

# Humanoid Robotics. Exercise Sheet 9 - Reachability maps and bipedal walking

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# 1 Exercise 17

Ex 17

a) 
$$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1,5 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0,3 \\ 0 & 0 & 1 \end{pmatrix}$$

b) 
$$V_1 = T_1(q_1) T_2(q_2) T_3(q_3) V_3$$

$$T_1(q_1) = \begin{pmatrix} \cos q_1 & -\sin q_1 & l \sin q_1 \\ \sin q_1 & \cos q_1 & -l \cos q_1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_2(q_2) = \begin{pmatrix} \cos q_2 & -\sin q_2 & l \sin q_2 \\ \sin q_2 & \cos q_2 & -l \cos q_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_3(q_3) = \begin{pmatrix} \cos q_3 & -\sin q_3 & l \sin q_3 \\ \sin q_3 & \cos q_3 & -l \cos q_3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_1(q_1) \cdot T_2(q_2) \cdot T_3(q_3) =$$

$$= \begin{pmatrix} \cos(q_1 - q_2 - q_3) & \sin(q_2 - q_1 - q_3) & l(\cos(q_3 - q_2 + q_1) + \sin(q_1 - q_2) + \sin q_1) \\ \sin(q_1 - q_2 + q_3) & \cos(q_1 - q_2 + q_3) & l(-\cos(q_1 - q_2 + q_3) - \cos(q_1 - q_2) + \cos q_1) \\ 0 & 0 & 1 \end{pmatrix}$$

Figure 1: Exercise 17

$$\begin{aligned}
c) \quad l &= 0.5 & q_1 &= \frac{\pi}{4} & q_3 &= \frac{\pi}{4} \\
\mathcal{V}_3 &= \begin{pmatrix} 0.1 \\ 0 \\ 1 \end{pmatrix} & q_2 &= \frac{\pi}{2}
\end{aligned}$$

$$\begin{pmatrix} \cos \frac{\pi}{2} & \sin 0 & 0.5 \left( \cos 0 + \sin -\frac{\pi}{4} + \sin \frac{\pi}{4} \right) \\ \sin 0 & \cos 0 & 0.5 \left( -\cos 0 - \cos -\frac{\pi}{4} + \cos \frac{\pi}{4} \right) \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & 0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{V}_1 = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & 0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{10\sqrt{2}}{20} \\ -0.5 \\ 1 \end{pmatrix}$$

Figure 2: Exercise 17

## 2 Exercise 18

- a) What observation can you make in the [0-1] second range?  
In first second we can observe, that Linearized Inverted Pendulum Model (LIPM) makes bad prediction, but Locally Linearized Inverted Pendulum Model (LLIPM) still has good results.
- b) What observation can you make in the [1-3] second range?  
In first 3 seconds we can observe, that both models cannot predict the angle after three seconds.
- c) Now change the initial angle to  $\theta = 1.2$  and repeat the experiment. Produce a second plot. In what way did the outcome change? Insert the plot in the pdf.  
Now both model cannot predict the angle even after a second.
- d) Still using  $C = 10$ , assume a 2D linear inverted pendulum model with an initial state of  $(x=-1.0, \dot{x} = 1.0; y = 1.0, \dot{y} = -1.0)$ . At what time is the pendulum going to reach a lateral position of  $y = 2.0$ ?  
Newer, because if it is in the state  $(-1, 1)$ , then the radius of the pendulum is  $\sqrt{2}$

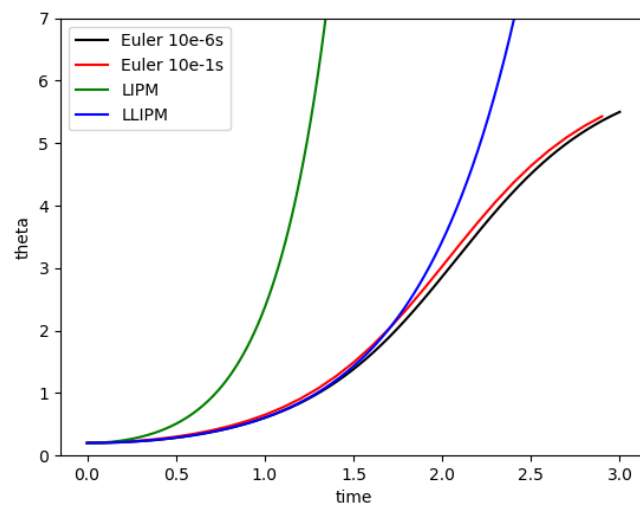


Figure 3:  $\Theta = 0.2$

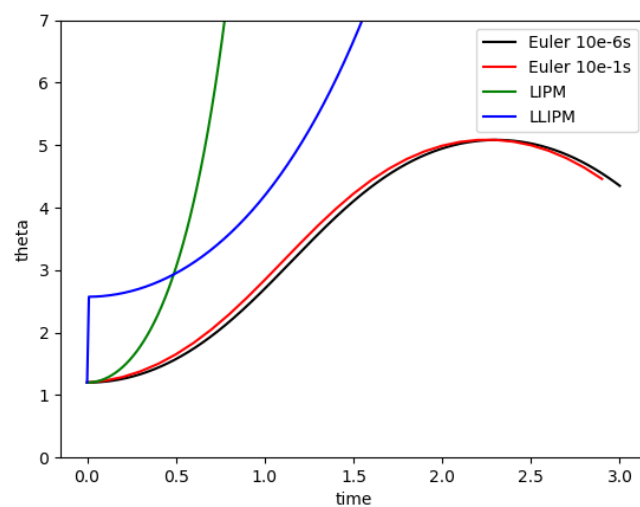


Figure 4:  $\Theta = 1.2$

### 3 Exercise 19

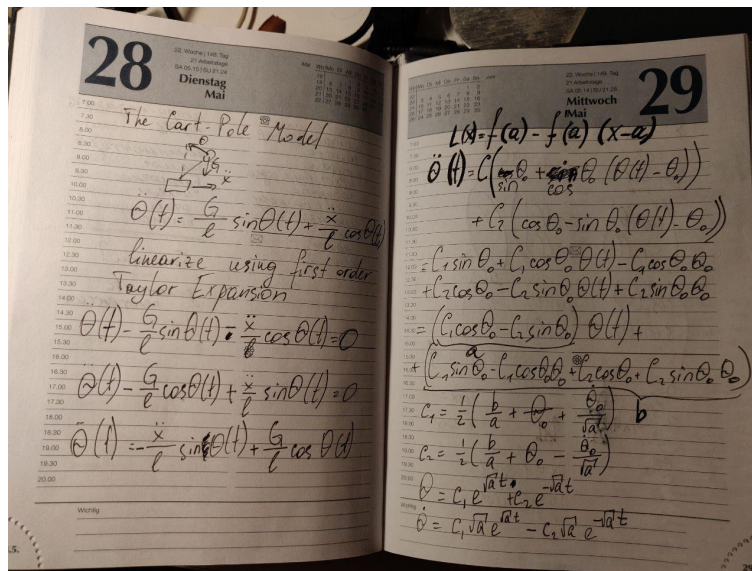


Figure 5: Exercise 19