Exam August

1. A person has a utility function given by $U(X) = -e^{-aX}$, where $\boldsymbol{a}$ is a constant. What is correct?
    a. The person is risk neutral
    b. The relative risk aversion coefficient is decreasing with increasing X
    c. The person has an increasing marginal utility
    d. The absolute risk aversion coefficient is increasing with increasing X
    e. The absolute risk aversion coefficient is constant with increasing X

Solution

$U(X) = -e^{-aX}$.

$U'(X) = -e^{-aX} = -ae^{-ax}$=> Decreasing utility with X

$U''(X) = -e^{-aX} = -a^2e^{-ax}$=> Decreasing marginal utility with X => concave utility function and thus risk averse (a and c is wrong)

$\boldsymbol{R_A(X)} = \frac{\boldsymbol{U''(X)}}{\boldsymbol{U'(X)}} = \frac{\boldsymbol{-a^2e^{-ax}}}{\boldsymbol{-ae^{-ax}}} = \boldsymbol{a}$ => Absolute risk aversion is constant with X (e is correct d is wrong)

$R_R(X) = xR_A(X) = \ xa$=> Relative risk aversion is increasing with X (b is wrong)

2. Based on the following information, which is true?
    The return of the risk-free rate is 3%. The return of the market is 7%, with a volatility (standard deviation) of 20%. The covariance between the return of the asset relative to that of the market is 8%.
    a. The asset's Beta is 0.4
    b. The market's risk premium is 10 %
    c. The cost of capital is 4 %
    d. The asset's return has an inverse relation to the market's return
    e. Based on CAPM the cost of equity capital is 11 %

Solution

Calculate Beta as $\frac{COV(market,asset)}{Var\ (market)}$=$\frac{0.08}{0.2^2} = \frac{0.08}{0.04} = 2$ (a is wrong)

Market risk premium is $\mathrm{E}(\mathrm{r_M} - \mathrm{r_f}\ )$= 0.04 (b is wrong)

Covariance is positive thus the asset return has a positive relation to market's return (d is wrong)

CAPM = $r_E = r_f + \beta\left[E(r_M - r_f\ \right]$ = 0.03 + 2*(0.07-0.03) = 0.03 + 0.08 = 0.11 = 11% (e is correct)

Cannot say anything of cost of capital as we don't know the cost of debt. In the case where here is no debt financing the cost of capital equals the cost of equity (thus c is wrong)

3. Regarding market imperfections, find the FALSE option.
   a. Taxes affect capital structure because interest payments are deductible from taxable income while dividends are not.
   b. Costs of financial distress affect capital structure because a firm in financial trouble may face extraordinary costs and/or decreased income from regular levels.
   c. Agency costs impact capital structure as the priorities of the managers are aligned with the good of the firm.
   d. Information asymmetry impacts capital structure as managers might hold inside information and conduct financing decisions accordingly.
   e. The problem of a firm financing through debt or stocks is similar to the problem of buying used cars: good cars will be underpriced, and bad cars overpriced.

4. Regarding the irrelevance theorem on dividends, find the FALSE option.
   a. Modigliani and Miller's proposition successfully explained the effects of dividend policy and capital structure decisions on investment.
   b. In perfect capital markets, investors can convert shares to cash and cash to shares independent of dividend policy.
   c. Modigliani and Miller's proposition explain the need to pay higher prices for dividend paying stocks.
   d. In perfect capital markets, dividend policy has no effect on the value of the firm.
   e. According to Modigliani and Miller's proposition, any price effects of dividends and capital structure would give arbitrage opportunities.

5. Select the option which is TRUE. If the market is efficient:
   a. Then no one can make money in the stock markets.
   b. Returns cannot be systematically increased without increasing risk.
   c. Excess returns on stock prices are autocorrelated.
   d. Prices first underreact to new information, then overreact to compensate.
   e. None of the above.

6. Select the option which is TRUE:
   a. The presence of survivorship and selection biases disprove the efficient market hypothesis.
   b. Data snooping can be used to prove the efficient market hypothesis.
   c. Anomalies such as the January effect and other calendar effects are evidence of the strong form of market efficiency.
   d. The joint problem hypothesis states that to test the market efficiency, one needs a return model, which also needs to be tested.
   e. None of the above.

7. Which of the following is NOT an input required by the Black-Scholes option pricing model?
    a. The expected volatility of the stock
    b. The expected return on the stock
    c. The risk-free interest rate
    d. The current stock price
    e. The exercise price

8. Choose the FALSE statement.
   In the State Preference Theory, a market is arbitrage-free if
    a. There exists an equivalent martingale measure.
    b. There is a state security with positive price for all states.
    c. The existing securities span all the states.
    d. There exists a positive pricing kernel.
    e. States a,b, and d are equivalent.

1. A person has an expected-value utility function based on the logarithm of wealth U(W) = a + b*ln(W), where W expresses the person's wealth. The person's current wealth is W_0
   (a) Is the person risk averse and how does the person's risk attitude change with increasing wealth?

   Solution: U'(W) = b/W, U''(W) = - b/W^2 => strictly negative second derivative thus concave utility function and the person is risk averse.

   Calculate Arrow-Pratt absolute risk aversion coefficient as: $ARA(W) = -\frac{U''(W)}{U(W)} = -\frac{-\frac{b}{W^2}}{\frac{b}{W}} = 1/W$ => risk aversion decreases with increasing wealth

   (b) The person has the opportunity to bet on a proposition that
      - has the probability p of receiving the double of the bet size and
      - (1-p) of returning nothing.

   What fraction of W_0 should be bet on this proposition?

