

Summary of TMA4300 - Spring 2023

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What did we do?

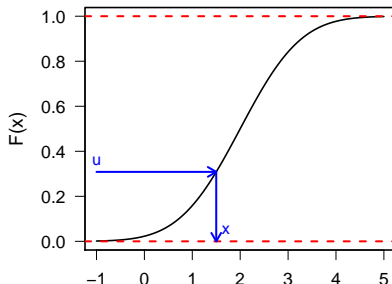
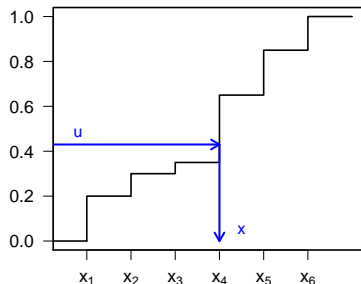
Summary of TMA4300

What did we do?

We had three blocks:

- ▶ Simulation
- ▶ Markov chain Monte Carlo and INLA
- ▶ Bootstrap and EM-algorithm

Block 1



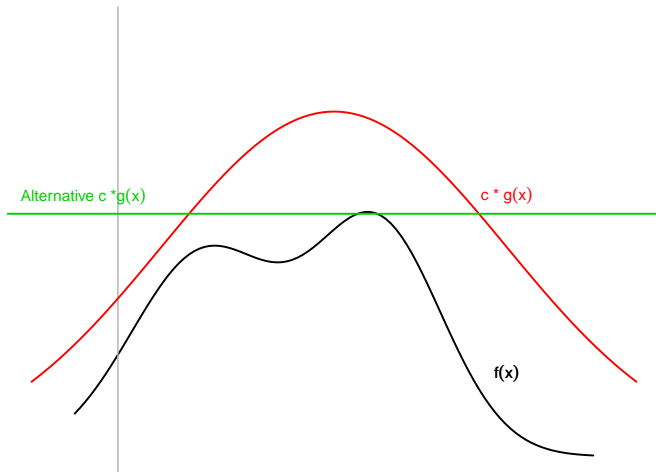
One important theorem to remember:

If $X \sim F(x)$ what is the distribution of $F(X)$?

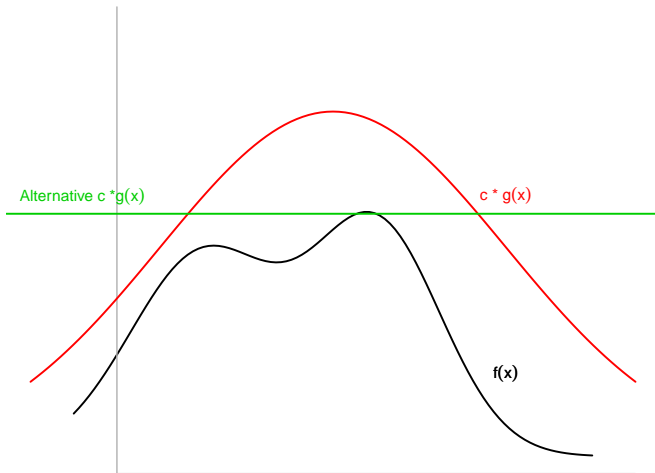
What else ...?

- ▶ Variable transformation
 - ▶ Univariate (e.g. location and scale transformation)
 - ▶ Bivariate (e.g. the Box-Muller algorithm)
 - ▶ Multivariate (e.g. multivariate Gaussian)
- ▶ Ratio-of-uniforms method
- ▶ Methods based on mixtures

Rejection Sampling: Do you remember this figure?

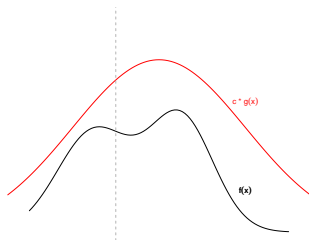


Rejection Sampling: Do you remember this figure?



Refinements: Make the envelope adaptive (different approaches)

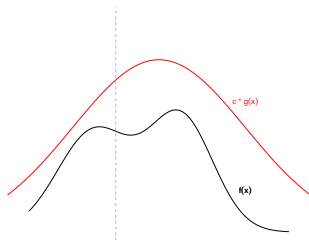
Rejection Sampling: When do we accept?



