Digital Image Processing: Chapter 4 Lab 2 Analysis of Image Filtering Using Fourier Transform Methods

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1 Introduction

Frequency domain filtering is on one of the methods used in digital Image processing to enhance or suppress specific details. In this report, we explore various low-pass and band-pass filtering techniques using the Discrete Fourier Transform (DFT). The four exercises implemented include:

- Ideal Low-Pass Filtering (ILPF)
- Gaussian Low-Pass Filtering (GLPF)
- Butterworth Low-Pass Filtering (BLPF)
- Gaussian Band-Pass Filtering (GBPF)

These filters are applied to the grayscale lena.png and house.png images, and observations are made on their impact on blurring, edge preservation, and artifact generation.

2 Exercise 1: Ideal Low-Pass Filtering

In this exercise, an ideal low-pass filter was applied to the Lena image with and without zero-padding.

2.1 Methodology

- Compute the Fourier Transform of the image.
- Generate a binary ideal low-pass filter with a cutoff radius of 15.
- Apply the filter in the frequency domain and reconstruct the image using the inverse Fourier Transform.
- Compare results with and without zero-padding.

2.2 MATLAB Code

```
A = imread('lena_gray_256.png');
figure, imagesc(A), axis image, colormap gray, colorbar;
B = zeros(2*size(A));
B(1:256, 1:256) = A;
FB = fftshift(fft2(B));
[x,y] = meshgrid(-256:255,-256:255);
z = sqrt(x.^2 + y.^2);
ILPF = (z < 15);
FFB = FB .* ILPF;
IFFB = real(ifft2(ifftshift(FFB)));
IFFB = IFFB(1:256,1:256);
figure, imagesc(IFFB), axis image, colormap gray, colorbar;
```

