Chapter 3: Modern Portfolio Theory - part 2

Exercises - solutions

1. (a) $r = .05 + 2.1(.15 - .05) = 0.26$

   (b)

	Market value	β	r
ZXco before	4.25	2.1	.26
new investment	4.25	0	.05
ZXco after	8.5	1.05	.155

   (c) It doesn't matter where the money comes from, only where it goes to. A risk free investment is just as valuable to the riskiest company on the stock exchange as it is to the safest company.

2. (a) The CAPM predicts that $\alpha = 0$; it does not predict any specific value for β but as a relative measure βs are usually in the range 0.25 to 2.0. Figure 1.3 in the book plots the β values for Nasdaq-100 companies.

   (b) The test statistic for a regression coefficient is the estimated value divided by its standard error. This statistic has a t-distribution and its critical values depend on the number of observations. You used 3 years of daily return data, i.e. several hundreds of observations, so that the difference between the t-distribution and the normal distribution is small. Using a 5% significance level from the normal distribution, the critical level is 1.96 (two-sided test). Without a formal test, a practical rule of thumb is that the t-value should be larger than two. For $\alpha: \; t_\alpha = 0.025/0.018 = 1.389$, i.e. below 2 so α is not significantly different from 0, as the CAPM predicts. For $\beta : t_\beta = 1.46/0.084 = 17.38$ far above 2, so β is significantly different from 0.

   (c) The β of the market portfolio is 1, so this stock is riskier than the market as a whole.

3. The following estimates and statistics can be produced with excel or with any statistical package, e.g. SPSS.

   (a) The following descriptive statistics, which can easily be calculated from the data in the spreadsheet, are needed later:

	Mean $\overline{r}_i$	St. dev. $\widehat{\sigma}_i$
AMZN	0.118	1.734
CSCO	0.191	1.728
LOGI	-0.077	2.516
IXIC	0.096	0.891

   The characteristic line is obtained by a time series regression of the stock's risk premium on the market risk premium (approximated by the index). We have a constant risk free rate, so we run regressions of stock returns on market returns. For each stock, we estimate the coefficients of $r_{it} = \widehat{\alpha}_i + \widehat{\beta}_i r_{mt} + \widehat{\varepsilon}_{it}$ where r_{it} are the returns of stock i in period t, r_{mt} are the returns on the index in period t, $\widehat{\alpha}_i$

and $\widehat{\beta}_i$ are coefficients to be estimated and ε_{it} is the error term. The results are:

	$\widehat{\alpha}$	$t(\widehat{\alpha})$	$\widehat{\beta}$	$t(\widehat{\beta})$	R^2
AMZN	0.01	0.16	1.08	10.47	.30
CSCO	0.09	0.98	1.04	9.99	.28
LOGI	-0.17	-1.11	0.95	5.62	.11

(b) The average risk free rate per day, $\overline{r}_f$, is approximately $\sqrt[252]{1.005} = 1$ so 0%. The Sharpe ratio is $(\overline{r}_i - \overline{r}_f)/\widehat{\sigma}_i$, so for:

AMZN
$$\frac{0.118 - 0}{1.734} = 0.068$$

CSCO
$$\frac{0.191 - 0}{1.728} = 0.111$$

LOGI
$$\frac{-0.077 - 0}{2.516} = -0.031$$

(c) The Treynor ratio is $(\overline{r}_i - \overline{r}_f)/\widehat{\beta}_i$, so for:

AMZN
$$\frac{0.118 - 0}{1.079} = 0.109$$

CSCO
$$\frac{0.191 - 0}{1.040} = 0.184$$

LOGI
$$\frac{-0.077 - 0}{0.951} = -0.081$$

(d) Jensen's alpha is $\overline{r}_i - (\overline{r}_f + \widehat{\beta}_i(\overline{r}_m - \overline{r}_f))$ so for:

AMZN
$$0.118 - (0 + 1.079 \times (0.096 - 0)) = 0.014$$

CSCO
$$0.191 - (0 + 1.04 \times (0.096 - 0)) = 0.091$$

LOGI
$$-0.077 - (0 + 0.951(0.096 - 0)) = -0.168$$

4. (a) Given the sensitivities of .75 and .7 we can calculate the portfolio return: $.075 + .75 \times .06 + .7 \times .03 = 0.141$ Together with the sensitivities we now have three equations for the three unknown portfolio weights:

$$\begin{aligned} .141 &= x_1 \times .18 + x_2 \times .15 + x_3 \times .12 \\ .75 &= x_1 \times 1.5 + x_2 \times .5 + x_3 \times .6 \\ .7 &= x_1 \times .5 + x_2 \times 1.5 + x_3 \times .3 \end{aligned}$$

The solution to this system is: $x_1 = 0.2$, $x_2 = 0.3$, $x_3 = 0.5$.

(b) The pure factor portfolios are found in the same way, because we know their sensitivities and returns:

$$\begin{aligned} .135 &= x_1 \times .18 + x_2 \times .15 + x_3 \times .12 \\ 1 &= x_1 \times 1.5 + x_2 \times .5 + x_3 \times .6 \\ 0 &= x_1 \times .5 + x_2 \times 1.5 + x_3 \times .3 \end{aligned}$$

Solution is: $[x_1 = 0.41,\ x_2 = -0.32,\ x_3 = 0.91]$

$$\begin{aligned} .105 &= x_1 \times .18 + x_2 \times .15 + x_3 \times .12 \\ 0 &= x_1 \times 1.5 + x_2 \times .5 + x_3 \times .6 \\ 1 &= x_1 \times .5 + x_2 \times 1.5 + x_3 \times .3 \end{aligned}$$

Solution is: $[x_1 = -0.590\,91, x_2 = 0.681\,82, x_3 = 0.909\,09]$

5. (a) The sensitivities b_1 and b_2 can be obtained by running a time series regression (first pass regression) of each fund's risk premium on appropriate indices of the the risk premiums of the of oil and fish industry:

$$r_{it} - r_{f\,t} = \widehat{\alpha}_i + \widehat{b}_{1i}(I_{oil,t} - r_{ft}) + \widehat{b}_{2i}(I_{fish,t} - r_{ft}) + \widehat{\varepsilon}_{it}$$

where r_{it} is the return of fund i in period t, r_{ft} is the risk free interest rate, $\widehat{b}_{1i}$ and $\widehat{b}_{2i}$ are the estimated sensitivities of fund i, I is the return index of the industry in the subscript, and $\widehat{\varepsilon}_{it}$ is a disturbance term.

(b) The risk free interest rate and the risk premiums are found by solving the 3 APT pricing relations $E(r_i) = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2}$ for the 3 unknowns lambda's:

$$\begin{aligned} \lambda_0 + \lambda_1 \times 1.2 + \lambda_2 \times 0.8 &= 0.172 \\ \lambda_0 + \lambda_1 \times 0.9 + \lambda_2 \times 1.4 &= 0.208 \\ \lambda_0 + \lambda_1 \times 0.5 + \lambda_2 \times 0.7 &= 0.136 \end{aligned}$$

which gives $\lambda_0 = 0.06,\ \lambda_1 = 0.04$, and $\lambda_2 = 0.08$. So the risk free interest rate is 6%, the risk premium of the oil industry is 4% and the risk premium of the fish industry is 8%.

(c) We can calculate the return of such a portfolio: $r_p = 0.06{+}1{\times}0.04{+}1{\times}0.08 = 0.18$ This must also be the weighted average return of your investment in the three mutual funds. Also, your portfolio's sensitivities for the risk factors must be the weighted average of the mutual funds' sensitivities. This gives us three equations that we can solve for the three unknown weights:

$$\begin{aligned} x_1 \times 0.172 + x_2 \times 0.208 + x_3 \times 0.136 &= 0.18 \\ x_1 \times 1.2 + x_2 \times 0.9 + x_3 \times 0.5 &= 1 \\ x_1 \times 0.8 + x_2 \times 1.4 + x_3 \times 0.7 &= 1 \end{aligned}$$

The solution is: $x_1 = 0.511\,11, x_2 = 0.355\,56$, and $x_3 = 0.133\,33$