CISS360: Computer Systems and Assembly Language
Assignment a08

Name: _____

Objectives

- Perform integer computations involving number representations in different bases without any restriction on size of the representation.

Some general instructions:

- Once you have unpacked the download, run `make` to check if your LaTeX is missing any files. If your `make` is successful (i.e. no error messages), you should see `main.pdf`. If there's an error, let me know ASAP.
- The main file is `main.tex`. The questions are in the files `q01.tex`, `q02.tex`, etc. You type up your answers in `q01s.tex`, `q02s.tex`, etc. These files are included in `main.tex` by the commands such as `\input{q01.tex}` (sort of like `#include`). So for sure you want to first look at `main.tex`. There are also files names `how-to-*.tex` that gives you examples on how to typeset certain computations in LaTeX. Look at the examples in these files and make full use of the LaTeX code for copy-paste-modify. As in all things, you want to work in "baby steps" – you want to do `make` frequently and check that the pdf generated is OK.
- I encourage discussion and whiteboarding and checking each others work in hangout. Or you can discuss on our discord server in the ciss360 channel. However when you write up the answer, it must be your own work. Sharing of LaTeX work is plagiarism will result in an immediate -1000% .
- Show all your work. All computations should be done by hand, i.e., do not use your TI calculators or any calculators. And then you typeset in LaTeX.
- As for LaTeX specific questions, again goto hangout or chat in discord. You can go to my website http://yliow.github.io and look for `latex.pdf` in the Tutorials section – but it's a lot easier to chat in hangout or discord.
- As usual submit using alex.

LaTeX

In general for regular English text, you just type regular English text. When you want to type math, you can type `$x = 1$` (i.e., enclose your math with `$` symbols). And this is what you see: $x = 1$. Notice that mathematical symbols are then in italics. Or, to emphasize your math you can also do `\[ x = 1 \]` (i.e., enclose in `\[ ... \]`). This is what it looks like:

$$x = 1$$

Notice that this math sentence is then centered in a separate paragraph.

Most of the LaTeX typesetting commands for this assignment is very basic. Most of the LaTeX commands in the files here should give you an idea of what to do. So just look at the LaTeX files here and copy-and-paste whatever you need. If you want more sophisticated LaTeX commands, you can look at my latex tutorial on my website for more information.

LaTeX is probably the most sophisticated scientific typesetting system in the world and is used by people in CS, math, physics, etc. It's now even used in general publishing. LaTeX is built on top of the original TeX language which was written by the world's most famous living computer scientist Donald Knuth. He is also writing the world's most famous multi-volume treatise on algorithms called The Art of Computer Programming.

How to write a conversion from base 10 another base

Problem: Convert 19 to base 2.
Answer:

2	19		
2	9	r	1
2	4	r	1
2	2	r	0
2	1	r	0
	0	r	1

Therefore $19 = 10011_2$.

Make sure you look at the LaTeX code:

```
\begin{longtable}{r|rrr}
  2 & 19 &   &   \\ \cline{2-4}
  2 &  9 & r & 1 \\ \cline{2-4}
  2 &  4 & r & 1 \\ \cline{2-4}
  2 &  2 & r & 0 \\ \cline{2-4}
  2 &  1 & r & 0 \\ \cline{2-4}
    &  0 & r & 1 \\ \cline{2-4}
\end{longtable}
Therefore $19 = 10011_{2}$.
```

The `&` is a separator of values in the table. The `\cline{2-4}` draws a line from column 2 to column 4.

When you need to do something similar to the computations below, just look for the LaTeX code in this file, copy-paste-and-modify.

How to write multi-column addition

Problem: Compute $423_8 + 636_8$.
Answer:

		1		1	
			4	2	3
+			6	3	6
		1	2	6	1

Therefore $423_8 + 839_8 = 1261_8$.

Make sure you look at the LaTeX code:

```
\begin{longtable}{ccccc}
  & 1 &   & 1 &   \\
  &   & 4 & 2 & 3 \\
+ &   & 6 & 3 & 6 \\ \hline
  & 1 & 2 & 6 & 1 \\ \hline
\end{longtable}
Therefore $423_{8} + 839_{8} = 1261_{8}$.
```

The `{ccccc}` in the LaTeX says there are 5 columns, `&` in the LaTeX are column separators for each row, the `\\` is a newline, the `\hline` draws a line through all columns.

(Note: Show the carries.)