Figure 1: Original Lena Image

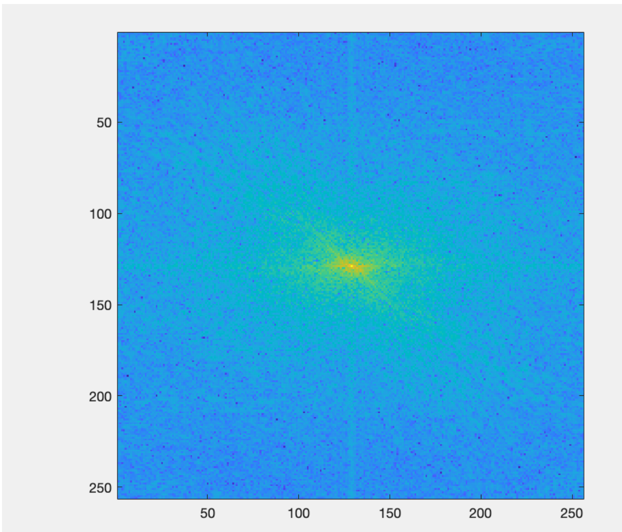

Figure 2: Colored Magnitude Spectrum

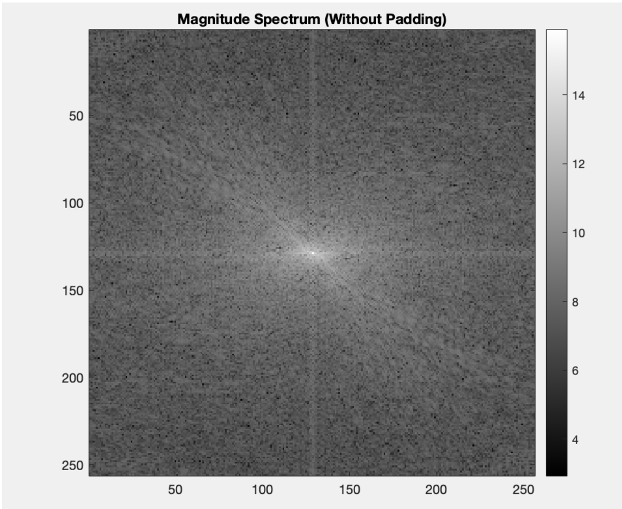

Figure 3: Magnitude Spectrum

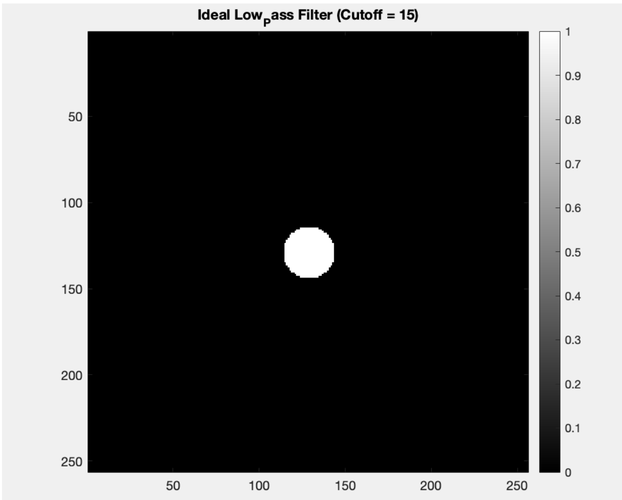

Figure 4: Filtered Spectrum Without Padding

2.3 Observations

- Without zero-padding, strong ringing artifacts are visible due to abrupt truncation in the frequency domain.
- With zero-padding, artifacts are reduced, and smoother filtering is observed.

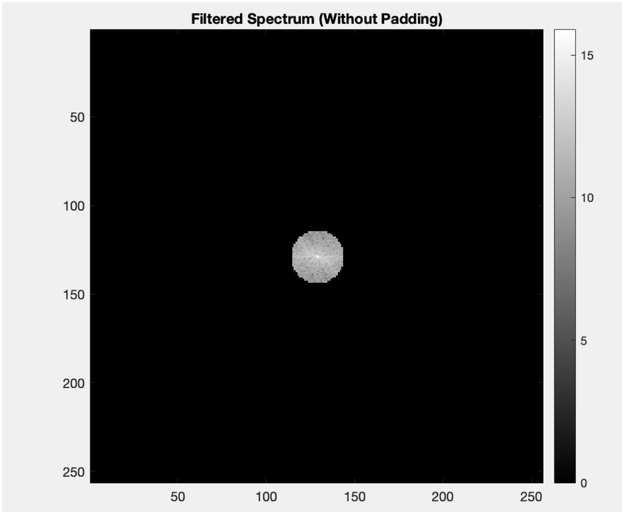

Figure 5: Ideal Low Pass Filter

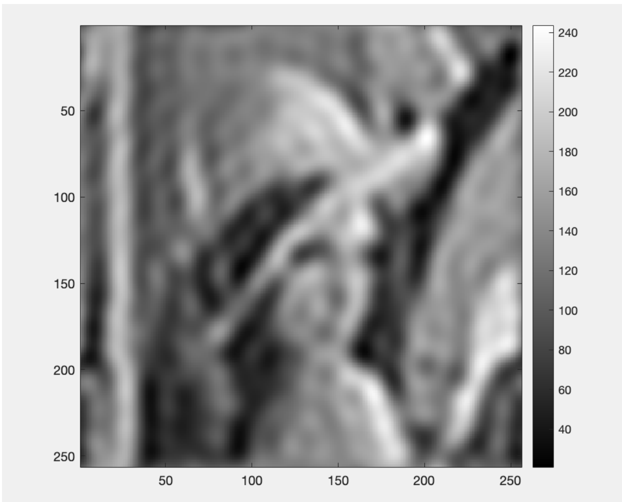

Figure 6: Ideal Low Pass Image of Lena

3 Exercise 2: Gaussian Low-Pass Filtering

Gaussian low-pass filtering was performed with different cutoff radii $D_0 = 30, 50, 100$.

