

# Øving 7

13.1:

2.

3. La  $z_1 = x_1 + iy_1$

$$z_2 = x_2 + iy_2$$

$$z = \frac{z_1}{z_2}$$

$$= \frac{x_1 + iy_1}{x_2 + iy_2}$$

$$= \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)}$$

$$= \frac{x_1 x_2 + y_1 y_2 + i x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2} \quad \text{OK}$$

$$\frac{26 - 18i}{6 - 2i} = \frac{26 \cdot 6 + (-18) \cdot (-2)}{36 + 4} + i \frac{6 \cdot (-18) - 26 \cdot (-2)}{36 + 4}$$

$$= \frac{24}{5} - i \frac{7}{5}$$

14.

$$z_1 = -2 + 5i$$

$$z_2 = 3 - i$$

$$\frac{z_1}{z_2} = \frac{-2 + 5i}{3 - i}$$

$$= \frac{(-2) \cdot 3 + (-5) \cdot 1}{9 + 1} + i \frac{3 \cdot (-5) - (-2) \cdot 1}{9 + 1}$$

$$= -\frac{11}{10} - i \frac{13}{10}$$

$$\frac{z_1}{z_2} = \frac{-2 + 5i}{3 - i}$$

$$= \frac{(-2) \cdot 3 + 5 \cdot (-1)}{9 + 1} + i \frac{3 \cdot 5 - (-2) \cdot (-1)}{9 + 1}$$

$$= -\frac{11}{10} + i \frac{13}{10}$$

$$= -\frac{11}{10} - i \frac{13}{10}$$

16.

$$z = x + iy$$

$$\operatorname{Im}\left(\frac{1}{z}\right) = \frac{1}{2i} \left(\frac{1}{z} - \frac{1}{\bar{z}}\right)$$

$$= \frac{1}{2i} \left(\frac{1}{x + iy} - \frac{1}{x - iy}\right)$$

$$= \frac{1}{2i} \left(-\frac{2iy}{x^2 + y^2}\right)$$

$$= -\frac{y}{x^2 + y^2}$$

$$\operatorname{Im}\left(\frac{1}{z^2}\right) =$$

13.2

1.

$$z = 1 + i$$

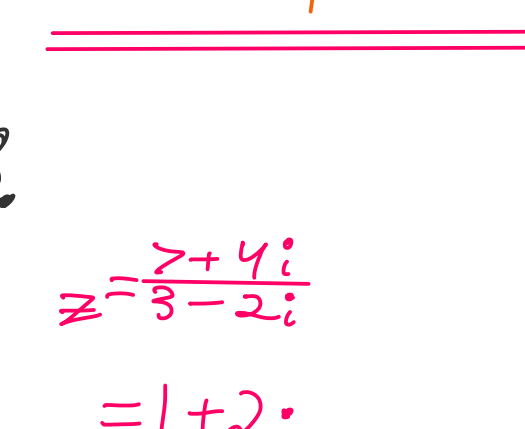
$$r = \sqrt{1^2 + 1^2}$$

$$= \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right)$$

$$= \frac{\pi}{4}$$

$$\Rightarrow z = \sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right)\right)$$



8.

$$z = \frac{7 + 4i}{3 - 2i}$$

$$= 1 + 2i$$

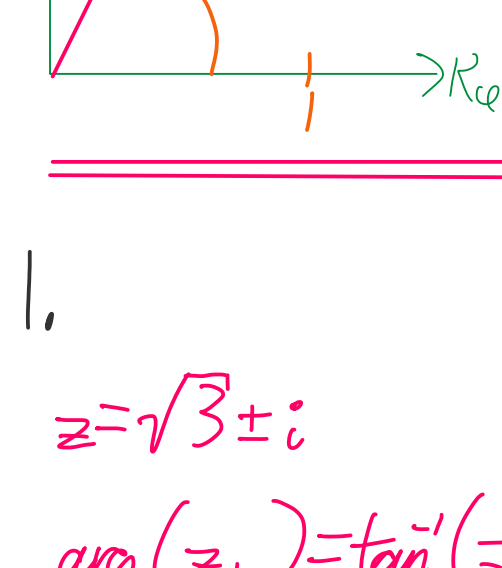
$$r = \sqrt{1^2 + 2^2}$$

$$= \sqrt{5}$$

$$\theta = \tan^{-1}\left(\frac{2}{1}\right)$$

$$= 63.43^\circ \approx 1.107$$

$$\Rightarrow z = \sqrt{5} \left(\cos(1.107) + i \sin(1.107)\right)$$



11.

