

## Summary &amp; Review

Exam 2016 problem 3 a)

Derive the full-conditionals of the components of  $(\alpha, \beta, \theta_1, \dots, \theta_n)$ .

Solution

Start by writing the posterior!

$$p(\alpha, \beta, \theta_1, \dots, \theta_n | y_1, \dots, y_n)$$

$$\propto [\text{Likelihood}] \times [\text{process model}] \times [\text{hyperpriors}]$$

$$= \left[ \prod_{i=1}^n p(y_i | \theta_i) \right] \times \left[ \prod_{i=1}^n p(\theta_i | \alpha, \beta) \right] \times [p(\alpha, \beta)]$$

$$= \left[ \prod_{i=1}^n \frac{\theta_i^{y_i}}{y_i!} e^{-\theta_i} \right] \times \left[ \prod_{i=1}^n \frac{\beta^\alpha}{\Gamma(\alpha)} \theta_i^{\alpha-1} e^{-\beta \theta_i} \right] \left[ a \cdot e^{-a\alpha} \cdot \frac{c^b}{\Gamma(b)} \beta^{b-1} e^{-c\beta} \right]$$

$$p(\theta_i | \alpha, \beta, \theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_n, y_1, \dots, y_n)$$

$$\propto \theta_i^{y_i} e^{-\theta_i} \times \theta_i^{\alpha-1} e^{-\beta \theta_i}$$

$$= \theta_i^{(\alpha+y_i)-1} e^{-(\beta+1)\theta_i}$$

$$\sim \text{Gamma}(\alpha + y_i, \beta + 1)$$