Let $x \sim g(x)$ and $u \sim \text{Unif}(0, 1)$ then

$$\begin{cases} u \leq \frac{1}{c} \frac{f(x)}{g(x)} & \text{keep} \\ u > \frac{1}{c} \frac{f(x)}{g(x)} & \text{reject} \end{cases}$$

Rejection Sampling: When do we accept?

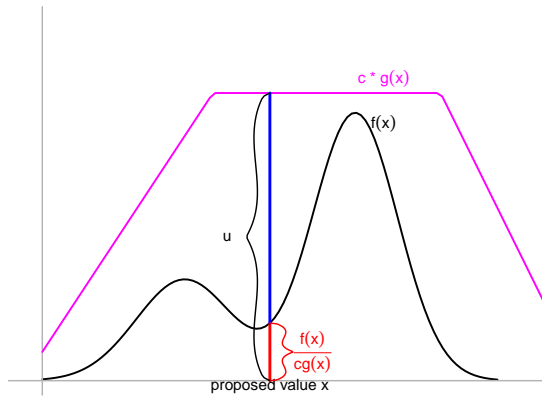


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- ▶ What is the overall acceptance probability?
- ▶ Do we need to know the normalizing constant of $f(\cdot)$?

Rejection sampling



Why do we want samples?

Often we would like to approximate a statistic that is difficult to compute directly.

Keywords:

- ▶ Monte Carlo integration
- ▶ Importance sampling
 - ▶ What is it?
 - ▶ When is it particularly usefull?

Monte Carlo approximation

We are interested in

$$\mu = E(h(x)) = \int h(x)f(x)dx; \quad x \sim f(x)$$

if we can sample $x_i \sim f(x)$, $i = 1, \dots, n$ (iid) then

$$\hat{\mu} = \sum_{i=1}^n h(x_i)$$

- ▶ Do you know how to compute the mean and variance of this estimator?

Importance sampling

We are interested in

$$\mu = E_f(h(x)) = \int h(x)f(x)dx$$

- ▶ If possible compute it analytically!
- ▶ If we can sample from $f(x)$ we can use Monte Carlo integration
- ▶ Possible alternative: Importance sampling
 - ▶ sample from auxiliary distribution $g(x)$ and re-weight
 - ▶ can be used as variance-reduction technique

Importance sampling

Let $x_1, \dots, x_n \sim g(x)$, and let $w(x_i) = \frac{f(x_i)}{g(x_i)}$, $i = 1, \dots, n$ then

$$\hat{\mu}_{IS} = \frac{\sum h(x_i)w(x_i)}{n}$$

- ▶ Unbiased
- ▶ Consistent
- ▶ Need to know the normalizing constant

$$\tilde{\mu}_{IS} = \frac{\sum h(x_i)w(x_i)}{\sum w(x_i)}$$

- ▶ Biased for finite n
- ▶ Consistent
- ▶ Self-normalizing

Bayesian inference

Basics:

- ▶ Posterior \propto Likelihood \times Prior
- ▶ Prior Choice
- ▶ Bayesian hierarchical models:
- ▶ Full-conditional distributions

Bayesian inference

Basics:

- ▶ Posterior \propto Likelihood \times Prior
- ▶ Prior Choice
- ▶ Bayesian hierarchical models:
 - ▶ Observation $\pi(\mathbf{y}|\mathbf{x})$
 - ▶ Latent process $\pi(\mathbf{x}|\theta)$
 - ▶ Hyperpriors $\pi(\theta)$
- ▶ Full-conditional distributions

Block 2: Two big topics

- ▶ Markov chain Monte Carlo
- ▶ Integrated Nested Laplace Approximations (INLA)

Markov chain Monte Carlo:

- ▶ What is the idea? What kind of Markov chain do we create?

Markov chain Monte Carlo:

- ▶ What is the idea? What kind of Markov chain do we create?
 - ▶ Construct a Markov chain that converges to the distribution of interest.

Markov chain Monte Carlo:

- ▶ What is the idea? What kind of Markov chain do we create?
- ▶ Why do we not use an approach from block 1?
- ▶ What kind of different MCMC techniques have we seen?

