Wikipendium

TIO4146: Finance for Science and Technology Students

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Tips

important pages in the book

P 118: Formelen til CAAR

P 154: Trade-off theories

P 174: Formelen til Project decisions's oppgavene

P 254: Option pricing in continuous time

Fundamentals

Time value of money

The value of money decreases as time passes by. Main two reasons:

- Humans are impatient and consumption now is guaranteed, while the circumstances may make consumption impossible in the future. Some human needs (like the need for food) must be fulfilled quickly.
- Unspent values can be invested and thus generate more value. One seed may generate hundreds of new seeds in a year if sown today.

Perpetuity

perpetuity = annuity with infinite number of payments, $PV = \frac{A}{r}$. For perpetuity with growth rate g ($r > g$) is $PV = \frac{A}{r-g}$

Accounting representation

Note that the accounting representation of a firm may be unsuitable to make financial descisions. The first mostly focuses on where values are tied up, while the latter focuses on the actual cash flows.

Depreciation

Fixed assets are normally depreciatied over a their lifetime. This makes sense as their aquirement only converts value from cash to fixed assets - the real loss of value happens as the assets age.

However, when determining whether to proceed with a project, the actual cashflows are more relevant, thus the entire cost is treated as a cash flow at the time of aquirement. The loss of value is reflected in a lower cash flow the opposite way around when the asset is sold.

Do however note that the depreciation is deducted when calculating taxes, and thus affects the cash flow through taxes!

Irrelevant information

The financial statements are required to show all aspects, even those not relevant to a decision, for example money already spent. This is of course irrelevant to an investment decision and should be disregarded.

Changes in capital

Financial statements often include working capital, but not the changes in working capital. These changes are in fact cash flows and must be included when calculating Net Present Value (NPV).

In other words, this capital is tied up in the project and thus does not generate interests (as it would otherwise do).

Utility

Assumptions made:

- People are greedy, so more is always better than less
- Each additional unit gives less utility than the previous
- Peoples preferences are well-behaved; their preferences form a partial order

This results in utility functions $U(W)$, which express the utility given by an amount W of resource (often wealth). Two common utility functions:

1. $U(W) = ln(W)$
2. $U(W) = \alpha + \beta W - \gamma W^2$

Note that W may be negative, although (1) may suggest otherwise. Also, (2) breaks down with large values of W (meaning it does not adhere to the assumptions).

Indifference curve

Given two resources, W_1 and W_2, the indifference curve shows which combinations gives the same utility, in other words the line defined by

$$U(W_1, W_2) = C$$

where C is a constant.

Risk aversion

The utility curves from the above assumptions also leads to risk aversion.

Example

If one expects W to be either 50kr or 150kr with a 50% probability of each, then

$$E(W) = 0.5 \cdot 50 + 0.5 \cdot 150 = 100$$

however, the expected utility assuming $U(W) = ln(W)$ is

$$E(U(W)) = 0.5 \cdot ln(50) + 0.5 \cdot ln(150) = 4.46$$

which is less than a guaranteed $W_1 = E(W) = 100$:

$$U(W_1) = ln(100) = 4.61$$

This follows from the non-linearity of the utility function. The amount W_2 such that

$$U(W_2) = E(U(W)) = 4.46$$

is the certainty equivalent of the risky W (in the above example $W_2 = 86,5$).

The difference $W - W_2 = 100 - 86,5 = 13,5$ is the risk premium.

Coefficients

The risk is dependent on the curvature (second derivative) of the utility function.

Arrow-Pratt absolute risk aversion coefficient:

$$APA(W) = -\frac{U''(W)}{U'(W)}$$

Corresponding relative risk aversion coefficient:

$$APA(W) = -W\frac{U''(W)}{U'(W)}$$

Efficient Market Hypothesis (EMH)

The Efficient Market Hypothesis states that financial markets are informationally efficient.

- Significant underperformance and insignificant overperformance don't contradict the EMH while the underreaction or overreaction to news do.
- The pre-event CAAR drifts do not test the EMH.
- The EMHs predicts that funds cannot consistently outperform the market, i.e. deviations from risk-adjusted expected returns are random.

It comes in three variants: the weak, the semi-strong and strong efficient market hypotheses.

Weak - historical

Market efficiency occurs when **all past price histories** are fully reflected in current prices.

Tests of technical analyses are often weak form if only using perceived patterns in plotted past prices.

Semi-strong - public

Under the semi-strong form of market efficiency current prices fully reflect **all publicly available information**; in addition to price histories this includes financial statements, surveys of investor sentiment, announcemnt, articles in the (financial) press, product-, industry- and macroeconomic data, etc.

