Øving 1

6.1

1.

$f(t) := 2t+8$

$F(s) = \mathcal{L}[f(t)](s)$

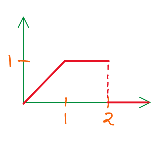
$= \mathcal{L}[2t+8](s)$

$= \int_0^\infty e^{-st}(2t+8)dt$

$= 2\int_0^\infty e^{-st} t dt + 8\int_0^\infty e^{-st} dt$

$= \frac{2}{s^2} + \frac{8}{s}$

12.



$f(t) := \begin{cases} t, & 0<t<1 \\ 1, & 1<t<2 \end{cases}$

$\Rightarrow f(t) = t(u(t-0)+u(t-1)) + (u(t-1)+u(t-2)) +$

$= tu(t-0) + u(t-1)(t+1) + u(t-2)$

$\mathcal{L}[f(t)](s) = \mathcal{L}[tu(t-0) + (t+1)u(t-1) + u(t-2)](s)$

$= e^{-0}\mathcal{L}[t] + e^{-s}\mathcal{L}[t] + e^{-2s}\mathcal{L}[1]$

$= \frac{1}{s^2} + \frac{e^{-s}}{s^2} + \frac{e^{-2s}}{s}$

$= \frac{1+e^{-s}}{s^2} + \frac{e^{-2s}}{s}$

23.

Hvis $\mathcal{L}[f(t)] = F(s)$, c positiv konstant

Vis: $\mathcal{L}[f(ct)] = \frac{F(\frac{s}{c})}{c}$

$F(s) = \mathcal{L}[f]$

$= \int_0^\infty e^{-st} f(t) dt$

26.

$F(s) := \frac{5s+1}{s^2-25}$

$\mathcal{L}^{-1}[F(s)] = \mathcal{L}^{-1}\left[\frac{5s+1}{s^2-25}\right]$

$= \mathcal{L}^{-1}\left[\frac{5s+1}{s^2-5^2}\right]$

$= \mathcal{L}^{-1}\left[\frac{5s+1}{(s+5)(s-5)}\right]$

$= \mathcal{L}^{-1}\left[\frac{A}{s+5} + \frac{B}{s-5}\right]$

$= \begin{cases} 5s+1 = A(s-5) + B(s+5) \\ \Rightarrow A = \frac{12}{5}, B = \frac{13}{5} \end{cases}$

$= \mathcal{L}^{-1}\left[\frac{12}{5} \cdot \frac{1}{s+5} + \frac{13}{5} \cdot \frac{1}{s-5}\right]$

$= \frac{12}{5} \cdot e^{-5t} + \frac{13}{5} \cdot e^{5t}$

36.

$f(t) := \sin(t)\cos(t)$

$\sin(t)\cos(t) = \sin(2t)\frac{1}{2}$

$\Rightarrow f(t) = \frac{\sin(2t)}{2}$

$\mathcal{L}[f(t)] = \frac{1}{2}\mathcal{L}[\sin(2t)]$

$= \frac{1}{2}\left(\frac{2}{s^2+4}\right)$

$= \frac{1}{s^2+4}$

40.

$F(s) := \frac{4}{s^2-2s-3}$

$s^2-2s-3 \Rightarrow \begin{cases} s_1 = \frac{2+\sqrt{4+12}}{2} = 3 \\ s_2 = \frac{2-4}{2} = -1 \end{cases}$

$\Rightarrow F(s) = \frac{4}{(s-3)(s+1)}$

$\mathcal{L}^{-1}[F(s)] = \mathcal{L}^{-1}\left[\frac{4}{(s-3)(s+1)}\right]$

$= \mathcal{L}^{-1}\left[\frac{A}{s-3} + \frac{B}{s+1}\right]$

$= \begin{cases} 4 = A(s+1) + B(s-3) \\ \Rightarrow A = -1, B = 1 \end{cases}$

$= \mathcal{L}^{-1}\left[-\frac{1}{s-3} + \frac{1}{s+1}\right]$

$= -e^{3t} + e^{-t}$

6.2

4.

$y'' + 9y = 10e^{-t}$

$y(0) = 0 = y'(0)$

1) $\mathcal{L}$-transf.

