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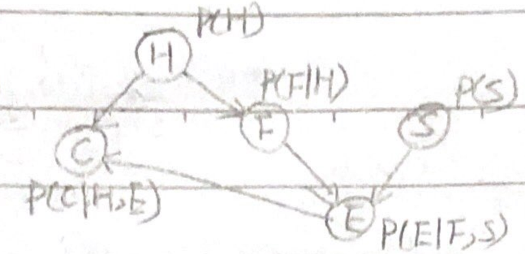
No.

Date

1.a)



b) It is not a polytree. Because it has no directed cycles.

c)

P(F|H)		
	F=1	F=0
H=1	x	$1-x$
H=0	y	$1-y$

d)

P(E|F,S)		
	E=1	E=0
F=1, S=1	1	0
F=1, S=0	1	0
F=0, S=1	0.5	0.5
F=0, S=0	0	1

e) $P(H|C) = \frac{P(H,C)}{P(C)} = \frac{\sum_{F,E,S} P(F|H)P(E|F,S)P(C|H,E)P(H)}{\sum_{H,F,E,S} P(F|H)P(E|F,S)P(C|H,E)P(H)}$

2.a) $P(a,\neg r) = \sum_{b,t,s} P(\neg r)P(b)P(t|\neg r,b,a)P(a|b)P(s|a)$

$= P(\neg r)\sum_b p(b)p(a|b)\sum_s p(s|a)\sum_t p(t|\neg r,b,a)$

$= 0.8 \times (p(b)p(a|b)+p(\neg b)p(a|\neg b))$

$= 0.8 \times 0.4 = 0.32$

b) $P(b,a) = \sum_{r,t,s} P(r)P(b)P(t|r,b,a)P(a|b)P(s|a)$

$= P(b)P(a|b)\sum_r P(r)\sum_t P(t|r,b,a)\sum_s P(s|\neg a)$

$= 0.4 \times 0.7 = 0.28$

c) Pruned nodes: R, T, S

d) $P(b|a) = \frac{P(b,a)}{P(a)} = \frac{0.28}{\sum_b P(b)P(a|b)} = \frac{0.28}{P(b)P(a|b)+P(\neg b)P(a|\neg b)} = \frac{0.28}{0.4} = 0.7$

3. $P(T|b) = \frac{P(T,b)}{P(b)} = \alpha \cdot P(T,b) = \alpha \sum_{r,a,s} P(r)P(b)P(t|r,b,a)P(a|b)P(s|a)$

$= \alpha\, p(b)\sum_r P(r)\sum_a P(a|b)P(t|r,b,a)\sum_s P(s|a)$

No.

Date . .

Eliminate S, replace $P(s|a)$ in the list by computing $m_S(a) = \sum_s P(s|a) = 1$

List: $P(b)$, $P(r)$, $P(t|r,b,a)$, $P(a|b)$, $P(s|a)$, $m_S(a)$

Eliminate A, $\sum_a P(t|r,b,a) \times P(a|b) \times m_S(a) = m_A(t,r,b)$

List: $P(b)$, $P(r)$, $m_A(t,r,b)$

Eliminate R, $\sum_r P(r), m_A(t,r,b) = m_R(t,b)$

List: $P(b)$, $m_R(t,b)$

Eliminate T, $\sum_t m_R(t,b) = m_T(b)$

List: $P(b)$, $m_T(b)$

For $m_A(t,r,b)$: $P(\boxed{T}|\boxed{R}, b=1, a) = \begin{bmatrix} 0.98 & 0.5 \\ 0.02 & 0.5 \end{bmatrix}$ $\quad P(\boxed{A}|b) = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$

$P(\boxed{T}|\boxed{R}, b=1, \neg a) = \begin{bmatrix} 0.88 & 0.4 \\ 0.12 & 0.6 \end{bmatrix}$

$m_A(t,r,b) = 0.7 \begin{bmatrix} 0.98 & 0.5 \\ 0.02 & 0.5 \end{bmatrix} + 0.3 \begin{bmatrix} 0.88 & 0.4 \\ 0.12 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.95 & 0.47 \\ 0.05 & 0.53 \end{bmatrix}$

For $m_R(t,b)$: $P(r) = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$, $m_R(t,b) = 0.2 \begin{bmatrix} 0.95 \\ 0.05 \end{bmatrix} + 0.8 \begin{bmatrix} 0.47 \\ 0.53 \end{bmatrix} = \begin{bmatrix} 0.566 \\ 0.434 \end{bmatrix}$

$m_T(b) = \sum_t m_R(t,b) = \begin{bmatrix} 0.566 \\ 0.434 \end{bmatrix}$

$P(T|b) = \alpha P(t,b) = \alpha P(b) m_T(b) = \begin{bmatrix} 0.566 \\ 0.434 \end{bmatrix}$

$\therefore P(t|b) = 0.566$, $P(\neg t|b) = 0.434$

4.a)

	MLE	LS
$\theta_a = P(a)$	$\frac{60}{144} = \frac{5}{12}$	$\frac{61}{146}$
$\theta_{b|a} = P(b|a)$	$\frac{33}{60} = \frac{11}{20}$	$\frac{34}{62} = \frac{17}{31}$
$\theta_{b|\neg a} = P(b|\neg a)$	$\frac{57}{84} = \frac{19}{28}$	$\frac{58}{86} = \frac{29}{43}$
$\theta_{c|a} = P(c|a)$	$\frac{50}{60} = \frac{5}{6}$	$\frac{51}{62}$
$\theta_{c|\neg a} = P(c|\neg a)$	$\frac{62}{84} = \frac{31}{42}$	$\frac{63}{86}$

No.

