Lecture 14 Week 13 27/03

The dependent bootstrap

Example (AR(1) model):

$$X_t = \mu + \varphi X_{t-1} + \varepsilon_t$$

$$|\varphi| < 1$$

$$\varepsilon_t \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$$

$$\hat{\theta} = (\hat{\mu}, \hat{\varphi}, \hat{\sigma}_\varepsilon^2)$$

$$\hat{\varepsilon}_t = X_t - (\hat{\mu} + \hat{\varphi} X_{t-1})$$

Block bootsrap:

$$\underbrace{X_1 - X_2 - X_3}_{\text{block } 1} - \underbrace{X_4 - X_5 - X_6}_{\text{block } 2} - \underbrace{X_7 - \cdots}_{\cdots}$$

bootstrap sample could be block 1, 2, 1:

$$\Rightarrow \underbrace{X_1 - X_2 - X_3}_{\text{block } 1} - \underbrace{X_4 - X_5 - X_6}_{\text{block } 2} - \underbrace{X_1 - X_2 - X_3}_{\text{block } 1}$$

Moving block bootstrap:

$$X_1 - X_2 - X_3 - X_4 - X_5 - X_6 - X_7 \quad \cdots$$

Block of blocks:

$$X = (X_1, X_2, X_3, \ldots, X_n)$$

$$Y = \Big\{ \underbrace{\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, \begin{pmatrix} X_2 \\ X_3 \end{pmatrix}, \begin{pmatrix} X_3 \\ X_4 \end{pmatrix}}_{\text{block 1}}, \underbrace{\begin{pmatrix} X_4 \\ X_5 \end{pmatrix}, \begin{pmatrix} X_5 \\ X_6 \end{pmatrix}, \begin{pmatrix} X_6 \\ X_7 \end{pmatrix}}_{\text{block 2}}, \underbrace{\begin{pmatrix} X_7 \\ X_8 \end{pmatrix}, \ldots}_{\ldots}, \begin{pmatrix} X_{n-1} \\ X_n \end{pmatrix} \Big\}$$

The lag 1 autocorrelation can be estimated as:

$$\hat{\varphi} = \frac{\sum_{t=1}^{n-1} (Y^*_{t,1} - \bar{X})(Y^*_{t,2} - \bar{X})}{\sum_{t=1}^{n} (X_t - \bar{X})^2}$$