Review: Bayesian Hierarchical models

Hierarchical models are an extremely useful tool in Bayesian model building.

Three parts:

- Observation model $\boldsymbol{y}|\boldsymbol{x}, \boldsymbol{\theta}_1$: Encodes information about observed data.
- The latent model $\boldsymbol{x}|\boldsymbol{\theta}_2$: The unobserved process.
- Hyperpriors for $\boldsymbol{\theta} = (\boldsymbol{\theta}_1,\ \boldsymbol{\theta}_2)$: Models for all of the parameters in the observation and latent processes.

Note: here we indicate the observed data by $\boldsymbol{y}$ while $\boldsymbol{x}$ and $\boldsymbol{\theta}$ are parameters

Bayesian Hierarchical models

Important Note:

When specifying a hierarchical model, **conditional independence is assumed whenever possible for all conditional dependencies left unspecified.**

Hierarchical Bayesian models - Tokyo rainfall example

Tokyo rainfall example from exercise 2

- y_t number of times daily rainfall in Tokyo ≥ 1 mm, $t = 1, \ldots, 366$
- τ_t logit probability of exceeding 1 mm $t = 1, \ldots, 366$
- n_t number of trials, $t = 1, \ldots, 366$
- $\pi(\tau_t) = \frac{1}{1+\exp(-\tau_t)}$

Hierarchical Bayesian models - Tokyo rainfall example

Tokyo rainfall example from exercise 2

- y_t number of times daily rainfall in Tokyo ≥ 1 mm, $t = 1, \ldots, 366$
- τ_t logit probability of exceeding 1 mm $t = 1, \ldots, 366$
- n_t number of trials, $t = 1, \ldots, 366$
- $\pi(\tau_t) = \frac{1}{1+\exp(-\tau_t)}$

Model:

$$y_t \mid \tau_t \sim \text{Bin}(n_t, \pi(\tau_t))$$

Hierarchical Bayesian models - Tokyo rainfall example

Tokyo rainfall example from exercise 2

- y_t number of times daily rainfall in Tokyo ≥ 1 mm, $t = 1, \ldots, 366$
- τ_t logit probability of exceeding 1 mm $t = 1, \ldots, 366$
- n_t number of trials, $t = 1, \ldots, 366$
- $\pi(\tau_t) = \frac{1}{1+\exp(-\tau_t)}$

Model:

$$y_t \mid \tau_t \sim \text{Bin}(n_t, \pi(\tau_t))$$

Prior for τ_t:

$$\tau_t = \tau_{t-1} + u_t, \quad u_t \overset{iid}{\sim} N(0, \sigma_u^2), \quad t = 2, \ldots, 366.$$

Hierarchical Bayesian models - Tokyo rainfall example

Tokyo rainfall example from exercise 2

- y_t number of times daily rainfall in Tokyo ≥ 1 mm, $t = 1, \dots, 366$
- τ_t logit probability of exceeding 1 mm $t = 1, \dots, 366$
- n_t number of trials, $t = 1, \dots, 366$
- $\pi(\tau_t) = \frac{1}{1+\exp(-\tau_t)}$

Model:

$$y_t \mid \tau_t \sim \text{Bin}(n_t, \pi(\tau_t))$$

Prior for τ_t:

$$\tau_t = \tau_{t-1} + u_t, \quad u_t \overset{iid}{\sim} N(0, \sigma_u^2), \quad t = 2, \dots, 366.$$

Hyper-prior on σ_u^2:

$$\sigma_u^2 \sim \text{Inv-Gamma}(\alpha, \beta)$$

Hierarchical Bayesian models - Tokyo rainfall example

Tokyo rainfall example from exercise 2

- y_t number of times daily rainfall in Tokyo ≥ 1 mm, $t = 1, \dots, 366$
- τ_t logit probability of exceeding 1 mm $t = 1, \dots, 366$
- n_t number of trials, $t = 1, \dots, 366$
- $\pi(\tau_t) = \frac{1}{1+\exp(-\tau_t)}$

Model:

$$y_t \mid \tau_t \sim \text{Bin}(n_t, \pi(\tau_t))$$

Prior for τ_t:

$$\tau_t = \tau_{t-1} + u_t, \quad u_t \overset{iid}{\sim} N(0, \sigma_u^2), \quad t = 2, \dots, 366.$$

Hyper-prior on σ_u^2:

$$\sigma_u^2 \sim \text{Inv-Gamma}(\alpha, \beta)$$

Use dependency graphs to visualize the conditional independence structure!

See notes