Lecture 4 17/01/23 Week 3.2

Rejection Sampling Variants, Monte Carlo, & Importance Sampling

Importance Sampling (Idea):

We want to estimate the expectation:

$$E_f[h(z)] = \int h(z) f(z) \, dz$$
$$\approx \frac{1}{N} \sum_{i=1}^{N} h(z^{(i)})$$

for $z^{(1)}, \dots, z^{(N)} \overset{iid}{\sim} f$. **But**: 1) f might be hard to sample from OR 2) $\frac{1}{N} \sum_{i=1}^{N} h(z^{(i)})$ might have high variance.

Instead, consider proposal density, g:

$$E_f[h(z)] = \int h(z) f(z) \, dz$$
$$= \int h(z) \frac{f(z)}{g(z)} g(z) \, dz$$
$$= E_g\left[h(z) \frac{f(z)}{g(z)}\right]$$

$$\Rightarrow \quad E_f[h(z)] \approx \frac{1}{N}\sum_{i=1}^{N} h(z^{(i)}) \frac{f(z^{(i)})}{g(z^{(i)})}$$

importance weights

for $z^{(1)}, \dots, z^{(N)} \overset{iid}{\sim} g.$