IN3140 Assignment 1

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Task 1 - Transformations

To find the Transformation matrix: T_T^B we can find T_W^B and T_T^W, then multiply them together, so: $T_T^B = T_W^B * T_T^W$. To make it easier, we can find $T_W^B => (T_B^W)^{-1}$, so first we find:

$$T_B^W = \begin{bmatrix} R_B^W & d_B^W \\ 0 & 1 \end{bmatrix}$$

Where R_B^W and d_B^W, is the Rotational matrix and distance matrix respectively relating World - frame, and the Base frame. The distance matrices are already given, so:

$$d_B^W = \begin{bmatrix} 250 \\ 650 \\ 1000 \end{bmatrix}$$

We find the rotational matrix, by figuring out around what axis (and by how much) we must rotate the World - frame to align it with the Base frame

If we want to aligne these to axises, we want to rotate World - frame θ around its Z-axis, then we want to rotate α around the **current** x-axis. Then we get:

$$R_B^W \implies R_{Z_W,\theta} * R_{X_{Current},\alpha}$$

We then fill in the "generic" rotation matrices for what axis we are rotating around:

$$R_B^W = \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\alpha & -S_\alpha \\ 0 & S_\alpha & C_\alpha \end{bmatrix}$$

We then fill in the angle of which we rotate by, which is $\frac{\pi}{2}$ about Z_W, then π about $X_{current}$.

$$R_B^W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$T_B^W = \begin{bmatrix} R_B^W & d_B^W \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 250 \\ 1 & 0 & 0 & 650 \\ 0 & 0 & -1 & 1000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We want to find the inverse of this matrix, which is given by:

$$(T_B^W)^{-1} = \begin{bmatrix} (R_B^W)^T & -(R_B^W)^T * d_B^W \\ 0 & 1 \end{bmatrix}$$

$$(R_B^W)^T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$-(R_B^W)^T * d_B^W = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 250 \\ 650 \\ 1000 \end{bmatrix} = \begin{bmatrix} -650 \\ -250 \\ 1000 \end{bmatrix}$$

We then get:

$$T_W^B = \begin{bmatrix} 0 & 1 & 0 & -650 \\ 1 & 0 & 0 & -250 \\ 0 & 0 & -1 & 1000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then we have to find T_T^W:

$$d_T^W = \begin{bmatrix} 1000 \\ 400 \\ 900 \end{bmatrix}$$

$$R_T^W = R_{y,\phi}$$

We rotate π about y_W. So the rotational matrix will be:

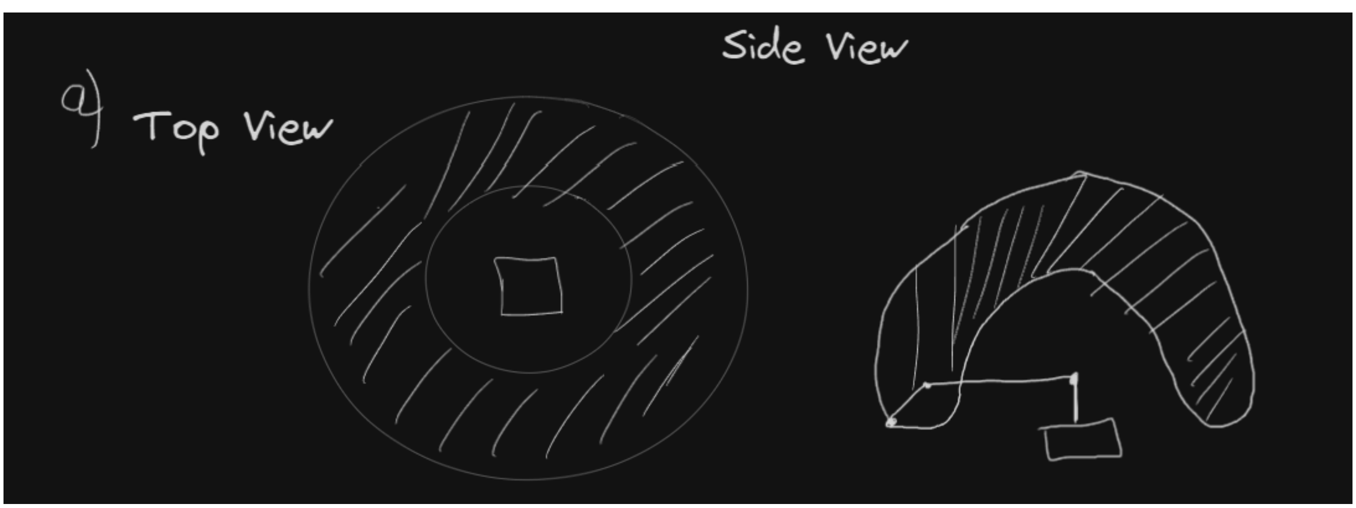
$$R_T^W = \begin{bmatrix} C_\phi & 0 & S_\phi \\ 0 & 1 & 0 \\ -S_\phi & 0 & C_\phi \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$T_T^W = \begin{bmatrix} -1 & 0 & 0 & 1000 \\ 0 & 1 & 0 & 400 \\ 0 & 0 & -1 & 900 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

