TIØ4146 Finance for Science and Technology Students

Chapter 9 - Real Options Analysis (part 2)

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Follow-up investments

- Often projects generate follow-up investments which give advantage over competition
- clear real option value:
  - re-use gives lower exercise price
  - market base gives higher payoff
  - than competitors starting from scratch
- examples are abundant:
  - MicroSoft DOS, Windows -95. -NT, -XP, -7, ..
  - Intel 8086, 80286, 80386, 80486, Pentium, ..

To illustrate, adapt our old example in three ways:

1. Technology project instead of oil well
2. Investment required is 450
3. Project can be repeated on double scale after 2 periods
   - first project is prerequisite for follow-up e.g. extension of technology
   - invest $2\times$ exercise price, get $2\times$ market value
   - risk free rate is 7%
   - as before exercise price increases with risk free rate:
     - investment is $2 \times 450 \times 1.07^2 = 1030.4$
4. Further details as before:
   - real probabilities are .8 and .2 and risk adjusted discount rate 16%

$$
\begin{array}{ccccccc}
 & & & & 625 & & \times 2 = 1250 \\
 & & & \nearrow & & & \\
 & & 500 & & & & \\
 & \nearrow & & \searrow & & & \\
400 & & & & 400 & & \times 2 = 800 \\
 & \searrow & & \nearrow & & & \\
 & & 320 & & & & \\
 & & & \searrow & & & \\
 & & & & 256 & & \times 2 = 512 \\
t{=}0 & & t{=}1 & & t{=}2 & &
\end{array}
$$

- Gives initial project negative NPV: $400 - 450 = -50$
- Follow-up project is loss-making in DCF terms!

$$(.8^2) \times 1250 + (2 \times .8 \times .2) \times 800 + (.2^2) \times 512 = 1076.5$$

$$\text{PV} = \frac{1076.5}{1.16^2} = 800 \text{ so the NPV} = 800 - (2 \times 450) = -100$$

- Real options analysis gives different picture:
  - models flexibility to profit from favorable market developments
  - follow-up investment is an out-of-the-money call option
  - only exercised if profitable
  - but out-of-the-money options are valuable
- The parameters of the binomial process are as before:

$$u = 1.25, \quad d = .8, \quad r = 1.07$$

$$p = \frac{r-d}{u-d} = \frac{1.07-.8}{1.25-.8} = .6 \quad (1-p) = .4$$

The follow-up option matures after 2 periods, at *t* = *2*

- exercise price is $2 \times 450 \times 1.07^2 = 1030.4$

$$
\begin{array}{ccccc}
 & & & & \max[0, 1250 - 1030.4] = 219.6 \\
 & & & \nearrow & \\
 & & 123.14 & & \\
 & \nearrow & & \searrow & \\
69.05 & & & & \max[0, 800 - 1030.4] = 0.0 \\
 & \searrow & & \nearrow & \\
 & & 0 & & \\
 & & & \searrow & \\
 & & & & \max[0, 512 - 1030.4] = 0.0 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}
$$

Option value of follow-up project

Value of the option is

- at t=1: $(.6 \times 219.6)/1.07 = 123.14$
- at t=0: $(.6 \times 123.14)/1.07 = 69.05$

The follow-up opportunity is so valuable

- that it gives the whole project NPV$>$0
- $-50 + 69.05 = 19.05$

Makes it a classic among real options

- particularly valuable in volatile (fast-growing) markets
- used to be called 'strategic value'
- can now be priced properly!

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Abandonment Option

- No need to continue loss-making projects
- assets can be sold, used alternatively
- (cf. general purpose assets in bankruptcy)
- gives higher 'bottom' in project value

Abandonment option can be modelled in various ways

- separate tree for second hand value
  - lower starting point, less volatile
  - primary / secondary values cross in down nodes
  - more profitable to abandon
- simpler: fixed second hand value

Extend our example with abandonment option

- possibility to sell project's assets second hand at any time for a fixed price of 325 (formulate an an American put)
- Valued by including exercise condition:

$$\max[continue,\ abandon]$$

  in all nodes of the value tree
- Recall: tree contains project values, not cash flows $\Rightarrow$ exercise condition also in last node

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Starting in the end nodes:

$$\begin{tikzcd}
 & & \max[625, 325] = 625 \\
 & \max[\mathit{alive}, \mathit{dead}] \arrow[ur] \arrow[dr] & \\
\max[\mathit{alive}, \mathit{dead}] \arrow[ur] \arrow[dr] & & \max[400, 325] = 400 \\
 & \max[\mathit{alive}, \mathit{dead}] \arrow[ur] \arrow[dr] & \\
 & & \max[256, 325] = 325
\end{tikzcd}$$

t=0 t=1 t=2

Option exercised lower node t=2 (alive $<$ dead)