Strong - private

The strong form claims that prices on traded assets reflect all information (even **private**), and prices instantly change to reflect any new information. Fund performance is generally considered a test of strong form efficiency, because the models, databases and strategies that funds use are not public information.

Project decisions

Often, you will need to consider a project proposal where company XYZ ventures into a new area of business. To figure out whether or not to go through with a project, you need to figure out whether or not the **NPV**, or Net Present Value, is positive. If $\mathbf{NPV} > 0$, go through with the project. Conversely, if $\mathbf{NPV} < 0$, do not go through with the project. Hence, making project decisions is an exercise of calculating **NPV**.

Typically, projects will cost a certain one-time, up-front amount of money C to be completed, and generate a perpetual yearly revenue of some amount R. If you have these two numbers, you can figure out **NPV** using the formula:

$$\mathbf{NPV} = -C + \frac{R}{\mathbf{WACC}}$$

Now all you have to do is to calculate the **WACC** (Weighted Average Cost of Capital) of the project.

Alternatively, the **APV** (Adjusted present value) could also be used for **NPV**, which is preferred to WACC in the case that the leverage ratio of the leveraged company is constantly changing during the duration of the investment.

Calculating Weighted Average Cost of Capital (WACC)

The formula for calculating Weighted Average Cost of Capital(WACC) is:

$$\mathbf{WACC} = \frac{D}{V} r_D (1 - T_C) + \frac{E}{V} r_E$$

D	Debt in dollars
E	Equity in dollars
V	Total value in dollars (i.e. debt + equity)
r_D	Cost of debt (typically interest rate of loan)
r_E	Cost of equity
T_C	Corporate tax rate

Unfortunately, r_E is impossible to know. Luckily, it can be estimated by looking at previous projects in the same business domain. One way to estimate r_E is to use the Capital Asset Pricing Model, or **CAPM**.

Calculating r_E using Capital Asset Pricing Model (CAPM)

The formula for **CAPM/Security Market Line** is

$$r_E = r_f + \beta(r_m - r_f)$$

r_E	Cost of equity
r_f	Risk-free interest rate
r_m	Market rate

β	Unsystematic risk
$(r_m - r_f)$	Market risk premium

the difference of SML (Security Market Line) and CML (Capital Market Line) lies in that: CML is pricing relation for efficient portfolios; while SML valid for all investments, incl. inefficient portfolios and individual stocks; CML prices all risks while SML only prices the systematic risk.

The formula for **CML/Capital Market Line** is $r_p = r_f + \frac{r_m - r_f}{\sigma_m}\sigma_p$

Levering and unlevering

When doing these project calculations, we need to make sure that we don't mistakenly assume that cost of equity is the same across different leverage degrees. To account for this, we need to un-lever and re-lever as necessary. Here are some formulas that help in doing this.

Miles-Ezzell WACC calculation

When debt of a project will be periodically rebalanced, you can use Miles-Ezzell.

$$WACC = r_a - \frac{D}{V} r_D T_C \frac{1 + r_a}{1 + r_d}$$

D	Debt in dollars
V	Total value in dollars
T_C	Corporate tax rate
r_a	Opportunity cost of capital (100% equity, i.e. unlevered)
r_d	Cost of debt

Modigliani-Miller (M&M) formula

The tax part $(1 - T_C)$ is left out when debt is continuously rebalanced.

$$r_E = r_A + (1 - T_C)(r_A - r_D)\frac{D}{E}$$

$$r_A = r_D(1 - T_C)\frac{D}{V - T_C D} + r_E \frac{E}{V - T_C D}$$

D	Debt in dollars
E	Equity in dollars
r_E	Cost of equity (levered)
r_A	Opportunity cost of capital (100% equity, i.e. unlevered)
T_C	Corporate tax rate
r_D	Cost of debt
V	Total value in dollars

M&M formula with β

$$\beta_E = \beta_A + (1 - T_C)(\beta_A - \beta_D)\frac{D}{E}$$

$$\beta_A = \beta_D(1 - T_C)\frac{D}{V - T_C D} + \beta_E \frac{E}{V - T_C D}$$

β_A can be converted to r_A by CAPM $r_A = r_f + \beta_A(r_m - r_f)$

Comparison based on existing actors in market

Sometimes we're asked if an entity should go ahead with a project or not, given some numbers about the project and some numbers about existing actors in the market.

To do this we calculate the opportunity cost of capital (OCC), or return on assets (r_a), for the existing projects.

