IN3050/IN4050 Mandatory Assignment 1: Traveling Salesman Problem

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Introduction

In this exercise, you will attempt to solve an instance of the traveling salesman problem (TSP) using different methods. The goal is to become familiar with evolutionary algorithms and to appreciate their effectiveness on a difficult search problem. You have to use Python to solve the assignment. You must write your program from scratch (but you may use non-EA-related libraries).

	Barcelona	**Belgrade**	**Berlin**	**Brussels**	**Bucharest**	**Budapest**
Barcelona	0	1528.13	1497.61	1062.89	1968.42	1498.79
Belgrade	1528.13	0	999.25	1372.59	447.34	316.41
Berlin	1497.61	999.25	0	651.62	1293.40	1293.40
Brussels	1062.89	1372.59	651.62	0	1769.69	1131.52
Bucharest	1968.42	447.34	1293.40	1769.69	0	639.77
Budapest	1498.79	316.41	1293.40	1131.52	639.77	0

Figure 1: First 6 cities from csv file.

Problem

The traveling salesman, wishing to disturb the residents of the major cities in some region of the world in the shortest time possible, is faced with the problem of finding the shortest tour among the cities. A tour is a path that starts in one city, visits all of the other cities, and then returns to the starting point. The relevant pieces of information, then, are the cities and the distances between them. In this instance of the TSP, a number of European cities are to be visited. Their relative distances are given in the data file, *european_cities.csv*, found in the zip file with the mandatory assignment.

(You will use permutations to represent tours in your programs. The **itertools** module in Python provides a permutations function that returns successive permutations, this is useful for exhaustive search)

Helper code for visualizing solutions

Here follows some helper code that you can use to visualize the plans you generate. These visualizations can **help you check if you are making sensible tours or not**. The optimization algoritms below should hopefully find relatively nice looking tours, but perhaps with a few visible inefficiencies.

In [2]:
```
import matplotlib.pyplot as plt
import numpy as np
%matplotlib inline
np.random.seed(57)
#Map of Europe
europe_map = plt.imread('map.png')

#Lists of city coordinates
city_coords = {
    "Barcelona": [2.154007, 41.390205], "Belgrade": [20.46, 44.79], "Berlin": [13.4
    "Brussels": [4.35, 50.85], "Bucharest": [26.10, 44.44], "Budapest": [19.04, 47.
    "Copenhagen": [12.57, 55.68], "Dublin": [-6.27, 53.35], "Hamburg": [9.99, 53.55
    "Istanbul": [28.98, 41.02], "Kyiv": [30.52, 50.45], "London": [-0.12, 51.51],
    "Madrid": [-3.70, 40.42], "Milan": [9.19, 45.46], "Moscow": [37.62, 55.75],
    "Munich": [11.58, 48.14], "Paris": [2.35, 48.86], "Prague": [14.42, 50.07],
    "Rome": [12.50, 41.90], "Saint Petersburg": [30.31, 59.94], "Sofia": [23.32, 42
    "Stockholm": [18.06, 60.33], "Vienna": [16.36, 48.21], "Warsaw": [21.02, 52.24]
```

In [3]:
```
#Helper code for plotting plans
#First, visualizing the cities.
import csv
with open("european_cities.csv", "r") as f:
    data = list(csv.reader(f, delimiter=';'))
    cities = data[0]

fig, ax = plt.subplots(figsize=(10, 10))
ax.imshow(europe_map, extent=[-14.56, 38.43, 37.697 + 0.3, 64.344 + 2.0], aspect="a

# Map (long, lat) to (x, y) for plotting
for city, location in city_coords.items():
    x, y = (location[0], location[1])
    plt.plot(x, y, 'ok', markersize=5)
    plt.text(x, y, city, fontsize=12)
```


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60
55
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45
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0
10
20
30
Norge
Finland
Bottenvika
Stockholm
Østersjøen
Estland
Saint Petersburg
Latvia
Nordsjøen
Danmark
Copenhagen
Litauen
Moscow
Storbritannia
Dublin
Irland
Hamburg
Berlin
Polen
Warsaw
Warszawa
Hviterussland
Nederland
London
Brussels
Tyskland
Belgia
Kiev
Kyiv
Prague
Paris
Tsjekkia
Munich
Vienna
Budapest
Ukraina
Østerrike
Ungarn
Moldova
Frankrike
Milan
Kroatia
Belgrade
Romania
Bucharest
Serbia
Svartehavet
Sofia
Italia
Rome
Roma
Bulgaria
Barcelona
Istanbul
Ankara
Portugal
Madrid
Tyrrenhavet
Spania
Lisboa
Hellas
Tyrkia

