Lecture 7 13/2 Week 7.1

More Bayesian Statistics, MCMC

Note (Detailed balance ⟹ stationary distribution):

detailed balance:

$$\pi(x) P(y|x) = \pi(y) P(x|y) \quad \forall\, x, y \in S$$

$$\Rightarrow \sum_{x \in S} \pi(x) P(y|x) = \sum_{x \in S} \pi(y) P(x|y)$$

$$= \pi(y) \underbrace{\sum_{x \in S} P(x|y)}_{=1}$$

$$= \pi(y) \quad \square$$

Note (choosing acceptance rate for MCMC):

We will choose α to satisfy detailed balance:

$$\pi(x) \cdot P(y|x) = \pi(y) \cdot P(x|y)$$

$$\pi(x) Q(y|x) \alpha(y|x) = \pi(y) Q(x|y) \alpha(x|y)$$

$$\frac{\alpha(y|x)}{\alpha(x|y)} = \frac{\pi(y) Q(x|y)}{\pi(x) Q(y|x)}.$$

Choose $\alpha(y|x) = \min\left(1, \dfrac{\pi(y)Q(x|y)}{\pi(x)Q(y|x)}\right)$. 2 cases:

<u>Case 1:</u> $\pi(y)Q(x|y) \geq \pi(x)Q(y|x)$

$$\Rightarrow \alpha(y|x) = 1, \quad \alpha(x|y) = \frac{\pi(x)Q(y|x)}{\pi(y)Q(x|y)}.$$

So,

$$\frac{\alpha(y|x)}{\alpha(x|y)} = \frac{\pi(y)Q(x|y)}{\pi(x)Q(y|x)}$$

$\Rightarrow$ detailed balance. ✓

<u>Case 2:</u> $\pi(y)Q(x|y) < \pi(y)Q(x|y)$

Same argument as in Case 1 $\Rightarrow$ detailed balance satisfied. ✓

Hence