Note: When estimating something for a given project only use the values belonging to that project. I.e. the r_E calculated for the existing actors/projects *is not* the r_E you should be using to calculate the proposed project's WACC.

See also figure 6.3 on page 174 of the book for a decision tree.

1. Estimate r_E for existing projects

Use the CAPM.

2. Calculate r_a and WACC for the market

If there's several existing entities or projects in the market for which you have been given either r_E or the needs to estimate it, calculate each of their r_a. The unweighted average is typically the r for the market, but make sure to look for such a statement in the problem text.

If debt is continuously rebalanced

Use either of the formulas below, depending on what information is available.

Unlevering:

$$r_a = r_E \cdot \frac{E}{V} + r_D \frac{D}{V}$$

M&M Formula without the tax part:

$$r_a = \frac{r_E + r_D \frac{D}{E}}{1 + \frac{D}{E}}$$

Use Miles-Ezzel or the definition of WACC to find it.

If debt is rebalanced periodically

$$r_E = r + (r - r_D)\frac{D}{E}(1 - \frac{T_C \cdot r_D}{1 + r_D})$$

Rewritten for r:

$$r = \frac{r_E + r_D \cdot y}{1 + y}$$

Where $y = \frac{D}{E} \cdot (1 - \frac{T_C \cdot r_D}{1 + r_D})$

Use Miles-Ezzel to find the WACC.

If debt is permanent & fixed & predetermined

$$r_a = r_D(1 - T_C)\frac{D}{V - T_C D} + r_E\frac{E}{V - T_C D}$$

Unlever using M&M (solved for r_a) remember to use the E and D from the company you got the r_E from

$$\mathbf{WACC} = r_a(1 - T_C\frac{D}{V})$$

Combined version of normal WACC and normal M&M. Use the new project's D / E values. Alternatively, use normal M&M + WACC instead of this shortcut formula

3. Calculate NPV using WACC

$$NPV = -Cost + \frac{CashFlow}{WACC}$$

Adjusted present value (APV)

- Step 1. Calculate the base case value of the project as if it is all equity financed (Un-levered) and without side-effects, the present base case value is dicsounted at OCC r_a.
- Step 2. Then, calculate all the side-effects, include tax advantage (from a given level of debt by discounting tax shield at the cost of debt r_d) and issue cost at t_0.
- In the end, the APV = PV of base case + PV of tax advantage – issue cost.

Option pricing

Options are financial contracts that give their holders the right, but not obligation, to buy or sell something on a future date (maturity date) at a price decided upon today. Options can be priced rationally using different models. European options (options that can only be exercised at the maturity date) should be priced using the Black-Scholes model, and American options (options that can be exercised at any time until the maturity date) should be priced using the Binomial options pricing model.The american option without exercising is *alive* while it turns to *dead* after exercise.

- Put:the right to sell at exercise price, expecting the price decrease.
- Call:the right to buy at exercise price, expecting the price increases.

Moneyness	Put	Call
In the money	$S_0 < X$	$S_0 > X$
At the money	$S_0 = X$	$S_0 = X$
Out of the money	$S_0 > X$	$S_0 < X$

- Payoff = Value of the Option = V_t
- Profit = Payoff - Future value of option cost = $V_t - V_0 e^{rt}$.

position & option	payoff at maturity

long a put	$max[X - S_0, 0]$
long a call	$max[S_0 - X, 0]$
short a put	$min[X - S_0, 0]$
short a call	$min[S_0 - X, 0]$

Black-Scholes model:contious time

Use this when you want to calculate the price of a European-style call option. Here are the formulas you will need. When calculating European puts, use $-N(-d_1)$ and $-N(-d_2)$ in the main formula instead of $N(d_1)$ and $N(d_2)$.

$$C(s,t) = N(d_1)S_0 - N(d_2)Xe^{-rT}$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}(\ln(\frac{S_0}{X}) + (r + \frac{\sigma^2}{2})T)$$

$$d_2 = \frac{1}{\sigma\sqrt{T}}(\ln(\frac{S_0}{X}) + (r - \frac{\sigma^2}{2})T)$$

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \mathrm{e}^{-\frac{z^2}{2}}\,\mathrm{d}z$$

d_2 can also be defined in terms of d_1: $d_2 = d_1 - \sigma\sqrt{T}$.

$N(x)$ is not trivial to calculate. It is equivalent to the cumulative distribution function for the normal distribution, which can be looked up in a table.

