

## The dependent bootstrap

Example (AR(1) model):

$$X_t = \mu + \phi X_{t-1} + \varepsilon_t$$

$$\uparrow$$

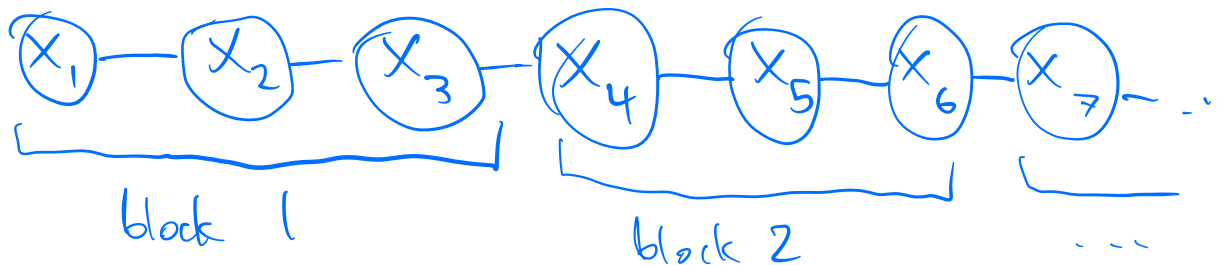
$$|\phi| < 1$$

$$\varepsilon_t \stackrel{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

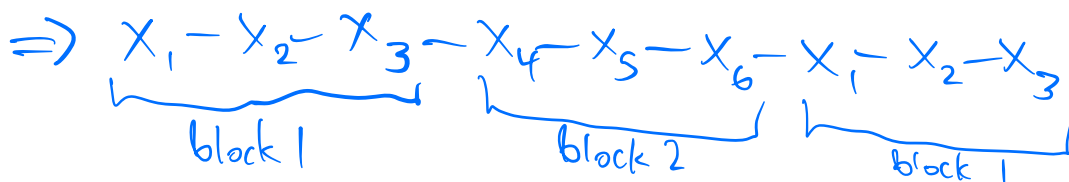
$$\hat{\theta} = (\hat{\mu}, \hat{\phi}, \hat{\sigma}_\varepsilon^2)$$

$$\hat{\varepsilon}_t = X_t - (\hat{\mu} + \hat{\phi} X_{t-1})$$

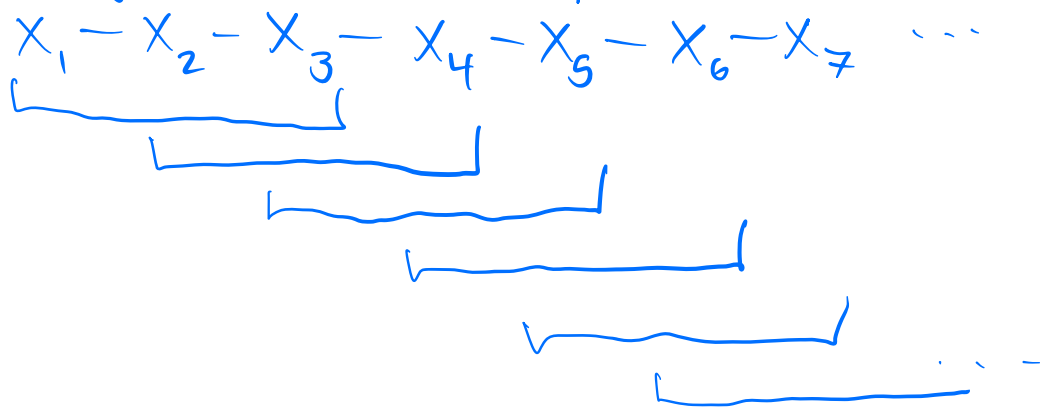
Block bootstrap:



bootstrap sample could be block 1, 2, 1:



Moving block bootstrap:



Block of blocks:

$$X = (X_1, X_2, X_3, \dots, X_n)$$

$$Y = \left\{ \underbrace{\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, \begin{pmatrix} X_2 \\ X_3 \end{pmatrix}, \begin{pmatrix} X_3 \\ X_4 \end{pmatrix}}_{\text{block 1}}, \underbrace{\begin{pmatrix} X_4 \\ X_5 \end{pmatrix}, \begin{pmatrix} X_5 \\ X_6 \end{pmatrix}, \begin{pmatrix} X_6 \\ X_7 \end{pmatrix}}_{\text{block 2}}, \underbrace{\begin{pmatrix} X_7 \\ X_8 \end{pmatrix}, \dots, \begin{pmatrix} X_{n-1} \\ X_n \end{pmatrix}}_{\dots} \right\}$$

The lag 1 autocorrelation can be estimated

as:

$$\hat{\rho} = \frac{\sum_{t=1}^{n-1} (Y_{t,1}^* - \bar{Y})(Y_{t,2}^* - \bar{Y})}{\sum_{t=1}^n (X_t - \bar{X})^2}$$