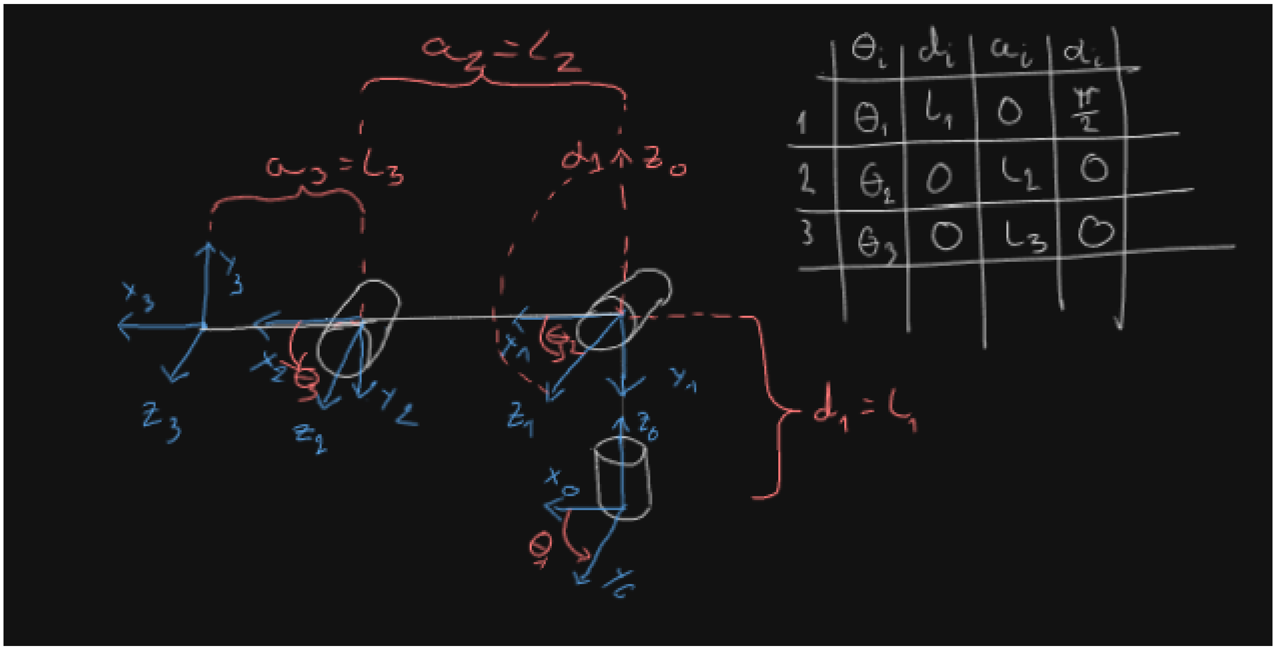
$$T_T^B = T_W^B * T_T^W = \begin{bmatrix} 0 & 1 & 0 & -650 \\ 1 & 0 & 0 & -250 \\ 0 & 0 & -1 & 1000 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 0 & 1000 \\ 0 & 1 & 0 & 400 \\ 0 & 0 & -1 & 900 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -250 \\ -1 & 0 & 0 & 750 \\ 0 & 0 & 1 & 100 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Task 2 - Transformations

a)



b)