In [6]:
```
#A method you can use to plot your plan on the map.
def plot_plan(city_order):
    fig, ax = plt.subplots(figsize=(10, 10))
    ax.imshow(europe_map, extent=[-14.56, 38.43, 37.697 + 0.3, 64.344 + 2.0], aspec

    # Map (long, lat) to (x, y) for plotting
    for index in range(len(city_order) - 1):
        current_city_coords = city_coords[city_order[index]]
        next_city_coords = city_coords[city_order[index+1]]
        x, y = current_city_coords[0], current_city_coords[1]
        #Plotting a line to the next city
        next_x, next_y = next_city_coords[0], next_city_coords[1]
        plt.plot([x, next_x], [y, next_y])

        plt.plot(x, y, 'ok', markersize=5)
        plt.text(x, y, index, fontsize=12)
    #Finally, plotting from last to first city
    first_city_coords = city_coords[city_order[0]]
    first_x, first_y = first_city_coords[0], first_city_coords[1]
    plt.plot([next_x, first_x], [next_y, first_y])
    #Plotting a marker and index for the final city
    plt.plot(next_x, next_y, 'ok', markersize=5)
    plt.text(next_x, next_y, index+1, fontsize=12)
    plt.show()
```

In [8]:
```
#Example usage of the plotting-method.
plan = list(city_coords.keys()) # Gives us the cities in alphabetic order
print(plan)
plot_plan(plan)
```

```
['Barcelona', 'Belgrade', 'Berlin', 'Brussels', 'Bucharest', 'Budapest', 'Copenhage
n', 'Dublin', 'Hamburg', 'Istanbul', 'Kyiv', 'London', 'Madrid', 'Milan', 'Moscow',
'Munich', 'Paris', 'Prague', 'Rome', 'Saint Petersburg', 'Sofia', 'Stockholm', 'Vie
nna', 'Warsaw']
```