3.1 MATLAB Code

```
A = imread('lena_gray_256.png');
B = zeros(2*size(A));
B(1:256, 1:256) = A;
FB = fftshift(fft2(B));
[x,y] = meshgrid(-256:255,-256:255);
sigma = 50;
GLPF = exp(-(x.^2 + y.^2) / (2 * sigma^2));
FFB = FB .* GLPF;
IFFB = real(ifft2(ifftshift(FFB)));
IFFB = IFFB(1:256,1:256);
figure, imagesc(IFFB), axis image, colormap gray, colorbar;
```

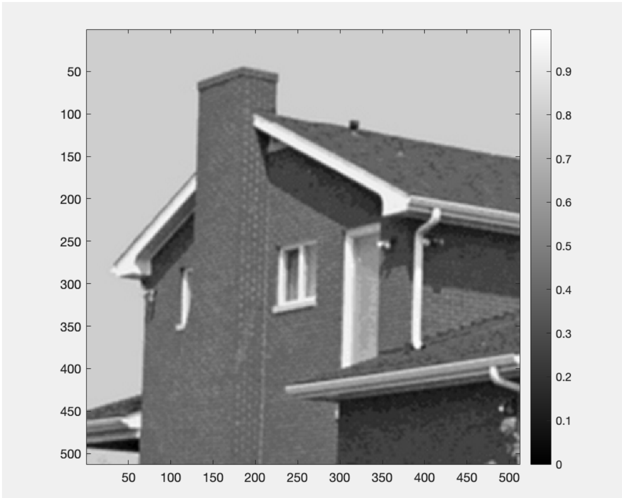

Figure 7: Original House Image

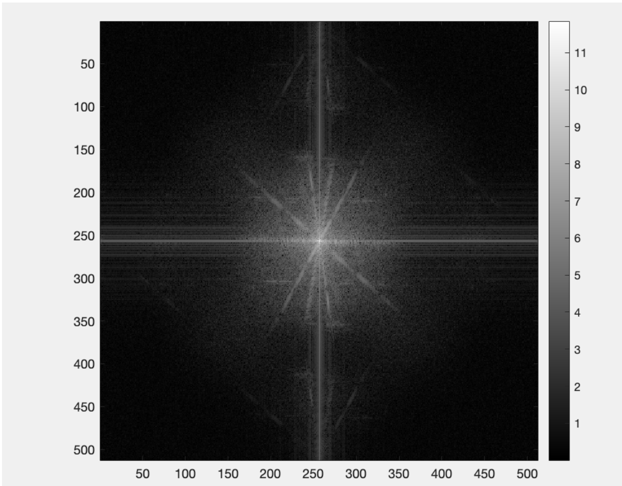

Figure 8: Magnitude Spectrum House Image

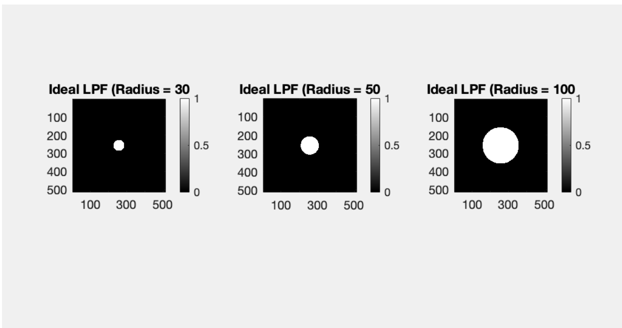

Figure 9: Ideal Low Pass

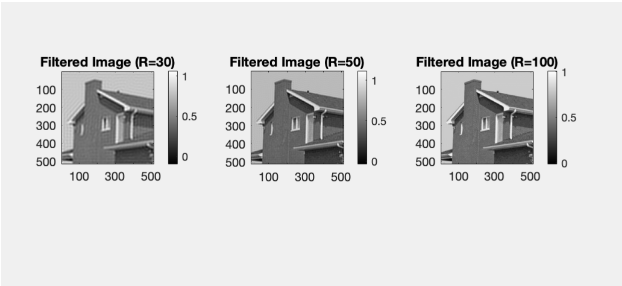

Figure 10: Filtered House Images

3.2 Observations

- Lower cutoff values ($D_0 = 30$) lead to strong blurring and loss of details.
- Higher cutoff values ($D_0 = 100$) retain more high-frequency details while reducing noise.
- Gaussian filters provide smoother transitions compared to ideal low-pass filters, reducing ringing artifacts.

4 Exercise 3: Butterworth Low-Pass Filtering

Butterworth low-pass filtering was applied for $D_0 = 30, 50, 100$, with order $n = 1$.

