$$
\begin{aligned}
g &= \sum_i \left\{ a\frac{V_i'}{V_i}(y_i-\mu_i) + \frac{(y_i-\mu_i)^2}{V_i} - \phi \right\} = 0 \\
g_\phi &= \sum_i \left\{ a_\phi \frac{V_i'}{V_i}(y_i-\mu_i) - 1 \right\}
\end{aligned}
$$

NB $\mathbb{E}g = 0$.

$$
\begin{aligned}
\mathbb{E}g_\phi &= -n \\
\mathbb{E}g^2 &= \mathbb{E}\sum_{ij} \left\{ a\frac{V_i'}{V_i}(y_i-\mu_i) + \frac{(y_i-\mu_i)^2}{V_i} - \phi \right\} \left\{ a\frac{V_j'}{V_j}(y_j-\mu_j) + \frac{(y_j-\mu_j)^2}{V_j} - \phi \right\} \\
&= \sum_i \left\{ a^2 \frac{V_i'^2}{V_i}\phi + 2a\frac{V_i'}{V_i^2}\kappa_{3i} + \frac{\kappa_{4i} + 3\phi^2 V_i^2}{V_i^2} - 2n\phi^2 + n\phi^2 \right\} + \sum_{i\neq j} \phi^2 \\
&= \sum_i \left\{ a^2 \frac{V_i'^2}{V_i}\phi + 2a\frac{V_i'}{V_i^2}\kappa_{3i} + \frac{\kappa_{4i}}{V_i^2} \right\} + 2n\phi^2
\end{aligned}
$$

It follows that

$$
\mathbb{G} = \left(\mathbb{E}g_\phi\right)^{-1} \mathbb{E}g^2 \left(\mathbb{E}g_\phi\right)^{-1} = \frac{2\phi^2}{n} + \frac{1}{n^2}\sum_i \left\{ a^2 \frac{V_i'^2}{V_i}\phi + 2a\frac{V_i'}{V_i^2}\kappa_{3i} + \frac{\kappa_{4i}}{V_i^2} \right\}
$$

For the exponential family, $\kappa_{3i} = \phi^2 V_i V_i'$ and $\kappa_{4i} = \phi^3(V_i'' V_i^2 + V_i'^2 V_i)$, so

$$
\begin{aligned}
\mathbb{G} &= \frac{2\phi^2}{n} + \frac{1}{n^2}\sum_i \left\{ a^2 \frac{V_i'^2}{V_i}\phi + 2a\frac{V_i'^2}{V_i}\phi^2 + \left(V_i'' + \frac{V_i'^2}{V_i} \right)\phi^3 \right\} \\
&= \frac{2\phi^2}{n} + \frac{\phi}{n^2}\left\{ (a+\phi)^2 \sum_i \frac{V_i'^2}{V_i} + \phi^2 \sum_i V_i'' \right\}
\end{aligned}
$$

Reminder

$$
\begin{aligned}
\mu &= \psi_\theta \\
V &= \psi_{\theta\theta} \\
V_\mu &= \psi_{\theta\theta\theta}/\mu_\theta = \kappa_3^\star/V \\
V_{\mu\mu}V + V_\mu^2 &= \kappa_4^\star/V
\end{aligned}
$$