   Solution: The person bet a fraction K = [0, 1] of its current wealth W0, so that the expected wealth is:
   In the bad case: Wb = (W0 – K) + 0 = W0 – K = W0-K
   In the good case: Wg = (W0 – K) + 2*K = W0+K

   The expected utility is thus:
   E[U(W)] = p* u(W0+K)+ (1-p) * u(W0-K)
   Since the person is expected to be utility maximizing it will choose K in order to maximize E[U(W)], thus differentiation wrt K
   E[U'(W(K))] = p* u'(W0+K)+ (1-p) * u'(W0-K) = 0
   Since U'(W) = b/W, and using the chain rule we get E[U'(W (K))] = $p * \frac{b}{\mathrm{W0+K}} + (1-p) * -\frac{b}{\mathrm{W0-K}}$ = 0 =>
   $p * b * (\mathrm{W_0} - \mathrm{K}) + (\mathrm{p}-1) * \mathrm{b} * (\mathrm{W_0} - \mathrm{K})$=0 and solving for K
   $K = W_0(2p-1)$,
   That is for $p \leq \frac{1}{2}$ the person will not bet on the proposition, and for p=1 the person will bet all its initial wealth

2. A company is considering investing in a project that generates income for 4 years starting 1 year from and that requires an initial investment now of €300 million. The investment is linearly depreciated (equal amounts each year). The value of the project at the end of year 4 is zero. The income is expected to be €280 million in Year 1 and to increase by 5 % each year. Production costs are 50 % of income and operating costs are 15 % of income. The project requires working capital of €30 million now and €35, €50, €50 million in the following three years. The working capital is liquidated by end of year 4. The tax rate is 22 %. The relevant cost of capital for the project is 10 %. Should the company accept the project or not? Support the answer with calculations and make additional assumptions if necessary.

Year	0	1	2	3	4
Income (+5% increase)		280.0	294.0	308.7	324.1
Production costs (50%)		-140.0	-147.0	-154.4	-162.1
Operating costs (15%)		-42.0	-44.1	-46.3	-48.6
Operating income		98.0	102.9	108.0	113.4
Depreciation		-75.0	-75.0	-75.0	-75.0
Income before tax		23.0	27.9	33.0	38.4
Tax @ 22%		-5.1	-6.1	-7.3	-8.5
Income after tax		17.9	21.8	25.8	30.0
Income after tax		17.9	21.8	25.8	30.0
Depreciation		75.0	75.0	75.0	75.0
Change in working capital	-30.0	-5.0	-15	0	50
Investment	-300.0				
Cash Flow	-330.0	87.9	81.8	100.8	155.0
PV of cash flows	329.1				
NPV	0				

The present value of the cash flow @ 10% discount rate is lower than the investment plus the necessary initial requirement of working capital, thus the investment should not be accepted (however the negative NPV is very low (= -€0.9 million) so the decision could be subject to discussions).

3. Consider a firm that has the option to abandon a certain project by selling it for a fixed price of 400 million NOK. Today, the future project value is estimated in 450 million NOK, which can increase 30% or decrease 20% every year. The free interest rate is 5%.
   (a) Present the value of the project in a two-period binomial tree if each period corresponds to one year.

   Solution:

t=0	**t=1**	**t=2**
		760.5
	585	
450		468
	360	
		288

   (b) Calculate the project value with abandonment option.

   Solution: Firstly we note that the risk neutral probabilities are given by

$$p = \frac{1.05-0.9}{1.3-0.8} = 0.5 \qquad \text{and} \qquad 1-p = 1-0.5 = 0.5$$

   The exercise condition for the abandonment option is max(continue, abandonment).

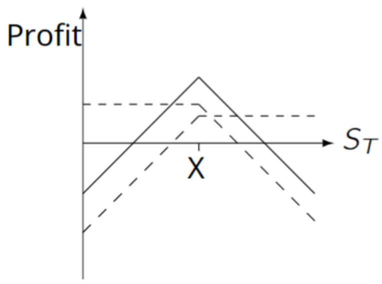
The continuation value in the lower node for t=1 is given by $\frac{0.5*400+0.5*468}{1.05} = 413.33$ and $\frac{0.5*760.5+0.5*468}{1.05} = 585$. The continuation value in the node t=0 is given by $\frac{0.5*585+0.5*468}{1.05} = 475.40$.

t=0	**t=1**	**t=2**
		Max(760.5, 400)=760.5
	Max(585, 400)=585	
Max(475.40,400)=475.4		Max(468, 400)=468
	Max(413.33, 400) = 413.33	
		Max(288, 400)=400

When we consider the abandonment option the project value is 475.4.

4. Consider the following two questions:
   (a) Suppose that you expect that the price of a stock will have small volatility in the future. What is the strategy that you could design to take advantage of this situation using options? Explain why we can use such a strategy in this case, including in your explanation the profit diagram of the strategy.

   Solution: We can use a short Straddle strategy. A short straddle is obtained combining a short put and a short call with the same exercise price. The profit diagram is the following:



(a) Short straddle

   The strategy is adequate when we believe that the stock price will end up near the strike price, because the profit of this strategy is positive in this case.

   (b) We saw in the lecturers that a bull spread could be obtained by buying a call option on the stock and selling a call option on the same stock with a higher exercise price. Can you create the same strategy buying and selling two put options with different exercise prices? If yes, explain how, present the profit diagram, including the profit of the strategy and the profit of each one of the puts you include in the strategy.

Solution: The bull spread is constructed by selling 1 put with exercise price x2 and buying 1 put with exercise price x1, with x1 < x2. We know that increasing the exercise price in a put results in an increase of its price. This means that we will receive more selling the put with exercise price x2 than what we have to pay to buy the put with exercise price x1. This justifies the payoffs of the puts in the figure below (dashed lines). Calculating the sum you get the bold line, which is the payoff of a bull spread.