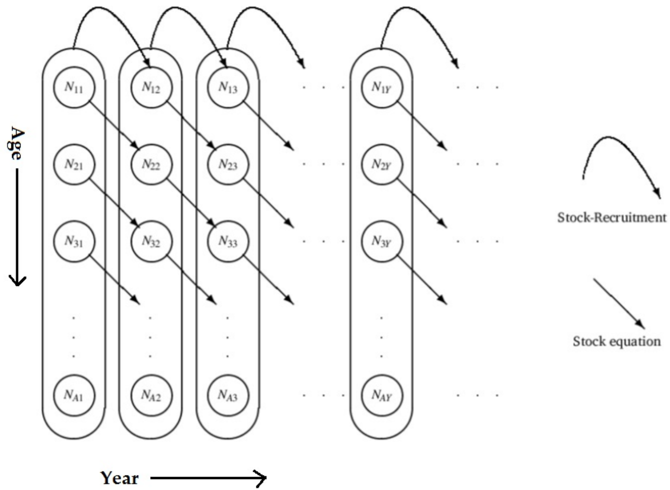
Explaining SAM

Olav Nikolai Breivik

Let $N_{a,y}$ be the number of fish at age a in year y.



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were

$$\log \mathbf{F}_y = \log \mathbf{F}_{y-1} + \boldsymbol{\xi}_y.$$

Observe:

$$\log C_{a,y} = \log\left(\frac{F_{a,y}}{F_{a,y}+M_{a,y}}(1-e^{-F_{a,y}-M_{a,y}})N_{a,y}\right) + \epsilon^{c}_{a,y}$$
$$\log I^{(s)}_y = \log(Q^{(s)}_a e^{-(F_{a,y}+M_{a,y})day^{(s)}/365} N_{a,y}) + \epsilon^{s}_{a,y}$$

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Assumes $\boldsymbol{\eta}_y$, $\boldsymbol{\xi}_y$ and $\boldsymbol{\epsilon}^{C}_{y}$ and $\boldsymbol{\epsilon}^{s}_{y}$ all Gaussian distributed.

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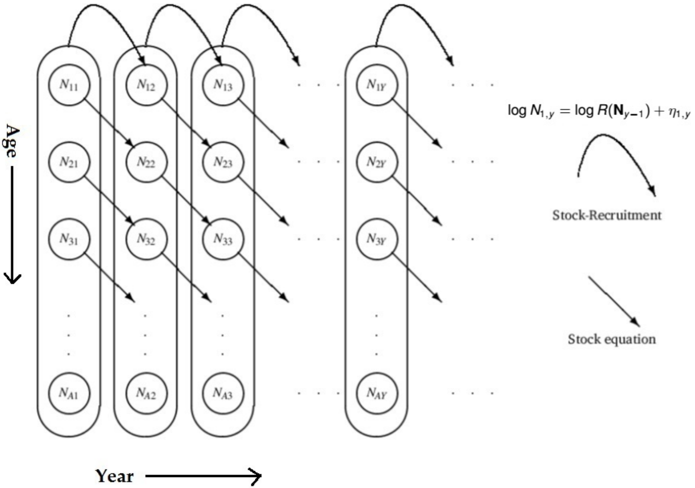

Recruitment

In SAM four types of recrutiment can be assumed:

- Random walk: $\log N_{1,y} \sim N(\log N_{1,y-1}, \sigma_r^2)$
- Ricker or Beverton-Holt: $\log N_{1,y} \sim N(f(SSB_{y-1}), \sigma_r^2)$
- Constant mean: $\log N_{1,y} \sim N(\mu_r, \sigma_r^2)$

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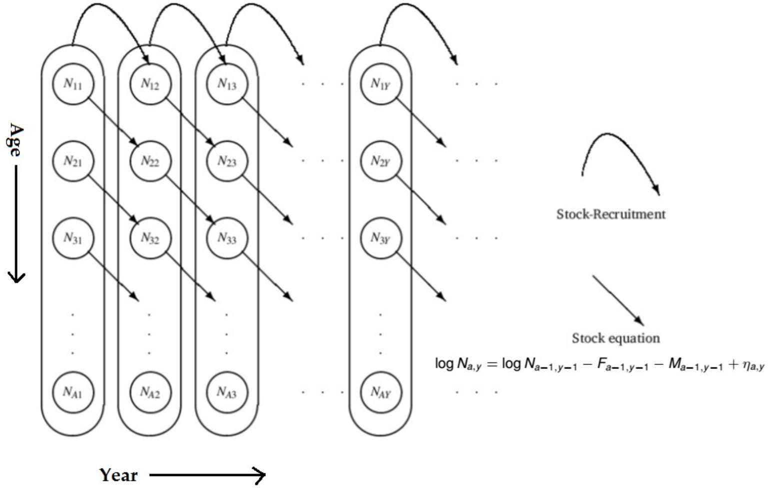

Fishing mortality F

In SAM it is typically assumed that:

$$\log \mathbf{F}_y = \log \mathbf{F}_{y-1} + \xi_y$$

were $\xi_y \sim N(0, \mathbf{\Sigma}_F)$.

- Several option of $\mathbf{\Sigma}_F$ are available.
- Note that this definition accommodate for time varying selectivity
- We can impose restrictions on **F** such that the fishing mortality is equal for some ages.