$s^2Y - sy(0) - y'(0) + 9Y = \frac{10}{s+1}$

$(s^2+9)Y = \frac{10}{s+1}$

$\Rightarrow Y = \frac{10}{(s^2+9)(s+1)}$

2) $\mathcal{L}^{-1}$-transf.

$\mathcal{L}^{-1}[Y] = \mathcal{L}^{-1}\left[\frac{10}{(s^2+9)(s+1)}\right]$

$= \left(\begin{array}{l} \frac{10}{(s^2+9)(s+1)} = \frac{As+B}{s^2+9} + \frac{C}{s+1} \\ 10 = (As+B)(s+1) + C(s^2+9) \\ 10 = 10 + 0s + 0s^2 \\ \quad = (B+9C) + (A+B)s + (A+C)s^2 \\ 10 = B+9C \\ 0 = A+B \\ 0 = A+C \\ \Rightarrow A=-1, B=1, C=1 \end{array} \right.$

$= \mathcal{L}^{-1}\left[\frac{-s+1}{s^2+9} + \frac{1}{s+1}\right]$

$= \mathcal{L}^{-1}\left[\frac{-s}{s^2+9} + \frac{1}{s^2+9} + \frac{1}{s+1}\right]$

$= -\cos(3t) + \frac{1}{3}\sin(3t) + e^{-t}$

13.

$y' - 6y = 0$

$y(-1) = 4$

$t = \tilde{t} - 1$

$\tilde{y}' - 6\tilde{y} = 0$

$\tilde{Y} = \mathcal{L}[\tilde{y}]$

$s\tilde{Y} - \tilde{y}(0) - 6\tilde{Y} = 0$

$(s-6)\tilde{Y} = 4$

$\tilde{Y} = \frac{4}{s-6}$

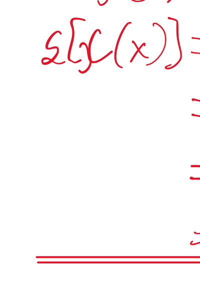
$\mathcal{L}^{-1}[\tilde{Y}] = 4\mathcal{L}^{-1}\left[\frac{1}{s-6}\right]$

$= 4e^{6\tilde{t}}$

$\Rightarrow y = 4e^{6(t+1)}$

6.3

8.



$f(x) := t^2, 1<t<2$

$\Rightarrow f(x) = t^2(u(t-1) - u(t-2))$

$\mathcal{L}[f(x)] = \mathcal{L}[t^2u(t-1) - t^2u(t-2)]$

$= e^{-s}\mathcal{L}[(t+1)^2] - e^{-2s}\mathcal{L}[(t+2)^2]$

$= e^{-s}\mathcal{L}[t^2+2t+1] - e^{-2s}\mathcal{L}[t^2+4t+4]$

$= e^{-s}\left(\frac{2!}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right) - e^{-2s}\left(\frac{2!}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right)$

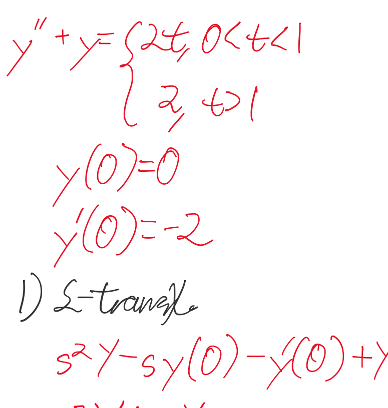
15.

$F(s) = \frac{e^{-2s}}{s^6}$

$\mathcal{L}^{-1}[F(s)] = u(t-2)\mathcal{L}^{-1}\left[\frac{1}{s^6}\right]$

$= u(t-2)\mathcal{L}^{-1}\left[\frac{5!}{s^6} \cdot \frac{1}{5!}\right]$

$= u(t-2)\frac{t^5}{5!}$



25.

$y'' + y = \begin{cases} 2t, & 0<t<1 \\ 2, & t>1 \end{cases}$

$y(0) = 0$

$y'(0) = -2$

1) $\mathcal{L}$-transf.

$s^2Y - sy(0) - y'(0) + Y = 2\mathcal{L}[t(u(t-0) - u(t-1)) - u(t-1)$

$s^2Y + 2 + Y =$