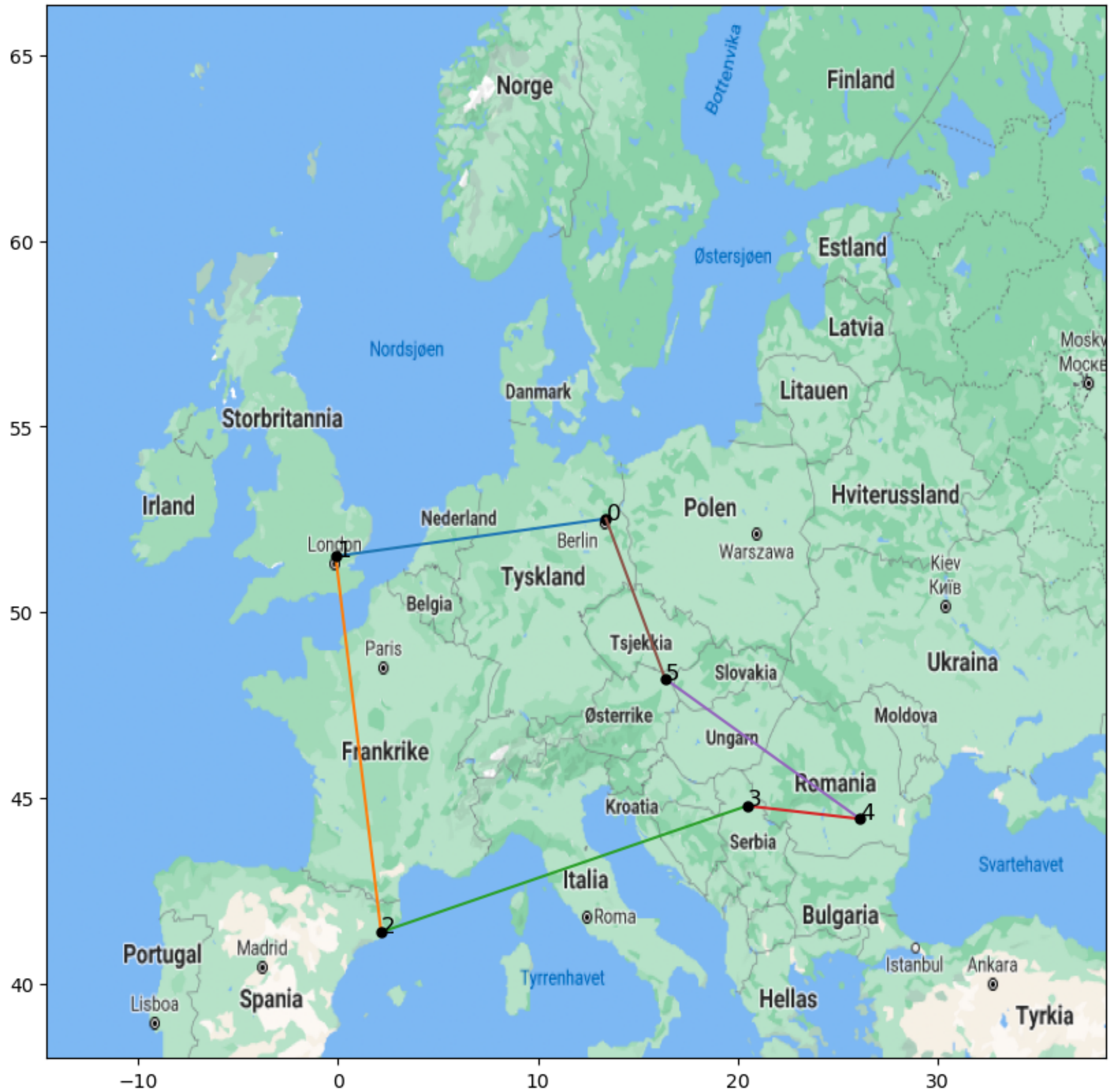
Exhaustive Search

First, try to solve the problem by inspecting every possible tour. Start by writing a program to find the shortest tour among a subset of the cities (say, **6** of them). Measure the amount of time your program takes. Incrementally add more cities and observe how the time increases. Plot the shortest tours you found using the plot_plan method above, for 6 and 10 cities.

Note: To get distances between cities, use the dictionary `data` created by reading the file `european_cities.csv`. *Do not* calculate distances based on the coordinates. The actual distances do not only depend on the differences in the coordinates, but also of the curvature of the earth. The distances available in `data` are corrected for this, and contain the actual true distances.

In [9]:
```
# Implement the algorithm here
from itertools import permutations
import time

distances = data[1::]

"""
Function for calculating the total distance of the tour
"""
def distance(tour):
    dst = 0
    for i in range(0,len(tour) - 1):#Iterate through all cities in each permutation

        dst += float(distances[data[0].index(tour[i])][data[0].index(tour[i+1])])

    dst += float(distances[data[0].index(tour[-1])][data[0].index(tour[0])]) #Add a

    return dst

"""
Function for finding the shortest tour through exhaustive search.
Takes a list of cities to find the tour through.
"""

def exhaustive_search(cities):
    perms = list(permutations(cities)) # Finds all permutations subset of cities.
    global_max = [None,None]

    for perm in perms: # Iterate through all permutations
        local_max = [perm,0]
        local_max[1] = distance(perm)

        if (global_max[0] == None):
            global_max[0] = local_max[0]
            global_max[1] = local_max[1]

        elif (local_max[1] < global_max[1]):
            global_max[0] = local_max[0]
            global_max[1] = local_max[1]

    return global_max
```

