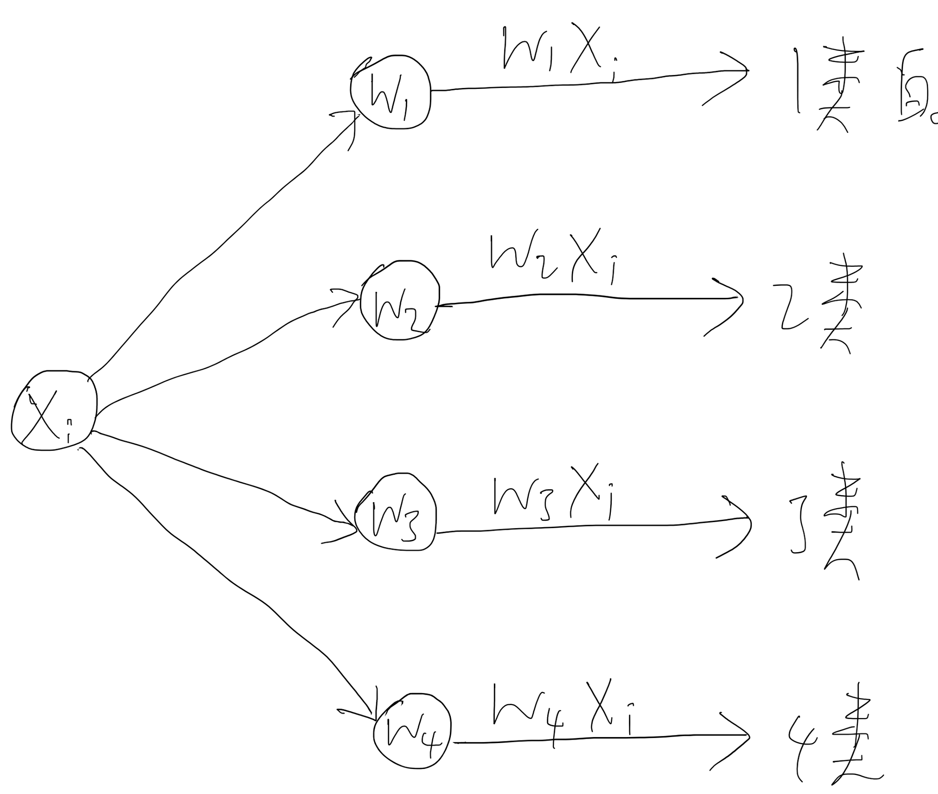
softmax回归

2020年8月14日 星期五　　下午8:29

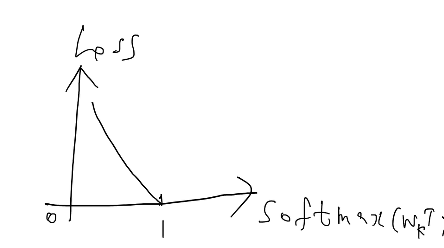
$$p(\underset{Y_c=1}{Y=c}|x) = softmax(w_c^T x) = \frac{\exp(w_c^T x)}{\sum_{i=1}^{C}\exp(w_i^T x)}$$ = 样本x是c类的概率



1类的概率为 $\frac{\exp(w_1^T x_i)}{\sum_{i=1}^{4}\exp(w_i^T x_i)} = softmax(w_1^T x_i)$

某样本x被误分类的损失 $Loss(x) = -\sum_{i=1}^{C} Y_i \log(softmax(w_i^T x))$ $(i=1,2,3,\cdots,C)$ $= -\underbrace{Y_k}_{\text{真实类别为k类}} \log(softmax(w_k^T x)) = -\log(softmax(w_k^T x))$

$$\begin{array}{c|c} \begin{array}{c} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ \vdots \\ Y_C \end{array} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} & \begin{bmatrix} 0.1 \\ 0.09 \\ 0.16 \\ 0.12 \\ 0.03 \\ \vdots \\ 0.04 \end{bmatrix} \begin{array}{c} s(w_1^T x) \\ s(w_2^T x) \\ s(w_3^T x) \\ s(w_4^T x) \\ s(w_5^T x) \\ \vdots \\ s(w_C^T x) \end{array} \end{array}$$



