Lecture 13 Week 12 21/03

The Bootstrap

Example (plug-in estimator):

$$E_F\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{EX^3 - 3\mu\sigma^2 - \mu^3}{\sigma^3}$$

$$E_{\hat{F}}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{E_{\hat{F}}X^3 - 3E_{\hat{F}}X \, Var_{\hat{F}}(X) - E^3_{\hat{F}}X}{\left(Var_{\hat{F}}(X)\right)^{3/2}}$$

$$= \frac{\sum_{i=1}^{n} x_i^3 \cdot \frac{1}{n} - 3\bar{x} \cdot \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 - \bar{x}^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2\right)^{3/2}}$$

this is our plug-in estimator for $E_F\left[\left(\frac{X-\mu}{\sigma}\right)^3\right]$.

* no distributional assumptions!

Example (Ideal bootstrap estimator for SE of sample mean):

$$\hat{\theta}^*(b) = \frac{1}{n}\sum_{i=1}^{n} X_i^{b*} = \overline{X}^{b*}$$

$$\mathrm{Var}_{\hat{F}}\left(\overline{X}^{b*}\right) = \mathrm{Var}_{\hat{F}}\left(\frac{1}{n}\sum_{i=1}^{n} X_i^*\right)$$

$$= \frac{1}{n^2}\sum_{i=1}^{n}\mathrm{Var}_{\hat{F}}\left(X_1^*\right)$$

$$= \frac{1}{n}\mathrm{Var}_{\hat{F}}\left(X_1^*\right)$$

$$= \frac{1}{n}E_{\hat{F}}\left[\left(X_1^* - E_{\hat{F}}[X_1^*]\right)^2\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 \cdot \frac{1}{n}$$

$$\boxed{= \frac{1}{n}\cdot\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$$