Chapter 7: Option pricing in discrete time - part 2

Exercises

1. On June 23, shares of ZX Co. were traded in Frankfurt for 97.70. Call options on this share with a time to maturity of one year and a strike price of 80 were traded for 23.20. Puts on the same stock with the same time to maturity and the same strike price traded for 4.16. The relevant interest rate for the options' lifetime was 3.15%.

   (a) Discuss the arbitrage opportunities on this market from a pricing point of view and from a practical point of view. Explicitly mention the assumptions on which arbitrage opportunities, or the lack thereof, rest.

2. 'Equivalent martingale' or 'risk neutral' probabilities play a very important role in derivatives pricing, so it is essential to have a good understanding of them. Below are a few statements about risk neutral probabilities. Explain for each statement why it is correct or incorrect.

   (a) Risk neutral probabilities are an alternative way of describing the likelihood that an event will happen.
   (b) The risk neutral probabilities are independent of the corresponding real probabilities.
   (c) Using risk neutral probabilities and discounting with the risk free interest rate means implicitly assuming that investors are risk neutral.
   (d) The risk neutral probabilities have the same values as the real probabilities would have if investors were risk neutral.
   (e) Risk neutral probabilities should be interpreted as prices, rather than probabilities.

3. In a financial market a stock is traded with a current price of 100. The price of the stock can either go up with 25% next period or go down with 20% next period. Risk free debt is available with an interest rate of 7%. Also traded are European options on the stock with an exercise price of 110 and a time to maturity of 2 (i.e. the option matures at end of the second period on the third moment).

   (a) Discuss the completeness and the arbitrage properties of this market
   (b) Calculate the value of the option assuming that:
      i. The stock pays no dividend
      ii. The stock pays out 25% of its value in dividends on the second moment (end of the first period)
      iii. Repeat step b(i) for an American call option
      iv. Repeat step b(ii) for an American call option
      v. Repeat step b(i) for a European put option
      vi. Repeat step b(ii) for a European put option

4. In a financial market a stock is traded with a current price of 50. Next period the price of the stock can either go up with 30% or go down with 25%. Risk free debt is available with an interest rate of 8%. Also traded are European options on the stock with an exercise price of 45 and a time to maturity of 1, i.e. they mature next period.

   (a) Calculate the price of a call option by constructing and pricing a replicating portfolio.

   (b) Calculate the price of a put option by constructing and pricing a replicating portfolio.

5. On page 202 of the book, a self-financing strategy is defined as a strategy that requires no extra cash along the way, i.e. all additional outlays must be part of the strategy. On page 212-213, the dynamic hedging portfolio of the two-period example is shown to give a perfect hedge with a self-financing strategy. Demonstrate that the dynamic hedge in this example is indeed self-financing. Hint: think of what self-financing means for rebalancing the portfolio.