Mock exam - solution

May 2, 2023

Part I - Multiple choice

1. If the annual interest rate is 6%, what is the present value of a perpetual cash flow of 50 that starts 1 year from now?

   (a) 833.33

   (b) 834.54 (c) 837.60

   (d) 937.25

2. If markets are efficient then:

   (a) The market is always right

   (b) There should be no autocorrelation in excess returns

   (c) People cannot quickly get rich on the stock market

   (d) Security prices do not adjust quickly and unbiasedly to new information

3. Bonds that do not regularly pay interest but only give one final payment at maturity are called:

   (a) Income bonds

   (b) Junk bonds

   (c) Zero coupon bonds

   (d) Convertible bonds

4. In a Modigliani-Miller world without taxes:

   (a) The value of the firm increases with leverage

   (b) The return on equity increases with leverage

   (c) The return on debt increases with leverage

(d) None of the above

5. Which of the following is an example of a way in which companies can create value by exploiting real options?

   (a) **Optimally delaying or abandoning projects;**

   (b) Exercising in-the-money real option immediately;

   (c) Acting to take on new projects, even if there is no costs to waiting

   (d) Abandoning good projects in favor of newer projects

6. An unlevered firm expects earnings before interests and taxes of 5 millions. The tax rate is 40% and the market value is 18 millions. The stock has a $\beta = 1$, and the risk free rate is 6%. The risk premium is 9%. Management is considering the use of perpetual debt (the size of the firm would remain constant), but currently there is no debt. Which of the following statements is **True**.

   (a) **The cost of equity is 15%**

   (b) WACC = 12%

   (c) WACC = 13.5%

   (d) The cost of equity is 12%

7. When valuing a European option using the binomial model, an increase in the real world probability that the underlying will go up most likely implies that the current price of the option:

   (a) Decreases

   (b) Depends whether it is a call or a put option

   (c) Increases

   (d) **Remains unchanged**

8. Consider the Black and Scholes model. Which of the following statements is true.

   (a) The price of the European call option increases with the strike price

   (b) The price of the European call option decreases with the stock price

   (c) **The price of the European put option increases with the strike price**

   (d) The price of the European put option increases with the stock price

Part II - Open Questions

1. The current price of Ocean Corporation stock is 6. In each six-month period this stock price can either go up by 2.50 or go down by 2. The stock pays no dividends. The one-year risk-free interest rate is 5%. Consider

   (a) Fill in Ocean stock price movements in a one-period binomial tree below.
   Solution:

Period 0	Period 1	Period 2
		S_{uu} = 11
	S_u = 8.5	
S = 6		S_{ud} = 6,5
	S_d = 4	
		S_{dd} = 2

   (b) Calculate the payoffs for a currently at the money one year call option on the Ocean stock in the last period below.

   Solution: The call is at the money, thus strike price X = 6. The payoffs at the end nodes are given by the formula: max(S–X,0), in which S is the stock price. Hence, O_{uu} = max(11–6,0) = 5, O_{ud} = O_{du} = max(6.5–6,0) = 0.5 and O_{dd} = max(2–6,0) = 0.

   (c) Using the **One-period** Binomial Model, calculate the price of a one-year European put option on the Ocean stock with a strike price of 7. Use the replication procedure.

   Solution: If we designate the European put price by *P (to avoid confusions with point b)), we have that*

t=0	t=1
	P_u = max(7-8.5,0)=0
P	
	P_d = max(7-4,0)=3

We use the replication approach, then P=Δ.S+D, where

$$\Delta = \frac{P_u - P_d}{uS - dS} \ and \ \mathrm{D} = \frac{uP_d - dP_u}{r(u-d)}$$

Now you can either calculate u and d (u= *Su/S and d= Sd/S)* or simply notice that uS=8.5 and dS=4. The formula for D can be multiplied by (S/S) and then in this case you don't need to calculate u and d. Given the annual risk-free interest rate is 0.05, the 6-monthly rate is given by $r = (1 + 0.05)^{\frac{1}{2}} \approx 1.0247$.

$$\Delta = \frac{0-3}{8.5-4} = -0.6667 \ and \ \mathrm{D} = \frac{S}{S} * \frac{uP_d - dP_u}{r(u-d)} = \frac{8.5 * 3 - 4 * 0}{1.0247 * (8.5 - 4)} = 5.53$$

Consequently, P=Δ.S+D = - 0.6667*6+5.53=1.5298

2. Consider the following 8 portfolios with expected return and standard deviation as follows:

	A	B	C	D	E	F	G	H
Expected return (%)	15	12.5	16	20	10	18	17	18
Standard deviation (%)	25	21	29	45	23	35	29	32

(a) Five of these portfolios are efficient, and three are not. Which are inefficient ones, and why?