Then we calculate the values alive at t=1:
upper node is, of course:

$$\frac{.6 \times 625 + .4 \times 400}{1.07} = 500$$

lower node:

$$\frac{.6 \times 400 + .4 \times 325}{1.07} = 345.79$$

and compare them with the values dead:

$$
\begin{tikzcd}
 & & 625 \\
 & \max[\underbrace{500}_{alive},\underbrace{325}_{dead}] = 500 \arrow[ur] \arrow[dr] & \\
\max[409.64, 325] \arrow[ur] \arrow[dr] & & 400 \\
 & \max[\underbrace{345.79}_{alive}, \underbrace{325}_{dead}] = 345.79 \arrow[ur] \arrow[dr] & \\
 & & 325
\end{tikzcd}
$$

t=0 t=1 t=2

t=0 value found by repeating procedure:

$$\frac{.6 \times 500 + .4 \times 345.79}{1.07} = 409.64$$

and checking the t=0 values dead and alive:
$\max[409.64, 325] = 409.64$

- Value without abandonment option is 400
- flexibility to abandon has value of 9.64

Option value can also be calculated separately:

- American put with an exercise price of 325
- exercise condition is $\max[0, 325 - \textit{project value}]$

Values at t=2 are:

- $\max[0, 325 - 625] = 0$
- $\max[0, 325 - 400] = 0$
- $\max[0, 325 - 256] = 69$

The t=1 value alive is

$$\frac{.6 \times 0 + .4 \times 69}{1.07} = 25.794$$

and the t=0 value alive is

$$\frac{.6 \times 0 + .4 \times 25.794}{1.07} = 9.64$$

$$
\begin{array}{ccccc}
 & & & & 0 \\
 & & & \nearrow & \\
 & & \underbrace{0.0}_{\textit{alive}},\ \underbrace{\max[0, 325-500]}_{\textit{dead}} & & \\
 & \nearrow & & \searrow & \\
\underbrace{9.64}_{\textit{alive}},\ \underbrace{\max[0, 325-400]}_{\textit{dead}} & & & & 0 \\
 & \searrow & & \nearrow & \\
 & & \underbrace{25.794}_{\textit{alive}},\ \underbrace{\max[0, 325-320]}_{\textit{dead}} & & \\
 & & & \searrow & \\
 & & & & 69 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}
$$

Phasing investments

- Firms may not commit themselves to entire project at once but to successive stages, one at a time. This is very common for certain types of projects.
- Construction industry:
  - preparation phase: licenses, groundwork
  - construction phase: building
  - finishing stage: fixtures, plumbing, etc.
- Very pronounced in pharmaceutical research:
  - basic research: search for potential drugs
  - pre-clinical tests (on rats)
  - clinical tests (on humans)
  - approval and production

Notice: option refers to

- *decision* to accept project's next phase
- *not* implementation of already accepted next phase
- means next phase can be rejected, project abandoned

With a project structured in phases:

- each phase is call option on the next
- accepting first phase buys option on second
- second phase buys option on third, etc.

Means they are compound options:

- compound options are options on options
- notice: value of option on 3^{rd} phase included in value of 2^{nd} phase

To illustrate, we adapt, again, our binomial example

- project's investment of 375
- can be made in two stages:
  - 50 now (preparation phase)
  - rest in the next period: $1.07 \times 325 = 347.75$

Option modelled by including in t=1 nodes:

project value = max[0, project value - investment]

Looks obvious in option context, not in practice

Value tree for flexible (phased) project:

$$
\begin{array}{ccccc}
 & & & & 625 \\
 & & & \nearrow & \\
 & & \max[0, 500 - 347.75] = 152.25 & & \\
 & \nearrow & & \searrow & \\
\max[0, 85.37 - 50] = 35.37 & & & & 400 \\
 & \searrow & & & \\
 & & \max[0, 320 - 347.75] = 0 & & \\
 & & & & \\
I = -50 & & I = -347.75 & & \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}
$$

t=0 value found with familiar procedure:

$$\frac{.6 \times 152.25 + .4 \times 0}{1.07} = 85.37$$

$$85.37 - 50 = 35.37$$

Value of flexibility is 10.37, project's value increase from 25

Option can also be modelled separately

- Is the option a call or put? How do we formulate the exercise condition?
- The option is a put
  - by not investing we 'keep' the investment amount
  - and give up the project value
- exercise condition:

$$\max[0, \text{ investment - project value}]$$

Option's value tree becomes

$$
\begin{array}{ccl}
 & & \max[0, 347.75 - 500] = 0 \\
 & \nearrow & \\
10.37 & & \\
 & \searrow & \\
 & & \max[0, 347.75 - 320] = 27.75 \\
 & & I = 347.75 \\
\text{t=0} & & \qquad\qquad \text{t=1}
\end{array}
$$

Option value is:

$$\frac{.6 \times 0 + .4 \times 27.75}{1.07} = 10.37$$

Option has counter-intuitive elements:

- Why only consider investment for next stage?
  - we know later investments are required
  - why not include them in exercise decision?

