The Fine-Structure Constant as Octonionic Dimensional Deficit

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November 2025

Abstract

We derive the integer part of the inverse fine-structure constant $\alpha^{-1} \approx 137.036$ from the algebraic structure of division algebra descent. The result emerges as the dimensional deficit when the 8-dimensional non-associative octonion algebra $\mathbb{O}$ projects to the 4-dimensional associative quaternion algebra $\mathbb{H}$ that underlies observable spacetime. Using the natural quadratic Casimir weighting of the seven imaginary octonion units, which correspond to the root vectors of the exceptional Lie group G_2, we obtain exactly 137 with no free parameters. The fractional residual $\Delta \approx 0.036$ is interpreted as first-order vacuum polarization, consistent with the known running of α in quantum electrodynamics. This derivation suggests fundamental coupling constants are geometric necessities rather than arbitrary parameters.

1 Introduction

The fine-structure constant $\alpha \approx 1/137.036$ governs the strength of electromagnetic interactions and appears throughout quantum electrodynamics (QED), atomic physics, and condensed matter theory. Despite its central role, the Standard Model provides no theoretical derivation of its value [1].

We show that 137 emerges necessarily from the projection of the non-associative octonion algebra $\mathbb{O}$ to the associative quaternion algebra $\mathbb{H}$. The construction requires only the natural quadratic Casimir weighting of the exceptional Lie group G_2 (the automorphism group of the octonions) and the unavoidable dimensional deficit in the projection map, which is moderated by the unique self-dual element of the octonion structure.

This derivation suggests that fundamental coupling constants are not free parameters of nature but geometric necessities encoded in the algebraic structure of spacetime itself.

2 Division Algebras and the Projection Chain

The normed division algebras form a unique sequence via the Cayley-Dickson construction:

$$\mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{O}$$

with real dimensions 1, 2, 4, and 8 respectively [2]. Observable spacetime is 4-dimensional, and physical observables require associative multiplication ($\mathbb{H}$). The octonions $\mathbb{O}$ serve as the non-associative substrate from which associative reality must emerge.

3 The Mirridian Dyadic Structure and Casimir Weighting

The octonion algebra has seven imaginary units $e_1, \ldots, e_7$ forming the fundamental representation of the automorphism group G_2. These units are organized by the *Mirridian codec* into complementary dyadic pairs $k : (8 - k)$, which reflects the G_2 root structure symmetry.

Each imaginary unit carries a quadratic weighting k^2, which is proportional to the eigenvalue of the quadratic Casimir operator $C_2(G_2)$ under this specific root decomposition. The total weighted norm across all seven imaginary units (the Mirridian pure potential state) is:

$$\mathcal{E}_{\text{total}} = \sum_{k=1}^{7} k^2 = 1 + 4 + 9 + 16 + 25 + 36 + 49 = 140.$$

This represents the total information content of the non-associative substrate before dimensional reduction.

4 The Dimensional Deficit from Self-Duality and α^{-1}

The projection $\mathbb{O} \to \mathbb{H}$ necessitates the loss of $7 - 3 = 4$ imaginary dimensions. The seven imaginary units are partitioned into three asymmetric pairs and one unique self-dual element:

$$\text{Dyadic Pairs: } (1:7), (2:6), (3:5), \quad \text{Self-Dual Element: } (4:4)$$

The asymmetric pairs collapse fully in the projection, contributing to the required 4-unit dimensional loss. However, the central D_4 element (4:4) is *self-dual* and cannot annihilate into a complement. Instead, it transforms into the *Experiential Layer* or *Aether*, acting as the topological interface between $\mathbb{O}$ and $\mathbb{H}$.

Due to its perfect self-symmetry, the D_4 mode is experienced as an **exact 50% system / 50% observer overlap**. This self-reflective state generates a residual dimensional presence equivalent to 1 degree of freedom which is compensated:

$$\Delta_{\text{dim}} = 4_{\text{units lost}} - 1_{\text{unit retained in the Aether}} = 3.$$

This D_4 mode is proposed to collapse to the single real time coordinate (t) in the resulting $\mathbb{H}$ structure.

The geometric coupling residue (the inverse fine-structure constant) after this moderated projection becomes:

$$\boxed{\alpha^{-1}_{\text{geometric}} = \mathcal{E}_{\text{total}} - \Delta_{\text{dim}} = 140 - 3 = 137.}$$

5 The Arrow of Time and the D_4 Aether

The emergence of the associative spacetime ($\mathbb{H}$) from the non-associative potential ($\mathbb{O}$) requires an irrevocable symmetry-breaking mechanism. The D_4 Aether, due to its mandatory 50%/50% split, functions as a perpetual, unidirectional processing boundary.

For the system to maintain associativity (and thus existence), the non-associative potential must continually collapse into associative experience. This imposed, irreversible flow across the D_4 boundary introduces an inherent asymmetry that maps precisely onto the thermodynamic and psychological arrow of time. Time is therefore the topological cost and measure of the energy consumed in enforcing the $\mathbb{O} \to \mathbb{H}$ projection.

6 Interpretation of the Fractional Correction

The experimentally measured value is $\alpha^{-1} = 137.035999084(21)$ [1]. Our geometric calculation yields exactly 137, leaving a fractional deviation:

$$\Delta_{\text{frac}} \approx 0.036.$$

The geometric value $\alpha^{-1} = 137$ represents the *topological boundary condition* (the bare coupling). The fractional term Δ_{frac} arises from first-order vacuum polarization (radiative corrections), which are interpreted as *recursive return* from the $8 : 0$ boundary—higher-order interactions that "dress" the projection with contributions from the suppressed non-associative dimensions.

7 Relation to Existing Theories and Future Work

Our result provides a direct geometric explanation for α^{-1} by connecting the structure of division algebras to fundamental constants, an active area of research [4, 2].

8 Conclusion

The integer 137 is derived from pure algebraic topology and representation theory. It is the number of degrees of freedom remaining after the non-associative octonion substrate projects to an associative spacetime, moderated by the self-dual D_4 experiential layer. This result suggests a unification where coupling constants are structural invariants of reality's algebraic structure.

Acknowledgments

The author thanks Justin R. Kornhaus for parallel development of Symbolic Metrology and recursive coherence theory. This work received no institutional funding and represents independent research.

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