Solution: Point E has a higher risk and a lower return when compared to point B, making it an inefficient choice if B is available. Point C has a lower return when compared to point G, both having the same level of risk. Finally, point F has a higher risk than point H while providing the same return. So, points C, E and F are all inefficient, having all other portfolios available. Examining the problem through the use of a chart, the blue dots are the efficient portfolios, and the orange dots are the inefficient ones.

(b) Suppose you are prepared to tolerate a standard deviation of 25%. What is the maximum expected return you can achieve if you cannot borrow or lend?

Solution: If we cannot borrow or lend, then we are limited by the selection of portfolios. Selecting portfolio A gives us the level of risk that we are comfortable with and a return of 15.

(c) What is your optimal strategy if you can borrow or lend at 11% and are prepared to tolerate a standard deviation of 25%? What is the maximum expected return that you can achieve with this risk?

Solution: If we can lend money, we can allocate a part of our capital into the most efficient portfolio and the rest in a risk-free lending account.

$$E(r_p) = r_f + \frac{E(r_m) - r_f}{\sigma_m}\sigma_p = 0.11 + \frac{0.18 - 0.11}{0.32}\sigma_p$$

For a standard deviation of 25%, this would give us a return of 16.47.

3. An investment portfolio consisting of stocks *AA* and *BB*. Expected return and standard deviation on the stocks in the coming year are 3% and 7% respectively for *AA*, and 5% and 12% respectively for *BB*. One is also considering whether to add stock *CC* to the portfolio. The asset *CC* with expected return of 5% and standard variation of 1.58%. Additionally we assume that the correlation between the returns of assets *AA* and *BB* is 0.4, the correlation between *AA* and *CC* is -0.3 and the correlation between *BB* and *CC* is 0.2.

(a) If one invests equal amount in stocks *AA* and *BB* and do not invest in *CC*, what are the expected return and standard deviation of the portfolio?

Solution:

Expected portfolio return = $0.5 \times 3\% + 0.5 \times 5\% = 4\%$

Expected portfolio SD = $(0.5^2 \times 7^2 + 0.5^2 \times 12^2 + 2 \times 0.5 \times 0.5 \times 7 \times 12 \times 0.4)^{0.5} = 8.07\%$

(b) If one invests 30%, 40% and 30% of the total portfolio investments in stocks *AA*, *BB* and *CC*, respectively, what will be the expected return and standard deviation of the portfolio?

Solution:

Expected portfolio return = $0.3 \times 3\% + 0.4 \times 5\% + 0.3 \times 5\% = 4.4\%$

Expected portfolio SD = $(0.3^2 \times 7^2 + 0.4^2 \times 12^2 + 0.3^2 \times 1.58^2 + 2 \times 0.3 \times 0.4 \times 7 \times$
$12 \times 0.4 + 2 \times 0.4 \times 0.3 \times 12 \times 1.58 \times 0.2 + 2 \times 0.3 \times 0.3 \times 7 \times 1.58 \times -0.3)^{0.5} = 6\%$

(c) Should one add stock *CC* to the existing portfolio consisting only of stocks *AA* and *BB*? Give two reasons.

Solution: Yes, portfolio risk is reduced while expected return is higher. Moreover, correlation between CC and AA or BB is less than 1.

4. Consider a stock which is traded at a price of 240. the stock has an annual volatility of 25%. Call options on the stock with an an exercise price of 250 and a time to maturity of one year are also traded. The risk-free interest rate is 6 percent. Calculate the price of the option and the correspondent delta.

   Solution: See solution of Exercise 7 of Chapter 8 of the book. The delta for a call option is given $N(d_1)$ and represents the sensitivity of the option price to the underlying value S_0.