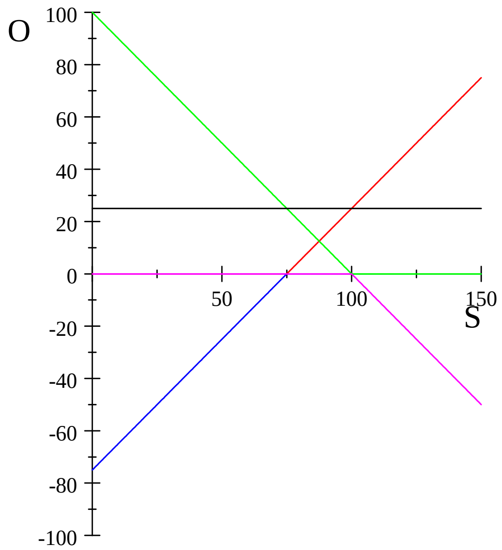
Chapter 7: Option pricing in discrete time - part 1

Exercises - solutions

1. (a) It may be helpful to calculate the options' pay-offs on different points in the interval:

Share price (S)	0	50	75	100	125	150
short put (X=75)	- 75	- 25	0	0	0	0
long put (X=100)	100	50	25	0	0	0
long call (X=75)	0	0	0	25	50	75
short call (X=100)	0	0	0	0	- 25	- 50
Total position (O)	+ 25	+ 25	+ 25	+ 25	+ 25	+ 25

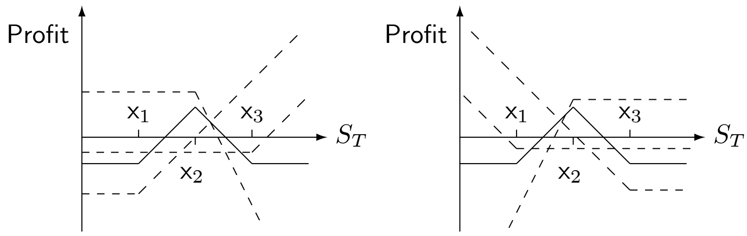
The positions are plotted in the figure below; the plotted functions are $\min[0, x - 75]$, $\max[0, 100 - x]$, $\max[0, x - 75]$, $\min[0, 100 - x]$ and their sum.



2. No, puts and calls do not cancel out, but buying a put is cancelled out by selling a put. Similarly, a short call is cancelled out by a long call. Also, the prices of otherwise identical at-the-money puts and calls are not the same, as is easily verified with the put-call parity: share + long put = long call + PV(X) or long put - long call = PV(X) - share. If the put and call have the same value, the share price has to be equal to the present value of the exercise price (so not the exercise price).

3. (a) A butterfly with calls consists of 1 long call with exercise price x1, 2 short calls with exercise price x2 and 1 long call with exercise price x3.

   (b) The same position with puts consists of 1 long put with exercise price x3, 2 short puts with exercise price x2 and 1 long put with exercise price x1. The positions are depicted in Figure 1 below.



(a) Using calls (b) Using puts

Figure 1: Profit diagrams for a butterfly spread

4. A butterfly spread involves buying 1 call with a low exercise price, selling two calls with a higher exercise price and buying 1 call with an even higher exercise price.

   (a) The initial investment required is:

	strike	price	amount
buy 1	460	20.75	-20.75
sell 2	480	11.75	23.50
buy 1	500	6.00	-6
total			-3.25

   (b) We first have to calculate the put prices using the put-call parity:

call	+PV(X)	-share	=price put
20.75	460/1.015=453.2	-462.50	=11.45
11.75	480/1.015=472.9	-462.50	=22.15
6	500/1.015=492.6	-462.50	=36.10

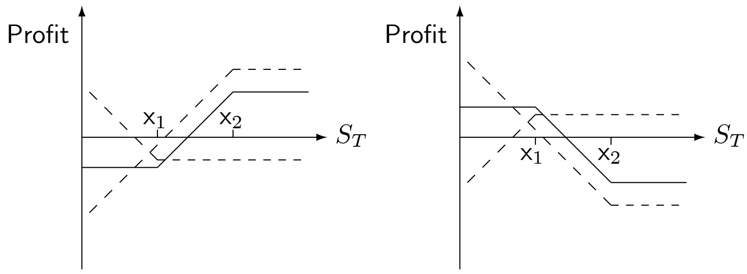
   Then we can set up the butterfly with puts:

	strike	price	amount
buy 1	500	36.10	-36.10
sell 2	480	22.15	44.30
buy 1	460	11.45	-11.45
total			-3.25

   which gives the same initial investment.