T	Time to maturity in fraction of interest-giving periods (e.g. T=90/365=0.247 if 90 days to maturity and interest is given for one year)
S_0	Spot price of the asset, i.e. what the asset costs now
X	Strike price, i.e. what can the option be bought/sold for at maturity
r	Risk-free rate (annual rate, continuously compounding)
σ	Volatility of returns of the asset

To macth discrete and continuous time volatility

$$u = e^{\sigma\sqrt{\delta t}}, d = e^{-\sigma\sqrt{\delta t}}$$

δt is small time interval.

to replicate a hedging portfolio with a option,portfolio=a fraction Δ of the stock + a risk free loan of D,

$$\Delta = \frac{O_u - O_d}{(u-d)S}$$

$$D = \frac{uO_d - dO_u}{(u-d)r}$$

Binomial Options Pricing Model (BOPM):discrete time

Use this when you want to calculate the price of an American-style option. Here are the formulas you will need.

$$p = \frac{e^{rt/n} - d}{u - d}, u = e^{\sigma^{\sqrt{t/n}}}, d = e^{-\sigma^{\sqrt{t/n}}}$$

p	probability of going up, also called risk neutral probability
u	a value multiplier for when an asset's value goes up
d	a value multiplier for when an asset's value goes down
σ	The underlying volatility
t	The time duration of all steps
n	The number of steps/moments.
r	Risk-free interest rate

