A Complete Geometric Derivation of the Fine-Structure Constant Residual via Harmonic Phase Variance and Mirror Scalar Projection

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Abstract

The fine-structure constant is traditionally treated as an empirical parameter whose numerical value arises from quantum electrodynamics and renormalization. In this work, we present a fully geometric, non-circular derivation of the fractional residual in the inverse fine-structure constant. The derivation proceeds from octonionic dimensionality, harmonic phase structure, and scalar observability constraints. We show that electromagnetic interaction measures a single normalized variance slice of a compressed reciprocal fiber—a mirror scalar—that houses the non-spatial octonionic degrees of freedom. This yields a parameter-free prediction for the scale of the residual, within a few percent of the CODATA value, with the remaining deviation identified as dynamical dressing rather than geometric deficiency. The result reframes the fine-structure constant as a projection thickness rather than a fundamental coupling.

1 Introduction

The numerical value of the fine-structure constant,

$$\alpha^{-1} \approx 137.035999177,$$

has long resisted derivation from first principles. While quantum electrodynamics (QED) successfully predicts radiative corrections and running behavior, the absolute low-energy value is typically treated as experimentally determined.

This work addresses a different question: whether the *fractional residual* beyond the integer scaffold can be derived geometrically, without perturbation theory, renormalization group flow, or circular dependence on α itself.

2 Octonionic Dimensional Scaffold

Octonionic algebra consists of one real scalar and seven imaginary units. Physical observables do not directly access this full structure. One degree of freedom is reserved for experiential or navigational coupling, leaving an effective harmonic phase manifold of dimension

$$N = 7.$$

This dimensionality fixes the fundamental phase increment

$$\theta = \frac{2\pi}{7}.$$

No empirical input is used; this follows purely from octonionic structure.

3 Mirror Scalar and Dimensional Compression

Projection into observable physics does not eliminate the remaining four non-spatial octonionic degrees of freedom. Instead, they are compressed into a single reciprocal scalar channel, hereafter referred to as the *mirror scalar*. This scalar does not represent an additional axis but a bundled fiber containing:

- triality-averaged structure,
- forward/backward traversal degeneracy,
- phase history,
- reciprocity information.

Formally, the projection takes the form

$$\mathbb{R}^7 \longrightarrow \mathbb{R}^3 \oplus \mathbb{R}_{\text{mirror}},$$

where $\mathbb{R}_{\text{mirror}}$ is one-dimensional in representation but multi-dimensional in content.

4 Scalar Observability and Phase Variance

Electromagnetism does not couple to phase directly. It is invariant under phase sign reversal, traversal direction, and triality orientation. Consequently, the lowest-order observable derived from phase is not θ itself, but its variance.

The unique scalar invariant associated with a phase angle is

$$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right).$$

Substituting the fundamental phase increment yields the raw harmonic shell variance

$$\Delta_{\text{raw}} = 2\sin^2\left(\frac{\pi}{7}\right) \approx 0.376510.$$

This quantity represents the total variance of the unobserved phase manifold.

5 Electromagnetic Projection

Electromagnetic interaction enforces two independent binary collapses:

1. Phase-sign invariance (charge conjugation symmetry),
2. Triality averaging over octonionic conjugate sectors.

Each contributes a factor of 1/2, yielding a total electromagnetic projection factor of

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

Thus,

$$\Delta_{\mathrm{EM}} = \frac{1}{4}\Delta_{\mathrm{raw}} = \frac{1}{2}\sin^2\left(\frac{\pi}{7}\right) \approx 0.0941275.$$

6 Four-Slice Observable Normalization

Observable physics presents four effective scalar channels:

$$\{x, y, z, |\psi|\},$$

corresponding to three spatial projections and one magnitude (mirror) scalar.

Normalizing by this four-slice structure yields

$$\Delta_{\mathrm{norm}} = \frac{1}{4}\Delta_{\mathrm{EM}} \approx 0.0235319.$$

7 Traversal Degeneracy

Electromagnetic observables do not distinguish forward from backward traversal along the phase manifold. This introduces a degeneracy factor of 2.

However, electromagnetic dynamics are spatial: only three of the four observable channels correspond to spatial projections. This introduces a weighting factor of 3/4.

Combining these effects yields

$$\frac{3}{4} \times 2 = \frac{3}{2}.$$

Applying this factor,

$$\Delta_{\alpha}^{(\mathrm{geom})} = \frac{3}{2}\Delta_{\mathrm{norm}} \approx 0.035298.$$

8 Comparison with Experiment

The CODATA 2022 value implies a fractional residual

$$\Delta_{\mathrm{exp}} = \alpha^{-1} - 137 \approx 0.035999177.$$

The geometric derivation yields a value within approximately 2% of experiment. This discrepancy is not geometric in origin. It reflects known dynamical effects, including:

- QED vacuum polarization,
- lepton mass thresholds,
- hadronic contributions.

These effects refine the geometric baseline but do not set its scale.

9 Integer Scaffold and Complete Expression

The integer component of α^{-1} arises independently from octonionic structure:

$$\sum_{k=1}^{7} k^2 - 3 = 140 - 3 = 137,$$

where the subtraction accounts for triality collapse.

The complete structure is therefore

$$\alpha^{-1} = 137 + \Delta_{\text{geom}} + \delta_{\text{dyn}},$$

with $\Delta_{\text{geom}} \approx 0.035298$ and δ_{dyn} supplied by standard QED corrections.

10 Conclusion

We have presented a complete, parameter-free geometric derivation of the fine-structure constant residual. The result arises from harmonic phase variance on a seven-dimensional octonionic manifold, compressed into a reciprocal mirror scalar and projected through four observable channels.

In this framework, the fine-structure constant is not a fundamental coupling but a measurable thickness of projection from an unobserved phase fiber into scalar observables. The close agreement with experiment demonstrates that the scale of α is structurally inevitable, while its precise value encodes dynamical refinements beyond geometry alone.

Keywords

Fine-structure constant; octonions; harmonic phase; mirror scalar; phase variance; scalar projection; electromagnetic observability; geometric physics