Lecture 9 2012 Week 8.1

More Gibbs, Blocking, Convergence

Example (Simple Linear Regression):

$$\pi(a,b,\tau\mid\vec{y}) \propto \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi/\tau}} \exp\left\{-\frac{\tau}{2}\left(y_i-(a+bx_i)\right)^2\right\}$$

$$\times \frac{1}{\sqrt{2\pi/\tau_a}} \exp\left\{-\frac{\tau_a}{2}a^2\right\} \times \frac{1}{\sqrt{2\pi/\tau_b}} \exp\left\{-\frac{\tau_b}{2}b^2\right\}$$

$$\times \frac{\beta^\alpha}{\Gamma(\alpha)} \tau^{\alpha-1} \exp\{-\beta\tau\}$$

$$\left(\pi(\vec{y}\mid a,b,\tau)\,\pi(a)\,\pi(b)\,\pi(\tau)\right)$$

a priori, independent.

To find $\pi(a\mid b,\tau,\vec{y})$, we will ignore multiplicative factors that don't involve a:

$$\pi(a\mid b,\tau,\vec{y}) \propto \prod_{i=1}^{n} \exp\left\{-\frac{\tau}{2}\left(y_i-(a+bx_i)\right)^2\right\}$$

$$\times \exp\left\{-\frac{\tau_a}{2}a^2\right\}$$

$$= \exp\left\{-\frac{\tau}{2}\sum_{i=1}^{n}\left(y_i-(a+bx_i)\right)^2 + -\frac{\tau_a}{2}a^2\right\}$$

$$= \exp\left\{-\frac{\tau}{2}\sum_{i=1}^{n}\left(a^2 - 2a(y_i - bx_i) + (y_i - bx_i)^2\right) - \frac{\tau_a}{2}a^2\right\}$$

$$= \exp\left\{-\frac{n\tau + \tau_a}{2}a^2 + \frac{\tau}{2}\cdot 2a\sum_{i=1}^{n}(y_i - bx_i) - \frac{\tau}{2}\sum_{i=1}^{n}(y_i - bx_i)^2\right\}$$

$$= \exp\left\{-\frac{n\tau+\tau_a}{2}\left(a^2 - \frac{\tau\cdot 2a}{n\tau+\tau_a}\sum_{i=1}^{n}(y_i - bx_i)\right)\right\}$$

$$= \underbrace{\exp\left\{-\frac{n\tau+\tau_a}{2}\left(a - \frac{\tau}{n\tau+\tau_a}\sum_{i=1}^{n}(y_i - bx_i)\right)^2\right\}}_{\text{core/kernel of } N\left(\frac{\tau}{n\tau+\tau_a}\sum_{i=1}^{n}(y_i - bx_i),\ \frac{1}{n\tau+\tau_a}\right)}$$

$$\pi(\tau \mid a, b, \vec{y}) \propto \left(\frac{1}{\sqrt{2\pi/\tau}}\right)^n \exp\left\{-\frac{\tau}{2}\sum_{i=1}^{n}\left(y_i - (a + bx_i)\right)^2\right\}$$

$$\times\ \tau^{\alpha-1}\exp\{-\beta\tau\}$$

$$\propto \tau^{(n/2+\alpha)-1}\exp\Bigg\{-\tau\underbrace{\left(\frac{1}{2}\sum_{i=1}^{n}\left(y_i-(a+bx_i)\right)^2 + \beta\right)}_{\text{a constant}}\Bigg\}$$

$$\Rightarrow \tau \mid a, b, \vec{y} \sim \text{Gamma}\left(\frac{n}{2}+\alpha,\ \frac{1}{2}\sum_{i=1}^{n}\left(y_i - (a+bx_i)\right)^2 + \beta\right)$$

Find $\pi(b \mid a, \tau, \vec{y})$ yourself!