TIØ4146 Finance for Science and Technology Students

Chapter 7 - Options as Securities

Carlos Miguel Dos Santos Oliveira
Ståle Størdal
Felipe Van de Sande Araujo

Norwegian University of Science and Technology

Option characteristics

Option positions

Arbitrage bounds on option prices

Options

Financial contracts that give their holders:

- the right, but not the obligation, to buy or sell something
- on a future date at a price determined today

Distinction between right and obligation, which gives the holder a choice, is the essential characteristic

Practical use of options is very old:

- oldest sources go back to Thales of Miletus (62?-546 BC), as related in Aristotle's (384-322 BC) work *Politics*
- in the 1600's options on rice were traded in Japan and options on tulips in The Netherlands

Use is very old, organised trade is not:

- First option exchange opened in 1973: Chicago Board of Options Exchange (CBOE)
- 1978: Standardized options trade in Europe (European Option Exchange (EOE), Amsterdam)

Enormous growth since, exchanges make trading easy by:

- operating clearinghouses:
  - transactions are properly effectuated
  - payment guaranteed (no counterparty risk)
- standardization of contracts w.r.t.
  - quantity: options on 100 shares, bonds, ounces gold
  - expiration dates
  - exercise prices

Example of standardization

- All Apple call and put options on Nasdaq maturing in:
  - February 2013 expire on the 16th
  - April 2013 expire on the 20th
- All these Apple call and put options on Nasdaq have exercise prices:
  - ranging from \$550 to \$670 in steps of \$5
  - but NO values in between
- Most of them are actively traded
  (situation mid-October 2012, price AAPL is $\pm$\$610)

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- Standard options are available on stocks, bonds, gold, silver, foreign currencies, stock-indices, etc.
- For special, large deals tailor made options can be negotiated with banks

Some option terminology:

- Call option
  - right to *buy* 'something' (the underlying)
  - at specified price (=*exercise* or *strike* price)
    - *on* a specified date (exercise date or maturity): *European* call option
    - *before* a specified date (exercise date or maturity): *American* call option
- Put option
  - right to *sell* 'something' (the underlying)
  - at specified price (=*exercise* or *strike* price)
    - *on* a specified date (exercise date or maturity): *European* put option
    - *before* a specified date (exercise date or maturity): *American* put option

Rights and obligations attached to options:

	Buyer (long position)	Seller (short position)
call options	right to buy	obligation to sell
put options	right to sell	obligation to buy

Exercise possibilities also called *style:*

- *American-style* and *European-style* options
- not geographical
- most traded options are American, also in Europe

There are many different kinds of options:

- ordinary 'plain vanilla' European and American options

Plus wide range of *exotic* options, e.g.:

- Bermudan options:
  American options with limited number of exercise dates
- Asian options
  payoff depends on average price underlying
- Barrier options
  payoff depends on whether underlying reaches threshold level
  - Knock-out options cease to exist if threshold is reached
  - Knock-in options come into existence
- Binary options
  cash-or-nothing call pays fixed amount if underlying ends in the money

Some more terminology:

- Price of an option is also called the *option premium*.
- To sell an option is also called to *write* an option
- *Moneyness* describes the value of an option if it would be exercised immediately:

moneyness	Call	Put
in the money	$underlying > strike$	$underlying < strike$
at the money	$underlying = strike$	$underlying = strike$
out of the money	$underlying < strike$	$underlying > strike$

Example of an option

- You have bought European call on a share of Apple, strike price \$600, maturity February 16
- Gives you the right, not the obligation, to buy that share on that date at that price
  - If share price of Apple on 16th of February $>$ \$600
    - you *exercise* the option (you have to *do* something)
    - and earn difference between share price and strike
  - If share price of Apple on 16th of February $<$ \$600
    - you will *not* exercise the option (do nothing): let it expire worthlessly.
- European put (long) gives you comparable right to sell
- With American options (long) you can do same things, but on any date before maturity

Main economic characteristics

- A long option is a *limited liability* investment:
  - gives the *right,* not the *obligation* to buy/sell
- Economically options represent *flexibility:*
  - possibility to choose best alternative
  - walk away from bad outcomes
- Also found in real investments (real options)
  - flexibility to change cash flows
  - profiting from opportunities, cutting off losses
  - DCF assumes passive, not flexible, position: accept cash flows as they come
  - DCF cannot handle flexibility well
- Options are almost always riskier than underlying values

Some more option characteristics:

- *redistribute risk* at market prices
- are *zero sum game:* one's losses are someone else's profits
- Reasons to use options:
  - to insure against
  - to profit from
  - to speculate
- Insure against e.g. a fall in stock price: buy a put:
  - gives 'bottom' in price
  - disadvantages: temporary and expensive
- Reverse position: selling a put
  - is like collecting insurance premiums without proper insurance
  - very risky: only use when really want to buy
- Speculation possibilities enhanced by leverage effect (need less money to control large positions)

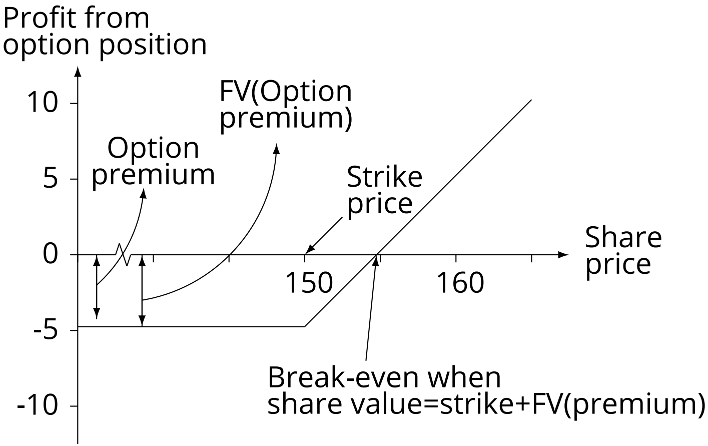
Example: speculating with options

- Mid-October call options on Apple shares, strike price 625, maturity November 17, cost $\pm\$25$
- Share price Apple same date is $\pm\$625$
- $625 buys you 1 share or 25 options
  - If, on November 17, the price of Apple share = $675
    - investment in share pays off $(675 - 625)/625 = 0.08$ or 8%
    - investment in options pays off $((25 \times (675 - 625)) - (25 \times 25))/625 = 1$ or 100% you have doubled your money
  - If on November 17 the price of Apple share = $610
    - investment in share pays off $(610 - 625)/625 = -0.024$ or -2.4%, you have 97.6% left
    - investment in options pays off $((25 \times 0) - (25 \times 25))/625 = -1$ or -100%, you have lost your whole investment

Option positions

Advantages and disadvantages of holding positions in options given in diagrams

- Usually depicted at maturity in:
  - *payoff diagrams* (or position diagrams) ignoring premium
  - *profit diagrams* including premium
- Option positions can be:
  - simple (or naked) option positions (1 option)
  - combined with other options and securities in:
    - strips, straps, straddles, spreads, butterflies, etc.



Profit diagram for a call, strike=150, $O_c = 4.5$

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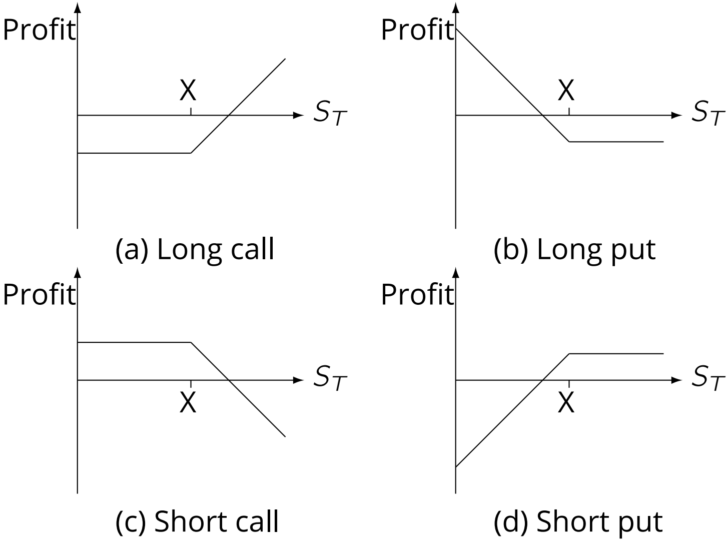
The profit diagram shows that:

- Option will be exercised if, at maturity:
  stock price > 150
- Option only earns money if:
  stock price $>$ exercise price + future value option premium
- Use future value to account for time value of money
  - owner could have earned interest on option premium
  - or borrowed money to buy option $\rightarrow$ has to pay back with interest