对真实类别的概率p = softmax($w_k^T x$)越接近1，损失越小

总损失 $J(\vec{W}) = \frac{1}{N}\sum_{i=1}^{N} Loss(x_i) = -\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{C} Y_j^{(i)}\log(softmax(w_j^T x_i))$

$$\begin{aligned}
\frac{\partial J(W)}{\partial w_t} &= -\frac{1}{N}\frac{\partial}{\partial w_t}\sum_{i=1}^{N}\sum_{j=1}^{C} Y_j^{(i)}\log\frac{e^{w_j^T x_i}}{\sum_{l=1}^{C} e^{w_l^T x_i}} \\
&= -\frac{1}{N}\frac{\partial}{\partial w_t}\sum_{i=1}^{N}\sum_{j=1}^{C} Y_j^{(i)}\left(w_j^T x_i - \log\sum_{l=1}^{C} e^{w_l^T x_i}\right) \\
&= -\frac{1}{N}\frac{\partial}{\partial w_t}\sum_{i=1}^{N}\left[Y_t^{(i)}\left(w_t^T x_i - \log\sum_{l=1}^{C} e^{w_l^T x_i}\right) + \sum_{j\neq t} Y_j^{(i)}\left(w_j^T x_i - \log\sum_{l=1}^{C} e^{w_l^T x_i}\right)\right] \\
&= -\frac{1}{N}\sum_{i=1}^{N}\left[Y_t^{(i)}\left(x_i - \frac{e^{w_t^T x_i}\cdot x_i}{\sum_{l=1}^{C} e^{w_l^T x_i}}\right) + \sum_{j\neq t} Y_j^{(i)}\left(0 - \frac{e^{w_t^T x_i} x_i}{\sum_{l=1}^{C} e^{w_l^T x_i}}\right)\right] \\
&= -\frac{1}{N}\sum_{i=1}^{N}\left(x_i Y_t^{(i)} - Y_t^{(i)}\frac{e^{w_t^T x_i} x_i}{\sum_{l=1}^{C} e^{w_l^T x_i}} - \sum_{j\neq t} Y_j^{(i)}\frac{e^{w_t^T x_i} x_i}{\sum_{l=1}^{C} e^{w_l^T x_i}}\right) \\
&= -\frac{1}{N}\sum_{i=1}^{N}\left(x_i Y_t^{(i)} - x_i\sum_{j=1}^{C} Y_j^{(i)}\cdot\frac{e^{w_t^T x_i}}{\sum_{l=1}^{C} e^{w_l^T x_i}}\right) \\
&= -\frac{1}{N}\sum_{i=1}^{N}\left(x_i Y_t^{(i)} - x_i\cdot\frac{e^{w_t^T x_i}}{\sum_{l=1}^{C} e^{w_l^T x_i}}\right) \\
&= -\frac{1}{N}\sum_{i=1}^{N} x_i\left(Y_t^{(i)} - \frac{e^{w_t^T x_i}}{\sum_{l=1}^{C} e^{w_l^T x_i}}\right) \\
&= -\frac{1}{N}\sum_{i=1}^{N} x_i\left(Y_t^{(i)} - softmax(w_t^T x_i)\right)
\end{aligned}$$

$\therefore\ W_{t\_new} = W_{t\_old} + \alpha\underbrace{\frac{1}{N}\sum_{i=1}^{N} x_i\left(Y_t^{(i)} - softmax(w_t^T x_i)\right)}$ （第t个类别的w的更新）

$\rightarrow 0.3[3,4] + 0.6[1,7] + \cdots + 0.2[5,2] = [0.3, 0.6, \cdots, 0.2]\begin{bmatrix} (3,4) \\ (1,7) \\ \vdots \\ (5,2) \end{bmatrix}$

正则化：

$$softmax(w_c^T x) = \frac{\exp(w_c^T x)}{\sum_{i=1}^{C}\exp(w_i^T x)}$$

$\frac{e^{(w_c-\psi)^T x}}{\sum_{l=1}^{C} e^{(w_l-\psi)^T x}} = \frac{e^{w_c^T x} e^{-\psi^T x}}{\sum_{l=1}^{C} e^{w_l^T x} e^{-\psi^T x}} = \frac{e^{w_c^T x}}{\sum_{l=1}^{C} e^{w_l^T x}}$，所有w都减去向量ψ后，结果不变