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2 Weak Formulation

In order to apply some of the results from the lecture, we need to derive the weak formulation of the given problem

$$\begin{aligned} \text{Find } u \in C^2(\Omega) : -\Delta u + u &= \cos(\pi x)\cos(\pi y) && \text{in } \Omega && (1)\\ \partial_n u &= 0 && \text{on } \partial\Omega. && (2) \end{aligned}$$

Multiplying with an arbitary $v \in C^2(\Omega)$ and integrating over Ω gives us

$$-\int_\Omega \Delta u v \,\mathrm{d}\boldsymbol{x} + \int_\Omega uv \,\mathrm{d}\boldsymbol{x} = \int_\Omega fv \,\mathrm{d}\boldsymbol{x}$$

where $f = \cos(\pi x)\cos(\pi y)$ and $\boldsymbol{x} = (x, y)$. Using Green's first formula and (2) we can obtain the weak formulation

$$\int_\Omega \nabla u \nabla v \,\mathrm{d}\boldsymbol{x} + \int_\Omega uv \,\mathrm{d}\boldsymbol{x} = \int_\Omega fv \,\mathrm{d}\boldsymbol{x}$$

From now on, we will denote the left hand side of the equation by $a(u, v)$, the right hand side by $F(v)$. Thus, we obtain the weak formulation

$$\text{Find } u \in H^1(\Omega) : \quad a(u, v) = F(v) \quad \forall v \in H^1(\Omega) \tag{3}$$

3 Existence and Uniqueness of a Solution