Separable F (used in herring assessment)

We can assume:

$$\log \mathbf{F}_y = \log \mathbf{U}_y + \log \mathbf{V}_y + \boldsymbol{\epsilon}_y^{(F)}$$

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$$\log \mathbf{U}_y = \rho_U \log \mathbf{U}_{y-1} + \boldsymbol{\alpha}_u + \boldsymbol{\epsilon}_y^{(U)}, \qquad \sum_a \log U_{a,y} = 1$$

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and

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and

$$\log V_y = \rho_V \log V_{y-1} + \alpha_V + \epsilon_y^{(V)}$$

were

$$\boldsymbol{\epsilon}_y^{(F)} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_F), \quad \boldsymbol{\epsilon}_y^{(U)} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_U), \quad \epsilon_y^{(V)} \sim N(\mathbf{0}, \sigma_V^2)$$

See Link for implementation in SAM ($\boldsymbol{\epsilon}_y^{(F)}$ excluded).

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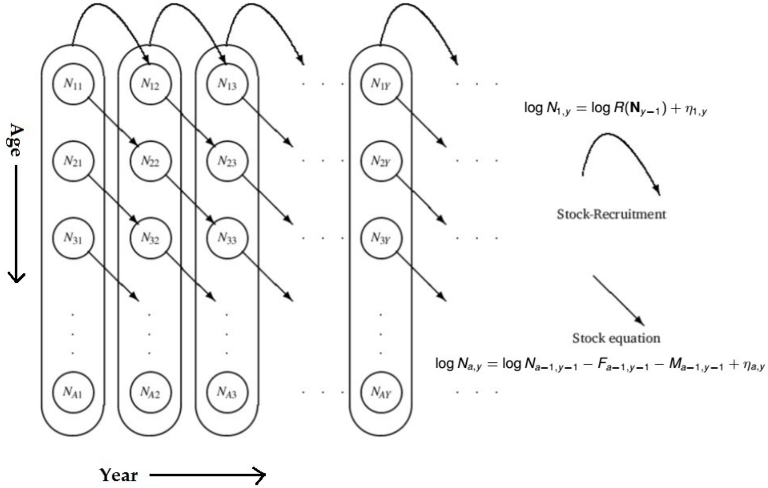

Observation equations

We observe catch:

$$\log C_{a,y} = \log\left(\frac{F_{a,y}}{F_{a,y}+M_{a,y}}(1-e^{-F_{a,y}-M_{a,y}})N_{a,y}\right) + \epsilon^{c}_{a,y}$$

Note

- $\frac{F_{a,y}}{F_{a,y}+M_{a,y}}$ is the proportion died in fishery
- $(1-e^{-F_{a,y}-M_{a,y}})N_{a,y}$ is the total amount of fish died

It is assumed $\epsilon^{c}_{y} \sim N(0, \boldsymbol{\Sigma}_C)$. Several option for $\boldsymbol{\Sigma}_C$ are available.

Observation equations

We observe indices:

$$\log I_{a,y}^{(s)} = \log(Q_a^{(s)} e^{-(F_{a,y}+M_{a,y})day^{(s)}/365} N_{a,y}) + \epsilon_{a,y}^{s}$$

- $Q_a^{(s)}$ is a catchability constant.
- $e^{-(F_{a,y}+M_{a,y})day^{(s)}/365} N_{a,y}$ is the abundance at survey time

It is assumed $\boldsymbol{\epsilon}_y^s \sim N(0, \boldsymbol{\Sigma}_s)$. Several option for $\boldsymbol{\Sigma}_s$ are available.

External covariance (used in the NSSS herring assessment)

We can assume that

$$\epsilon_y^C \sim N(0, c^C \mathbf{R}_y^C)$$
$$\epsilon_y^I \sim N(0, c^s \mathbf{R}_y^s),$$

were $\mathbf{R}_y^C$ and $\mathbf{R}_y^s$ are provided outside of the assessment model. Further is c^C and c^s estimated inside the assessment model.

This structure is implemented in SAM.

- See testmore/nscodcovar on the SAM GitHub page for an example Link

External covariance (used in the NSSS herring assessment)

- In the herring assessment, smoothed estimates of $\mathbf{R}^c$ and $\mathbf{R}^s$ are provided.

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Smoothed estimates of e.g. $\mathbf{R}^c$ are provided with the following procedure outside of SAM:

Let $v_{a,y}$ be estimated variance of $C_{a,y}$. By assuming that

$$\log v_{a,y} = \alpha + \beta \log(C_{a,y}) + \epsilon_{a,y}^{(s)}$$

and that $C_{a,y}$ is log-normal, we have that

$$\sigma_{a,y}^2 = \log(e^{\alpha} C_{a,y}^{\beta-2} + 1) \tag{1}$$

is an estimate of the variance of the log-catch. Let $\mathbf{R}_y^c = \boldsymbol{\sigma}_y' \mathbf{I} \boldsymbol{\sigma}_y$.

Link between mean and variance inside SAM

- Link between the mean and variance can be included inside SAM
  - Implemented in a development version of SAM .
  - By using this approach we can accommodate for that the CV's are typically smaller for larger catches or indices.