4.1 MATLAB Code

```
A = imread('lena_gray_256.png');
B = zeros(2*size(A));
B(1:256, 1:256) = A;
FB = fftshift(fft2(B));
[x,y] = meshgrid(-256:255,-256:255);
D = sqrt(x.^2 + y.^2);
D0 = 50; n = 1;
BLPF = 1 ./ (1 + (D / D0).^(2 * n));
FFB = FB .* BLPF;
IFFB = real(ifft2(ifftshift(FFB)));
IFFB = IFFB(1:256,1:256);
figure, imagesc(IFFB), axis image, colormap gray, colorbar;
```

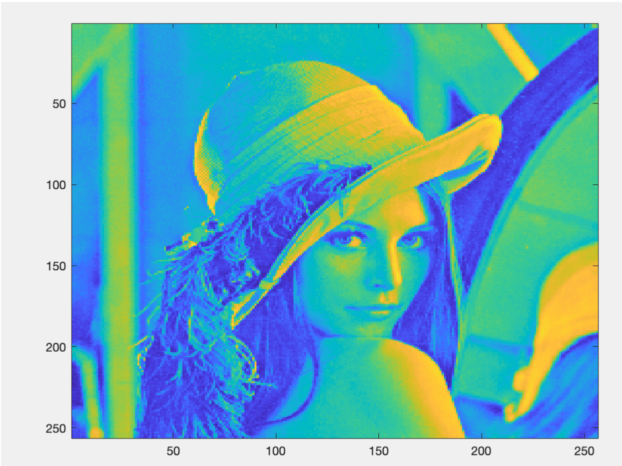

Figure 11: Colored lena Image

Figure 12: Original lena Image

Figure 13

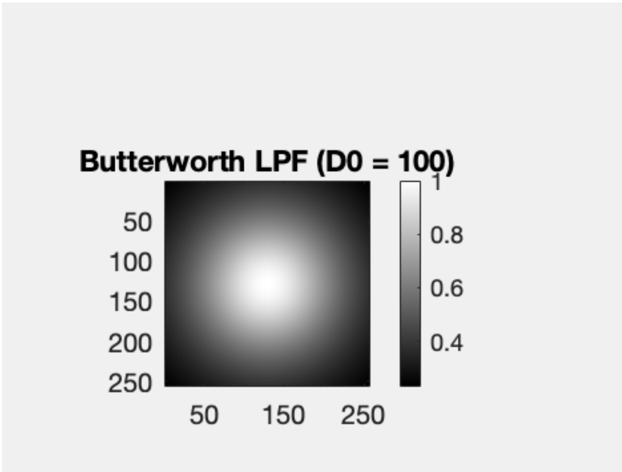

Figure 14: figure

4.2 Observations

- Butterworth filters provide a more gradual transition between passed and attenuated frequencies, reducing artifacts.
- Increasing D_0 allows more high-frequency content, reducing blurring effects.
- Compared to Gaussian LPF, Butterworth filtering allows better control over the sharpness of frequency cut-off.

5 Exercise 4: Gaussian Band-Pass Filtering

Gaussian band-pass filters were applied with multiple center frequencies D_0 and bandwidths W to extract different characteristics from the image.

