TMA4265 Stochastic Modelling: Exercise 4

Week 37

Problem 1

Consider the Markov chain $\{X_n : n = 0, 1, \ldots\}$ with transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 0.50 & 0.50 & 0 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 & 0 & 0 \\ 0.25 & 0.25 & 0.25 & 0.25 & 0 & 0 \\ 0.25 & 0 & 0.25 & 0.25 & 0 & 0.25 \\ 0 & 0 & 0 & 0 & 0.50 & 0.50 \\ 0 & 0 & 0 & 0 & 0.50 & 0.50 \end{bmatrix}.$$

a) Is the Markov chain reducible or irreducible? If it is reducible, specify its equivalence classes.

b) Calculate the period of each state.

c) Which states are transient and which states are recurrent? Are there any absorbing states?

Problem 2

Let $\{X_n : n = 0, 1, \ldots\}$ be a Markov chain with finite state space $\{0, 1, \ldots, N\}$. Show that if the Markov chain is aperiodic and irreducible, then it is regular and recurrent.

Problem 3

Consider the discrete-time Markov chain with transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.05 & 0.8 & 0.15 \\ 0 & 0.1 & 0.9 \end{bmatrix}.$$

a) Is the Markov chain reducible or irreducible?

b) Calculate the limiting probability of each state.

c) Write code to simulate the Markov chain, and verify the answer in b) numerically.

Problem 4

We throw a fair coin repeatedly, and denote each outcome by H (heads) or T (tails). Let n denote the n-th throw in a sequence, and define a stochastic process $\{X_n : n = 0, 1, \ldots\}$ by

$$X_n = \text{"The number of consecutive throws of H, including throw } n\text{"}, \quad n = 0, 1, \ldots.$$

a) Explain why this is a Markov process. Specify the state space, and find the transition probabilities.

b) Is this Markov chain irreducible?

c) For each state, calculate its period.

d) Show that the states are recurrent.

e) Calculate the long run proportion of time that the sequence of throws ends in three or more heads.