Chapter 8: Black and Scholes option pricing - part 2

Exercises - solutions

1. Holders of a long call have a claim on the upward potential of stock, but they don't pay for it until maturity, if at all (the option may also expire out of the money). The possibility to delay payment is more valuable the higher the interest rate is (the holder can earn interest over the exercise price). Holders of a long put can sell the stock at the fixed exercise price in the future. The value of a future payment decreases with the interest rate.

2. The put-call parity states:
$$put = call + PV(X) - S$$
If the stock price increases with 1, the call increases (by definition) with $\Delta_c \times 1 = \Delta_c$. The $PV(X)$ is unaffected by changes in the stock price. So the right hand side of the equation changes with
$$\Delta_c \times 1 + 0 - 1 = \Delta_c - 1$$
This must be the delta of the put: $\Delta_p = \Delta_c - 1$.

3. Economically, holding an extremely far in the money call option is equivalent to holding a share that is not yet paid for, i.e. $S - PV(X)$. Differences in volatility (practically) do not matter any more as the options are (almost) certain to be exercised anyway. The options in left hand figure are paid for on the same date, so their PV(X) are the same and, consequently, they have a common value as a function of the stock price. The options in right hand figure are not paid for on the same date, so their PV(X) are different and they do not have a common value as a function of the stock price.
Technically, as the stock price S get larger and larger, both the option delta N(d_1) and the probability of exercise N(d_2) approach 1. The Black and Scholes price then approaches $O_{c,0} = S_0 - Xe^{-rT}$. This price is independent of σ, but not of T, hence options with different volatilities converge to a common value, but options with different maturities do not.

4. An option position that has limit downside risk and that profits from increasing stock prices is the bull spread. If it is set up with puts it has a positive initial balance, i.e. it requires no net investment today. A put bull spread has either a negative or no pay-off at maturity, so it must be set up such that it has no pay-off at maturity if investors A's expectations are realized. This can be done by combining a short put with exercise price €105 with a long put with exercise price €100. If the investor's expectations are realized the stock will end on or above €105 and both options will expire out-of-the-money. The investor will then have earned the (future value of the) initial balance of the option premiums. The long put costs:
$$d_1 = \frac{\ln(S_0/X) + (r + \frac{1}{2}\sigma^2) \times T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

The input data are:
$S = 100$, $X = 100$, $\sigma = 35\%$, $r = 5\%$, $T = 0.25$

$$d_1 = \frac{\ln(100/100) + (0.05 + 0.5 \times 0.35^2) \times 0.25}{0.35\sqrt{0.25}} = 0.15893$$

$$d_2 = 0.15893 - 0.35\sqrt{0.25} = -0.01607$$

$\mathrm{NormalDist}(-0.15893) = 0.43686$
$\mathrm{NormalDist}(0.01607) = 0.50641$
So the put price is

$$O_p = Xe^{-rT}N(-d_2) - S_0N(-d_1) = 100e^{-0.05\times 0.25} \times 0.50641 - 100 \times 0.43686 = 6.33$$

The short put brings in:

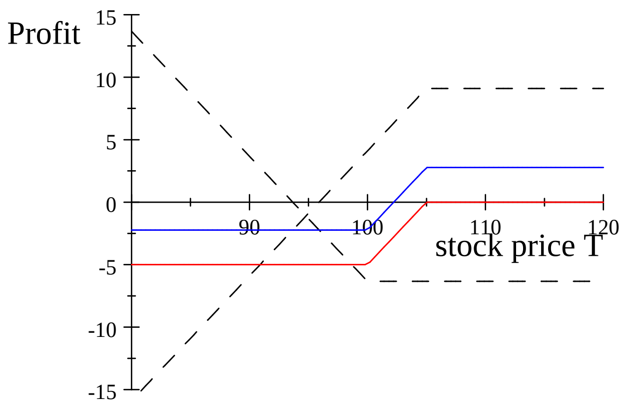
$$d_1 = \frac{\ln(100/105) + (0.05 + 0.5 \times 0.35^2) \times 0.25}{0.35\sqrt{0.25}} = -0.11987$$

$$d_2 = -0.11987 - 0.35\sqrt{0.25} = -0.29487$$

$\mathrm{NormalDist}(0.11987) = 0.5477$
$\mathrm{NormalDist}(0.29487) = 0.61595$
so the put price is:

$$105e^{-0.05\times 0.25} \times 0.61595 - 100 \times 0.5477 = 9.10$$

and the initial balance of the option premiums is $9.10 - 6.33 = 2.77$ and its future value: $2.77e^{0.25\times 0.05} = 2.80$. That is the maximum profit at maturity and it is obtained when the share prices ends at €105 or higher.
The minimum profit occurs if the share price ends at €100 or lower. At €100, the short option will be exercised and costs $100 - 105 = -5$ so the net profit is $2.80 - 5 = -2.20$. Below €100, the additional costs of the short put are cancelled out by the long put, which is also exercised. The position is depicted below.



