Lecture 8 14/2 Week 7.2

MCMC: Metropolis-Hastings and Gibbs

Note (MCMC for continuous distributions):

Instead of $P(y|x)$, we will write

$$p(\theta, \phi) = \begin{cases} q(\theta, \phi)\alpha(\theta, \phi), & \theta \neq \phi, \\ 1 - \int_{-\infty}^{\infty} q(\theta, \phi)\alpha(\theta, \phi)\, d\phi, & \theta = \phi. \end{cases}$$

For $A \subseteq S$,

$$p(\theta, A) = \int_A q(\theta, \phi)\alpha(\theta, \phi)\, d\phi + I\{\theta \in A\}\left[1 - \int_{-\infty}^{\infty} q(\theta, \phi)\alpha(\theta, \phi)\, d\phi\right]$$

$$\alpha(\theta, \phi) = \min\left\{1, \frac{\pi(\phi)q(\phi, \theta)}{\pi(\theta)q(\theta, \phi)}\right\}.$$

Gibbs Sampling (acceptance rate = 1):

$$Q(y^j \mid x^j_{i-1}, \vec{x}^{-j}_{i-1}) = \pi(y^j \mid \vec{x}^{-j}_{i-1})$$

$$\alpha(\vec{y} \mid \vec{x}_{i-1}) = \min\left\{ \frac{\pi(\vec{y})}{\pi(\vec{x}_{i-1})} \cdot \frac{Q(x^j_{i-1} \mid \vec{y}^{-j})}{Q(y^j \mid \vec{x}^{-j}_{i-1})}, 1 \right\}$$

$$= \min\left\{ \frac{\pi(y^j \mid \vec{y}^{-j})\,\cancel{\pi(\vec{y}^{-j})}}{\pi(x^j_{i-1} \mid \vec{x}^{-j}_{i-1})\,\cancel{\pi(\vec{x}^{-j}_{i-1})}} \cdot \frac{\pi(x^j_{i-1} \mid \vec{y}^{-j})}{\pi(y^j \mid \vec{x}^{-j}_{i-1})}, 1 \right\}$$

$$(\vec{y}^{-j} = \vec{x}^{-j})$$

$$= 1$$