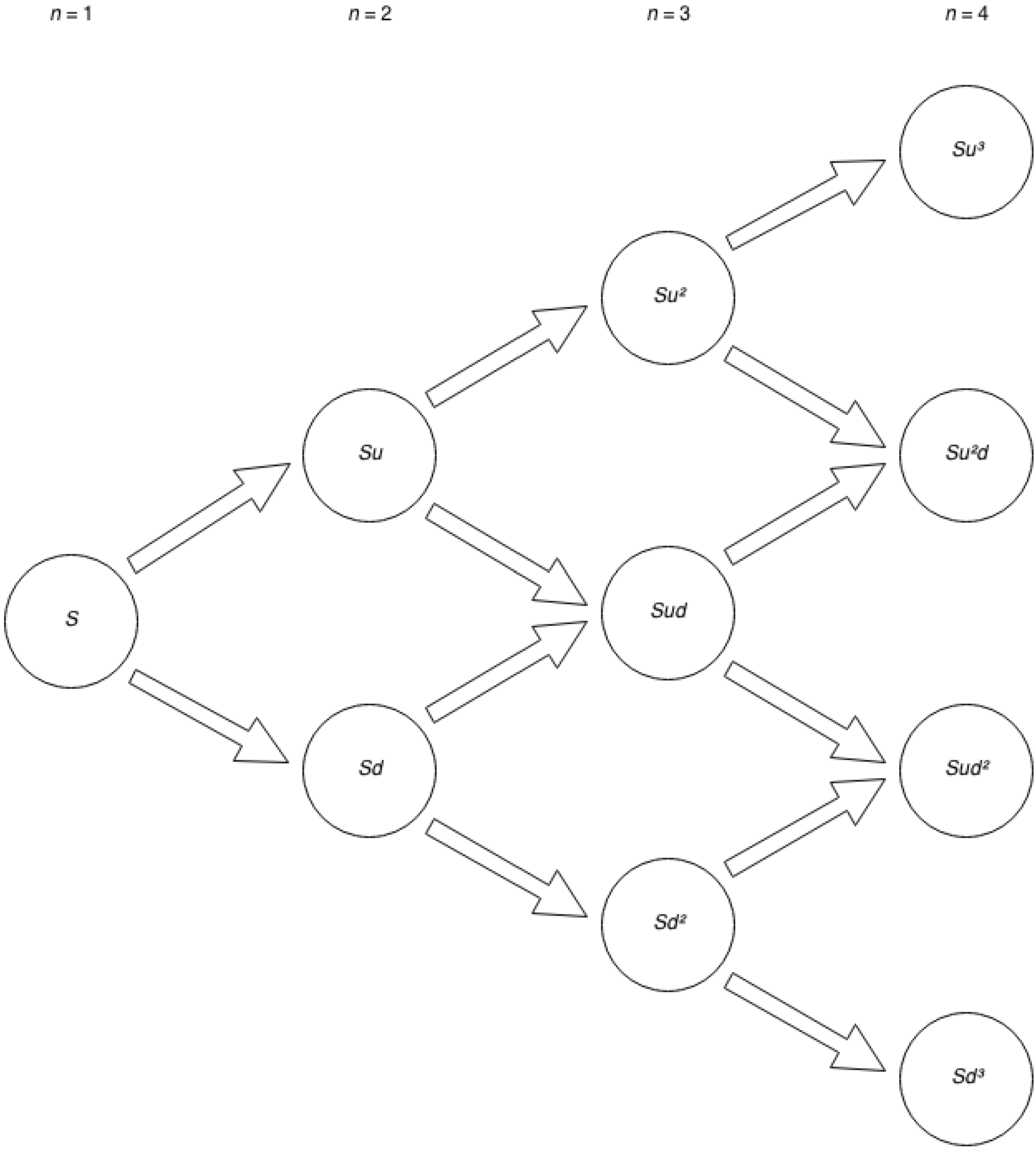
$e^{rt/n}$ can be replaced with r

to replicate a hedging portfolio with a option,portfolio=a fraction Δ of the stock + a risk free loan of D,

