

# TIØ4146 Finance for Science and Technology Students

## Chapter 9 - Real Options Analysis (part 1)

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## **Investment opportunities as options**

### **The option to defer**

# Real Options Analysis

The essential economic characteristic of options is:

*the flexibility to exercise or not*

- ▶ possibility to choose best alternative
- ▶ walk away from bad outcomes

Investments in real assets have flexibility - projects can be:

- ▶ delayed or speeded up
- ▶ made bigger or smaller
- ▶ abandoned early or extended beyond original life-time, etc.

- ▶ Real options are options where underlying value is a real asset
- ▶ not a financial asset as stock, bond, currency

Flexibility in real investments means:

- ▶ changing cash flows along the way:
- ▶ profiting from opportunities, cutting off losses

Discounted cash flow (DCF) calculation cannot handle flexibility:

- ▶ assumes passive, not flexible, position
- ▶ accepts cash flows as they come

# Real Options vs Financial Options

- ▶ Financial option: the option' owner has the right but not the obligation to buy or sell depending on the option a certain underlying.
- ▶ Real option: the company has right, but not obligation, to undertake a new project.

Determinant	Stock option	Real option
underlying	stock	project revenue
strike	exercise price	investment
time to maturity	maturity	license validity
volatility	stock $\sigma$	price volatility
interest rate	$r_f$	$r_f$

Some examples:

Call options	Put options
delay	default
expand	contract
extend	abandon
re-open	shut down

# Challenges

Option character of real options can be limited:

- ▶ Real options can be less clearly defined:
  - ▶ underlying value may be project in planning
  - ▶ no clear time to maturity
  - ▶ no clear exercise price
- ▶ Input data may be difficult to obtain
  - ▶ underlying not traded - value?
  - ▶ volatility of underlying even more difficult
  - ▶ exercise possibilities may be unclear
- ▶ Underlying project may be difficult to replicate

We look away from practical obstacles

- ▶ assume real options can be valued as financial ones
  - ▶ assume market is, at least locally, complete
  - ▶ and prices are arbitrage free
  - ▶ means projects and their options can be replicated



# The option to defer

- ▶ A very common real option
- ▶ usually discussed as possibility to postpone a project
- ▶ but notice proper option formulation:
  - ▶ is NOT decision to postpone accepted project
  - ▶ but postponement of decision to accept a project until more information is available

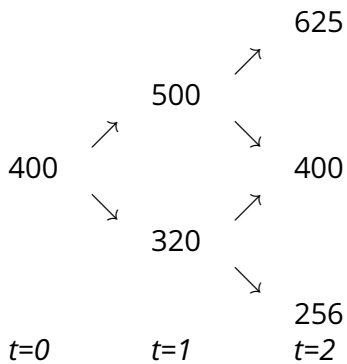
So valuing the option to defer means

- ▶ *valuing the opportunity to do a project!*
- ▶ This potentially includes the value of the project.

# Example

Re-use our old example and re-define it as real option

- ▶ Project is an oil well to be taken in production
- ▶ Size is accurately measured, only price uncertainty
- ▶ Values represent future production from the well, properly discounted (not cash flows)
- ▶ Value depends on oil price
- ▶ develops over time as binomial lattice
- ▶ after two periods price uncertainty is resolved  
⇒ stable values



Binomial tree for the value of an oil well

### Further details:

- ▶ risk free interest rate is 7%.
- ▶ Real probability of upward movement is 80%,
- ▶ Risk adjusted discount rate for oil production from wells like this is 16%.
- ▶ Investment necessary to bring well into production is 375, amount increases with the risk free rate over time
- ▶ Investment is irreversible
- ▶ Production profitable from the start:  $400 - 375 = 25$ , a positive net present value.
- ▶ Firm has exclusive one-period license to develop the well  
⇒ can defer decision to develop with one period

License gives firm a real option:

- ▶ has the right, but no obligation, to develop the well
- ▶ firm has flexibility, or future decision making opportunity, to profit from real option

Project has the ingredients that make options valuable:

- ▶ **Time**  
without time, becomes now-or-never decision as in DCF
- ▶ **Uncertainty**  
without uncertainty, situation in 1 year exactly same as today

Reformulate project as option:

- ▶ License gives the right to 'buy' the oil in the well
  - ▶  $\Rightarrow$  option is a call
- ▶ by paying development costs
  - ▶  $\Rightarrow$  exercise price = 375.
- ▶ License expires in 1 year
  - ▶  $\Rightarrow$  option has a time to maturity of 1 year
- ▶ Volatility follows from binomial process
- ▶ interest rate = 7%

All ingredients necessary to value the option

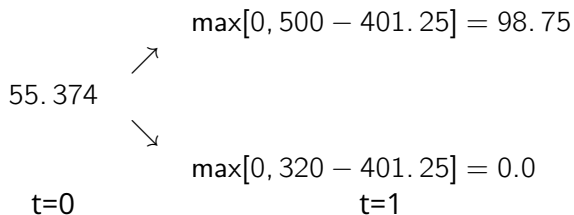
The parameters of binomial process are:

$$u = 1.25, \quad d = .8, \quad r = 1.07$$

$$p = \frac{r - d}{u - d} = \frac{1.07 - .8}{1.25 - .8} = .6 \quad (1 - p) = .4$$

Begin with values at maturity, i.e. at  $t = 1$

- ▶ option value is  $\max[0, S - X]$
- ▶ exercise price is  $375 \times 1.07 = 401.25$ 
  - ▶  $O_u = \max[0, 500 - 401.25] = 98.75$
  - ▶  $O_d = \max[0, 320 - 401.25] = 0.0$



Value of the option is:

$$O = \frac{.6 \times 98.75 + .4 \times 0}{1.07} = 55.374$$



# How can option to defer be more valuable than project itself?

- ▶ Project very profitable in up node but loss making in down node
- ▶ both included in  $t_0$  value of 25.

