IN3140 Assignment 4

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Task 1

a)

For this task i will use the τ_2 expression from assignment 3:

$$\tau_2 = m_2 L_2^2 \ddot{\theta}_2 + m_2 L_2^2 \dot{\theta}_1^2 c_2 s_2 + m_2 g L_2 c_2$$

We want to make this equation independent of all other joints, from the new model we see that the base joint and its link has been removed, we also see that what used to be joint 3 is now not there, so we set all variables associated with those to 0. Those variables are: $\theta_1, \dot{\theta}_1, \ddot{\theta}_1$, $\theta_3, \dot{\theta}_3, \ddot{\theta}_3$, when we set these to 0 we end up with:

$$\tau_2 = m_2 L_2^2 \ddot{\theta}_2 + m_2 g L_2 c_2$$

Where, m_2, L_2, θ_2 is now: m, L, θ:

$$\tau = mL^2\ddot{\theta} + mgL\cos(\theta)$$

b)

If we Laplace transform the given equation, we get:

$$u(t) = Js^2\theta(t) + Bs\theta(t) + D(t)$$

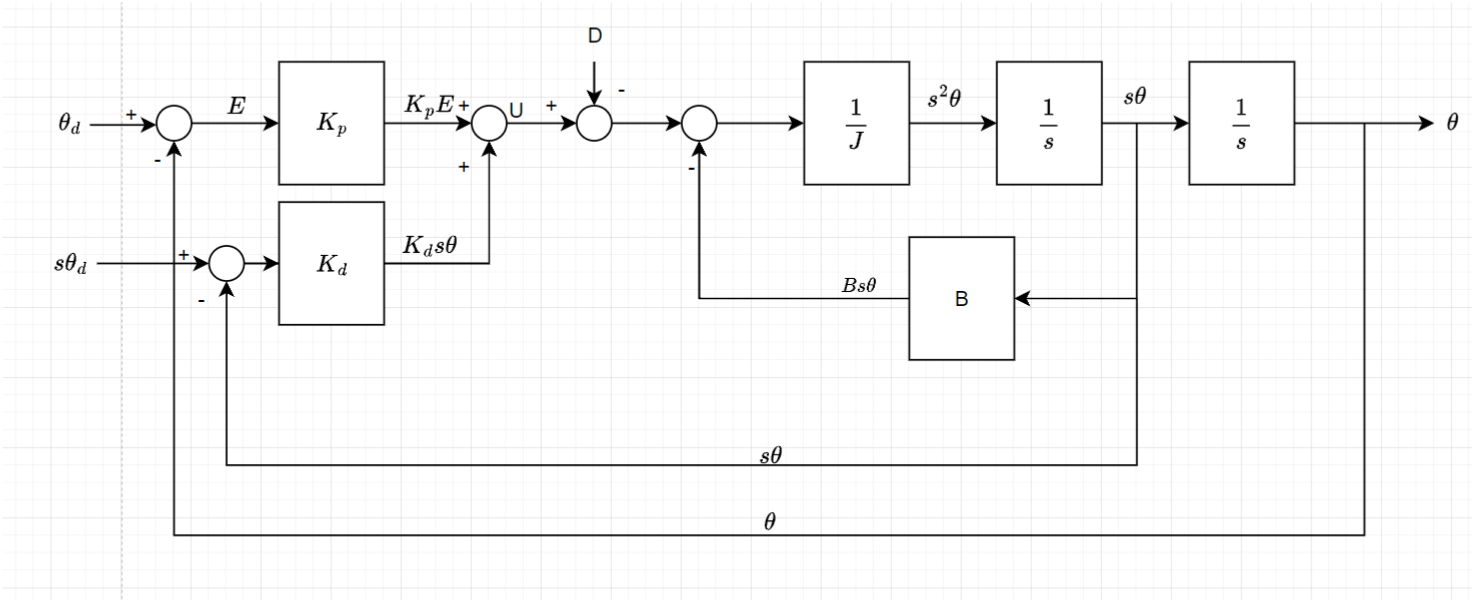
From the equation we derived in a, we get the laplace transformation:

$$\tau = mL^2 s^2 \theta(t) + mgLcos(\theta(t))$$

Where J is the first term of the equation above and D (the disturbance) is the second term, which are the gravitational forces acting on the arm.

c)

Block diagram for system with PD-controller



We have the controller equation for the PD-controller:

$$U_{controller} = Kp(\theta_d - \theta) - K_d s\theta$$

And the dynamic equation for the system:

$$U_{sys} = Js^2\theta + D$$

Since B = 0.