5.1 MATLAB Code

```
A = imread('lena_gray_256.png');
B = zeros(2*size(A));
B(1:256, 1:256) = A;
FB = fftshift(fft2(B));
[x,y] = meshgrid(-256:255,-256:255);
D = sqrt(x.^2 + y.^2);
D0 = 100;
W = 50;
GBPF = exp(-((D.^2 - D0^2).^2) / (D0^2 * W^2));
FFB = FB .* GBPF;
IFFB = real(ifft2(ifftshift(FFB)));
IFFB = IFFB(1:256,1:256);
figure, imagesc(IFFB), axis image, colormap gray, colorbar;
```

5.2 Observations

- Lower D_0 values enhance low-frequency details and smooth structures.
- Higher D_0 values retain sharp edges and high-frequency textures.
- Narrow bandwidths (small W) isolate specific frequency bands, while wider bandwidths preserve a larger range of details.
- Band-pass filtering allows selective enhancement of image features based on frequency components.

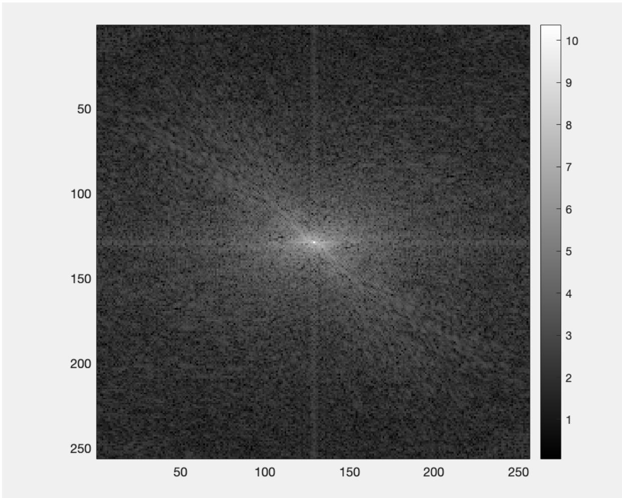

Figure 15: Magnitude Spectrum of lena.png

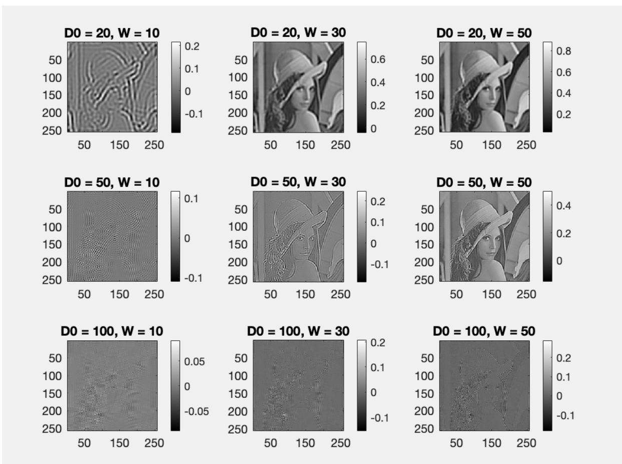

Figure 16: Filtered lena Images with different D and W values

6 Conclusion

This report explored different frequency domain filtering methods using Fourier Transform techniques. The following key conclusions were drawn:

- Ideal low-pass filters provide sharp cutoffs but introduce significant ringing artifacts.
- Gaussian and Butterworth low-pass filters offer smoother transitions, reducing unwanted artifacts.
- Band-pass filtering enables selective enhancement of image components, making it useful for texture analysis.
- Zero-padding improves the performance of filtering by reducing edge artifacts.
- Additional experiments using `lena_color.png` and `house.png` with Gaussian blurring and noise addition demonstrated the sensitivity of frequency domain methods to image content and degradation.

These techniques are widely used in applications such as denoising, edge enhancement, and feature extraction in image processing.