Normally, profit diagrams less detailed but reflect option premium

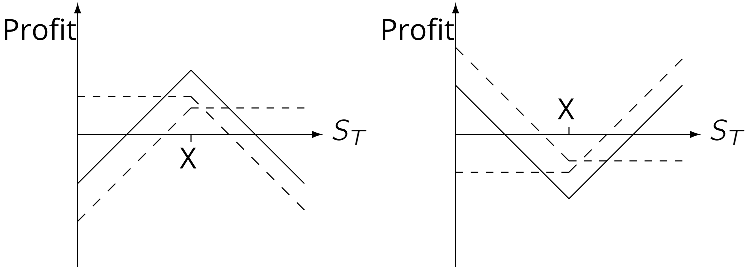


Profit from simple option positions as a function of the share price at maturity, S_T, and the strike price, X

Combined option positions

Straddles are combinations of options that are constructed as bets on volatility

- *Long straddle* is long put + long call with same strike
  - profits from large price changes
  - e.g. important news expected, but nature of the news (good or bad) unknown
- *Short straddle* is short put + short call with same strike
  - profits from small price changes
  - no news expected, collect double premium, but possibly large loss if expectation is wrong
  - (Nick Leeson sold those before Kobe earthquake)

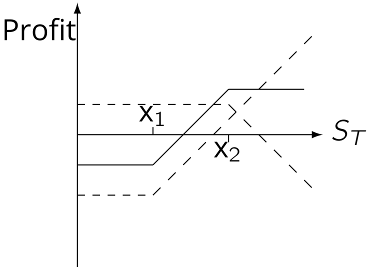


(a) Short straddle (b) Long straddle

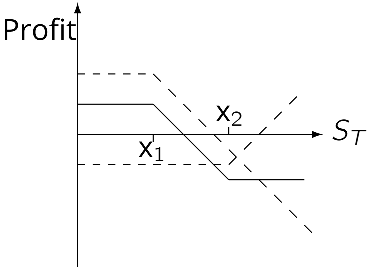
Profit diagrams for straddles

Spreads are limited bets on stock price movements

- A *bull spread* bets on increasing stock price
  - long call and short call with higher strike on same stock
  - short call cheaper $\Rightarrow$ initial balance is negative
- A *bear spread* bets on decrease in stock price
  - long and short positions reversed, lower strike sold, higher strike bought
  - initial balance of option premiums positive
- Payoffs limited on up- and downside
- Price and riskiness vary with moneyness:
  - out of the money calls $\Rightarrow$ cheap, low prob. of payoff
  - in the money calls $\Rightarrow$ less risky, more expensive



(a) Bull spread



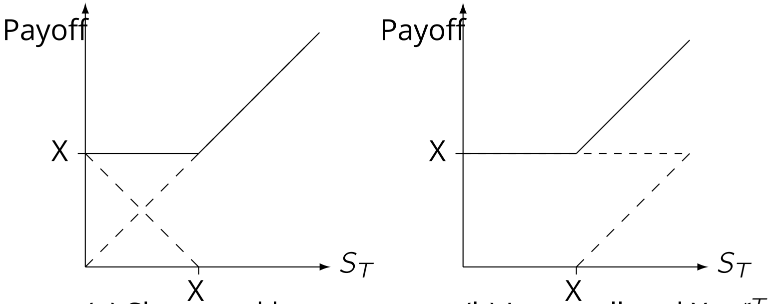
(b) Bear spread

Profit diagrams for spreads

Next picture shows 2 *payoff* diagrams

$\Rightarrow$ option premia not depicted, only values at maturity

1. Payoff of a share and long put
    - the protective put we saw earlier
    - gives a floor in combined position
2. Payoff of a call and the PV(strike) invested risk free
    - risk free investment gives floor in position
    - upward potential from long call



(a) Share and long put (b) Long call and Xe^{-rT}

Payoff diagrams for the put-call parity

Payoffs at maturity are the same:

- $\Rightarrow$prices of combinations have to be the same

$$\begin{aligned} share + long\ \ put &= long\ \ call + pv(x) \\ long\ \ put &= long\ \ call + pv(x) - share \end{aligned}$$

Expression for put, called *put-call parity*

- only valid for European options on stocks that don't pay dividends
- parity relation invalidated by:
  - early exercise
  - dividend payments

Bounds on option prices

- Simple arbitrage arguments limit the range of option prices
- Only assumption made is greedy investors $\Rightarrow$ bounds are independent of pricing model
- But all pricing models must stay within these bounds to be acceptable
- Bounds formulated for stock options:
  - S is stock price, X is strike price, T maturity
  - have wider validity
- Some bounds formally proven with arbitrage portfolio
- Most should be intuitively clear, and should give good intuition of option prices

Bound 1 A call option cannot be worth more than the stock

Intuition: Call gives right to buy stock, cannot be worth more than stock itself, obviously!