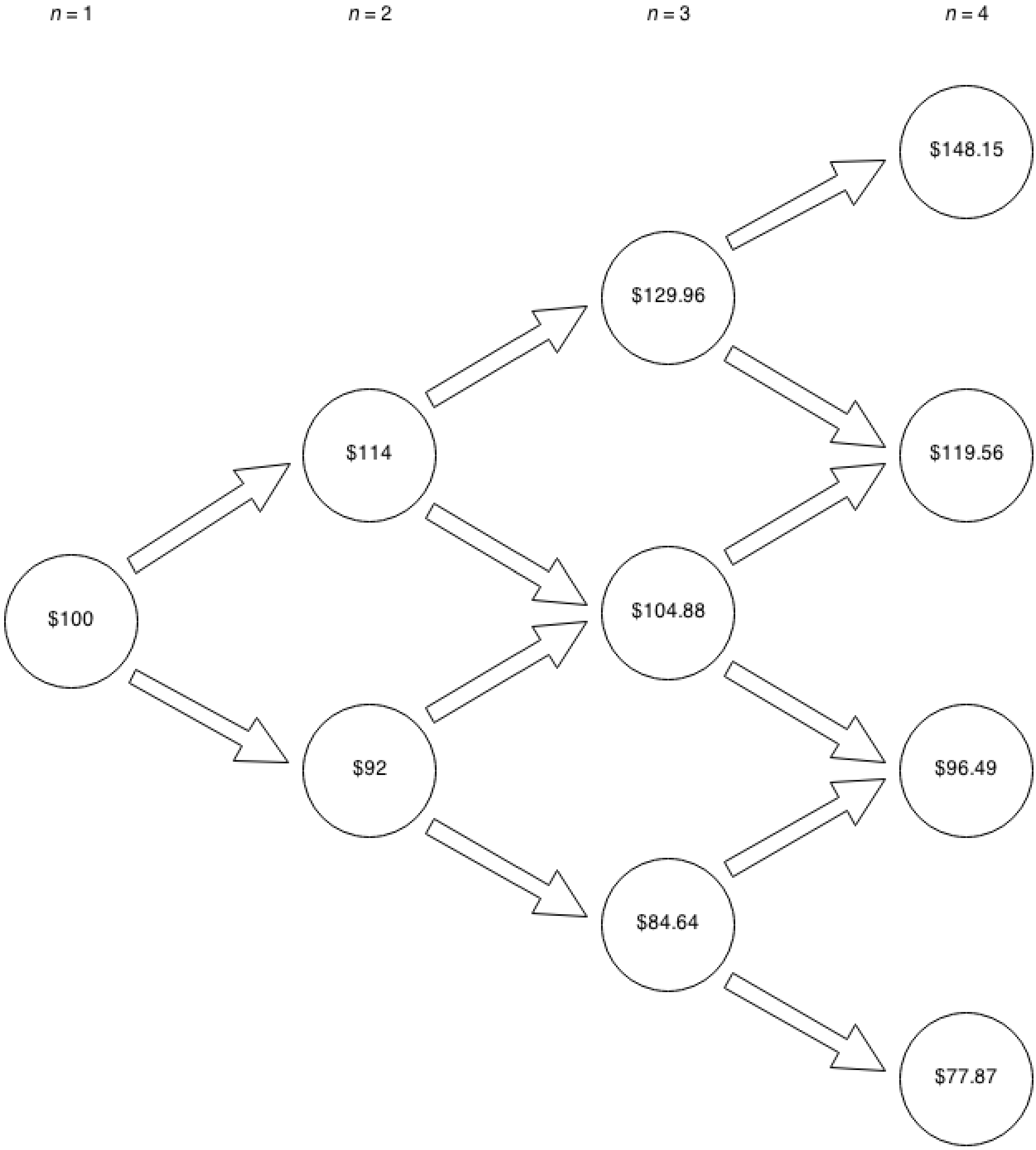
$$\Delta = \frac{O_u - O_d}{(u-d)S}$$

$$D = \frac{uO_d - dO_u}{(u-d)r}$$

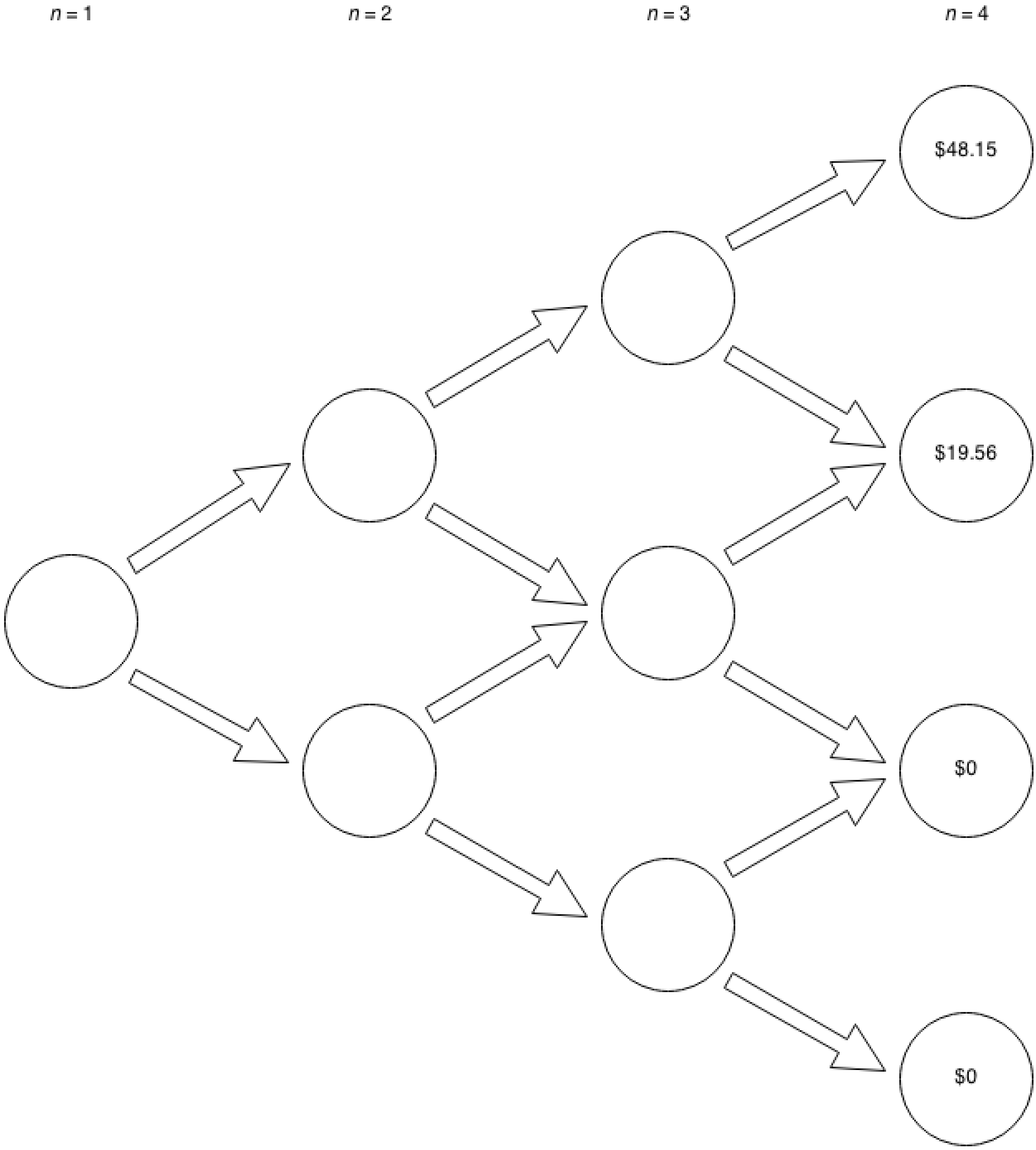
First, make a binomial lattice with n steps. S is the asset price at the current time, and u and d are the value multipliers from the formula above:



Then you want to fill in the numbers in all the nodes of the lattice with dollar (or euro, or whatever) values. Here I have just made up some numbers for the sake of example: $S = \$100, u = 1.14, d = 0.92$.