For subtraction you don't have to show the borrows, because it would be too messy:

	1	2	3
−	1	0	8
		1	5

How to write multi-column multiplication

Problem: Compute $123_8 \times 456_8$.
Answer:

$$
\begin{array}{cccccc}
 & & & 1 & 2 & 3 \\
\times & & & 4 & 5 & 6 \\
\hline
 & & & 7 & 6 & 2 \\
 & & 6 & 3 & 7 & \\
+ & 5 & 1 & 4 & & \\
\hline
 & 6 & 0 & 7 & 5 & 2
\end{array}
$$

Therefore $123_8 \times 456_8 = 60752_8$

Take a look at the LaTeX code:

```
\begin{longtable}{cccccc}
         &   &   & 1 & 2 & 3 \\
$\times$ &   &   & 4 & 5 & 6 \\ \hline
         &   &   & 7 & 6 & 2 \\
         &   & 6 & 3 & 7 &   \\
  +      & 5 & 1 & 4 &   &   \\ \hline
         & 6 & 0 & 7 & 5 & 2 \\ \hline
\end{longtable}
Therefore $123_{8} \times 456_{8} = 60752_{8}$
```

The `{cccccc}` means there are 6 columns (with values centered). The `&` is a separator of column values. The `\\` is newline. The `\hline` is to draw a horizontal line.

Note that for this problem, there are actually two multicolumn additions. You don't have to show the work for these multicolumn additions.

How to write fast conversions

Problem: Convert 1011110_2 to base 4.
Answer:

$$\begin{aligned} 1011110_2 &= (01_2|01_2|11_2|10_2)_4 \\ &= (1_4|1_4|3_4|2_4)_4 \\ &= 1132_4 \end{aligned}$$

Therefore $1011110_2 = 1132_4$.

In general when doing a sequence of aligned computations, you do this

$$\begin{aligned} a + b + c &= d \times e \cdot f \\ &= g_h + i \\ &= j + k^l \end{aligned}$$

Notice that the = symbols are aligned. Take a look at the LaTeX code:

```
\begin{align*}
a + b + c &= d \times e \cdot f \\
          &= g_{h} + i \\
          &= j + k^{l}
\end{align*}
```

The `&` is the alignment character and the `\\` is newline. The aligned computations must be enclosed in `\begin{align*}` and `\end{align*}`.

Q1. Convert the following

(a) 1980 to base 2
(b) 1980 to base 5
(c) 1980 to base 8
(d) 1980 to base 16

(a) Answer:

2	1980		
2	990	r	0
2	495	r	0
2	247	r	1
2	123	r	1
2	61	r	1
2	30	r	1
2	15	r	0
2	7	r	1
2	3	r	1
2	1	r	1
2	0	r	1

Therefore $1980 = 11110111100_2$

(Add more lines if necessary.)

(b) Answer:

5	1980		
5	396	r	0
5	79	r	1
5	15	r	4
5	3	r	0
5	0	r	3

Therefore $1980 = 30410_?$

(c) Answer:

$1\times2^{10}+1\times2^9+1\times2^8+1\times2^7+0\times2^6+1\times2^5+1\times2^4+1\times2^3+1\times2^2+0\times2^1+0\times2^0$

$(1\times2^1+1\times2^0)(2^3)^3+(1\times2^2+1\times2^1+0\times2^0)(2^3)^2+(1\times2^2+1\times2^1+1\times2^0)(2^3)^1+(1\times2^2+0\times2^1+0\times2^0)(2^3)^0$

$3\times8^3+6\times8^2+7\times8^1+4\times8^0$

Therefore $1980=3674_8$

(Hint: Use (a).)

(d) Answer:

$1\times2^{10}+1\times2^9+1\times2^8+1\times2^7+0\times2^6+1\times2^5+1\times2^4+1\times2^3+1\times2^2+0\times2^1+0\times2^0$

$(0\times2^3+1\times2^2+1\times2^1+1\times2^0)(2^4)^2+(1\times2^3+0\times2^2+1\times2^1+1\times2^0)(2^4)^1+(1\times2^3+1\times2^2+0\times2^1+0\times2^0)(2^4)^0$

$7\times16^2+11\times16^1+12\times16^0$
Therefore $1980=7BC_{16}$

Q2. Convert the following

(a) 110010_2 to base 10
(b) 110010_2 to base 4
(c) 110010_2 to base 8
(d) 110010_2 to base 16

(a) Answer:

$$\begin{aligned} 110010_2 &= 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 \\ &= 32 + 16 + 2 \\ &= 50 \end{aligned}$$

Therefore $110010_2 = 50$

(b) Answer:

$$\begin{aligned} 110010_2 &= (1 \times 2^1 + 1 \times 2^0)(2^2)^2 + (0 \times 2^1 + 0 \times 2^0)(2^2)^1 + (1 \times 2^1 + 0 \times 2^0)(2^2)^0 \\ &= 3 \times 4^2 + 0 \times 4^1 + 2 \times 4^0 \\ &= 302_4 \end{aligned}$$

Therefore $110010_2 = 302_4$.