Put bull spread, the options are dashed. the profit in blue and pay-off at maturity in red

5. The call option delta increases with the stock price, all other things equal. As the stock price increases, the option becomes more likely to be exercised and becomes more like a stock that has not yet been paid for. Ultimately, when the option is so far in the money that it is certain to be exercised the call option delta becomes 1. Conversely, when the stock price falls and the call is farther and farther out of the money, it becomes less and less likely to be exercised. Ultimately, when the option is so far out of the money that it is certain not to be exercised the call option delta becomes 0. The call has lost its value, and is no longer sensitive to (small) changes in the stock price.

6. (a) We use the put-call parity to construct a synthetic put and check for any mispricing:

$$long\ put = long\ call + PV(X) - share\ price$$

First we calculate the PV(X) using the appropriate NIBOR rate and period:
NHY nov.5: $620e^{-.02235\times 2/12} = 617.69$
NHY nov.5: $680e^{-.02235\times 2/12} = 677.47$
NHY feb.6: $620e^{-.02337\times 5/12} = 613.99$
NHY feb.6: $680e^{-.02337\times 5/12} = 673.41$
ORK jan.6: $240e^{-.02313\times 4/12} = 238.16$
NSG dec.5: $100e^{-.0229\times 3/12} = 99.429$
We construct a synthetic put by buying a call, putting PV(X) in the bank and selling the share. So we use the ask price for the call and the bid price for the share.

Synthetic and market option prices

			call		share	syn.	put option	
Ticker	T	X	ask	PV(X)	bid	put	bid	ask
NHY	nov.5	620	70.00	617.69	677.00	10.69	9.25	10.00
"	"	680	30.25	677.47	677.00	30.72	29.00	31.25
"	feb.6	620	83.75	613.99	677.00	20.74	19.00	20.75
"	"	680	47.00	673.41	677.00	43.41	41.50	44.25
ORK	jan.6	240	26.75	238.16	259.00	5.91	5.00	5.50
NSG	dec.5	100	11.00	99.429	108.75	1.68	2.85	3.35

The rows for NHY nov.5-620, ORK jan.6 and NSG dec.5 in Table 4 show that the prices of the synthetic puts for these options lie outside the bid-ask spread on the market. But the first two are not arbitrage opportunities: you can buy a synthetic put NHY nov.5-620 at 10.69 or an ordinary put at 10, but you cannot sell at 10.69, only at 9.25. The same applies to ORK jan.6. They are outside the bid-ask spread, but on wrong side from an arbitrage point of view. The NSG option is outside the bid-ask spread on the other side and that offers an arbitrage opportunity: we can buy the synthetic put at 1.68 and sell the ordinary put at 2.85. This gives an arbitrage profit of 1.17. If we manage to close a million of these contracts we have become millionaires overnight.
Such arbitrage opportunities are in practice not open to investors who get their information from a newspaper and on closer examination it appears that we made a typing error in Table 2: the bid-ask prices for NSG are 106.75 and 107.5. With the proper stock price, the price of the synthetic put becomes:
call + PV(X) - share = 11+99.429-106.75=3.679
The price of the synthetic put now lies outside the bid-ask spread on the other side and the arbitrage opportunity has disappeared. We can make sure by checking the relation the other way around by constructing a synthetic short put. Then we write a call, borrow pv(X) and buy the share. That costs: -10 + (-99.429) + 107.5 = -1.929 i.e. brings in 1.929. Note that we use the bid price for the call and the ask price for the share. If we sell the put on the market we get 2.85, so the synthetic price is on the wrong side of the bid-ask spread from a an arbitrage point of view.

(b) Differences between implied and observed prices can occur because we apply the put-call parity, which is only valid for European options on non-dividend paying stocks, to traded American options on stocks that may pay dividends. Further, price differences can occur because of nonsynchronous trading. The prices in newspapers are generally closing prices, but we do not know when the last trade of the day took place. If the last option trade was at 13.00 hours and the last stock trade at

15.00 hours, the option trade could be based on a different stock price than the one we read in the newspaper.