Example: Put option of European style when the stock pays out dividends

Let's consider the book's example:

$S = 400$ (stock current price)

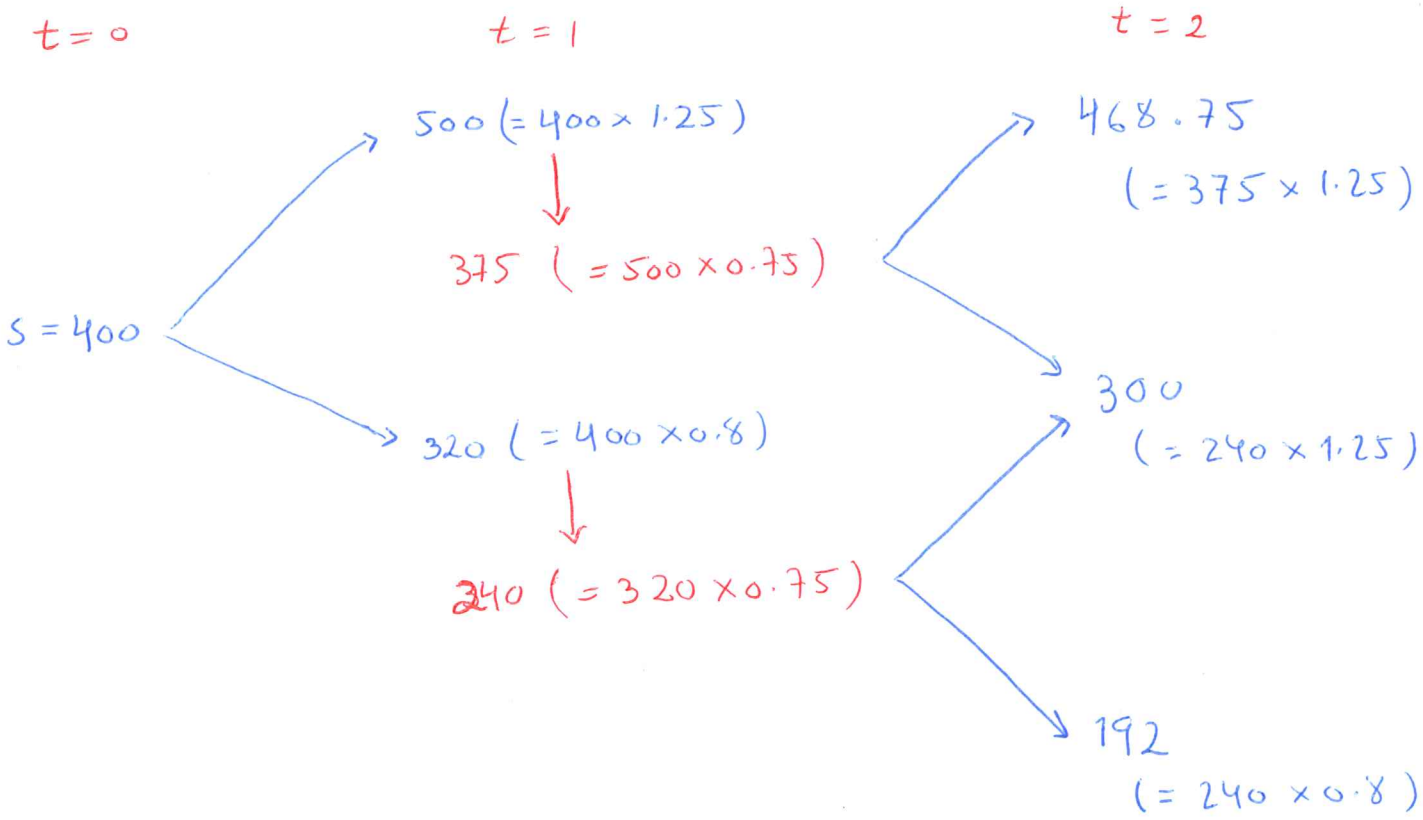
$u = 1.25$ (up factor)

$d = 0.8$ (down factor)

