

Kandidatnr: 10009. Problem 1

(i) $M \in \mathbb{R}^{n \times n}$

$I - M$ non-singular

$$\begin{aligned} \|(I - M)^{-1} - I\| &= \max_{\|x\|=1} \|(I - M)^{-1} - I\|x\| \\ &= \max_{\|x\|=1} \|(I - M)^{-1}x - x\| \\ &\stackrel{\Delta}{=} \max_{\|x\|=1} \|x\| \end{aligned}$$

(ii) $(1 + \|M\|)^{-1} \leq \|(I - M)^{-1}\|$

Let $\|x_1\| = 1$ s.t. $\|A\| = \|Ax_1\|$

Then $x_2 = Ax_1 \cdot \frac{1}{\|A\|} = 1$

$$\begin{aligned} (1 + \|M\|)^{-1} &= \frac{1}{1 + \|M\|} \\ &= \frac{\|x_1\|}{1 + \|M\|} \\ &= \|(I - M)^{-1} (I - M) \cdot \frac{x_1}{1 + \|M\|}\| \\ &\leq \|(I - M)^{-1} x_2\| \\ &\leq \|(I - M)^{-1}\| \|x_2\| \\ &= \|(I - M)^{-1}\| \end{aligned}$$
