TIØ4146 Finance for Science and Technology Students

Chapter 9 - Real Options Analysis (part 1)

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Investment opportunities as options

The option to defer

Real Options Analysis

The essential economic characteristic of options is:

the flexibility to exercise or not

- possibility to choose best alternative
- walk away from bad outcomes

Investments in real assets have flexibility - projects can be:

- delayed or speeded up
- made bigger or smaller
- abandoned early or extended beyond original life-time, etc.

- Real options are options where underlying value is a real asset
- not a financial asset as stock, bond, currency

Flexibility in real investments means:

- changing cash flows along the way:
- profiting from opportunities, cutting off losses

Discounted cash flow (DCF) calculation cannot handle flexibility:

- assumes passive, not flexible, position
- accepts cash flows as they come

Real Options vs Financial Options

- Financial option: the option' owner has the right but not the obligation to buy or sell depending on the option a certain underlying.
- Real option: the company has right, but not obligation, to undertake a new project.

Determinant	Stock option	Real option
underlying	stock	project revenue
strike	exercise price	investment
time to maturity	maturity	license validity
volatility	stock σ	price volatility
interest rate	r_f	r_f

Some examples:

Call options	Put options
delay	default
expand	contract
extend	abandon
re-open	shut down

Challenges

Option character of real options can be limited:

- Real options can be less clearly defined:
  - underlying value may be project in planning
  - no clear time to maturity
  - no clear exercise price
- Input data may be difficult to obtain
  - underlying not traded - value?
  - volatility of underlying even more difficult
  - exercise possibilities may be unclear
- Underlying project may be difficult to replicate

We look away from practical obstacles

- assume real options can be valued as financial ones
  - assume market is, at least locally, complete
  - and prices are arbitrage free
  - means projects and their options can be replicated

The option to defer

- A very common real option
- usually discussed as possibility to postpone a project
- but notice proper option formulation:
  - is NOT decision to postpone accepted project
  - but postponement of decision to accept a project until more information is available

So valuing the option to defer means

- *valuing the opportunity to do a project!*
- This potentially includes the value of the project.

Example

Re-use our old example and re-define it as real option

- Project is an oil well to be taken in production
- Size is accurately measured, only price uncertainty
- Values represent future production from the well, properly discounted (not cash flows)
- Value depends on oil price
- develops over time as binomial lattice
- after two periods price uncertainty is resolved
  $\Rightarrow$ stable values

$$\begin{tikzcd}[row sep=small]
 & & 625 \\
 & 500 \arrow[ur] \arrow[dr] & \\
400 \arrow[ur] \arrow[dr] & & 400 \\
 & 320 \arrow[ur] \arrow[dr] & \\
 & & 256 \\
\textit{t=0} & \textit{t=1} & \textit{t=2}
\end{tikzcd}$$

Binomial tree for the value of an oil well

Further details:

- risk free interest rate is 7%.
- Real probability of upward movement is 80%,
- Risk adjusted discount rate for oil production from wells like this is 16%.
- Investment necessary to bring well into production is 375, amount increases with the risk free rate over time
- Investment is irreversible
- Production profitable from the start: 400 - 375 = 25, a positive net present value.
- Firm has exclusive one-period license to develop the well
  $\Rightarrow$ can defer decision to develop with one period

License gives firm a real option:

- has the right, but no obligation, to develop the well
- firm has flexibility, or future decision making opportunity, to profit from real option

Project has the ingredients that make options valuable:

- **Time**
  without time, becomes now-or-never decision as in DCF
- **Uncertainty**
  without uncertainty, situation in 1 year exactly same as today

Reformulate project as option:

- License gives the right to 'buy' the oil in the well
  - $\Rightarrow$ option is a call
- by paying development costs
  - $\Rightarrow$ exercise price = 375.
- License expires in 1 year
  - $\Rightarrow$ option has a time to maturity of 1 year
- Volatility follows from binomial process
- interest rate = 7%

All ingredients necessary to value the option

The parameters of binomial process are:

$$u = 1.25, \quad d = .8, \quad r = 1.07$$

$$p = \frac{r-d}{u-d} = \frac{1.07 - .8}{1.25 - .8} = .6 \quad (1-p) = .4$$

Begin with values at maturity, i.e. at *t = 1*

- option value is $max[0, S - X]$
- exercise price is $375 \times 1.07 = 401.25$
  - $O_u = \max[0, 500 - 401.25] = 98.75$
  - $O_d = \max[0, 320 - 401.25] = 0.0$

$$
\begin{array}{ccc}
 & & \max[0, 500 - 401.25] = 98.75 \\
 & \nearrow & \\
55.374 & & \\
 & \searrow & \\
 & & \max[0, 320 - 401.25] = 0.0 \\
t=0 & & t=1
\end{array}
$$

Value of the option is:

$$O = \frac{.6 \times 98.75 + .4 \times 0}{1.07} = 55.374$$

How can option to defer be more valuable than project itself?

