Exam May 31

1. Which one of the following statements is **not** correct?
    a. Concave utility functions imply that each additional unit of a good gives less utility than its predecessor
    b. The more concave an individual's utility function is, the more risk averse is the individual
    c. The certainty equivalent is the risk premium that persons require to accept the risk of an uncertain outcome instead of a certain one.
    d. The introduction of a financial market that gives the opportunity to move consumption back and forth in time, and where deals are done free of charge, has the consequence that most individuals are better off and nobody worse.
    e. Fisher separation implies that investment and consumption decisions can be separated

2. You believe that a company will be worth 100 Euros per share one year from now. How much should you be willing to pay for one share today if the risk-free rate is 3 %, the expected return on the market is 8 % and the company's Beta is 2.0?
    a. 88.5 Euro
    b. 94.3 Euro
    c. 86.2 Euro
    d. 90.1 Euro
    e. 82.0 Euro

3. Select the option which is TRUE. Regarding models of market efficiency:
    a. In a fair game, the result never deviates from the expected value.
    b. In a Martingale the properly discounted future values are equal to the present value.
    c. Random walk processes have the property of memory preservation.
    d. Random walk, Martingale and fair game, all consider only the expected result.
    e. None of the above.

4. Select the option which is TRUE.
    a. The efficient market hypothesis states that the market price is always right.
    b. Large fluctuation in stock prices means that the market is inefficient.
    c. If the markets are efficient, then all stocks are similar in regard to profit opportunities.
    d. People that make money trading stock prove that the market is inefficient.
    e. None of the above.

5. Those are characteristics of common bonds, except:
    a. Bonds offer no property rights.
    b. Bonds are permanent investments.
    c. Bonds can be market traded.
    d. Bonds can be underlying for derivatives.
    e. Bonds have fixed maximum return.

6. Those are characteristics of common stocks, except:
    a. Stocks may offer property rights.
    b. Stocks are permanent investments.
    c. Stocks can be market traded.
    d. Stocks can be underlying for derivatives.
    e. Stocks have fixed maximum return.

7. Which of the following is NOT a real option?
    a. A stock option
    b. An abandonment option
    c. An investment timing option
    d. An expansion option
    e. A follow-up investment option

8. Which of the following will decrease the value of a call option?
    a. An increase in the time to maturity
    b. An increase in the stock price
    c. An increase in the stock's volatility
    d. An increase in the exercise price
    e. An increase of the interest rate

1. Consider two alternatives, A and B, with the following expected returns and corresponding probability distributions

$$A = \begin{cases} 1 \; with \; prob. = \; 0.5 \\ 9 \; with \; prob. = 0.5 \end{cases}$$

$$B = \begin{cases} 4 \; with \; prob. = \; 0.99 \\ 9 \; with \; prob. = 0.01 \end{cases}$$

You want to choose between the two alternatives.
Determine your choice using:

   i. Mean-Variance efficiency

Solution: Calculate expected return as

E[rA] = 1x0.5 + 9x0.5 = 5
E[rB] = 4x0.99 + 9x0.01 = 4.05

Calculate the variance:

A: $\sigma^2(r_A) = 0.5 \times (1-5)^2 + 0.5 \times (9-5)^2 = 16$
B: $\sigma^2(r_B) = 0.99 \times (4-4.05)^2 + 0.01 \times (9-4.05)^2 = 0.2475$

The standard deviation is

$\sigma(r_A) = 4$
$\sigma(r_B) = 0.4974$

Return pr risk unit:

A: 5/4 =1.25
B: 4.05/0.4974 = 8.14

Alternative B offers the highest return pr risk unit.

Other answers admitted:

- Using the variance:

Return per risk unit:

A: 5/16 =0.3125
B: 4.05/0.2475 = 16.36

Alternative B offers the highest return pr risk unit.

- Considering the risk-taking perspective:

$$CV(r_A) = \frac{\sigma(r_A)}{\mathrm{E}[r_A]} = \frac{4}{5} = 0.2 \text{ and } CV(B) = \frac{\sigma(r_B)}{\mathrm{E}[r_B]} = \frac{0.4974}{4.05} = 0.1228$$

So, the answer changes with changing point of views which is in terms of risk or return.

   ii. Maximizing expected utility, using the utility function $u(x) = \sqrt{x}$

Solution:

$$A: E[u_A] = 0.5 \times \sqrt{1} + 0.5 \times \sqrt{9} = 0.5 + 1.5 = 2.00$$

$$B: E[u_B] = 0.99 \times \sqrt{4} + 0.01 \times \sqrt{9} = 1.98 + 0.03 = 2.01$$

Alternative B yields the highest expected utility, which should be the choice.

This calculation should be corrected as we should use 9 instead of 81.

2. Consider a publicly traded company and a market index that have performed as follows over the past four years:

Year	Price (beginning - end of year)	Dividend	Market index (beginning – end of year	Market dividend
2019	25 – 27	1.00	100 – 105	3.05 %
2020	27 – 29	1.00	105 – 110	3.00 %
2021	29 – 32	1.50	110 – 120	2.95 %
2022	32 – 33	1.50	120 – 125	2.80 %

The relevant risk-free rate over the period is 3 % per year.

(a) From the information given above - calculate the company's Beta and interpret the result.