In [11]:
```
# Pick a random subset of cities, and call exhaustive search on the subset.
test_cities6 = np.random.choice(cities, 6, replace=False)
test_cities10 = np.random.choice(cities, 10, replace=False)

answ6 = exhaustive_search(test_cities6)
answ10 = exhaustive_search(test_cities10)
plot_plan(answ6[0])
plot_plan(answ10[0])
```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60
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Norge
Bottenvika
Finland
Østersjøen
Estland
Latvia
Nordsjøen
Danmark
Litauen
Storbritannia
Irland
Polen
Hviterussland
Nederland
Berlin
Warszawa
London
Tyskland
Belgia
Kiev
Київ
Paris
Tsjekkia
Slovakia
Ukraina
Østerrike
Moldova
Ungarn
Frankrike
Romania
Kroatia
Serbia
Svartehavet
Italia
Roma
Bulgaria
Portugal
Madrid
Istanbul
Ankara
Lisboa
Spania
Tyrrenhavet
Hellas
Tyrkia
0
1
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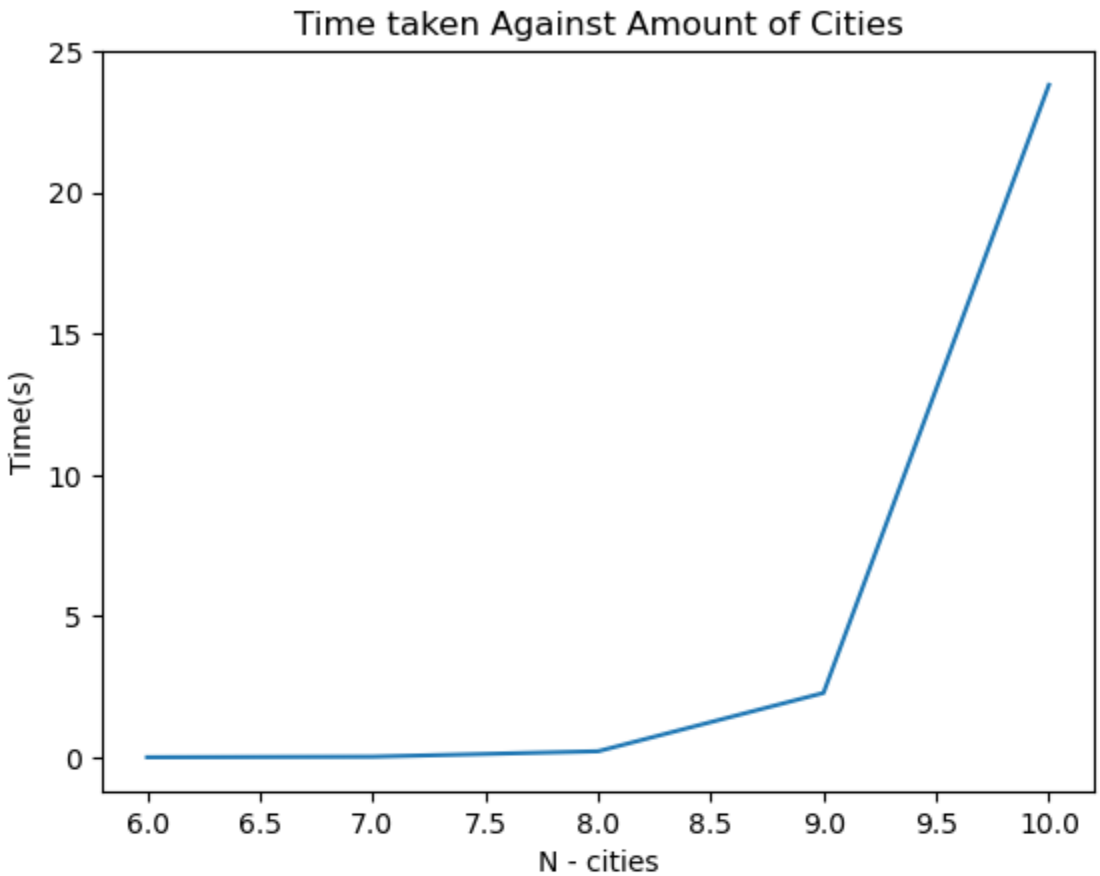
5

In [12]:

```
# Take time for for 6 upto 10 cities and plot the time against amount of cities to
# the algorithm increases.

time_exh = []
n_cities = [6,7,8,9,10]


for i in n_cities:
    test_cities = np.random.choice(cities, i, replace=False)
    start = time.time()
    answ = exhaustive_search(test_cities)
    end = time.time()
    time_exh.append((end - start))

plt.xlabel("N - cities")
plt.ylabel("Time(s)")
plt.title("Time taken Against Amount of Cities")
plt.plot(n_cities, time_exh)
```

Out[12]: [<matplotlib.lines.Line2D at 0x28bdba25190>]



What is the shortest tour (i.e., the actual sequence of cities, and its length) among the first 10 cities (that is, the cities starting with B,C,D,H and I)? How long did your program take to find it? Calculate an approximation of how long it would take to perform exhaustive search on all 24 cities?

In [13]:
```
# Answer

test_cities = cities[:10]

start = time.time()
print(f"Shortest tour among the first 10 citiest: {exhaustive_search(test_cities)}")
end = time.time()
print(f"Time exhaustive search used to find shortest tour: {end - start}")
```

```
Shortest tour among the first 10 citiest: [('Copenhagen', 'Hamburg', 'Brussels', 'D
ublin', 'Barcelona', 'Belgrade', 'Istanbul', 'Bucharest', 'Budapest', 'Berlin'), 74
86.309999999999]

Time exhaustive search used to find shortest tour: 16.275661945343018
```

Answer to Questions

What is the shortest tour (i.e., the actual sequence of cities, and its length) among the first 10 cities (that is, the cities starting with B,C,D,H and I)?

- The shortest tour among the first ten cities are: ('Bucharest', 'Belgrade', 'Budapest', 'Berlin', 'Copenhagen', 'Hamburg', 'Brussels', 'Dublin', 'Barcelona', 'Istanbul').

How long did your program take to find it?

- Time exhaustive search used to find shortest tour: 15.03 [seconds]

Calculate an approximation of how long it would take to perform exhaustive search on all 24 cities?

- For 10 cities, the we get 3,628,800 permutations, that took ca 15 seconds, so $\frac{1}{241920}$ seconds per permutation. For 24 cities, we get $6.204484e+23$ permutations. Which in a rough approximation, will give us a time of: $\frac{1}{241920} * 6.204484e+23 = 2.6e+18$ s. Which is: $8.2e+10$ years.