This is now the finished asset price lattice. Now we want to make a new lattice for the option price. It should have as many steps as the asset price lattice, but all nodes should be empty except for the right-most leaf nodes. These nodes should contain the option value at that point, calculated as $S_n - K$, i.e. the asset price for the corresponding node in the asset lattice minus the strike price. Of course, if this expression is less than $0, the value of the option is truncated to zero, since excercising options are, well, optional. Anyway, for our example, with $K = \$100$, this will yield an option price lattice that looks like this:



Now, all that is remaining is to calculate the values of the empty nodes from right to left. The value in each node, C_{node}, can be calculated from its two direct child nodes (to the right) by using this nice formula:

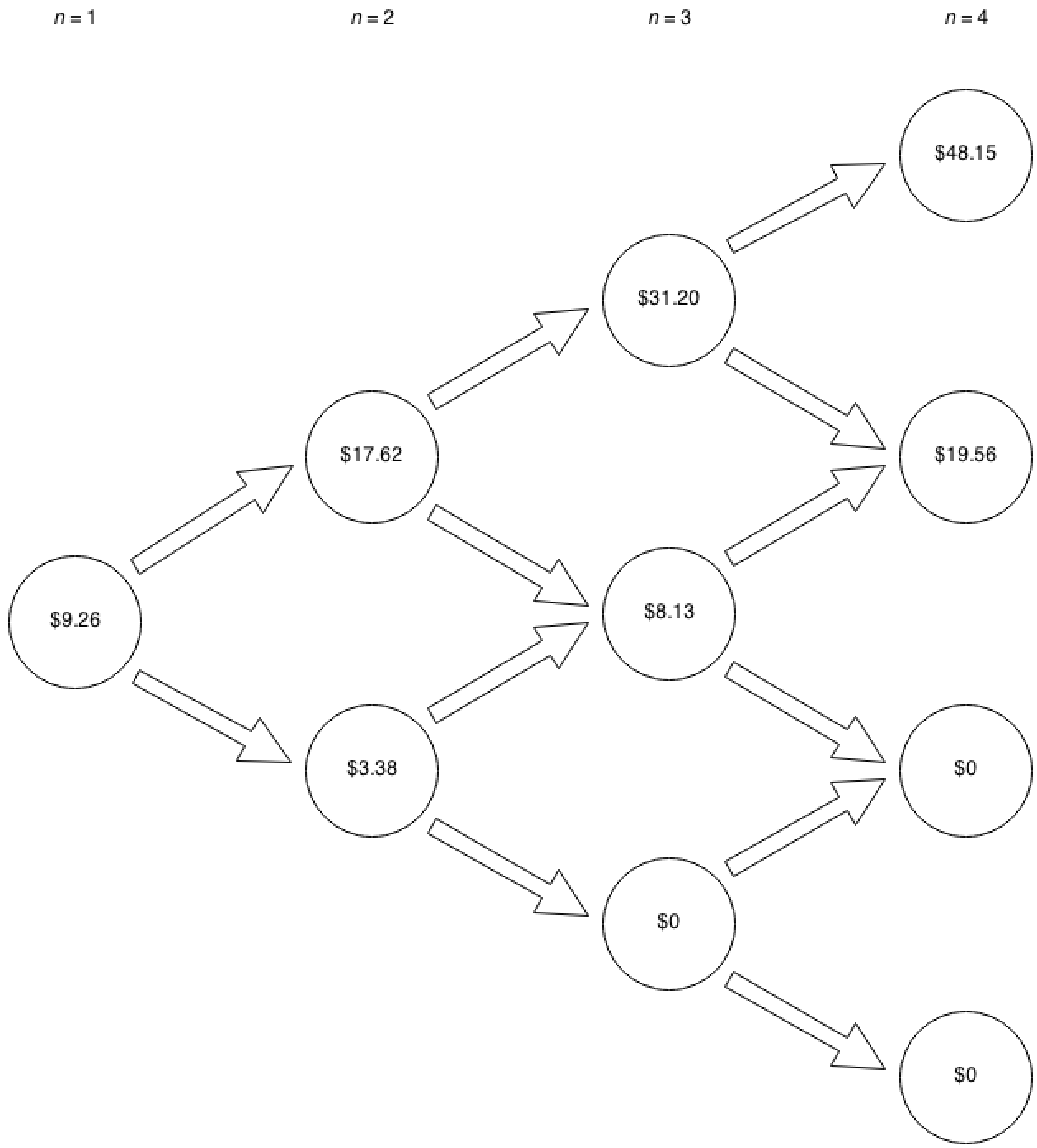
$$C_{\text{node}} = e^{-r\Delta t}(pC_{\text{up child}} + (1-p)C_{\text{down child}})$$

Or you could use this formula:

$$C_{\text{node}} = \frac{pC_{\text{up child}} + (1-p)C_{\text{down child}}}{(1+r)\Delta t}$$

Note it uses $r+1$ because it assumes r is an actual percentage expressed as a real number (i.e. $r \in [0,1]$).

