Øving 10

3.7

10 $y'' - 4y' + 5y = 0$

$r^2 - 4r + 5 = 0$

$r = \frac{4 \pm \sqrt{16 - 20}}{2}$

$= \frac{4 \pm \sqrt{-4}}{2}$

$= \frac{4 \pm 2i}{2}$

$= 2 \pm i$

$b = 2, \quad \omega = 1$

$y = Ae^{2t}\cos(t) + Be^{2t}\sin(t)$

14. $\begin{cases} y'' + 10y' + 25y = 0 \\ y(1) = 0 \\ y'(1) = 2 \end{cases}$

$r^2 + 10r + 25 = 0$

$(r+5)^2 = 0$

$r_1 = -5, \quad r_2 = -5$

$y = (A + Bx)e^{-5x}$

$y' = Be^{-5x} - 5(A + Bx)e^{-5x}$

$(A + B)e^{-5} = 0$

$Be^{-5} - 5(A + B)e^{-5} = 2$

$B=-A$

$Be^{-5}=2$

$B=2e^5$, $A=-2e^5$

$y=(-2e^5+2e^5)e^{-5x}$

$=(-2+2)e^{5-5x}$

16.

17.

24. $\begin{cases} y''+4y=0 \\ y(0)=2 \\ y'(0)=-5 \end{cases}$

$r^2+4=0$

$r=\frac{\pm\sqrt{0-16}}{2}$

$=\pm\frac{4i}{2}$

$=\pm 2i$

$b=0$, $w=2$

$y=A\cos(2t)+B\sin(2t)$

$y'=-2A\sin(2t)+2B\cos(2t)$

$A=2$

$2B=-5$

$B=-\frac{5}{2}$

$$y = 2\cos(2t) - \frac{5}{2}\sin(2t)$$

Periode:

$$\omega = 2 \Rightarrow T = \frac{2\pi}{2} = \pi$$

Amplitude:

$$\sqrt{A^2 - B^2} = \sqrt{4 + \frac{25}{4}}$$

$$= 10{,}25$$

Frekvens:

$$\frac{2}{2\pi} H_z = \frac{1}{\pi} H_z$$

18.5

2. $y^{(4)} - 2y'' + y = 0$

$r^4 - 2r^2 + 1 = 0$

$(r^2 - 1)^2 = 0$

$((r+1)(r-1))^2 = 0$

$r = \pm 1$

$y = C_1 e^t + C_2 t e^t + C_3 e^{-t} + C_4 t e^{-t}$

8. $x^2 y'' - xy' - 3y = 0$

$y = x^r$

$y' = r x^{r-1}$

$y'' = r(r-1) x^{r-2}$

$x^2 r(r-1) x^{r-2} - x r x^{r-1} - 3x^r = 0$

$r(r-1)x^r - rx^r - 3x^r = 0$

$r(r-1)x^r - rx^r - 3x^r = 0$

$(r(r-1) - r - 3)x^r = 0$

$(r^2 - r - r - 3)x^r = 0$

$(r^2 - 2r - 3)x^r = 0$

$(r+1)(r-3)x^r = 0$

$r_1 = -1, \; r_2 = 3$

$y = C_1 x^{-1} + C_2 x^3$

19.