Øving 9

1. $\iint_D \ln(x)\,dA$

$2x+2y=5,\ xy=1$

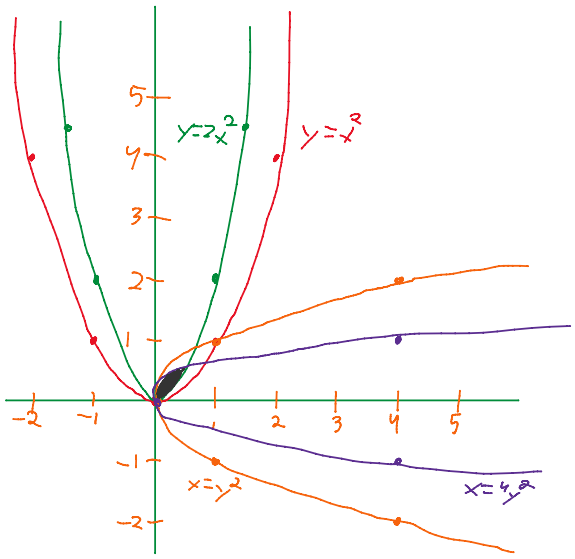
$\Rightarrow 2 \le x \le \frac{1}{2},\ 2 \le y \le \frac{1}{2}$

$\int_2^{\frac{1}{2}}\int_2^{\frac{1}{2}} \ln(x)\,dx\,dy = \int_2^{\frac{1}{2}} [x\ln(x)-x]_2^{\frac{1}{2}}\,dy$

$\approx \int_2^{\frac{1}{2}} (-0.233)\,dy$

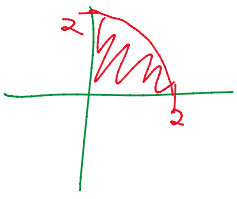
≈ 0.35

2(a)



(b)

3. $D=\{x^2+y^2+z^2 \le 2 :\ x \ge 0\ \ y \ge 0\ \ z \ge 0\}$



$x = \rho\sin(\phi)\cos(\theta)$

$y = \rho\sin(\phi)\sin(\theta)$

$z = \rho\cos(\phi)$

$0 \le \rho \le 2$

$0 \le \phi \le \frac{\pi}{2}$

$0 \le \theta \le \frac{\pi}{2}$

$$\iiint_D 2z\,dV = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^2 2\rho\cos(\phi)\cdot\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^2 2\rho^3\cos(\phi)\sin(\phi)\,d\rho\,d\phi\,d\theta$$

$$= 2\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\cos(\phi)\sin(\phi)\int_0^2\rho^3\,d\rho\,d\phi\,d\theta$$

$$= 2\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\cos(\phi)\sin(\phi)\,4\,d\phi\,d\theta$$

$$= \left[\begin{array}{l} u = \sin(\phi) \\ d\phi = \frac{du}{u'} \end{array}\right]$$

$$= 8\int_0^{\frac{\pi}{2}}\int_0^1 u\,du\,d\theta$$

$$= 8\int_0^{\frac{\pi}{2}}\frac{1}{2}\,d\theta$$

$$= 4\int_0^{\frac{\pi}{2}}d\theta$$

$$= 2\pi$$

4.(a) $y = x^2$

$y + z = 1$

$z = 0$

$0 \le z \le 1$

$0 \le y \le 1$

$-1 \le x \le 1$

$$\int_0^1\int_0^1\int_{-1}^1 dV = \int_0^1\int_0^1 2\,dy\,dz$$

$$= \int_0^1 2\,dz$$

$$= 2$$

(b) $x^2 + y^2 + z^2 \le 5$

(b) $x^2+y^2+z^2\leq 5$

$\delta(x,y,z)=2x^2+2y^2+2z^2+1$

$0\leq\theta\leq 2\pi$

$0\leq\phi\leq\pi$

$0\leq\rho\leq\sqrt{5}$

$x=\rho\cos(\theta)\sin(\phi)$

$y=\rho\sin(\theta)\sin(\phi)$

$z=\rho\cos(\phi)$

$\delta(\rho,\theta,\phi)=2(\rho)^2+1$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{5}}(2(\rho)^2+1)\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta=2\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{\sqrt{5}}\rho^4+\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$$

$$=2\int_0^{2\pi}\int_0^{\frac{\pi}{2}}625+\frac{125}{3}\sin(\phi)\,d\phi\,d\theta$$

$$=2\int_0^{2\pi}1250\pi+0\,d\theta$$

$$=$$

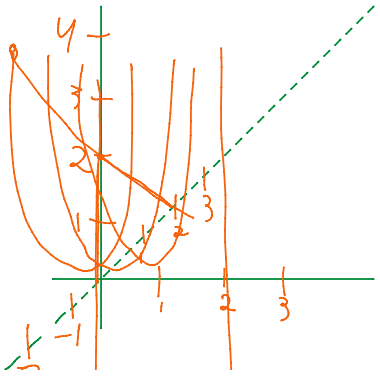
5. $\int_{\mathbb{R}^2}\frac{1}{(1+x^2+y^2)^5}dA$

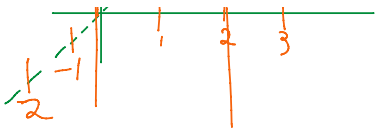
6(a) $z=y^2$

$z=2-y$

$x=0$

$x=2$





$0 \le z \le 4$

$-2 \le y \le 2$

$0 \le x \le 2$

$$\int_0^4 \int_{-2}^{2} \int_0^z dV = \int_0^4 \int_{-2}^{2} 2\,dy\,dz$$

$$= \int_0^4 8\,dz$$

$$= 32$$

(b)

7. $\iint_D x\cos(y)\,dA$

$y = 1 - x^2$

$0 \le x \le 1$

$1 \ge y \ge 0$

$$\int_1^0 \int_0^1 x\cos(y)\,dx\,dy = \int_1^0 \cos(y)\left[\frac{x^2}{2}\right]_0^1 dy$$

$$= \frac{1}{2}\int_1^0 \cos(y)\,dy$$

$$= \frac{1}{2}[\sin(y)]_1^0$$

$=$

(b) $\iiint_D e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\,dV$

$1 \le x^2 + y^2 + z^2 \le 4$

$z \ge 0$