Continuing our example, assuming $r = 5\%, t = 1$, we get:



The root (leftmost) node of the lattice is the binomial option price, $9.26, which is what we wanted to calculate!

Matching discrete and continuous time volatility

$$u = e^{\sigma\sqrt{\delta t}}, d = e^{-\sigma\sqrt{\delta t}}, p = \frac{e^{r\delta t} - d}{u - d},$$

small time interval of american step δt. Like 1 year with 2 step then $\delta t = 0.5$

Put-Call Parity

Put-call parity expresses the relationship between the price of a European call option and a European put option. The relationship is as expressed in this equation:

$$C - P = D(F - X)$$

C	Current price of a call

P	Current price of a put
D	Discount rate (typically risk-free interest or similar)
F	Forward price of the asset
X	Strike price

Option positions

Short straddle

A good strategy when you expect the stock price to have low volatility. The short straddle will give maximum profit if the price stays the same as today.

You sell a put option and sell a call option (short put and short call) with an *at-the-money* exercise price. This option position has no upfront cost, as you only sell two options. The downside is that if the stock price changes a lot in the future, there is a possibility for infinite losses.

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Long straddle

You buy a put option and buy a call option (long put and long call). This is a good strategy if you expect high volatility in the stock price.

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Butterfly spread

Buy two call options – one at an *in-the-money* exercise price and one *out-of-money*. Then sell two call options for an *at-the-money* exercise price. This has an upfront cost of the cost of the two long options minus the cost of the two short options. The advantage of a butterfly spread over a short straddle is the reduced risk in case of an increased stock price. The downside compared to the short straddle is a lower maximum profit.

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Collar

Also called "Split-Strike Conversion"

Buy an out-of-the-money put option and sell an out-of-the-money call option (long put and short call). It reduces volatility because it 'cuts off' the high and low tails of the stock's returns.The short call caps the possible gains from the stock (provides a ceiling), but it generates cash. The cash is used to increase the return of the position and to finance the put, which insures against a (large) loss of stock (provides a floor). So a part of the upware potential is given up to reduce the downside risk, resulting is lower volatility.

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Trade-off Theory of Capital Structure

The trade-off theory of capital structure states that as the debt/equity ratio increases, there is a trade-off between the interest tax shield and bankruptcy, causing an optimum capital structure. The theory implies that the marginal benefit of further increases in debt declines as debt increases, while the marginal cost increases as the debt increases. Informally, this means that as the cost of debt goes up, a good strategy is to decrease the debt/equity ratio, and that as the benefits go up (for instance by tax increases) the debt/equity ratio should increase.

According to Trade-off Theory of optimal structure,

- All changes should be in the direction of the optimun and lead to an increase in firm value, e.g. a positive CAAR (Cumulative Average Abnormal Return), a positive price reaction.
- General purpose assets can easily be redeployed by other companies (a high re-sale value) and lose little value in a bankruptcy/financial distress, they have low bankruptcy costs and such assets can carry much debt. Firm-specific assets have limited or no value to other firms, and they have high bankruptcy costs and can carry little debt.
- Rented and leased equipment offer no security for loans.
- Firms should use some debt because of its tax benefits, thus zero leverage firms have a sub-optimal capital structure.

Real options analysis

An investment project in real assets is valued as an option if

- the exercise price of the option is the investment cost.
- the underlying value of the option is the project's revenue.

American Put Option: The sell-back option to abandon a project at any point of its lifetime and sell its assets in the second hand market

European Call Option: In the binomial model, the option to defer an investment project with one period.

Misc definitions

Here are some different definitions that might come in handy.

Cumulative abnormal return (CAR)

Cumulative abnormal return (CAR) Sum of the differences between the expected return on a stock (systematic risk multiplied by the realized market return) and the actual return often used to evaluate the impact of news on a stock price.

Annual percentage rates on loans (APRs)

Annual percentage rates indicate the rate of return set by the bank (lender) on a financial instrument. The borrower (debtor) agrees to pay a set interest rate over time. It is calculated on a yearly basis, with 12 months and 365 days as the standard timeframe. APR rates are a point of reference when seeking to calculate the total cost of borrowing money. Common examples are mortages and consumer loans.

More information on non collateralized loans: https://www.xn--forbruksln-95a.no/

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