A Holographic Phase Interpretation of Dyadic Translation, Scalar Projection, and Temporal Direction

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Abstract

We present a unified framework in which dyadic systems of the form $(i{:}j)$, constrained by a conserved total $i + j = N$, are reinterpreted as discrete samplings of a single continuous phase invariant $\theta \in S^1$. Within this model, dyadic translation corresponds to rotational motion on a closed phase manifold, scalar quantities arise as projections of phase, and octave closure removes artificial endpoints. Varying N does not produce distinct systems but instead changes the sampling resolution of the same underlying structure, yielding a holographic description across scale. This perspective provides a natural explanation for scalar reduction, the emergence of a directional arrow of time, and the apparent absence of additional spatial dimensions, which are shown to be projection effects rather than missing degrees of freedom.

1 Introduction

Dyadic representations appear across mathematics, physics, and symbolic systems in contexts where paired quantities evolve while maintaining a conserved total. Such systems are often interpreted linearly, with implied endpoints, asymmetries, or irreversible progression.

This work proposes that dyadic systems are more accurately understood as *phase-based representations* of a closed invariant structure. Linear interpretations are shown to arise from discrete sampling of an underlying cyclic object rather than from intrinsic directionality or expansion.

2 Dyadic Systems and Conserved Quantities

We define a dyadic state as an ordered pair

$$(i{:}j), \quad i, j \in \mathbb{Z}_{\geq 0}, \quad i + j = N,$$

where N is a conserved quantity.

For fixed N, the dyadic ladder is

$$\{(0{:}N), (1{:}N-1), \ldots, (N{:}0)\}.$$

The value of N does not evolve under translation and therefore does not constitute a dynamic degree of freedom.

3 Dyadic Translation as Discrete Rotation

Define the translation operators

$$T_+ : (i{:}j) \mapsto (i+1{:}j-1), \qquad T_- : (i{:}j) \mapsto (i-1{:}j+1).$$

These operators preserve $i + j = N$ and generate motion along the dyadic ladder.

Introduce the phase mapping

$$\theta = \frac{2\pi}{N}(i - j).$$

Under this mapping, dyadic translation becomes

$$\theta \mapsto \theta + \frac{2\pi}{N},$$

demonstrating that translation is equivalent to discrete rotation on a phase circle.

4 Octave Closure and Removal of Endpoints

Identifying

$$0 \equiv N$$

closes the dyadic ladder into a cyclic group $\mathbb{Z}_N$.

Consequences include:

- $(0{:}N)$ and $(N{:}0)$ correspond to the same phase point,
- balance states are phase crossings rather than terminal positions,
- expansion and collapse are opposite phases of the same cycle.

Thus, the dyadic system forms a closed octave rather than a bounded interval.

5 Scalar Invariants as Phase Projections

A single scalar invariant describing dyadic state is

$$\Phi = \frac{i - j}{i + j}.$$

This scalar is antisymmetric about balance, varies uniformly per translation step, and encodes directional information via its sign. Importantly,

$$\Phi = \cos(\theta),$$

demonstrating that scalar quantities arise as projections of phase rather than as independent dimensions. Apparent scalar collapse therefore reflects projection, not loss of structure.

6 Holographic Scaling and Intrinsic Interpolation

The phase invariant θ exists independently of N. Each dyadic system $0{:}N$ corresponds to a discrete sampling

$$\theta_k^{(N)} = \frac{2\pi k}{N}, \quad k \in \mathbb{Z}_N.$$

Changing N alters only the sampling resolution, not the underlying structure. Dyadic systems are therefore holographic across scale: each contains a complete representation of the same phase cycle at different granularity.

Because all dyadic states lie on the intrinsic phase manifold S^1, interpolation between states must respect this geometry. Linear interpolation in coordinate space generally leaves the manifold. The correct interpolation is instead geodesic motion on the circle, given by spherical linear interpolation (SLERP):

$$\theta(t) = \theta_0 + t(\theta_1 - \theta_0).$$

SLERP thus follows directly from the holographic phase structure: it preserves phase coherence across scales and ensures that interpolation is invariant under changes in N.

7 The Arrow of Time

Within this framework, the arrow of time emerges as an orientation choice on the phase circle. Directionality arises when:

- a traversal direction is selected,
- phase states are discretely ordered,
- memory encodes prior phase positions.

Time is therefore not a separate dimension but a *projected phase axis*. Irreversibility is a consequence of discretization, ordering, and projection rather than a property of the invariant phase itself, which remains fully reversible.

8 Dimensional Interpretation

The familiar "3D + time" structure emerges naturally as a representational effect:

- spatial dimensions correspond to orthogonal phase projections,
- time corresponds to ordered traversal along one selected phase axis,
- scalar quantities arise from phase collapse,
- apparently missing dimensions are unexpressed phase degrees suppressed by projection.

No dimensions are absent; they are simply not represented in the chosen coordinate system.

9 Conclusion

Dyadic systems, scalar quantities, and temporal direction can be unified within a single holographic phase framework. Translation is rotation, collapse is projection, and dimensionality is representation-dependent. This approach removes artificial endpoints, explains scale invariance across dyadic systems, and provides a structurally economical account of time and dimensional emergence without introducing additional assumptions.

Keywords

Dyadic systems; phase invariants; holography; scalar projection; time emergence; dimensional reduction; octave closure