Hill Climbing

Then, write a simple hill climber to solve the TSP. How well does the hill climber perform, compared to the result from the exhaustive search for the first **10 cities**? Since you are dealing with a stochastic algorithm, you should run the algorithm several times to measure its performance. Report the length of the tour of the best, worst and mean of 20 runs (with random starting tours), as well as the standard deviation of the runs, both with the **10 first cities**, and with all **24 cities**. Plot one of the the plans from the 20 runs for both 10 cities and 24 cities (you can use plot_plan).

In [17]:
```
# Implement the algorithm here

def hill_climb(perms):
    count = 0
    rnd = np.random.randint(1,(len(perms)-2))
    best = [perms[rnd], distance(perms[rnd])]


    for i in range(500):
        count += 1
        left =  [perms[perms.index(best[0]) - 1], distance(perms[perms.index(best[0
        right =  [perms[perms.index(best[0]) + 1], distance(perms[perms.index(best[

        if (left[1] < right[1]):
            if (left[1] < best[1]):
                best = left
        elif (right[1] < best[1]):
            best = right
        elif (best[1] < right[1] and best[1] < left[1]):
            return best

    return best
perm_10 = list(permutations(test_cities))

start = time.time()
answ = hill_climb(perm_10)
end = time.time()

print(f"Time for Hill Climb: First ten cities: {(end - start)}")

plot_plan(answ[0])

# Running hill climb for 20 runs to find mean, best and worst solutions, for 10 and
sol_10 = np.zeros(20) # List to store answers for 10 first cities
sol_24 = np.zeros(20) # List to store answers for all cities
perm_24 = list(permutations(cities))

for i in range(20):
    sol_10[i] = hill_climb(perm_10)[1]
    sol_24[i] = hill_climb(perm_24)[1]

print(f"First 10 cities: Mean: {np.mean(sol_10)} Worst: {np.max(sol_10)} Best: {np.
print(f"All Cities: Mean: {np.mean(sol_24)} Worst: {np.max(sol_24)} Best: {np.min(s
```

```
Time for Hill Climb: First ten cities: 1.7877795696258545
```


```
---------------------------------------------------------------------------
MemoryError                               Traceback (most recent call last)
Cell In[17], line 36
     34 sol_10 = np.zeros(20) # List to store answers for 10 first cities
     35 sol_24 = np.zeros(20) # List to store answers for all cities
---> 36 perm_24 = list(permutations(cities))
     38 for i in range(20):
     39     sol_10[i] = hill_climb(perm_10)[1]

MemoryError:
```

Answer to Questions

How well does the hill climber perform, compared to the result from the exhaustive search for the first 10 cities?

- In terms of time taken, hill climb is faster than exhaustive search, atleast for larger amount of cities, but in terms of answer correctness, hill climb often comes with a worse solution than exhaustive search does.

#

#

Genetic Algorithm

Next, write a genetic algorithm (GA) to solve the problem. Choose mutation and crossover operators that are appropriate for the problem (see chapter 4.5 of the Eiben and Smith textbook). Choose three different values for the population size. Define and tune other parameters yourself and make assumptions as necessary (and report them, of course).

For all three variants: As with the hill climber, report best, worst, mean and standard deviation of tour length out of 20 runs of the algorithm (of the best individual of last generation). Also, find and plot the average fitness of the best fit individual in each generation (average across runs), and include a figure with all three curves in the same plot in the report. This means that the x-axis should be the generations over time and the y-axis should be the average (over the 20-runs) fitness of the best gene in that generation. Conclude which is best in terms of tour length and number of generations of evolution time.