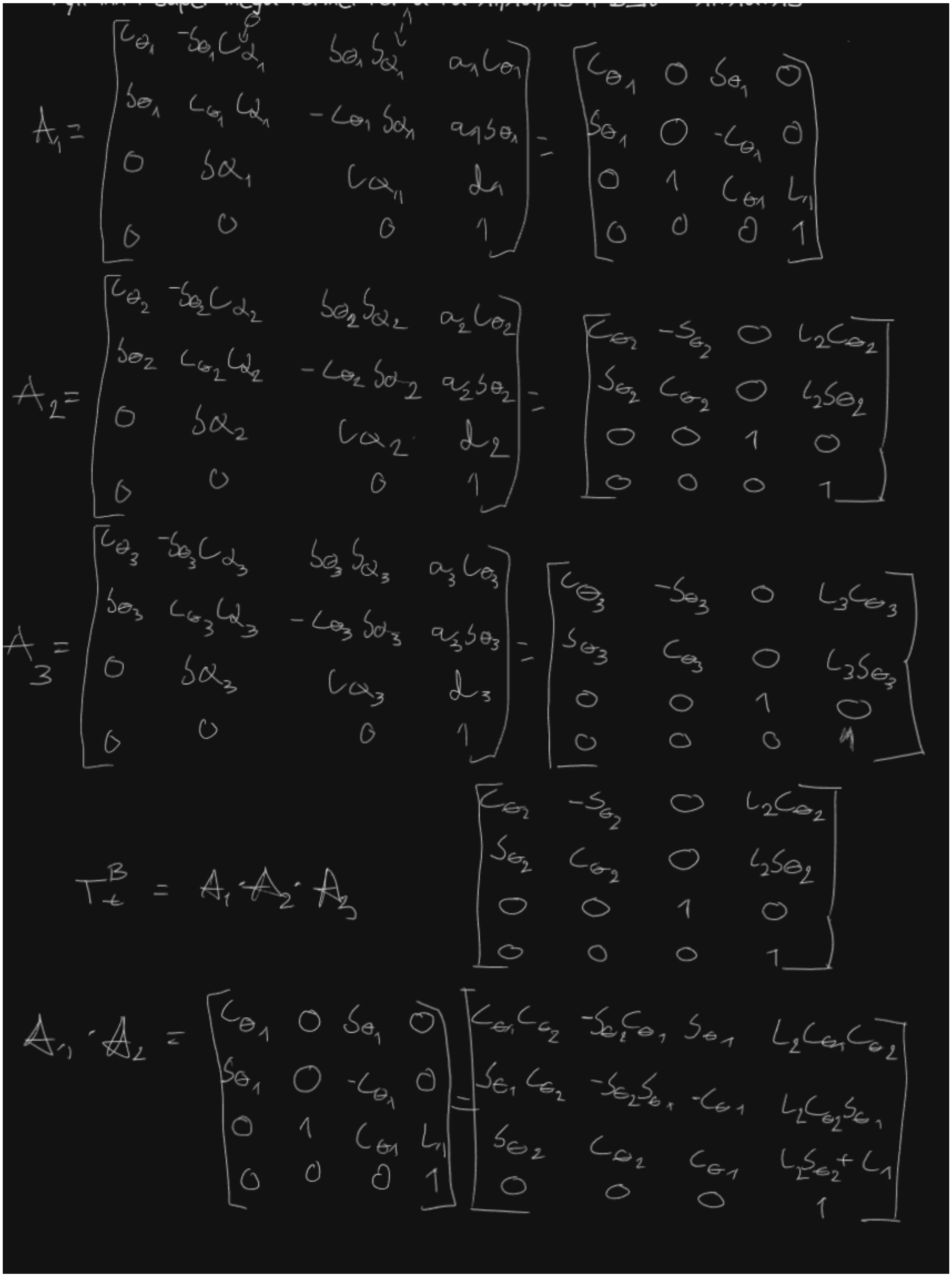


c)

