fredag 11. desember 2020 08:50

1. $A = \begin{pmatrix} 1 & 2 & 0 & 10 \\ 0 & 1 & 0 & 3 \\ -1 & 1 & 1 & 1 \\ -1 & 2 & 0 & 3 \end{pmatrix}$

$$\det(A) = a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33} + a_{43}C_{43}$$

$$= 0 - 0 + 1 \cdot \begin{vmatrix} 1 & 2 & 10 \\ 0 & 1 & 3 \\ -1 & 2 & 3 \end{vmatrix} - 0$$

$$= 1 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} - 0 - 1 \cdot \begin{vmatrix} 2 & 10 \\ 1 & 3 \end{vmatrix}$$

$$= -3 + 4$$

$$= 1$$

Invertibel

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 10 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ -1 & 2 & 0 & 3 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \\ |+R_1 \\ |+R_1 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 10 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 11 & 1 & 0 & 1 & 0 \\ 0 & 4 & 0 & 13 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} |-2R_2 \\ \\ |-3R_2 \\ |-4R_2 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 4 & 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -4 & 0 & 1 \end{array}\right) \begin{array}{l} |-4R_4 \\ |-3R_4 \\ |-2R_4 \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -3 & 14 & 0 & -4 \\ 0 & 1 & 0 & 0 & -3 & 13 & 0 & -3 \\ 0 & 0 & 1 & 0 & -1 & 5 & 1 & -2 \\ 0 & 0 & 0 & 1 & 1 & -4 & 0 & 1 \end{array}\right)$$

$$\underline{\underline{A^{-1} = \begin{pmatrix} -3 & 14 & 0 & -4 \\ -3 & 13 & 0 & -3 \\ -1 & 5 & 1 & -2 \\ 1 & -4 & 0 & 1 \end{pmatrix}}}$$

2. $A = \begin{pmatrix} 1 & a & 1 & 2 \\ 2 & 1 & 6 & 1 \\ 3 & 2 & 7 & c \end{pmatrix}$

$\vec{b} \in \text{span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right\}$

$A\vec{z} = \vec{b}$

$\left(\begin{array}{cccc|c} 1 & a & 1 & 2 & b_1 \\ 2 & 1 & 6 & 1 & b_2 \\ 3 & 2 & 7 & c & b_3 \end{array}\right) \begin{array}{l} \\ -2R_1 \\ -3R_1 \end{array}$

$\begin{pmatrix} 1 & a & 1 & 2 & b_1 \\ 0 & 1-2a & 4 & -3 & b_2-2b_1 \\ 0 & 2-3a & 4 & c-6 & b_3-3b_1 \end{pmatrix}$

3 $A=\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & a & 3 & 3 \\ 1 & 2 & 2 & b \\ 3 & 6 & c & 6 \end{pmatrix}$

nullity $=2 \Rightarrow 2$ frie ukjente

$A\vec{x}=\vec{0}$

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & a & 3 & 3 \\ 1 & 2 & 2 & b \\ 3 & 6 & c & 6 \end{pmatrix} \begin{array}{l} \\ \mid -2R_1 \\ \mid -R_1 \\ \mid -3R_1 \end{array}$$

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & a-4 & 1 & 3 \\ 0 & 0 & 1 & b \\ 0 & 0 & c-3 & 6 \end{pmatrix}$$

$a=4,\ c=5,\ b=3$ gir

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 6 \end{pmatrix} \begin{array}{l} \\ \\ \mid -R_2 \\ \mid -2R_2 \end{array}$$

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\text{nullity}(A)=2=4-2 \Rightarrow \text{rank}(A)=2$

$\underline{\underline{a=3,\ c=5,\ b=3 \text{ gir nullity}(A)=2}}$

$x_4 = s$

$x_3 = -3s$

$x_2 = t$

$x_1 = -2t + 3s$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2t+3s \\ t \\ -3s \\ s \end{pmatrix} = t\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ -3 \\ 1 \end{pmatrix}$$

Basisen til nullrommet til A er $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ -3 \\ 1 \end{pmatrix} \right\}$

4. $\vec{u}=(1,1,2,1)$, $\vec{v}=(1,3,2,1)$

$\langle \vec{x},\vec{u}\rangle = 0 = \langle \vec{x},\vec{v}\rangle$

Bruker G.S.