$r = 1.04$

$X = 375$

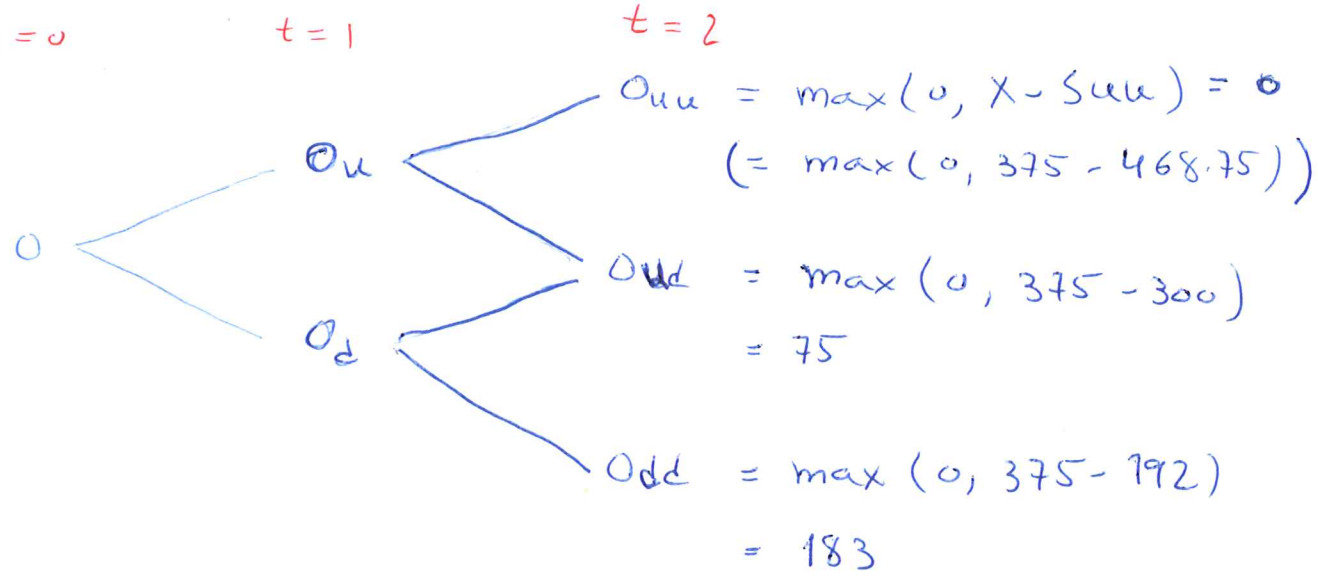
The stock pays out 25% of its value at t_1



$$p = \frac{r - d}{u - d} = 0.6$$

$$(1 - p) = 0.4$$

Let O be the price of the European put option



$$O_u = \frac{0 \times 0.6 + 75 \times 0.4}{1.07} = 28.04$$

$$O_d = \frac{75 \times 0.6 + 183 \times 0.4}{1.07} = 110.47$$

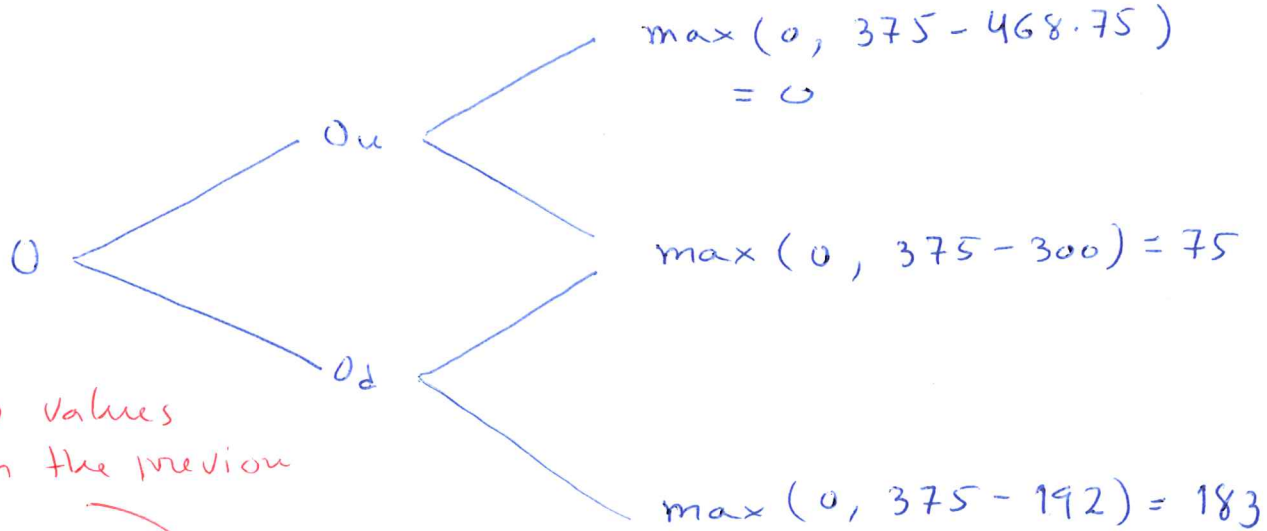
$$O = \frac{28.04 \times 0.6 + 110.47 \times 0.4}{1.07} = 57.02$$

The option value is now greater when compared with the option value for the stock that does not pay dividends (16.63)

• American put option

→ We saw that the owner of an American call option wants to receive the dividends, then the option is exercised right before the dividends are paid.

→ The owner of an American put option has the right to sell the stock at any time up to the maturity. If he/she exercise it when the dividends are paid, it will be right after the dividends are paid (and not before)



These two values come from the previous example. It is the value to continue

$O_u = \max(28.04, 375 - 375) = 28.04$

$O_d = \max(110.47, 375 - 240) = 135$

$O = \max(\text{dead}, \text{alive}) = 66.19$

$\text{dead} = 375 - 400 = -25$

$\text{alive} = \frac{28.04 \times 0.6 + 135 \times 0.4}{1.07} = 66.19$