We use the formula:

$$A_i = \begin{bmatrix} C_{\theta_i} & -S_{\theta_i}C_{\alpha_i} & S_{\theta_i}S_{\alpha_i} & a_iC_{\theta_i} \\ S_{\theta_i} & C_{\theta_i}C_{\alpha_i} & -C_{\theta_i}S_{\alpha_i} & a_iS_{\theta_i} \\ 0 & S_{\alpha_i} & C_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And put in the values from the table on the sketch for each joint, then we multiply: $T_t^B = A_1 * A_2 * A_3$, i will paste my hand written solution:



When we multiply the last matrix we end up with, these are simplyfied terms:

$$T_t^B = \begin{bmatrix} C_{\theta_1} * C_{\theta_2+\theta_3} & -C_{\theta_1} * S_{\theta_2+\theta_3} & S_{\theta_1} & C_{\theta_1} * (l_2 * C_{\theta_2} + l_3 * C_{\theta_2+\theta_3}) \\ S_{\theta_1} * C_{\theta_2+\theta_3} & -S_{\theta_1} * S_{\theta_2+\theta_3} & -C_{\theta_1} & S_{\theta_1} * (l_2 * C_{\theta_2} + l_3 * C_{\theta_2+\theta_3}) \\ S_{\theta_2+\theta_3} & C_{\theta_2+\theta_3} & C_{\theta_1} & l_1 * l_2 * S_{\theta_2} + l_3 * S_{\theta_2+\theta_3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Task 3 - Forward Kinematics II

First find the angles correct for how i defined my theta angles: For θ_1 i don't have to move the x-axis to align with how the exercise wants to. For θ_2 i have to move the x-axis -30 degrees around z1. For θ_3 we also have to move it 45 degrees.

Inserting the angles and the length for the links into the matrix from task 2, we get the Transformation matrix:

$$T_t^B = \begin{bmatrix} \frac{\sqrt{6}-\sqrt{2}}{4} & -\frac{\sqrt{6}+\sqrt{2}}{4} & 0 & 323.9 \\ 0 & 0 & -1 & 0 \\ \frac{\sqrt{6}-\sqrt{2}}{4} & \frac{\sqrt{6}+\sqrt{2}}{4} & 1 & 25.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P^T = T_B^T * P^B = (T_T^B)^{-1} * P^B$, i have already shown earlier how to inverse the transformational matrix, so i will just set up the answer for the point.

$$P^T = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 323.9 \\ 0 \\ 25.1 \end{bmatrix} = \begin{bmatrix} 0 \\ 323.9 \\ 25.1 \end{bmatrix}$$

Task 4 - Inverse Kinematics

a)

The two most common ways of deriving inverse kinematics are the analytic approach and the geometrical approach. The analytic approach uses the expression for the position of the tooltip in the base frame, we can then find expressions on how we should rotate and move the arm to get to the position of the tooltip.
The geometric approach is based on viewing the arm from side and top view and then finding inverse kinematics equations off of geometry on how the arm can move.
I plan on using the analytic approach, because we already have the expressions from task 3 to find the position of the tooltip.

b)

So we have the expressions for the position of the tooltip:

$$x = C_{\theta_1} * (l_2 * C_{\theta_2} + l_3 * C_{\theta_2+\theta_3})$$
$$y = S_{\theta_1} * (l_2 * C_{\theta_2} + l_3 * C_{\theta_2+\theta_3})$$
$$z = l_1 * l_2 * S_{\theta_2} + l_3 * S_{\theta_2+\theta_3}$$

The L values never change in the robot we are using here , so we must find expressions for $\theta_0, \theta_2, \theta_3$:

$$\frac{y}{x} = \frac{S_{\theta_1} * (l_2 * C_{\theta_2} + l_3 * C_{\theta_2+\theta_3})}{C_{\theta_1} * (l_2 * C_{\theta_2} + l_3 * C_{\theta_2+\theta_3})} \Longrightarrow \frac{y}{x} = \frac{sin(\theta_1)}{cos(\theta_1)}$$

$$\theta_1 = arctan(\frac{y}{x})$$

Didn't have time to finish the rest of the equations and the net sub-exercise.