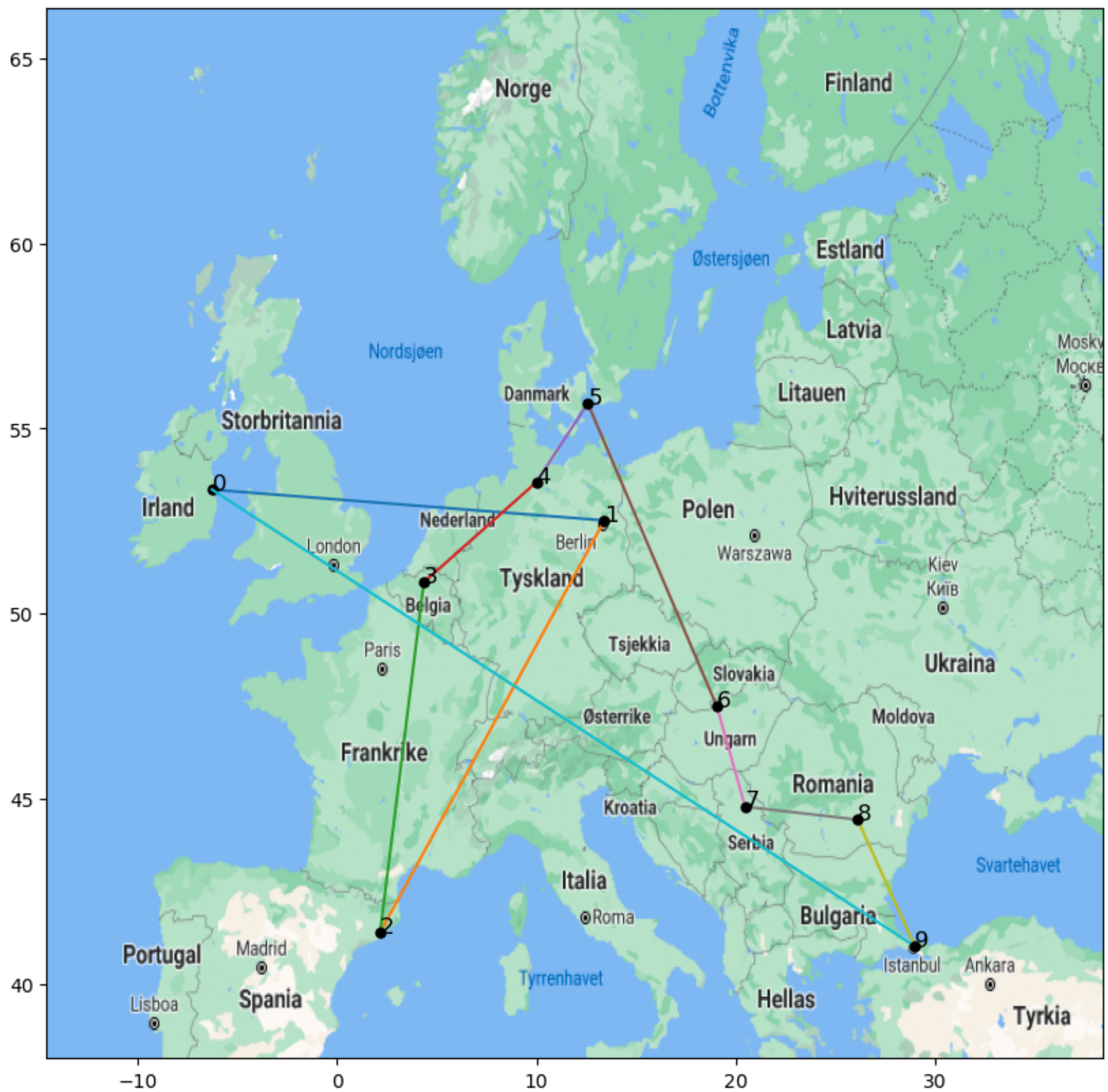
Finally, plot an example optimized tour (the best of the final generation) for the three different population sizes, using the plot_plan method.

In [92]:
```
import random
import statistics
# Implement the algorithm here

# I choose swap mutation, so we will get 'some' change in mutation, but not too muc
def pmx_pair(P1,P2):
    cross_c1_1 = random.randint(0,len(P1)-1)
    cross_c1_2 = random.randint(cross_c1_1, len(P1)-1)
    cross_c2_1 = random.randint(0,len(P1)-1)
    cross_c2_2 = random.randint(cross_c2_1, len(P1)-1)
    c1 = [None]*len(P1)
    c2 = [None]*len(P1)

    # Copy segment of P1 into Child 1 and 2
    c1[cross_c1_1:cross_c1_2+1] = P1[cross_c1_1:cross_c1_2+1]
    c2[cross_c2_1:cross_c2_2+1] = P1[cross_c2_1:cross_c2_2+1]

    # Find a place for the elements in P2 that are within the cross segment, but is
    for i in range(cross_c1_1, cross_c1_2 + 1):          # Loop over cross segment
        if (P2[i] not in c1):                            # If element in P2 within
            j = P2.index(P1[i])

            while (c1[j] != None):
                j = P2.index(P1[j])
            c1[j] = P2[i]

    # In the still-empty places within the child, we copy the corresponding value f
    for i in range(len(c1)):
        if (c1[i] == None):
            c1[i] = P2[i]


    # Same procedures, but for child 2.
    for i in range(cross_c2_1, cross_c2_2 + 1):
        if (P2[i] not in c2):
            j = P2.index(P1[i])

            while (c2[j] != None):
                j = P2.index(P1[j])
            c2[j] = P2[i]

    for i in range(len(c2)):
        if (c2[i] == None):
            c2[i] = P2[i]


    return c1, c2

def GA(pop, generations):
    popu = list(pop)

    fittest_gen = []

    for g in range(generations):
        for i in range(len(popu)):
            popu[i] = list(popu[i])
        # Calculate fitness of each solution
        weights = []
```

