac{2z\pm\sqrt{(2z)^2-4}}{2}$
$=\frac{2z\pm 2\sqrt{z^2-1}}{2}$
$=z\pm\sqrt{z^2-1}$

$\Rightarrow iw=\ln\left(z+\sqrt{z^2-1}\right)$

$\Rightarrow w=\cos^{-1}(z)=-i\ln\left(z+\sqrt{z^2-1}\right)$