Real option analysis values the flexibility to avoid losses in down node:

- ▶ wait a period, see how value develops:
  - ▶ If oil price goes up, develop well and profit from opportunity
  - ▶ If oil price goes down, do not develop and avoid loss
- ▶ Don't lose much by waiting one period
  - ▶ oil is still in well
  - ▶ license still valid

That is essence of real option valuation!

# What is wrong with DCF valuation of flexibility?

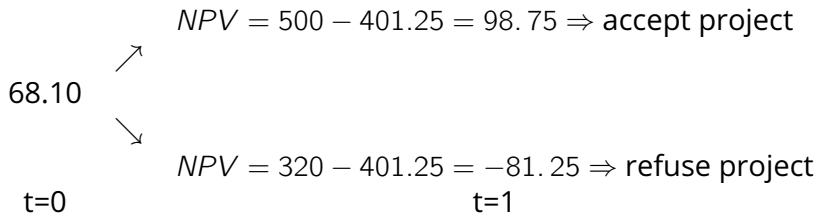
It can be argued:

- ▶ DCF can capture value of flexibility
- ▶ by calculating NPV as if project started one period later
- ▶ decide to abandon project if NPV is negative
- ▶ gives same 98.75 in upper node and 0 in lower node

Result is a **decision tree**

Decision trees are analysed by:

- ▶ weighting branches with real probabilities of .8 and .2
- ▶ discounting expected value at risk adjusted discount rate of 16%



Value of the project opportunity is:

$$\frac{.8 \times 98.75 + .2 \times 0}{1.16} = 68.103$$

Different from 55.374 we found with real options analysis!

1. How can we determine which value is correct?
2. Answer in modern finance: *by making a replicating portfolio*
3. Assume payoff structure in tree can be constructed in market
  - ▶ here we need assumption of locally complete market

Option's delta and  $D$  are:

$$\Delta = \frac{O_u - O_d}{(u - d)S} = \frac{98.75 - 0}{500 - 320} = 0.5486 \text{ and}$$

$$D = \frac{uO_d - dO_u}{(u - d)r} = \frac{1.25 \times 0 - .8 \times 98.75}{1.25 \times 1.07 - .8 \times 1.07} = -164.07$$

At time  $t_1$  portfolio pays off either:

$$(0.5486 \times 500) - 164.07 \times 1.07 = 98.745 \text{ or}$$

$$(0.5486 \times 320) - 164.07 \times 1.07 = 0$$

So payoff pattern is replicated.

Value of portfolio now is

$$(0.5486 \times 400) - 164.07 = 55.37$$

Real option value correct:

- ▶ no rational investor pays 68.10
- ▶ for payoff pattern that can be replicated for 55.37

## Where does discounted cash flow approach go wrong?

- ▶ The error we made was:
  - ▶ applying risk adjusted discount rate for oil production from the well
  - ▶ to the opportunity to develop the well.
- ▶ The **opportunity to do a project** seldom has same risk as **project itself**,
- ▶ precisely because flexibility embedded in opportunity is used to change the risk:
  - ▶ upward potential is enhanced
  - ▶ downside risk is reduced

Once well is in production, then:

- ▶ fortunes tied to oil price
- ▶ proper discount rate for cfl's and values is 16%
- ▶ Values move through time with uncertainty of
  - ▶ up factor of 1.25
  - ▶ down factor of 0.8

Opportunity to do project has much larger uncertainty

- ▶ moves through time from 25 at  $t_0$ 
  - ▶ to either 98.75  $\Rightarrow$  up factor 3.95
  - ▶ or 0  $\Rightarrow$  down factor 0.

In principle, correct option value can be calculated using:

- ▶ real probabilities of 0.8 and 0.2
- ▶ risk adjusted discount rate

But that rate must be calculated from replicating portfolio:

- ▶  $\Delta S = 0.5486 \times 400 = 219.44$  in twin security S
- ▶ risk free loan of -164.07.

Gives weighted average portfolio return of:

$$\frac{219.44}{219.44 - 164.07} \times 0.16 + \frac{-164.07}{219.44 - 164.07} \times 0.07 = 0.427$$

i.e. > 2.5% discount rate for the project



Discounting exp. payoff with this rate gives correct value:

$$\frac{0.8 \times 98.75 + 0.2 \times 0}{1.427} = 55.36$$

However, with  $\Delta S$  and  $D$ , we already know the option value:

$$O = \Delta S + D = 219.44 - 164.07 = 55.37$$

Boils down to calculating rate *given* option value:

$$55.37 = (0.8 \times 98.75)/r_{adj} \Rightarrow r_{adj} = 1.427$$

Moreover:

- ▶  $\Delta$  and  $D$  likely to differ between nodes
- ▶ must be calculated for each node in tree
- ▶ makes correct use of decision trees highly impractical