Date

$\theta_{d\|b,c} = P(d\|b,c)$	$\frac{20}{65} = \frac{4}{13}$	$\frac{21}{67}$
$\theta_{d\|\neg b,c} = P(d\|\neg b,c)$	$\frac{26}{47}$	$\frac{27}{49}$
$\theta_{d\|b,\neg c} = P(d\|b,\neg c)$	$\frac{11}{25}$	$\frac{12}{27} = \frac{4}{9}$
$\theta_{d\|\neg b,\neg c} = P(d\|\neg b,\neg c)$	$\frac{4}{7}$	$\frac{5}{9}$

b) i) $W_{bs_1} = P(b|a,c,d)$ or $P(b|a,c,\neg d) = \frac{P(b,a,c,d)}{P(a,c,d)}$ or $\frac{P(b,a,c,\neg d)}{P(a,c,\neg d)}$

$= \frac{\cancel{P(a)}\cdot P(b|a)\cdot \cancel{P(c|a)}\cdot P(d|b,c)}{\sum_b \cancel{P(a)}\cdot P(b|a)\cdot \cancel{P(c|a)}\cdot P(d|b,c)}$ or $\frac{\cancel{P(a)}\cdot P(b|a)\cdot \cancel{P(c|a)}\cdot P(\neg d|b,c)}{\sum_b \cancel{P(a)}\cdot P(b|a)\cdot \cancel{P(c|a)}\cdot P(\neg d|b,c)}$

$= \frac{P(b|a)\cdot P(d|b,c)}{P(b|a)P(d|b,c)+P(\neg b|a)P(d|\neg b,c)}$ or $\frac{P(b|a)P(\neg d|b,c)}{P(b|a)P(\neg d|b,c)+P(\neg b|a)P(\neg d|\neg b,c)}$

≈ 0.405 or 0.654

$\because 0.654 > 0.405$

$\therefore W_{bs_1} = 0.654$

$W_{\neg bs_1} = 0.346$

$W_{ds_1} = P(d|a,b,c)$ or $P(d|a,\neg b,c) = \frac{P(a,b,c,d)}{P(a,b,c)}$ or $\frac{P(a,\neg b,c,d)}{P(a,\neg b,c)}$

$= \frac{\cancel{P(a)}\cdot \cancel{P(b|a)}\cdot \cancel{P(c|a)}\cdot P(d|b,c)}{\sum_d \cancel{P(a)}\cdot \cancel{P(b|a)}\cdot \cancel{P(c|a)}\cdot P(d|b,c)}$ or $\frac{\cancel{P(a)}\cdot \cancel{P(\neg b,a)}\cdot \cancel{P(c|a)}\cdot P(d|\neg b,c)}{\sum_d \cancel{P(a)}\cdot \cancel{P(\neg b,a)}\cdot \cancel{P(c|a)}\cdot P(d|\neg b,c)}$

$= P(d|b,c)$ or $P(d|\neg b,c)$

≈ 0.308 or 0.553

$\because 0.553 > 0.308$

$\therefore W_{ds_1} = 0.553$

$\therefore W_{\neg ds_1} = 0.447$

No.

Date

ii) $P(a) = \frac{62}{146}$ $P(b|a) = \frac{33+1+0.654}{62} \approx 0.559$

$P(b|\neg a) = \frac{57}{84}$ $P(c|a) = \frac{51}{62}$ $P(c|\neg a) = \frac{62}{84}$

$P(d|b,c) = \frac{20+0.553}{65+0.654} \approx 0.313$ $P(d|\neg b,c) = \frac{26+0.553}{47+0.346} \approx 0.561$

$P(d|b,\neg c) = \frac{11}{26}$ $P(d|\neg b,\neg c) = \frac{4}{7}$

iii) $W_{bs_1} = \frac{0.559 \times 0.313}{0.559 \times 0.313 + 0.441 \times 0.561}$ or $\frac{0.559 \times 0.687}{0.559 \times 0.687 + 0.441 \times 0.439}$

≈ 0.414 or 0.665

$\because 0.665 > 0.414$

$\therefore W_{bs_1} = 0.665$

$\therefore W_{\neg bs_1} = 0.335$

$W_{ds_1} = 0.313$ or 0.561

$\because 0.561 > 0.313$

$\therefore W_{ds_1} = 0.561$

$\therefore W_{\neg ds_1} = 0.439$