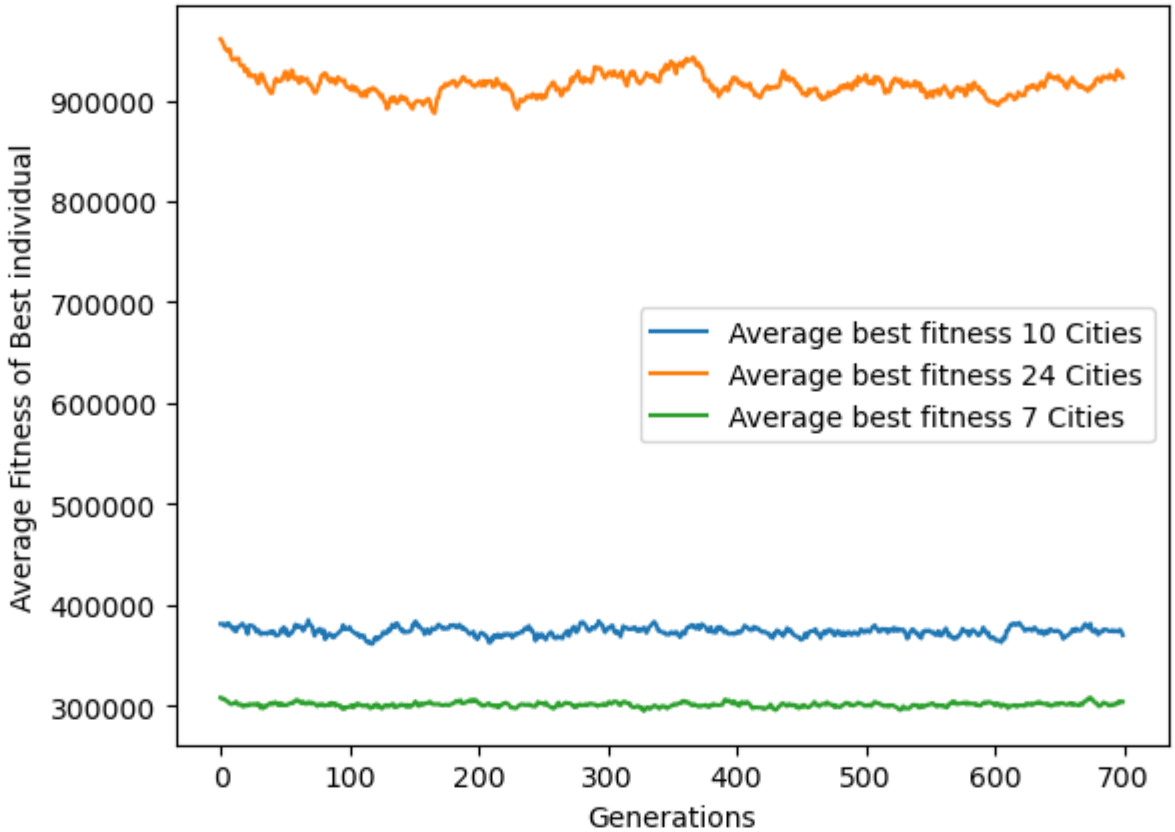
```
    for i in range(len(popu)):
        weights.append(distance(popu[i]))

    total_weight = sum(weights)

    for i in range(len(weights)):
        weights[i] = (total_weight-weights[i])

    # Parent selection, select an even number of parents
    k = int(len(popu)/2)
    if (k % 2 != 0):
        k += 1
    if i == 0:
        parents = popu[:k]
    else:
        parents = random.choices(popu, weights = weights, k = k)

    # Mutating and crossover
    count = 0
    while (count < len(parents)):
        c1, c2 = pmx_pair(parents[count], parents[count + 1])
        popu.append(c1)
        popu.append(c2)
        count += 2

    #Swap mutation
    for p in popu:
        if (random.random() <= .25): #Choise of mutation rate, is chosen on a w
            i1 = random.randint(0, len(p)-1)
            i2 = random.randint(0, len(p)-1)
            p[i1], p[i2] = p[i2], p[i1]

    # Select the survivors for the next generation
    weights = []
    for i in range(len(popu)):
        weights.append(distance(popu[i]))

    total_weight = sum(weights)

    for i in range(len(weights)):
        weights[i] = (total_weight-weights[i]) # Fitness

    fittest_gen.append(max(weights))
    popu = random.choices(popu, weights = weights, k = 20)

# Calculating weights again to find best solution after GA
weights = []
for i in range(len(popu)):
    weights.append(distance(popu[i]))

total_weight = sum(weights)

for i in range(len(weights)):
    weights[i] = (total_weight-weights[i])

# Choose best as 'max' of weights because of how weights are defined, max weigh
best = popu[weights.index(max(weights))]

return best, distance(best), fittest_gen
```

```
pop_10 = [cities[:10]]*20
pop_24 = [cities]*20
pop_7 = [cities[11:19]]*20

# Uses GA to find shortest route of first 10 cities. And takes the time
start = time.time()
best_10, d_best_10, fittest_gen_10 = GA(pop_10, 700)
end = time.time()

time_10 = (end-start)

start = time.time()
best_24, d_best_24, fittest_gen_24 = GA(pop_24, 700)
end = time.time()

time_24 = (end-start)

plot_plan(best_10)
print(f"Time taken 10 cities: {time_10}s. 24 Cities: {time_24}s")

generations = [i for i in range(700)] # For plot
fittest_all_10 = []
fittest_all_24 = []
fittest_all_7 = []

# Run the GA 20 times to find mean, best, etc.
pop_10_best = []
pop_24_best = []
pop_7_best = []
for i in range(20):
    best_10, d_best_10, fittest_gen_10 = GA(pop_10, 700)
    best_24, d_best_24, fittest_gen_24 = GA(pop_24, 700)
    best_7, d_best_7, fittest_gen_7 = GA(pop_7, 700)
    pop_10_best.append(d_best_10)
    pop_24_best.append(d_best_24)
    pop_7_best.append(d_best_7)
    fittest_all_10.append(fittest_gen_10)
    fittest_all_24.append(fittest_gen_24)