(c) Answer:

$$\begin{aligned} 110010_2 &= (1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0)(2^3)^1 + (0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0)(2^3)^0 \\ &= 6 \times 8^1 + 2 \times 8^0 \\ &= 62_8 \end{aligned}$$

Therefore $110010_2 = 62_8$.

(d) Answer:

$$\begin{aligned} 110010_2 &= (1 \times 2^2 + 1 \times 2^1)(2^4)^1 + (0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0)(2^4)^0 \\ &= 3 \times 16^1 + 2 \times 16^0 \\ &= 32_16 \end{aligned}$$

Therefore $110010_2 = 32_{16}$.

Q3. Convert the following

(a) 172_8 to base 10
(b) 172_8 to base 2
(c) 172_8 to base 7
(d) 172_8 to base 16

(a) Answer:

$$\begin{aligned} 172_8 &= 1 \cdot 8^2 + 7 \cdot 8^1 + 2 \cdot 8^0 \\ &= 64 + 56 + 2 \\ &= 122_{10} \end{aligned}$$

Therefore $172_8 = 122_{10}$.

(b) Answer:

$$\begin{aligned} 172_8 &= 1 \cdot (2^3)^2 + 7 \cdot (2^3)^1 + 2 \cdot (2^3)^0 \\ &= (1 \cdot 2^0)(2^3)^2 + (1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0)(2^3)^1 + (0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0) \cdot (2^3)^0 \\ &= 1111010_2 \end{aligned}$$

Therefore $172_8 = 1111010_2$.

(c) Answer:

7	122		
7	17	r	3
7	2	r	3
2	0	r	2

Therefore $172_8 = 233_7$

(d) Answer:

$$
\begin{aligned}
172_8 &= 1 \cdot (2^3)^2 + 7 \cdot (2^3)^1 + 2 \cdot (2^3)^0 \\
&= (1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0)(2^4)^1 + (1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0) \cdot (2^4)^0 \\
&= 7 \cdot 16 + 10 \cdot 16^0 \\
&= 7A_{16}
\end{aligned}
$$

Therefore $172_8 = 7A_{16}$.

Q4. Perform the following operations without first converting the numbers to base 10. Leave the answers in the given base.

(a) $423_8 + 267_8$
(b) $423_8 - 267_8$
(c) $423_8 \times 267_8$

(a) Answer:

$$\begin{array}{rcccc} & & 1 & 1 & \\ & & 4 & 2 & 3 \\ + & & 2 & 6 & 7 \\ \hline & & 7 & 1 & 2 \\ \hline \end{array}$$

Therefore $423_8 + 267_8 = 712_8$.

(b) Answer:

$$\begin{array}{rcccc} & & 4 & 2 & 3 \\ - & & 2 & 6 & 7 \\ \hline & & 1 & 3 & 4 \\ \hline \end{array}$$

Therefore $423_8 - 267_8 = 134_8$.

(c) Answer:

$$\begin{array}{rcccccc} & & & & 4 & 2 & 3 \\ \times & & & & 2 & 6 & 7 \\ \hline & & & 3 & 6 & 0 & 5 \\ & & 3 & 1 & 6 & 2 & \\ + & 1 & 0 & 4 & 6 & & \\ \hline & 1 & 4 & 2 & 2 & 2 & 5 \\ \hline \end{array}$$

Therefore $423_8 \times 267_8 = 142225_8$.

Q5. Perform the following operations without first converting the numbers to base 10. Leave the answers in the given base.

(a) $110001_2 + 101011_2$
(b) $110001_2 - 101011_2$
(c) $110001_2 \times 101011_2$

(a) Answer:

$$\begin{array}{rcccccccc} & & & & & & 1 & 1 & \\ & & & 1 & 1 & 0 & 0 & 0 & 1 \\ + & & & 1 & 0 & 1 & 0 & 1 & 1 \\ \hline & & 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{array}$$

Therefore $110001_2 + 101011_2 = 1011100_2$

(b) Answer:

$$\begin{array}{rcccccc} & 1 & 1 & 0 & 0 & 0 & 1 \\ - & 1 & 0 & 1 & 0 & 1 & 1 \\ \hline & 0 & 0 & 0 & 1 & 1 & 0 \end{array}$$