(b) Year	Company	Market
R_2019	(27-25+ 1)/25 = 0.120=12.00%	105-100)/100 + 3.05% = 8.05%
R_2020	(29-27+1)/27 = 0.111 = 11.11%	110-105)/105 + 3.00% = 7.76%
R_2021	(32-29+1,5)/29 = 0.155 = 15.52%	(120-110)/110 + 2.95% = 12.04 %
R_2022	(33-32+1,5)/32 = 0.078 = 7.81 %	125-120)/120 + 2.80% = 6.97 %
Ave (R)	0.1161	0.0870
St.dev = √∑(xi-x)^2/n-1	0.03167	0.02271
Cov = ∑(xi-x)(yi-y)/n-1	0.0006617	
Beta	1.28 (1.22)	

Average R is here calculated as arithmetic average. (Geometric or logarithmic averages can be accepted)

St.dev and covariance are calculated based sample (population (divide by n could also be accepted) as well as arithmetic return (based on logarithmic return could also be accepted)

Beta based on logarithmic return in parentheses

A security that has a beta higher than 1 should show a large rise in price when there is an upward movement in the market and has a large drop in price in case of a downward movement. These fluctuations may cause a considerable amount of uncertainty about the return of this security, and greater risk associated with it. Therefore, a high beta security is also a high-risk security.

(b) Shares are considered permanent investments and their valuation is commonly based on an infinite stream of dividend payments that are assumed to grow over time. The company expects at end of 2022 to pay a dividend of 2.00 after one year and further expects its dividend growth rate to be 5 % in the future. What should be a fair price of the traded company's share at Dec 31, 2022?

Solution: Cost of equity capital from CAPM = $r_i = r_f + \beta \times (r_m + r_f) = 3\% + 1.28 \times (8.7\% - 3\%) = 10.3\%$

Value of equity under constant growth: $P_0 = \frac{Div_1}{r-g} = \frac{2.00}{10.3\%-5\%} = 37.7$

If Beta and average return is calculated from log returns in (a) this could accepted as ¾ correct (answer is then 42.5), however note in Textbook ch 3, A2 it is argued that an unbiased estimate of the asset's future value is found by compounding the present value forward at the arithmetic average rate of return.

3. The Corporation ABC has a current stock price of 15 per share. The volatility of the returns is 30% and the risk-free interest rate is 4%. Consider an European call option on the stock with a strike price equal to 15 and with maturity of one year.
   (a) Assume that the stock pays no dividends. Calculate the option price and the option delta. Explain the meaning of the option delta.

   Solution: To use the Black-Scholes formula we have to calculate d_1 and d_2.

   $$d_1 = \frac{ln(S_0/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} = 0.2833 \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T} = -0.0167$$

   Then the cumulative probabilities are given by $N(d_1)$ =0.612 and $N(d_2) = 0.493$. Applying the Black-Scholes formula we get

   $$O = SN(d_1) - Xe^{-rT}N(d_2) = 2.075$$

   Note that the result can slightly vary if we round differently the probabilities.

   The option delta is given by $N(d_1)$ =0.612

   The delta represents the sensitivity of the option price for a single unit change in the underlying value S.

   (b) Assume now that the stock pays dividends of 2 after 6 months. Calculate know the new option delta.

Solution: Firstly, we can see that the present value of the paid interest is $2e^{-0.04*1/2} \approx 1.96$. Then the adjusted stock price is S=15-1.96=13.04 and we can proceed as before

$$d_1 = \frac{ln(S_0/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} = -0.183 \text{ and } N(d_1) = 0.427$$

Alternative solution: If one assumes two dividend payments, the present value of the paid interest is $2e^{-0.04*1/2} + 2e^{-0.04*1} \approx 3.88$. Then the adjusted stock price is S=15-3.88=11.12 and we can proceed as before

$$d_1 = \frac{ln(S_0/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} = -0.71 \text{ and } N(d_1) = 0.239$$

4. A given stock can go up by 15% or down by 10% every year. Both outcomes are equally likely. There are no payment of dividends. The yearly risk-free rate is 5% and the current stock price is 100.
   (a) Present the stock price movement in a two-period binomial tree assuming that each period corresponds to one year.

   Solution:

t=0	**t=1**	**t=2**
		132.25
	115	
100		103.5
	90	
		81

   (b) Consider a European put option on the stock. Calculate the risk neutral probabilities and the payoffs for the last period, assuming that the strike price is 100.

   Solution: The risk neutral probabilities are given by

$$p = \frac{1.05-0.9}{1.15-0.9} = 0.6 \qquad \text{and} \qquad 1-p = 1-0.6 = 0.4$$

The payoffs of the put option are given by $max(0, X-S)$. Then we get $O_{dd} = max(0{,}100-81) = 19, O_{du} = max(0{,}100-103.5) = 0$, and $O_{uu} = max(0{,}100 - 132.25) = 0$.

   (c) Calculate the price of a European put option with a strike price of 100 using the binomial valuation model.

   Solution: The option price doing the following calculation:

$$O = \frac{0.6^2 * 0 + 2 * 0.6 * 0.4 * 0 + 0.4^2 * 19}{1.05^2} = 2.757$$

   There are alternative approaches to get the solution:

   1) Using a replication strategy;
   2) Constructing the binomial tree and solving it in a backward way.