Chapter 3: Modern Portfolio Theory - part 1

Exercises

1. Stock A has a return of 10% and a standard deviation of 15%. Stock B has a return of 14% and a standard deviation of 20%. Their correlation coefficient is -1. Which combination of A and B gives the lowest variance? How high is that variance?

2. Mrs Pennymoney has invested 50% of her money in a risk free bank account and 50% in an index fund that invests in all shares on the stock exchange. The risk free interest rate is 5% and the expected return on the market index is 15% with a standard deviation of 20%.

   (a) Is Mrs Pennymoney's portfolio mean-variance efficient?
   (b) What is the expected return on the portfolio?
   (c) What is the portfolio standard deviation?
   (d) What is the portfolio β (relative to the market as a whole)?

3. Mr Poundmoney has invested 150% of his money in an index fund that invests in all shares on the stock exchange. The risk free interest rate is 5% and all investors can borrow and lend unlimited amounts at this rate. The expected return on the market index is 15% with a standard deviation of 20%.

   (a) How can Mr Poundmoney invest more than 100% of his money in the index fund?
   (b) Is Mr Poundmoney's portfolio mean-variance efficient?
   (c) What is the expected return on the portfolio?
   (d) What is the portfolio standard deviation?
   (e) What is the portfolio β (relative to the market as a whole)?

4. Ms Stonemoney has invested 50% of her money in ADC (Aker Dissolution Company) and 50% in NAC (Northern Aluminium Company). ADC has an expected return of 20% and a standard deviation of 40% and NAC has an expected return of 12.5% and a standard deviation of 25%. A statistical analysis of the returns in the dissolution and the aluminium industries over the period 2004-2007 (which was a period of stable growth) has shown that the two industries are statistically independent (i.e. uncorrelated - correlation coefficient is 0).

   (a) Is Ms Stonemoney's portfolio mean-variance efficient?
   (b) The correlation coefficient between the returns in the dissolution and the aluminium industries was measured over a period of stable growth. How does this correlation coefficient change in times of financial crises, such as the 2007-2008 credit crunch?
   (c) What is the expected return on the portfolio?
   (d) What is the portfolio standard deviation?

5. You decide to invest your money in a stock portfolio consisting of 60% Cisco Systems and 40% Amazon.com. Using the data in Table 3.7 in the book you find that Cisco has an annual standard deviation of 0.363 and Amazon of 0.34. The correlation coefficient between the returns of both stocks is 0.34

   (a) Calculate the variance and standard deviation of this portfolio.

   (b) Calculate the relative contribution of each stock to this portfolio's variance.

   (c) Calculate the β of each stock relative to this two-stock portfolio. Check you results.

   (d) Now calculate portfolio risk (standard deviation) and return using some different values for the weights and plot the results in risk-return space. Use the returns in Table 3.7 in the book (Cisco 7.5%, Amazon 12.5%)

   (e) The graph you plotted under (d) gives you a good idea what the minimum variance portfolio looks like, but can you calculate its properties exactly? What weights give the portfolio its minimum variance? What are this portfolio's standard deviation and return?

   (f) How would the graph you plotted under (d) look if Cisco and Amazon were perfectly positively correlated?

6. Recall your uncle Bob's portfolio problem in Tables 3.7 and 3.8. On the next birthday party, uncle Bob boasts that he has optimized his portfolio of 5 stocks and he demonstrates the calculation of the minimum variance portfolio using your spreadsheet. As Table 3.8 shows, the minimum variance portfolio has an expected return of 9.2% and a standard deviation of 23.2%. Your aunt Agatha is so impressed by the demonstration that she wants to invest her money in this minimum variance portfolio. You have collected some additional data about the risk free interest rate, which is 2.5%, and the expected return and risk of a broad index that represents the stock market as a whole. There are several index funds that follow this index very closely; they have an expected return of 12% and a standard deviation of 24%.

   (a) Taking your additional information into account, is aunt Agatha's proposed portfolio mean-variance efficient?

   (b) If not, how much extra return can she get with a portfolio that has the same standard deviation as the minimum variance portfolio in Table 3.8? What is the composition of that portfolio?

   (c) Similarly, how much risk can she avoid with a portfolio that has the same return as the minimum variance portfolio in Table 3.8? What is the composition of that portfolio?

7. After being rebuked by his wife for spending too much time on his stock portfolio, uncle Bob considers reducing his portfolio to only two stocks, 50% in his familiar Logitech and 50% in one of the other four stocks in Table 3.7. Which of the four stocks should he choose if he wants to:

   (a) maximize the expected return of his portfolio?

   (b) minimize the risk (standard deviation) of his portfolio?