Therefore $110001_2 - 101011_2 = 110_2$

(c) Answer:

$$\begin{array}{rcccccccccccc} & & & & & & 1 & 1 & 0 & 0 & 0 & 1 \\ \times & & & & & & 1 & 0 & 1 & 0 & 1 & 1 \\ \hline & & & & & & 1 & 1 & 0 & 0 & 0 & 1 \\ & & & & & 1 & 1 & 0 & 0 & 0 & 1 & \\ + & & & & 0 & 0 & 0 & 0 & 0 & 0 & & \\ & & & 1 & 1 & 0 & 0 & 0 & 1 & & & \\ & & 0 & 0 & 0 & 0 & 0 & 0 & & & & \\ & 1 & 1 & 0 & 0 & 0 & 1 & & & & & \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \end{array}$$

Q6. Perform the following operations without first converting the numbers to base 10:

(a) $3EA_{16} + 2EF_{16}$
(b) $3EA_{16} - 2EF_{16}$
(c) $3EA_{16} \times 2EF_{16}$

(a) Answer:

$$\begin{array}{cccc} & 1 & 1 & \\ & 3 & E & A \\ + & 2 & E & F \\ \hline & 6 & D & 9 \end{array}$$

Therefore $3EA_{16} + 2EF_{16} = 6D9_{16}$

(b) Answer:

$$\begin{array}{cccc} & 3 & E & A \\ - & 2 & E & F \\ \hline & 0 & F & B \end{array}$$

Therefore $3EA_{16} - 2EF_{16} = FB_{16}$

(c) Answer:

$$\begin{array}{ccccc} & & 3 & E & A \\ \times & & 2 & E & F \\ \hline & 3 & A & B & 6 \\ 3 & 6 & C & C & \\ + \; 7 & C & 4 & & \\ \hline B & 6 & B & 7 & 6 \end{array}$$

Therefore $3EA_{16} \times 2EF_{16} = B6B76_{16}$

Q7. Perform the following operations without first converting the numbers to base 10. The following are numbers in base 17 where the symbols (alphabet) of the number system is 0,1,2,3,...,9,A,B,C,D,E,F,G where these numbers in base 10 are 0,1,2,3,...,9,10,11,12,13,14,15,16 respectively

(a) $7GA_{17} + 2EF_{17}$
(b) $7GA_{17} - 2EF_{17}$
(c) $7GA_{17} \times 2EF_{17}$

(a) Answer:

$$\begin{array}{cccc} & & 1 & 1 & \\ & & 7 & G & A \\ + & & 2 & E & F \\ \hline & & A & E & 8 \end{array}$$

Therefore $7GA_{17} + 2EF_{17} = AE8_{17}$

(b) Answer:

$$\begin{array}{cccc} & & 7 & G & A \\ - & & 2 & E & F \\ \hline & & 5 & 1 & C \end{array}$$

Therefore $7GA_{17} - 2EF_{17} = 512_{17}$

(c) Answer:

$$\begin{array}{cccccc} & & & 7 & G & A \\ \times & & & 2 & E & F \\ \hline & & 6 & 0 & A & E \\ & 6 & 9 & 3 & 4 & \\ + & E & F & 3 & & \\ \hline 1 & 4 & D & 6 & E & E \end{array}$$

Therefore $7GA_{17} \times 2EF_{17} = 14D6EE_{17}$

Q8. You're a software consultant selling software to the people of Ljutia of planet Jwercx. And boy oh boy, they have this annoying practice of using several bases at the same time!!! Their president Jabba wanted to pay

$$8356 + 100111_2 - 124_5 \times \mathrm{AE2}_{16}$$

in US dollars. Your (unforgiving) boss told you not to accept anything below US$5000. Exactly how much is President Jabba offering? Are you going to accept his/her/its offer? Is he/she/it a crook? (You only need to answer the first question.)

Answer:

-100259

Q9. You are a software engineer newly posted to planet Khais and this is your first day of work. You are attending a meeting where the Khais are discussing some software engineering metrics and someone (someit?) wrote

$$(35 + 24) \times 21 = 1501$$

on the board. You gasp and quickly realize that *they are not using base 10*. (Now you know why John was laughing when he sent you off at the intergalactic transporter ...)

People are going down the row giving comments on this computation (fortunately you are sitting at the end of the table so you're the last to give comments). You quickly look at their hands - they don't have fingers. That was the reason why no one shook your hand when you arrived. So that doesn't help.

With beads of sweat rolling down your face, you try frantically to work out the base they use in order to make some intelligent comments. You do want to keep your job right?