    fittest_all_7.append(fittest_gen_7)

average_fit_10 = []
average_fit_24 = []
average_fit_7 = []
for i in range(len(fittest_all_10[0])):
    average_fit_10.append(np.mean([fittest_all_10[k][i] for k in range(20)]))
    average_fit_24.append(np.mean([fittest_all_24[k][i] for k in range(20)]))
    average_fit_7.append(np.mean([fittest_all_7[k][i] for k in range(20)]))

plt.plot(generations, average_fit_10, label = "Average best fitness 10 Cities")
plt.plot(generations, average_fit_24, label = "Average best fitness 24 Cities")
plt.plot(generations, average_fit_7, label = "Average best fitness 7 Cities")
plt.xlabel("Generations")
plt.ylabel("Average Fitness of Best individual")
plt.legend()
plt.show()

print("Out of 20 runs of 3 sizes of population these are the results: \n")
print(f"Best 10: {min(pop_10_best)} Best 24: {min(pop_24_best)} Best 7: {min(pop_7_
print(f"Worst 10: {max(pop_10_best)} Worst 24: {max(pop_24_best)} Worst 7: {max(pop
```

```
print(f"Worst 10: {max(pop_10_best)} Worst 24: {max(pop_24_best)} Worst 7: {max(pop
print(f"Mean 10: {np.mean(pop_10_best)} Mean 24: {np.mean(pop_24_best)} Mean 7: {np
print(f"Standard dev 10: {np.std(pop_10_best)} Std 24: {np.std(pop_24_best)} Std 7:
```




```
Time taken 10 cities: 0.1953105926513672s. 24 Cities: 0.5289833545684814s
```




```
Out of 20 runs of 3 sizes of population these are the results:

Best 10: 8363.05 Best 24: 26005.64 Best 7: 8218.24

Worst 10: 11536.34 Worst 24: 32260.260000000002 Worst 7: 10383.199999999999

Mean 10: 10096.157500000001 Mean 24: 28956.161 Mean 7: 9244.143999999998

Standard dev 10: 872.8217280170963 Std 24: 1832.0486120703788 Std 7: 511.7578045755
6255
```

Among the first 10 cities, did your GA find the shortest tour (as found by the exhaustive search)? Did it come close?

For both 10 and 24 cities: How did the running time of your GA compare to that of the exhaustive search?

How many tours were inspected by your GA as compared to by the exhaustive search?

Among the first 10 cities, did your GA find the shortest tour (as found by the exhaustive search)? Did it come close?

- No, the GA did not find the best solution, but it was close, and it probably would if i optimized som parameters.

For both 10 and 24 cities: How did the running time of your GA compare to that of the exhaustive search?

- Time taken 10 cities: 0.18848729133605957s. 24 Cities: 0.5677394866943359s, so the GA is much faster than the exhaustive search.

How many tours were inspected by your GA as compared to by the exhaustive search?

- My hill climb algorithm didnt work for 24 cities because of the way i implemented it, by using permutations. So i cant compare this properly. And i dont have time to fix it.

In [ ]:

```

```