Kandidatnr: 10009. Problem 3

α is root to $f(x)=0$

$x_{n+1}=x_n-3\frac{f(x_n)}{f'(x_n)}$ converges to α

Convergence:

Let $g(x_n)=x_n-3\frac{f(x_n)}{f'(x_n)}$

$|x_k-\alpha|\leq\varepsilon_k$

$\lim_{k\to\infty}\frac{\varepsilon_{k+1}}{\varepsilon_k^q}=\mu$

$\mu>0,\ q=2$

$\lim_{k\to\infty}\frac{|x_{k+1}-\alpha|}{|x_k-\alpha|^2}=\lim_{k\to\infty}\frac{|g(x_k)-g(\alpha)|}{|x_k-\alpha|^2}$

$=\lim_{k\to\infty}\frac{|x_k-3\frac{f(x_k)}{f'(x_k)}-\alpha+3\frac{f(\alpha)}{f'(\alpha)}|}{|x_k-\alpha|^2}$

$=\left[\lim_{k\to\infty}3\frac{f(x_k)}{f'(x_k)}=3\frac{f(\alpha)}{f'(\alpha)}=0,\ f'(\alpha)\neq0\right]$

$=\lim_{k\to\infty}\frac{|x_k-\alpha|}{|x_k-\alpha|^2}$

$=\lim_{k\to\infty}$

$g'(x_n)=1-3\left(\frac{f(x_n)}{f'(x_n)}\right)'$

know when $n\to\infty$, $\left(\frac{f(x_n)}{f'(x_n)}\right)'\to0$

$g'(\alpha)=1-3\cdot0$

$=1$