What is the base?

Answer:

8

Q10. You have just finished a major project on Planet We and the CEO is sending everyone to a one week vacation on Planet Ew. Not knowing the history of the two planets, you agreed. While enjoying your vacation and wondering about the curious names of the planets, war erupts because the Weians and Ewians cannot agree on the following problem: If a and b are digits from $0, 1, ..., 7$, then when is the following true:

$$ab_{10} = ba_8$$

(In the above, ab_{10} is a 2-digit base 10 number while ba_8 is a 2 digit base 8 number.)

This is serious. The last time someone casually brought up this ancient puzzle of the two worlds, angry words flew between the planets and within hours, nuclear war broke out and all lifeforms on the two planets were annihilated except for one-cell amoebas. That was about 2 trillion years ago. In order to save the two planets (and yourself) you have to find all the solutions to the above problem. It is projected that the wars will escalate to nuclear proportions within one hours.

Explain your work in complete and proper English and mathematical sentences.

Answer:

the answer is $a = 0, b = 0$, because whenever we expand both we get $10a+b = 8b+a$ and then we get the relationship between a and b we get $9a = 7b$ so $b = \frac{9}{7}a$ the only value of a that can give an integer value is $a = 7$ and that would make $b = 9$ and b cant be 9 because our constraint is between $0 - 7$ so $a = b = 0$

Instructions

In `main.tex` change the email address in

```
\renewcommand\AUTHOR{jdoe5@cougars.ccis.edu}
```

yours. In the bash shell, execute "`make`" to recompile `main.pdf`. Execute "`make v`" to view `main.pdf`. Execute "`make s`" to create `submit.tar.gz` for submission.

For each question, you'll see boxes for you to fill. You write your answers in `main.tex` file. For small boxes, if you see

```
1 + 1 = \answerbox{}.
```

you do this:

```
1 + 1 = \answerbox{2}.
```

`answerbox` will also appear in "true/false" and "multiple-choice" questions.

For longer answers that needs typewriter font, if you see

```
Write a C++ statement that declares an integer variable name x.
\begin{answercode}
\end{answercode}
```

you do this:

```
Write a C++ statement that declares an integer variable name x.
\begin{answercode}
int x;
\end{answercode}
```

`answercode` will appear in questions asking for code, algorithm, and program output. In this case, indentation and spacing is significant. For program output, I do look at spaces and newlines.

For long answers (not in typewriter font) if you see

```
What is the color of the sky?
\begin{answerlong}
\end{answerlong}
```

you can write

```
What is the color of the sky?
\begin{answerlong}
The color of the sky is blue.
\end{answerlong}
```

For students beyond 245: You can put LaTeX commands in `answerlong`.

A question that begins with "T or F or M" requires you to identify whether it is true or false, or meaningless. "Meaningless" means something's wrong with the statement and it is not well-defined. Something like "$1+_2$" or "$\{2\}^{\{3\}}$" is not well-defined. Therefore a question such as "Is $42 = 1+_2$ true or false?" or "Is $42 = \{2\}^{\{3\}}$ true or false?" does not make sense. "Is $P(42) = \{42\}$ true or false?" is meaningless because $P(X)$ is only defined if X is a set. For "Is $1 + 2 + 3$ true or false?", "$1 + 2 + 3$" is well–defined but as a "numerical expression", not as a "proposition", i.e., it cannot be true or false. Therefore "Is $1 + 2 + 3$ true or false?" is also not a well-defined question.

When writing results of computations, make sure it's simplified. For instance write 2 instead of $1 + 1$. When you write down sets, if the answer is $\{1\}$, I do not want to see $\{1, 1\}$.

When writing a counterexample, always write the simplest.

Here are some examples (see `instructions.tex` for details):

1. T or F or M: $1 + 1 = 2$ ..... T
2. T or F or M: $1 + 1 = 3$ ..... F
3. T or F or M: $1+^2 =$ ..... M
4. $1 + 2 =$ 3
5. Write a C++ statement to declare an integer variable named `x`.
   ```
   int x;
   ```
6. Solve $x^2 - 1 = 0$.

   Since $x^2 - 1 = (x - 1)(x + 1)$, $x^2 - 1 = 0$ implies $(x - 1)(x + 1) = 0$. Therefore $x - 1 = 0$ or $x = -1$. Hence $x = 1$ or $x = -1$.

7. Which is true? C
   (A) $1 + 1 = 0$
   (B) $1 + 1 = 1$
   (C) $1 + 1 = 2$
   (D) $1 + 1 = 3$
   (E) $1 + 1 = 4$