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Proof.

if $O_c > S$, writing covered call is arbitrage opportunity:

- selling the call and buying the stock
  - gives positive cash flow now: $O_c - S > 0$
  - and at maturity or exercise:
    - either strike price (if exercised)
    - or stock value (if not exercised)
    - both

      0
- positive cash flow now + later is money machine

$\square$

Bound 2 A put option cannot be worth more than the strike price

Proof/intuition

- Put gives the right to sell stock for the strike price
- cannot be worth more than strike

Bound 3 A European put cannot be worth more than the present value of the strike price

Proof/intuition

- Put at maturity not worth more than strike
- European put cannot be exercised early
- hence value now cannot be higher than pv(strike)

Bound 4 The minimum value of a European call option, O_c, on a stock that pays no dividends is $\max[0, S - PV(X)]$

If $0 < O_c < S - PV(X)$ this arbitrage strategy exists:

- buy the call, short sell the stock and lend PV(X)
- costs $-O_c + S - PV(X) > 0$ if bound is violated
- payoff of the option at maturity:
  - $(S_T - X)$ if exercised (i.e. if $S_T > X$)
  - 0 if not exercised (i.e. if $S_T < X$)
- payoff of the short stock at maturity: $-S_T$
- payoff of lending at maturity: X
- Total payoff:
  - $(S_T - X) - S_T + X = 0$ if exercised
  - $0 - S_T + X > 0$ if not exercised

Arbitrage strategies usually summarized in tables:

Proof.
If $0 < O_c < S - PV(X)$
then following arbitrage possibility exists:
buy call, shortsell stock and lend PV(strike)

	Now	At expiration	
		$S_T > X$	$S_T < X$
Buy call	$-O_c$	$+(S_T - X)$	0
Sell stock	$+S$	$-S_T$	$-S_T$
Lend $PV(X)$	$-PV(X)$	X	X
Total position	> 0	0	$-S_T + X > 0$

$\square$

Positive cash flow now and either no or positive cash flow later is money machine

Bound 5 The minimum value of an American call option on a stock is $\max[0, S - X]$

Proof/intuition

- American call can be exercised immediately, which gives $S - X \Rightarrow$ option value cannot be less

Bound 6 An American call option is worth at least as much as a comparable European call option

Proof/intuition

- With an American call you do everything that you can do with a European call, plus exercise early
- the right to exercise early cannot have negative value

Together, these imply an exercise bound on American call options:

Bound 7 An American call option on a stock that pays no dividends will not be exercised before maturity

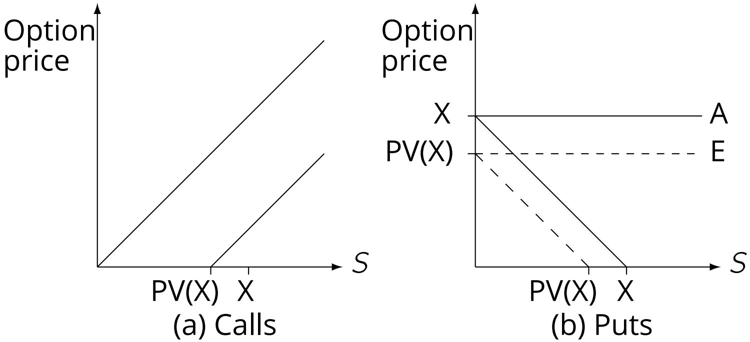
Proof.
By the previous bounds:

- American call $\geq$ European call $\geq \max[0, S - PV(X)]$
- If exercised now, American call is $\max[0, S - X]$
- since $X > PV(X) \Rightarrow [S - X] < [S - PV(X)]$

$\square$

Intuition:

- Exercise now $\Rightarrow$ paying now $\Rightarrow$ give up interest on X, end up with same share, + accept risk that price falls below X
- Option worth more alive than dead, so sell, don't exercise



Arbitrage bounds on option prices before maturity, non-dividend paying stocks; A=American, E=European

Figure also shows why option pricing modelling is difficult:

- Option prices cannot be negative
  - limited liability investment
- We know one call option price: (similar set for puts)
  - if stock price drops to zero, cannot become > 0 later
  - call option price then also has to be zero
- Means option price cannot be linear function of underlying
  - slope $> 45^\circ$ would cross upper limit
  - slope $< 45^\circ$ would cross lower limit
  - slope of 45° would mean option price = stock price for all strikes, maturities, etc.
- Early models failed to stay in bounds

Black and Scholes provided first correct model