Chapter 7: Option pricing in discrete time - part 1

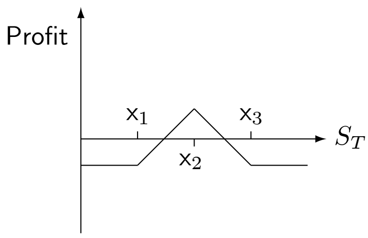
Exercises

1. You have done some option trading and you now hold the following option contracts: you have written (sold) a put with an exercise price of 75, you have bought a put with an exercise price of 100, you have bought a call with an exercise price of 75 and you have written a call with an exercise price of 100. All options are on the same underlying share and have the same maturity.

   (a) Construct the payoff diagram (not the profit diagram) for your total position over the share price interval from 0 to 150.

2. It is sometimes said that the prices of at-the-money puts and calls on the same underlying and with the same maturity have to be the same, since simultaneously buying and selling cancels out. Is this reasoning correct? Assume European options on non dividend paying stocks.

3. The option position depicted in the figure below is known as a butterfly spread.



   (a) Work out a combination of call options that produces the butterfly spread in the figure above.

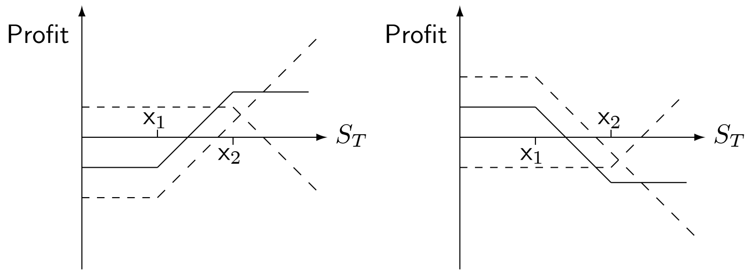
   (b) Work out a combination of put options that produces the butterfly spread in the figure above.

4. On November 1^{th}, 3 months European call options on a share ZXco with an exercise price of 460 cost 20.75. The same options with an exercise price of 480 resp. 500 cost 11.75 resp. 6. The share price of ZXco is 462.5 and the 3 months risk free interest rate is 1.5%.

   (a) Calculate the initial investment that is required to set up a butterfly spread.

   (b) Show that the same butterfly spread set up with put options requires the same initial investment.

5. The figure below repeats the profit diagrams for option positions known as spreads that was used in the main text.



(a) Bull spread (b) Bear spread

(a) Construct the spreads in this figure using put options.

(b) What happens to the initial investment (initial balance of the option premiums) required to set up the position, compared to the same position in calls? Hint: look at the prices of options in relation to the exercise prices.

6. The numerical example of state-preference theory in the book uses the following payoff matrix Ψ:

$$\Psi = \begin{bmatrix} 4 & 1 & 2 \\ 5 & 7 & 4 \\ 6 & 10 & 16 \end{bmatrix}$$

for 3 investment opportunities Y_1, Y_2, Y_3 (columns) in 3 future states of the world (rows). For example, Y_1 pays off 4 in state 1, 5 in state 2 and 6 in state 3. The values of Y_1, Y_2, Y_3 are calculated as:

$$v = \begin{bmatrix} 4.5 & 5.25 & 5.5 \end{bmatrix}$$

the state prices are calculated as:

$$v\Psi^{-1} = \begin{bmatrix} 0.298 & 0.419 & 0.202 \end{bmatrix}$$

and the risk free interest rate as 8.8%.

(a) Calculate the equivalent martingale probability for each of the states.

(b) Demonstrate that the values of Y_1, Y_2, Y_3 can be reproduced using the risk neutral valuation formula.

(c) Demonstrate that return equalization obtains under the equivalent martingale probability measure.

(d) Demonstrate that the martingale property obtains under the equivalent martingale probability measure.