Øving 10

14.3:

2.

$|z-1-i| = \frac{\pi}{2}$

$\oint_C \frac{z^2}{z^2-1} dz = \oint$

$|z-1-i| = |x+iy-1-i|$

$= \sqrt{(x-1)^2+(y-1)^2}$

$= \frac{\pi}{2}$

$(x-1)^2+(y-1)^2 = \frac{\pi^2}{4}$ inneholder $z=1$

$\Rightarrow \oint_C \frac{1}{z-1} \cdot \frac{z^2}{z+1} dz = 2\pi \left[\frac{z^2}{z+1}\right]_{z=1}$

$= \pi$

13.

$\oint_C \frac{z}{z^2+4z+3} dz$

$C: \ |z+1| = 2$

$z^2+4z+3 = (z+1)(z+3)$

$\oint_C \frac{z}{(z+1)(z+3)} dz = \oint_C \frac{A}{z+1} + \frac{B}{z+3} dz$

$= \Big\{$
$z = A(z+3)+B(z+1)$
$z = Az + Bz$
$1 = A+B$
$A = 1-B$
$0 = 3A+B$
$= 3(1-B)+B$
$= 3-3B+B$
$= 3-2B$
$B = \frac{3}{2}$
$A = -\frac{1}{2}$

$= \oint_C -\frac{1}{2(z+1)} dz + \oint_C \frac{3}{2(z+3)} dz$

= [Ikke anal. i $z=-1$]

$= \oint_C \frac{1}{z+1} \cdot \left(-\frac{1}{2}\right) dz + \oint_C \frac{3}{2(z+3)} dz$

$= 2\pi i \cdot \left(-\frac{1}{2}\right) + 0$

$= -\pi i$

18.

$\oint_C \frac{\sin(z)}{4z^2-8iz} dz$

14.4:

2.

$\oint_C \frac{z^6}{(2z-1)^6} dz$

$f(z) = z^6$

$z_0 = \frac{1}{2}$

$\Rightarrow \oint_C \frac{f(z)}{(z-\frac{1}{2})^6} dz = \frac{2\pi i}{5!} \cdot f^{(5)}(z_0)$

$= \frac{2\pi i}{5!} \cdot 6!\, z \Big|_{z=\frac{1}{2}}$

$= 12\pi i \cdot \frac{1}{2}$

$= 6\pi i$

7.

$\oint_C \frac{\cos(z)}{z^{2n+1}} dz, \ n = 0, 1, \ldots$

$z_0 = 0$

$f(z) = \cos(z)$

$\oint_C \frac{\cos(z)}{z^{2n+1}} dz = \frac{2\pi i}{(2\pi)!} \cdot f^{(2n)}(z_0)$

16.

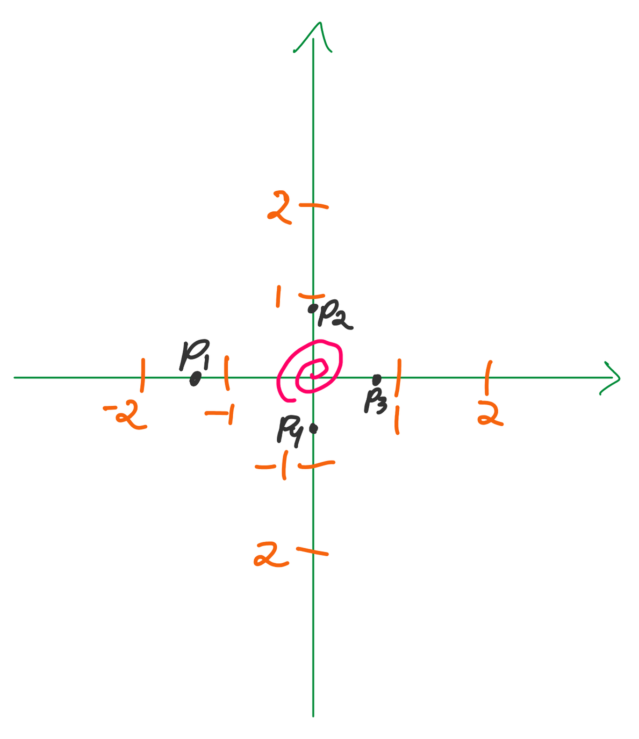
$\oint_C \frac{e^{4z}}{z(z-2i)^2} dz$

15.1

17.

$\sum_{n=2}^{\infty} \frac{(-i)^n}{\ln(n)}$

$\left[= \underbrace{-\frac{1}{\ln(2)}}_{p_1} + \underbrace{\frac{i}{\ln(3)}}_{p_2} + \underbrace{\frac{1}{\ln(4)}}_{p_3} - \underbrace{\frac{i}{\ln(5)}}_{p_4} - \ldots \right]$



Ser at den konvergerer

18.

$\sum_{n=1}^{\infty} n^2 \left(\frac{i}{4}\right)^n$

$\left|\frac{z_{n+1}}{z_n}\right| = \left|(n+1)^2 \left(\frac{i}{4}\right)^{n+1} / n^2 \left(\frac{i}{4}\right)^n\right|$

$= \frac{i}{4} \cdot \left|\frac{n^2+2n+1}{n^2}\right|$

$= \frac{i}{4} \cdot \underbrace{\left|1+\frac{2}{n}+\frac{1}{n^2}\right|}_{\to 1} \xrightarrow{n\to\infty} \frac{i}{4} = L < 1$

$\Rightarrow$ Konvergerer

15.2

5.

Vet at

$\lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = R$

10.

$\sum_{n=0}^{\infty} \frac{(z-2i)^n}{n^n}$

$a_n = \frac{1}{n^n}$

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{1}{n^n} / \frac{1}{(n+1)^{n+1}}\right|$

$= \left|\frac{(n+1)^{n+1}}{n^n}\right| \xrightarrow{n\to\infty} \infty$

Konvergere ikke $\Rightarrow R = 0$

14.

$\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(n!)^2} z^{2n}$