Project 4: Optimal vaccination of an SIR epidemic model

$$
\begin{aligned}
\frac{dS(t)}{dt} &= \upsilon N - (\upsilon + u(t))S(t) - \frac{\beta I(t)S(t)}{N}, && S(0) = S_0 \geq 0, \\
\frac{dI(t)}{dt} &= \frac{\beta I(t)S(t)}{N} - (\gamma + \upsilon)I(t), && I(0) = I_0 \geq 0, \\
\frac{dR(t)}{dt} &= \gamma I(t) - \upsilon R(t) + u(t)S(t), && R(0) = R_0 \geq 0.
\end{aligned} \tag{2}
$$

S: susceptible, never been infected

I: infected

R: recovered, immune for life

υ: birth, death rate, γ recover rate

u: percentage of suceptible vaccinated per unit of time

$$
J(u) = \int_0^{t_{\text{end}}} \left[A_1 S(t) + A_2 I(t) + \frac{1}{2} \tau u^2(t) \right] dt
$$

(CP) $\min J(u)$ s.t. (SIR), $u \in U_{ad}$