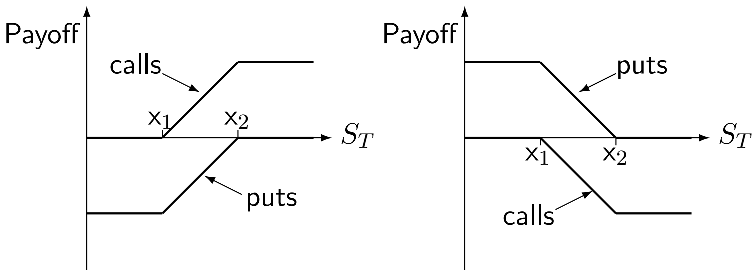
5. (a) The bull spread is constructed by selling 1 put with exercise price x2 and buying 1 put with exercise price x1. In the bear spread the positions are reversed, i.e. a put with exercise prise x1 is sold and a put with exercise price x2 is bought. The positions are depicted in Figure 2 below

   (b) The initial balances of option premiums (initial investments) are reversed compared with the same positions constructed with calls. The price of a put increases with the exercise price (call prices decrease). In a bull spread constructed with puts, the more expensive put with a high exercise price is sold and the cheaper one is bought so that the initial balance is positive (negative investment). On the other hand, the payoff at maturity is either zero or negative. In a bear spread constructed with puts the more expensive option is bought so that the initial balance is negative (initial investment required). However, the payoff at maturity is either zero or positive.



(a) Bull spread (b) Bear spread

Figure 2: Profit diagrams for spreads



(a) Bull spread (b) Bear spread

Figure 3: Payoff diagrams of spreads

The pay-offs at maturity at depicted in the payoff diagram below. The differences are summarized in the table below.

	Bull	**Bear**
Calls	buy low X (costs more)	sell low X (costs more)
decrease	sell high X (costs less)	buy high X (costs less)
with X	price < 0	price > 0
	payoff at maturity > 0	payoff at maturity < 0
Puts	buy low X (costs less)	sell low X (costs less)
increase	sell high X (costs more)	buy high X (costs more)
with X	price > 0	price < 0
	payoff at maturity < 0	payoff at maturity > 0

6. (a) Equivalent martingale probabilities can be calculated as standardized state prices: $0.298 + 0.419 + 0.202 = 0.919$ and

$$\frac{0.298}{0.919} = 0.324 \quad \frac{0.419}{0.919} = 0.456 \quad \frac{0.202}{0.919} = 0.220$$

Alternatively, we can use the definition in equation (7.8) in the book where they are calculated as compounded state prices: $p_i = \psi_i(1 + r_f)$ and:
$0.298 \times 1.088 = 0.324$
$0.419 \times 1.088 = 0.456$
$0.202 \times 1.088 = 0.220$

(b) Using the risk neutral valuation formula the value of Y_1 can be calculated as:

$$\frac{0.324 \times 4 + 0.456 \times 5 + 0.22 \times 6}{1.088} = 4.5$$

and that of Y_2 as:

$$\frac{0.324 \times 1 + 0.456 \times 7 + 0.22 \times 10}{1.088} = 5.25$$

and that of Y_3 as:

$$\frac{0.324 \times 2 + 0.456 \times 4 + 0.22 \times 16}{1.088} = 5.5$$

(c) Under the equivalent martingale probability measure the expected value of Y_1 at $t = 1$ is $0.324 \times 4 + 0.456 \times 5 + 0.22 \times 6 = 4.896$ so that its expected return is: $4.896/4.5 = 1.088$. For Y_2 the expected value at $t = 1$ is $0.324 \times 1 + 0.456 \times 7 + 0.22 \times 10 = 5.716$ so that its expected return is: $5.716/5.25 = 1.0888$ and for Y_3 the expected value at $t = 1$ is $0.324 \times 2 + 0.456 \times 4 + 0.22 \times 16 = 5.992$ so that its expected return is: $5.992/5.5 = 1.0895$. We see that the returns are equal, except for some rounding. The risk adjusted returns are 10%, 13% and 16%, respectively.

(d) The martingale property means that the properly discounted expected future values equal the present values. Under the equivalent martingale probability measure the proper discount rate is the risk free interest rate, so the martingale property follows from the calculations under (b) and (c): $4.896/1.088 = 4.5$, $5.716/1.088 = 5.2537$ and $5.992/1.088 = 5.5074$