Øving 11

4.6.1

$r = 3$

$\mu = 1000$

$f_Y(t) = \frac{\lambda^r}{\Gamma(r)} t^{r-1} e^{-\lambda t}$

$\Gamma(r) = \int_0^\infty t^{r-1} e^{-t}\, dt$

r Heltall $\Rightarrow \Gamma(r) = (r-1)!$

$f_Y(t) = \frac{\lambda^r}{(r-1)!} t^{r-1} e^{-\lambda t}$

$\lambda = \frac{1}{\mu} = 0.001$

$f_Y(t) = \frac{0.001^3}{2} t^2 e^{-0.002t}$

5.2.1

$n = 8$

$x_1 = 1$

$x_2 = 0$

$x_3 = 1$

$x_4 = 1$

$x_5 = 0$

$x_6 = 1$

$x_7 = 1$

$x_8 = 0$

$p_x(b;\theta) = \theta^b (1-\theta)^{1-b}$

$b = 0, 1$

$0 < \theta < 1$

$L(\theta) = \prod_{i=1}^{n} p_x(b_i;\theta)$

$= \prod_{i=1}^{8} \theta^{b_i} (1-\theta)^{1-b_i}$

$= (p_x(1;\theta))^5 (p_x(0;\theta))^3$

$= (\theta^1 (1-\theta)^0)^5 (\theta^0 (1-\theta)^1)^3$

$= \theta^5 (1-\theta)^3$

$= \theta^5 (-\theta^3 + 3\theta^2 - 3\theta + 1)$

$= \theta^5(-\theta^3+3\theta^2-3\theta+1)$

$= -\theta^8+3\theta^7-3\theta^6+\theta^5$

$\frac{d}{d\theta}L(\theta) = -8\theta^7+21\theta^6-18\theta^5+5\theta^4$

$\frac{d}{d\theta}L(\theta) = 0$

$-8\theta^7+21\theta^6-18\theta^5+5\theta^4 = 0$

$\theta^4(-8\theta^3+21\theta^2-18\theta+5) = 0$

$-8\theta^3+21\theta^2-18\theta+5 = 0$

$-(\theta-1)^2(8\theta-5) = 0$

~~$\theta=0$~~, ~~$\theta=1$~~, $\theta = \frac{5}{8}$

$\underline{\theta = \frac{5}{8}}$

5.2.6 $f_Y(y;\theta) = \frac{\theta}{2\sqrt{y}}e^{-\theta\sqrt{y}}$, $y \geq 0$

$n = 4$

$y_1 = 6.2$

$y_2 = 7.0$

$y_3 = 2.5$

$y_4 = 4.2$

$L(\theta) = \prod_{i=1}^{n} f_Y(y_i;\theta)$

$= \prod_{i=1}^{4} \frac{\theta}{2\sqrt{y_i}}e^{-\theta\sqrt{y_i}}$

$= \frac{\theta^4}{2^4\sqrt{6.2}\sqrt{7.0}\sqrt{2.5}\sqrt{4.2}}e^{-\theta(\sqrt{6.2}+\sqrt{7.0}+\sqrt{2.5}+\sqrt{4.2})}$

$= \frac{\theta^4}{16\sqrt{455.7}}e^{-\theta(\sqrt{6.2}+\sqrt{7.0}+\sqrt{2.5}+\sqrt{4.2})}$

$\ln(L(\theta)) = 4\ln(\theta) - \ln(16\sqrt{455.7}) - \theta(\sqrt{6.2}+\sqrt{7.0}+\sqrt{2.5}+\sqrt{4.2})$

$\frac{d}{d\theta}\ln(L(\theta)) = \frac{4}{\theta} - 0 - (\sqrt{6.2}+\sqrt{7.0}+\sqrt{2.5}+\sqrt{4.2})$

$\frac{d}{d\theta}\ln(L(\theta)) = 0$

$\frac{4}{\theta} - (\sqrt{6.2}+\sqrt{7.0}+\sqrt{2.5}+\sqrt{4.2}) = 0$

$\underline{\theta = \frac{4}{\sqrt{6.2}+\sqrt{7.0}+\sqrt{2.5}+\sqrt{4.2}}}$

5.2.12 $f_Y(y;\theta) = \frac{2y}{\theta^2}$, $0 \leq y \leq \theta$

$L(\theta) = \prod_{i=1}^{n} f(y_i;\theta)$

5.2.12 $f_Y(y;\theta) = \frac{2y}{\theta^2}, \quad 0 \le y \le \theta$

$$L(\theta) = \prod_{i=1}^{n} f_Y(y_i;\theta)$$

$$= \prod_{i=1}^{n} \frac{2y_i}{\theta^2}$$

$$= 2^n \frac{\prod_{i=1}^{n} y_i}{\theta^{2n}}$$

$$\frac{d}{d\theta} L(\theta) = -4n \frac{\prod_{i=1}^{n} y_i}{\theta^{2n+1}}$$

$$\frac{d}{d\theta} L(\theta) = 0$$

$$-4n \frac{\prod_{i=1}^{n} y_i}{\theta^{2n+1}} = 0$$

Eksamen Juni 2007

4. $X \sim N(m, 0.01^2)$

$m < 0.99 \Rightarrow$ undervektig

(a) Så lenge maskinen fungerer som den skal, vil ikke den forrige pakken påvirke den neste.

$m = 1.00$ kg

$$P(0 \le X \le 0.99) = P\left(\frac{0-1}{0.01} \le \frac{X-1}{0.01} \le \frac{0.99-1}{0.01}\right)$$

$$= P(-100 \le Z \le -1)$$

$$= P(Z \le -1) - P(Z \le -100)$$

$$= 0.1587 - 0$$

$$= 0.1587$$

(b)

Eksamen desember 2010

2. $A \sim N(36.80, 0.25^2)$

$B \sim N(37.00, 0.25^2)$

C = Skøytebaner A har siste indre sving

$C \sim N(37, 0.25^2)$

D = Skøytebaner B har siste indre sving

$D \sim N(37.20, 0.25^2)$

$$P(A > D) = P\left(\frac{A - 36.80}{0.25} > \frac{D - 37.20}{0.25}\right)$$

$$=$$