We set these equal to eachother, then solve for θ:

$$U_{controller} = U_{sys}$$
$$Kp(\theta_d - \theta) - K_d s\theta = Js^2\theta + D$$

When solved for θ we get the transfer function:

$$\theta = \frac{K_p\theta_d - D}{Js^2 + K_d s + K_p}$$

d)

Since we are dealing with a second order, critically damped system with a natural frequency of 6 we can use the formula:

$$s^2 + 2\xi ws + w^2 = 0$$

Because we know its a critically damped system we set ξ to 1, and we know that $w = 6$ we can then find the value for s.

$$s^2 + 12s + 36 = 0$$

From this equation we get that s = -6. Now we use the denominator from the transfer function and get it on the form of a general/critically damped system, and set this equal to 0:

$$Js^2 + K_d s + K_p = 0$$

$$s^2 + \frac{K_d}{J}s + \frac{K_p}{J} = 0$$

From the equation for critically damped systems, we get the relations:

$$2w = \frac{K_d}{J} \Rightarrow K_d = 2wJ = 2wmL^2$$

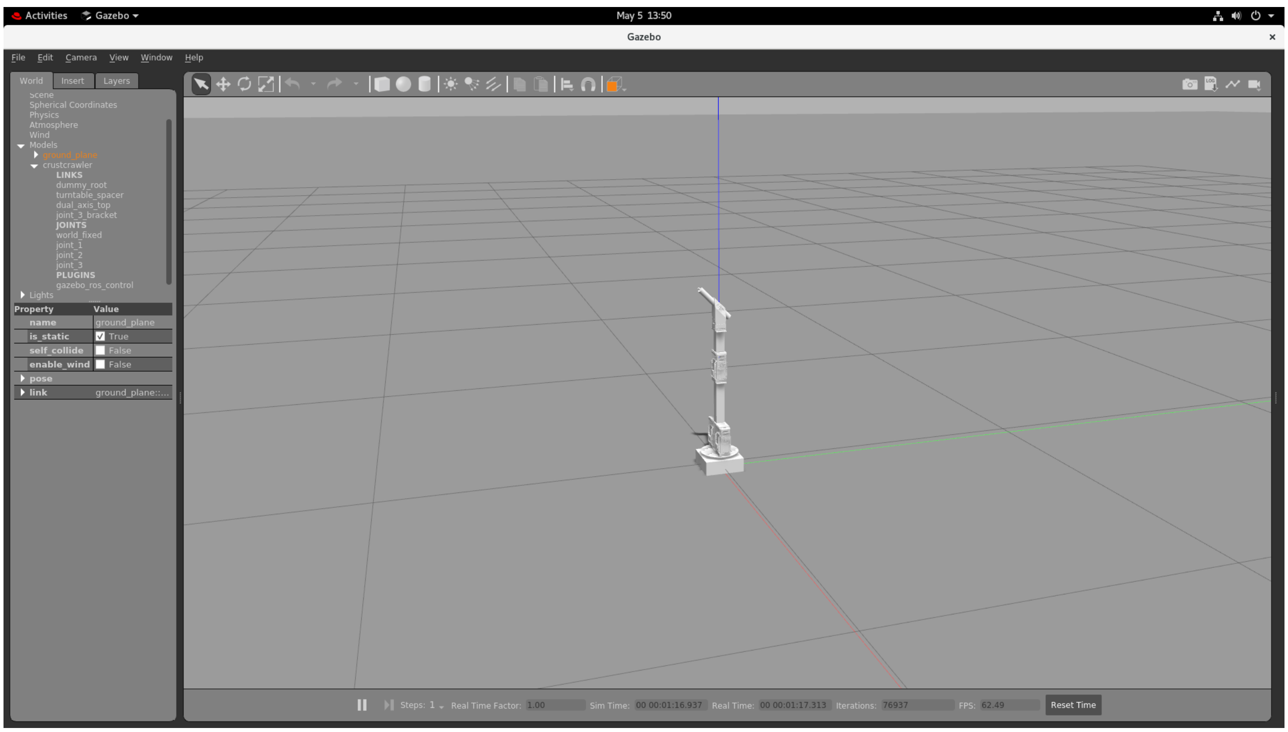
$$w^2 = \frac{K_p}{J} \Rightarrow K_p = Jw^2 = mL^2w^2$$

