Exericse 1

(1) Simulate and plot 50 timesteps of the following model,

$$x_{t+1} = w_t + w_{t-1}, \quad x_1 = 0, \quad w_t \sim \mathcal{N}(0,1) \text{ are iid.}$$

Plot the theoretical and sample autocorrelation functions of $(x_t)_{t\geq 1}$. Using Property 1.2 in the book, assess the peaks in the sample autocorrelation. Repeat this process for larger samples such as 1000 and 10000. Derive and plot 95% confidence intervals for x_t. Check numerically if x_t appears stationary. What does the initial condition x_1 have to be in order for x_t to be (a snippet of) a stationary process $\{x_t\}_{t\in\mathbb{Z}}$?

(2) Simulate and plot 50 timesteps of the following model,

$$x_{t+1} = 0.5x_t + w_t, \quad x_1 = 0, \quad w_t \sim \mathcal{N}(0,1) \text{ are iid.}$$

Plot the theoretical and sample autocorrelation functions of $(x_t)_{t\geq 1}$. Derive and plot 95% confidence intervals for x_t. Check numerically if x_t is stationary. What does the initial condition x_1 have to be in order for x_t to be (a snippet of) a stationary process $\{x_t\}_{t\in\mathbb{Z}}$?

(3) Problem 1.6 in the textbook.

(4) Problem 1.8 in the textbook.

(5) Sample data from the following bivariate time series. Compute the sample autocorrelations and the cross-correlation for x and y and plot along with expected intervals from Properties 1.2 and 1.3. Consider these results and compare with the prominent peaks of the autocorrelations in univariate cases above.

$$x_{t+1} = 0.5x_t + 0.3y_t + w_t, \quad x_1 = 0, \quad w_t \sim \mathcal{N}(0,1) \text{ are iid.}$$
$$y_{t+1} = 0.2x_{t-5} + 0.4y_t + z_t, \quad x_1 = 0, \quad z_t \sim \mathcal{N}(0,1) \text{ are iid.}$$

(6) Let $X = (X_1, \ldots, X_n)$ be an $\mathbb{R}^n$-valued random variable, and Y be an $\mathbb{R}$-valued random variable. Express the best linear predictor of Y as MX, where $M \in \mathbb{R}^{1\times n}$ is a matrix.