$\vec{x}_1=\vec{u}=(1,1,2,1)$

$\vec{x}_2 = \vec{v} - \frac{\langle \vec{v},\vec{x}_1\rangle}{\|\vec{x}_1\|^2}\vec{x}_1 = (1,3,2,1) - \frac{9}{7}(1,1,2,1) = \left(-\frac{2}{7}, \frac{12}{7}, -\frac{4}{7}, -\frac{2}{7}\right)$

$\tilde{\vec{x}}_1 = \frac{\vec{x}_1}{\|\vec{x}_1\|} = \frac{1}{\sqrt{7}}(1,1,2,1)$

$\tilde{\vec{x}}_2 = \frac{\vec{x}_2}{\|\vec{x}_2\|} = \frac{7}{2\sqrt{42}}\left(-\frac{2}{7}, \frac{12}{7}, -\frac{4}{7}, -\frac{2}{7}\right) = \frac{1}{\sqrt{42}}(-1,6,-2,-1)$

$\vec{x} \in \underline{\underline{\left\{\frac{1}{\sqrt{7}}(1,1,2,1), \frac{1}{\sqrt{42}}(-1,6,-2,-1)\right\}}}$

5 $h=(0,0,0)$, $t=(5+3\sqrt{2},\sqrt{2},4\sqrt{2})$

$\sqrt{1^2+0^2+0^2}=1$ ✓

$\sqrt{0^2+x^2+x^2}=1 \Rightarrow$

$\sqrt{2x^2}=1 \Rightarrow$

$\sqrt{2}x=1 \Rightarrow$

$x=\pm\frac{1}{\sqrt{2}}$ ✓

$\sqrt{y^2+0^2+y^2}=1 \Rightarrow$

$\sqrt{2y^2}=1 \Rightarrow$

$y=\pm\frac{1}{\sqrt{2}}$

Velger positiv retning for x, y

$(5+3\sqrt{2},\sqrt{2},4\sqrt{2})=a(1,0,0)+b(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})+c(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}})$

$5+3\sqrt{2}=a+\frac{1}{\sqrt{2}}c$

$\sqrt{2}=\frac{1}{\sqrt{2}}b$

$4\sqrt{2}=\frac{1}{\sqrt{2}}b+\frac{1}{\sqrt{2}}c$

$$\begin{pmatrix} 1 & 0 & \frac{1}{\sqrt{2}} & 5+3\sqrt{2} \\ 0 & \frac{1}{\sqrt{2}} & 0 & \sqrt{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 4\sqrt{2} \end{pmatrix} \begin{matrix} \\ \\ \mid -R_2 \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & \frac{1}{\sqrt{2}} & 5+3\sqrt{2} \\ 0 & \frac{1}{\sqrt{2}} & 0 & \sqrt{2} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 3\sqrt{2} \end{pmatrix} \begin{matrix} \mid -R_3 \\ \\ \\ \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \sqrt{2} \\ 0 & 0 & \frac{1}{\sqrt{2}} & 3\sqrt{2} \end{pmatrix} \begin{matrix} \\ \mid \cdot\sqrt{2} \\ \mid \cdot\sqrt{2} \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \end{pmatrix}$$

$a=5, b=2, c=6$

Magnus må ta 5 steg i retning $(1,0,0)$, 2 steg i retning $(0,1,1)$ og 6 steg i retning $(1,0,1)$

6. $P_1=(0,0),\ P_2=(2,1),\ P_3=(-1,1)$

$Q_1=(0,-4),\ Q_2=(0,0),\ Q_3=(3,-1)$

Ser at: $T_A\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$T_A\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}3\\-1\end{pmatrix}$

$T_A\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}0\\-4\end{pmatrix}$

$A\left(T\begin{pmatrix}1\\0\end{pmatrix}, T\begin{pmatrix}0\\1\end{pmatrix}\right)$:

- $\begin{pmatrix}1\\0\end{pmatrix}=a\begin{pmatrix}2\\1\end{pmatrix}+b\begin{pmatrix}-1\\1\end{pmatrix}$

$\Rightarrow 1=2a-b$

$0=a+b$

$\Rightarrow b=-\frac{1}{3} \Rightarrow a=\frac{1}{3}$

$T\begin{pmatrix}1\\0\end{pmatrix}=T\left(\frac{1}{3}\begin{pmatrix}2\\1\end{pmatrix}-\frac{1}{3}\begin{pmatrix}-1\\1\end{pmatrix}\right)$

$=\frac{1}{3}T\begin{pmatrix}2\\1\end{pmatrix}-\frac{1}{3}T\begin{pmatrix}-1\\1\end{pmatrix}$

$=\frac{1}{3}\begin{pmatrix}3\\-1\end{pmatrix}-\frac{1}{3}\begin{pmatrix}0\\-4\end{pmatrix}$

$=\frac{1}{3}\begin{pmatrix}3\\-1\end{pmatrix}-\begin{pmatrix}0\\-4\end{pmatrix}$

$=\frac{1}{3}\begin{pmatrix}3\\3\end{pmatrix}$

$=\begin{pmatrix}1\\1\end{pmatrix}$

- $\begin{pmatrix}0\\1\end{pmatrix}=a\begin{pmatrix}2\\1\end{pmatrix}+b\begin{pmatrix}-1\\1\end{pmatrix}$

$\Rightarrow 0=2a-b$

$1=a+b$

$\Rightarrow b=\frac{2}{3} \Rightarrow a=\frac{1}{3}$

$T\begin{pmatrix}0\\1\end{pmatrix}=T\left(\frac{1}{3}\begin{pmatrix}2\\1\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-1\\1\end{pmatrix}\right)$

$=\frac{1}{3}T\begin{pmatrix}2\\1\end{pmatrix}+\frac{2}{3}T\begin{pmatrix}-1\\1\end{pmatrix}$

$=\frac{1}{3}\begin{pmatrix}3\\-1\end{pmatrix}+\frac{2}{3}\begin{pmatrix}0\\-4\end{pmatrix}$

$=\begin{pmatrix}1\\-\frac{1}{3}\end{pmatrix}+\begin{pmatrix}0\\-\frac{8}{3}\end{pmatrix}$

$=\begin{pmatrix}1\\-3\end{pmatrix}$

$A=\begin{pmatrix}1 & 1\\1 & -3\end{pmatrix}$

$$\underline{\underline{T_A(\vec{x})=\begin{pmatrix}1 & 1\\1 & -3\end{pmatrix}\vec{x}}}$$

7. $\frac{x^2}{4}-\frac{y^2}{16}=1$

8.(a) $A+A^T$ er diagonaliserbar $\Leftrightarrow$ $A+A^T$ har n lin. uavh. egenvektorer

$A+A^T=B$, B symmetrisk

Skal vise B har n lin. uavh. egenvektorer $\Rightarrow$ B er diagonaliserbar

B har n lin uavhengige egenvektorer $\{\vec{p}_1, \vec{p}_2, \dots, \vec{p}_n\}$

$\Rightarrow$ $B\vec{p}_i = \lambda_i \vec{p}_i$ for $\lambda_i \in \mathbb{R}$

La $P = \{\vec{p}_1, \vec{p}_2, \dots, \vec{p}_n\}$, $D = \begin{pmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{pmatrix}$

$\Rightarrow$ $P^{-1}BP = D$ ▭

(b) $A = PDP^T$

9. $\lambda = b$, $\lambda = -b$

$A\vec{x} = \lambda\vec{x}$, $A\vec{x} = -\lambda\vec{x}$

$\|A\vec{x}\| = \|\lambda\vec{x}\| = \|(-\lambda)\vec{x}\| = |\lambda|\ \|\vec{x}\| = |-\lambda|\ \|\vec{x}\|$

$|\lambda| = |b|$

$\|A\vec{x}\| = |b|\ \|\vec{x}\|$ ▭