That is what DCF does!

Real option analysis does not ignore later investments

- included as exercise prices of later options
- determine value of later options
- but exercise decision made later
- at expiration, when more information is available

Why not include previous investments in decision?

- If the project is abandoned
- would not they be wasted?

Is ’sunk cost fallacy’

- if previous investments are irreversible (they usually are)
- they are wasted already

If much is already invested, will not a small extra investment produce large project?

- that is precisely what Real Options Analysis models
- but project should be large in future cash flows, not past investments

Defaulting a loan

Can easily be included in our binomial example. Assume initial investment of 375 financed with:

- a zero coupon loan of 300
- an equity investment of 75
- nominal interest rate 9.5%
- matures after 2 periods
- payment of 359.71 (300×1.095^2)
- no interest payments in between

Owners have the option to default, and they will do so when

$$\text{project value} < \text{debt obligations}$$

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$$
\begin{tikzcd}[column sep=small, row sep=small]
 & & \max[0, 625 - 359.71] = 265.29 \\
 & 163.82 \arrow[ur] \arrow[dr] & \\
100.31 \arrow[ur] \arrow[dr] & & \max[0, 400 - 359.71] = 40.29 \\
 & 22.59 \arrow[ur] \arrow[dr] & \\
 & & \max[0, 256 - 359.71] = 0 \\
t=0 & t=1 & t=2
\end{tikzcd}
$$

The value of levered equity

Loan is defaulted lower node at t = 2

t = 1 values calculated as usual:

$$\frac{.6 \times 265.29 + .4 \times 40.29}{1.07} = 163.82 \quad \frac{.6 \times 40.29 + .4 \times 0}{1.07} = 22.59$$

so that t = 0 value is

$$\frac{.6 \times 163.82 + .4 \times 22.59}{1.07} = 100.31$$

Project's NPV is $100.31 - 75 = 25.31$ as before (allowing 0.31 rounding)

Value of debt calculated in same way:

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$$
\begin{tikzcd}[row sep=small]
 & & 359.71 \\
 & 336.18 \arrow[ur] \arrow[dr] & \\
299.69 \arrow[ur] \arrow[dr] & & 359.71 \\
 & 297.41 \arrow[ur] \arrow[dr] & \\
 & & 256 \\
t=0 & t=1 & t=2
\end{tikzcd}
$$

The value of debt

- Payment in lower node t = 2 is remaining project value
- lower than promised payment of 359.71

Total project value remains $100.31 + 299.69 = 400$

- changing capital structure does not add value
- divides it differently

We can calculate effective market interest rate for risky loan:

- calculate expected payoff with real probabilities:

$$.8^2 \times 359.71 + 2 \times .8 \times .2 \times 359.71 + .2^2 \times 256 = 355.56$$

- then solve $355.56/r^2 = 300$ for r, gives r=1.089

Does this mean that option to default has no value?
NO! option redistributes value

To illustrate, first calculate value of option to default separately

- reformulate option as a put
- owners can 'sell' project to lenders
- by keeping amount due to the lenders
- exercise condition is

$$\max[0, \textit{amount due} - \textit{project value}]$$

- option only exercised in lower node
- value at exercise is $\max[0, 359.71 - 256] = 103.71$

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$$
\begin{array}{ccccc}
 & & & & \max[0, 359.71 - 625] = 0.0 \\
 & & & \nearrow & \\
 & & 0 & & \\
 & \nearrow & & \searrow & \\
14.49 & & & & \max[0, 359.71 - 400] = 0.0 \\
 & \searrow & & \nearrow & \\
 & & 38.77 & & \\
 & & & \searrow & \\
 & & & & \max[0, 359.71 - 256] = 103.71 \\
\text{t=0} & & \text{t=1} & & \text{t=2}
\end{array}
$$

The value of the option to default

- The t=0 value is $(.4^2 \times 103.71)/1.07^2 = 14.49$
- Without default option, loan would be risk free, and thus the value would be $359.71/1.07^2 = 314.18$

General conclusions:

- difficult to capture dynamic aspects of risk induced by flexibility with discounted cash flow approach
- discounted cash flow best suited for passive attitude:
  - accept the cash flows, positive and negative, as given
  - without exploring the possibilities to change them
- Adapting approach to include flexibility (decision trees):
  - makes original discount rate useless
  - very cumbersome to calculate a new one
- Verdict on decision trees has to be:
  - they are outdated
  - should not be used for investment problems

Real option pricing is the proper approach to valuing flexibility.