- Project very profitable in up node but loss making in down node
- both included in t_0 value of 25.

Real option analysis values the flexibility to avoid losses in down node:

- wait a period, see how value develops:
  - If oil price goes up, develop well and profit from opportunity
  - If oil price goes down, do not develop and avoid loss
- Don't lose much by waiting one period
  - oil is still in well
  - license still valid

That is essence of real option valuation!

What is wrong with DCF valuation of flexibility?

It can be argued:

- DCF can capture value of flexibility
- by calculating NPV as if project started one period later
- decide to abandon project if NPV is negative
- gives same 98.75 in upper node and 0 in lower node

Result is a **decision tree**
Decision trees are analysed by:

- weighting branches with real probabilities of .8 and .2
- discounting expected value at risk adjusted discount rate of 16%

$$
68.10 \quad
\begin{array}{l}
\nearrow \; NPV = 500 - 401.25 = 98.75 \Rightarrow \text{accept project} \\[2em]
\searrow \; NPV = 320 - 401.25 = -81.25 \Rightarrow \text{refuse project}
\end{array}
$$

t=0 t=1

Value of the project opportunity is:

$$
\frac{.8 \times 98.75 + .2 \times 0}{1.16} = 68.103
$$

Different from 55.374 we found with real options analysis!

1. How can we determine which value is correct?
2. Answer in modern finance: *by making a replicating portfolio*
3. Assume payoff structure in tree can be constructed in market
   - here we need assumption of locally complete market

   Option's delta and D are:

$$\Delta = \frac{O_u - O_d}{(u-d)S} = \frac{98.75 - 0}{500 - 320} = 0.5486 \text{ and}$$

$$D = \frac{uO_d - dO_u}{(u-d)r} = \frac{1.25 \times 0 - .8 \times 98.75}{1.25 \times 1.07 - .8 \times 1.07} = -164.07$$

 NTNU

At time t_1 portfolio pays off either:

$$(0.5486 \times 500) - 164.07 \times 1.07 = 98.745 \text{ or}$$
$$(0.5486 \times 320) - 164.07 \times 1.07 = 0$$

So payoff pattern is replicated.
Value of portfolio now is

$$(0.5486 \times 400) - 164.07 = 55.37$$

Real option value correct:

- no rational investor pays 68.10
- for payoff pattern that can be replicated for 55.37

Where does discounted cash flow approach go wrong?

- The error we made was:
  - applying risk adjusted discount rate for oil production from the well
  - to the opportunity to develop the well.
- The opportunity to do a project seldom has same risk as project itself,
- precisely because flexibility embedded in opportunity is used to change the risk:
  - upward potential is enhanced
  - downside risk is reduced

Once well is in production, then:

- fortunes tied to oil price
- proper discount rate for cfl's and values is 16%
- Values move through time with uncertainty of
  - up factor of 1.25
  - down factor of 0.8

Opportunity to do project has much larger uncertainty

- moves through time from 25 at t_0
  - to either 98.75 $\Rightarrow$ up factor 3.95
  - or 0 $\Rightarrow$ down factor 0.

In principle, correct option value can be calculated using:

- real probabilities of 0.8 and 0.2
- risk adjusted discount rate

But that rate must be calculated from replicating portfolio:

- $\Delta S = 0.5486 \times 400 = 219.44$ in twin security S
- risk free loan of -164.07.

Gives weighted average portfolio return of:

$$\frac{219.44}{219.44 - 164.07} \times 0.16 + \frac{-164.07}{219.44 - 164.07} \times 0.07 = 0.427$$

i.e.$> 2.5\times$ discount rate for the project

Discounting exp. payoff with this rate gives correct value:

$$\frac{0.8 \times 98.75 + 0.2 \times 0}{1.427} = 55.36$$

However, with ΔS and D, we already know the option value:

$$O = \Delta S + D = 219.44 - 164.07 = 55.37$$

Boils down to calculating rate *given* option value:

$$55.37 = (0.8 \times 98.75)/r_{adj} \Rightarrow r_{adj} = 1.427$$

Moreover:

- Δ and D likely to differ between nodes
- must be calculated for each node in tree
- makes correct use of decision trees highly impractical