Markov chain Monte Carlo:

- ▶ What is the idea? What kind of Markov chain do we create?
- ▶ Why do we not use an approach from block 1?
- ▶ What kind of different MCMC techniques have we seen?
- ▶ Is the algorithm working at all?

Elements of a MCMC algorithm

- ▶ Target distribution $\pi(x)$: Given by the problem
- ▶ Proposal distribution $Q(y|x)$: Chosen by the user
- ▶ Acceptance probability $\alpha(y|x)$: Derived in order to fulfill the detailed balance condition

Some keywords

detailed balance condition, Metropolis-within-Gibbs, random-walk proposal, burn-in, convergence diagnostics, mixing, effective sample size, ...

Integrated nested Laplace approximations

- ▶ What is the idea?
- ▶ For which models does it work?
- ▶ What are the main “ingredients”
- ▶ Potential advantages over MCMC

Integrated nested Laplace approximations

- ▶ What is the idea?
 - ▶ Deterministic approximation instead of simulations
 - ▶ Focus on marginal posterior instead of joint posterior
- ▶ For which models does it work?
- ▶ What are the main “ingredients”
- ▶ Potential advantages over MCMC

Integrated nested Laplace approximations

- ▶ What is the idea?
- ▶ For which models does it work?
 - ▶ Latent Gaussian Models (with Markov dependence structure)
- ▶ What are the main “ingredients”
- ▶ Potential advantages over MCMC

Integrated nested Laplace approximations

- ▶ What is the idea?
- ▶ For which models does it work?
- ▶ What are the main “ingredients”
 - ▶ Conditional probability

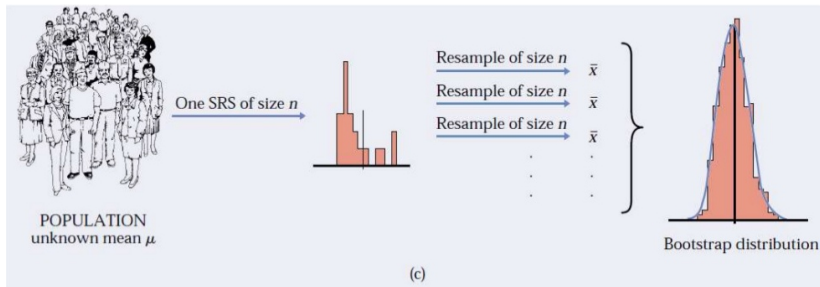
$$\pi(x|z) = \frac{\pi(x, z)}{\pi(z)} \Rightarrow \pi(z) = \frac{\pi(x, z)}{\pi(x|z)}$$

- ▶ Laplace Approximation
 - ▶ 2nd order Taylor approximation of non-Gaussian responses
- ▶ Potential advantages over MCMC

Block 3

- ▶ Bootstrap
- ▶ EM algorithm

Bootstrap

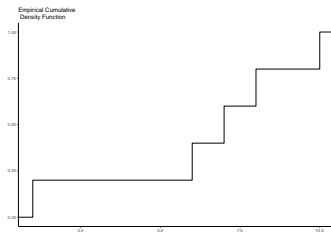


Bootstrap - Some keywords

► Empirical distribution function

Data: 1, 7, 8, 10, 6

Empirical distribution function:



Bootstrap - Some keywords

- ▶ Empirical distribution function
- ▶ Plug-in principle
- ▶ Bootstrap sample
 - ▶ Non-parametric
 - ▶ Parametric
- ▶ Bootstrapping more complex data structures - Regression
 - ▶ Bootstrap residuals
 - ▶ Paired-Bootstrap - Bootstrap time series
 - ▶ Model based bootstrap
 - ▶ Block-bootstrap

EM-algorithm

- ▶ Goal? Basic idea? What are the steps?
- ▶ Field of applications: mixture models, censored data, missing data, hidden models, ...

The exam - June 8, 2023

- ▶ Digital School Exam

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- ▶ <https://i.ntnu.no/wiki/-/wiki/English/Digital+school+exam+-+for+students>

Permitted exam support materials: C

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- ▶ 1 yellow stamped A5 sheet with handwritten notes and equations (both sides, 7th floor of Sentralbygg 2)

GOOD



LUCK!