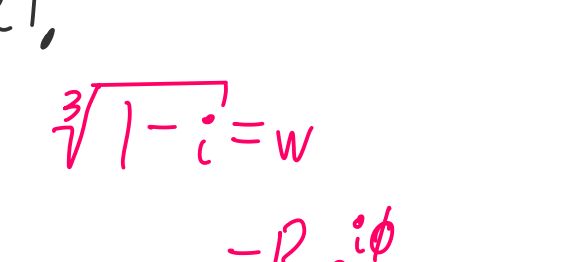
$$z = \sqrt{3} \pm i$$

$$\arg(z_1) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

$$\arg(z_2) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$$

$$= -\frac{\pi}{6}$$



21.

$$\sqrt[3]{1 - i} = w$$

$$= R e^{i\phi}$$

$$w^3 = 1 - i$$

$$= R e^{i\theta}$$

$$R = \sqrt{2}$$

$$\theta = \tan^{-1}(-1)$$

$$= -\frac{\pi}{4}$$

$$w^3 = \sqrt{2} e^{-i\pi/4}$$

$$R^3 = \sqrt{2}$$

$$3\phi = -\frac{\pi}{4} + k \cdot 2\pi, k = 0, 1, 2$$

$$\sqrt[3]{1 - i} \in \{\sqrt{2} e^{-i\pi/12}, \sqrt{2} e^{i5\pi/12}, \sqrt{2} e^{i9\pi/12}\}$$

25.

$$\sqrt[4]{i} = w$$

$$= R e^{i\phi}$$

$$w^4 = i$$

$$= e^{i\pi/2}$$

$$R^4 = 1$$

$$4\phi = \frac{\pi}{2} + k \cdot 2\pi$$

$$\sqrt[4]{i} \in$$

13.3

6.

$$\operatorname{Re}\left(\frac{1}{z}\right) < 1$$

$$\operatorname{Re}\left(\frac{1}{z}\right) = \operatorname{Re}\left(\frac{x - iy}{1 + z\bar{z}}\right)$$

$$= \frac{x}{1 + z\bar{z}}$$

$$= \frac{x}{x^2 + y^2 + 1} < 1$$

$$\Rightarrow x < x^2 + y^2 + 1$$

$$\Rightarrow -x^2 + x < y^2$$

15.

$$f(z) = \begin{cases} |z|^2 \operatorname{Im}\left(\frac{1}{z}\right), & z \neq 0 \\ 0, & z = 0 \end{cases}$$

$$f(0) = 0$$

$$z_0 = 0$$

$$|z - 0| < \delta$$

$$\Rightarrow |f(z) - 0| = |z|^2 \operatorname{Im}\left(\frac{1}{z}\right)$$

$$= |z|^2 \cdot \frac{-y}{|z|^2} = -y$$

$$= -y$$

$$\lim_{z \rightarrow 0} (-y) = 0 = f(0)$$

16.

$$f(x) = \begin{cases} \frac{\operatorname{Im}(z^2)}{|z|^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

$$f(0) = 0$$

$$z_0 = 0$$

$$|z - 0| < \delta$$

$$\Rightarrow |f(z) - 0| = \left| \frac{\operatorname{Im}(x^2 - y^2 + 2xyi)}{|z|^2} \right|$$

$$= \frac{2xy}{|z|^2}$$

$$=$$

18.

$$\left(\frac{z-i}{z+i}\right)' = \frac{(z'-1) \cdot (z+i) - (z-i)(z'+1)}{(z+i)^2}$$

$$= \frac{z'z + z'i - z - z'z' - z + iz' + i}{(z+i)^2}$$

$$= \frac{2z'i - 2z'}{(z+i)^2}$$