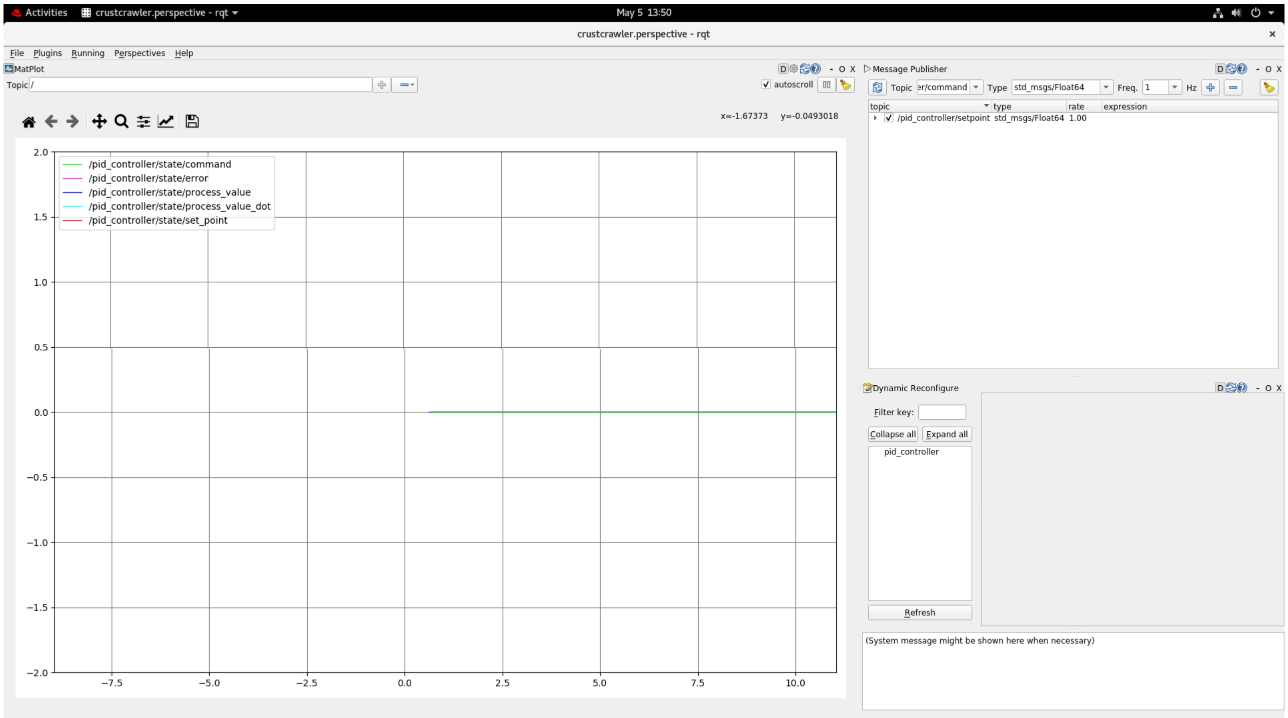
We have not been given values for L, $m = 0.413kg$, $w = 6$, im gonna use the link lengths from assignment 2: $L = 0.3583m$, then we get the K_p and K_d values:

$$K_d = 2 * 6 * 0.413 * 0.3583^2 = 0.63625$$

$$K_p = 0.413 * 0.3583^2 * 6^2 = 1.9087$$

Task 2

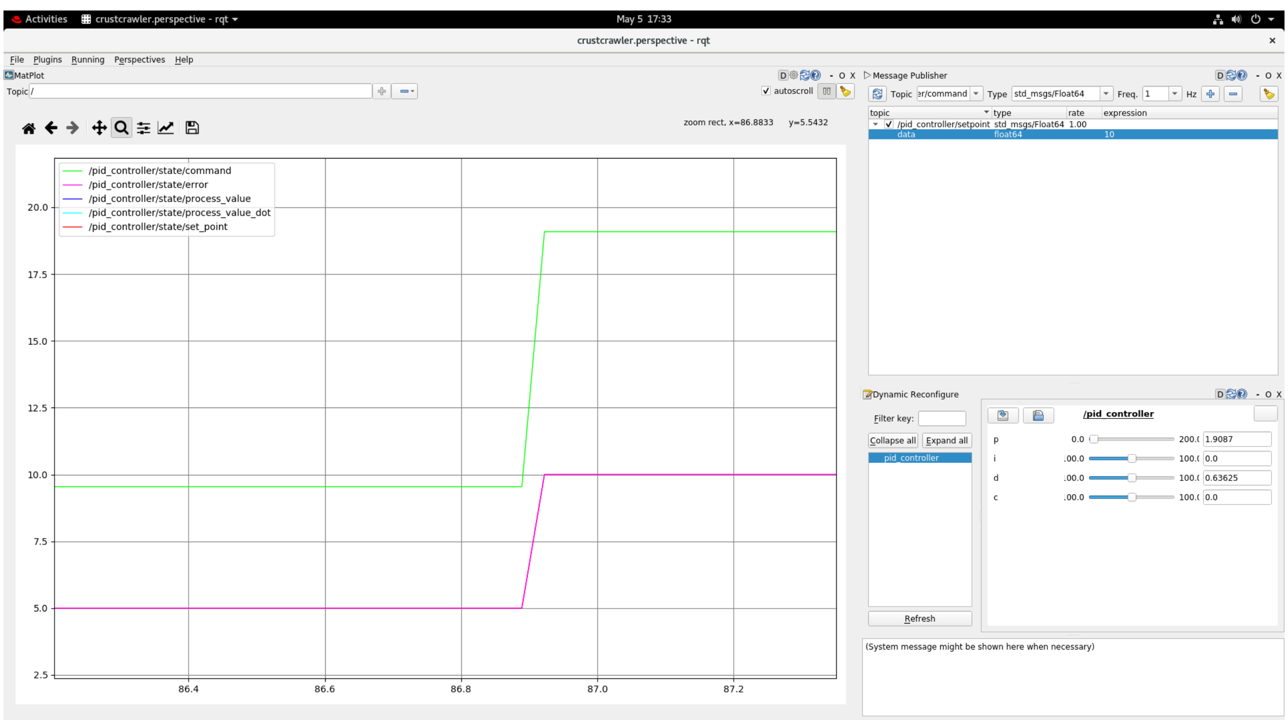




Task 3

a)

When implementing the PD-controller, and running the plot program/simulation, i cant seem to get the plots that i am expecting, i also try to print out the value of velocity_theta it is always 0, even when changing setpoint, so i think i am doing something wrong when it comes to the simulation, since the positions doesn't seem to change, and i didn't have time to try to fix it.



b)

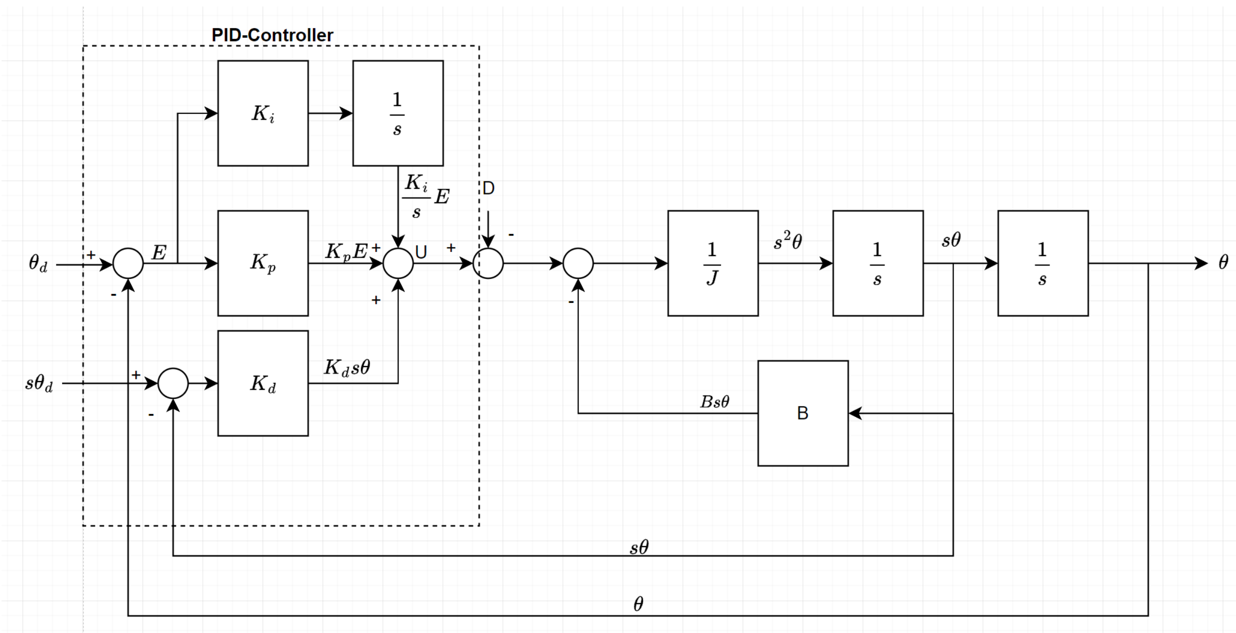
Since i can't really see anything from the plots, this subtask is hard to do.

c)

Since my plots doesn't really work, i cant really say anything on this, but if the steady state error were too large, you could implement a PID-controller instead of a PD-controller. The integral term of the PID-controller can help giving the controller the extra push if it stops too early because of the derivative term, of course if you choose the right K_i.

d)

I add a integral term to the PD-controller, the block diagram now looks like this:



Task 4

I tried implementing the Ziegeler-Nichol's first method, but since i cant get proper results when trying to get simulate it, therefore i couldn't get the proper value for T_k or K_{pk}, which means i can't get the values for K_p and T_i unless there is something i have misunderstood, my attempt is atleast in the python file.

Task 5

As i understood it this was not mandatory to do? And since i couldn't get proper results on the previous tasks i cant do this one.