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Note: This TN is a modified copy of: Reference Frames for the PRISMA Formation Flying Mission.

1. INTRODUCTION

This note describes the transformation between reference frames employed in the InnoSat ACS. Emphasis is given to a rigorous transformation chain relating GPS measurements and InnoSat star camera data. It is recommended to adopt the Earth-Mean Equator of J2000 (EME2000) system as the nominal inertial reference system for both the Attitude Control System (ACS) and for integrating the equations of motion within the GPS-based orbit determination system. Differences between the nominal reference frames and their actual realizations are discussed.

1.1 LIST OF SYMBOLS

$\boldsymbol{N}$	Nutation matrix
$\boldsymbol{P}$	Precession matrix
$\boldsymbol{R}_x$	Elementary rotation matrix describing the transformation of a vector into a system that arises from a right handed rotation around the *x*-axis
$\boldsymbol{R}_y$	Elementary rotation matrix describing the transformation of a vector into a system that arises from a right handed rotation around the *y*-axis
$\boldsymbol{R}_z$	Elementary rotation matrix describing the transformation of a vector into a system that arises from a right handed rotation around the *z*-axis
t	Time
$\boldsymbol{T}$	Transformation matrix from EME2000 to WGS84 system
x_p, y_p	Angular coordinates of the rotation axis w.r.t. a reference pole
$\boldsymbol{x}$	Cartesian position vector
z, ϑ, ζ	Precession angles
$\Delta\psi, \Delta\varepsilon$	Nutation angles
ε	Mean obliquity of the ecliptic
$\dot{\boldsymbol{x}}$	Cartesian velocity vector
$\boldsymbol{\Theta}$	Earth rotation matrix
Θ	True Sidereal Time
$\bar{\Theta}$	Mean Sidereal Time
$\boldsymbol{\Pi}$	Polar motion matrix
$\omega_{\oplus}$	Earth angular velocity

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For clarity, the explicit representations of the elementary rotation matrices are given below:

$$\boldsymbol{R}_x(\phi)=\begin{pmatrix}1 & 0 & 0\\ 0 & +\cos\phi & +\sin\phi\\ 0 & -\sin\phi & +\cos\phi\end{pmatrix}\quad \boldsymbol{R}_y(\phi)=\begin{pmatrix}+\cos\phi & 0 & -\sin\phi\\ 0 & 1 & 0\\ +\sin\phi & 0 & +\cos\phi\end{pmatrix}\quad \boldsymbol{R}_z(\phi)=\begin{pmatrix}+\cos\phi & +\sin\phi & 0\\ -\sin\phi & +\cos\phi & 0\\ 0 & 0 & 1\end{pmatrix}$$

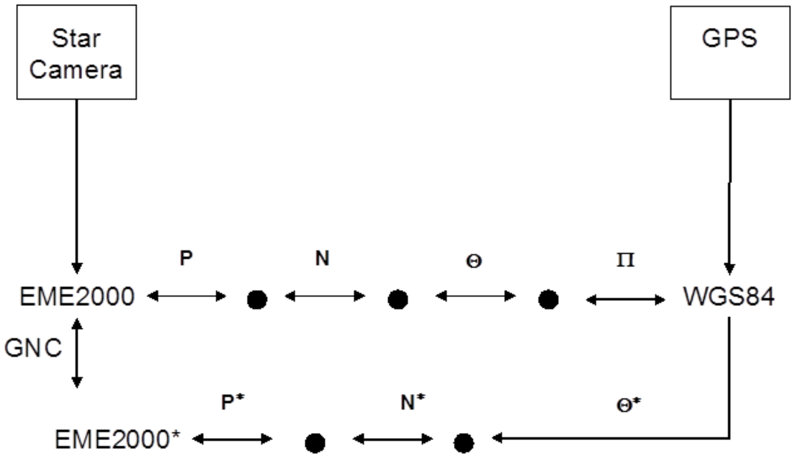

Figure 1-1: Reference system overview

2. STANDARD EPOCHS AND TIME SYSTEM

GPS system time is adopted as time system for the applications onboard InnoSat. It differs from International Atomic Time (TAI) by a constant offset of -19 s and matched Universal Time Coordinated (UTC) when it was introduced in January 1980. The standard epoch is 6.0 January 1980 GPS Time (Julian Date JD(GPS) 2 444 244.5) which serves as origin for the GPS week count. The GPS week starts Sunday 0.00 GPS Time and the first week, starting at standard epoch, is assigned the week number (WN) 0 [1]. The WN for arbitrary dates is given by

$$\mathrm{WN}=\left\lfloor\frac{\mathrm{JD(GPS)}-2\,444\,244.5}{7}\right\rfloor. \tag{1}$$

Within a GPS week, times are specified in seconds past the start of the week, yielding values between zero and 604 800 s. Within InnoSat, time synchronization is nominally based upon a One-Pulse-per-Second (1PPS) signal and associated messages of the onboard GPS receiver.

3. EARTH-CENTERED REFERENCE FRAMES

The reference frames relevant for InnoSat are specified in the sequel with a focus to the well-established systems like EME2000 and WGS84. An overview is given in Figure 1-1.

3.1 EARTH MEAN EQUATOR AND EQUINOX OF DATE (EME2000)

The International Celestial Reference Frame (ICRF) provides a realization of the International Celestial Reference System (ICRS) using a catalog of distant VLBI radio sources [2]. The ICRF matches the EME2000 system, which has previously been employed in planetary ephemerides and star catalogs, on a level of some tens of milli-arcseconds. EME2000 is an inertial system which is aligned with the *mean* equator and equinox at the reference epoch J2000 (1.5 Jan. 2000). Its *z*-axis is parallel to the mean rotation axis of the Earth and the *x*-axis points into the direction of the mean vernal equinox, i.e. the ascending node of the

Earth's mean orbital plane on the mean equator, at the fixed epoch J2000. Here, the term "mean" indicates that oscillations of the equator and ecliptic with periods of 2 weeks to 18.6 years (i.e. nutation) have been averaged.

3.2 WORLD GEODETIC SYSTEM 1984 (WGS84)

The WGS84 system is a commonly adopted realization of an Earth-fixed reference system from a set of well-established ground station coordinates. Its axes are aligned with the adopted International Reference Pole and Meridian that are fixed with respect to the surface of the Earth. WGS84 can, at a level of a few centimeters, be considered as a particular implementation of an Inertial Terrestrial Reference Frame (ITRF).

3.3 POSITION TRANSFORMATION

A position vector in the WGS84 frame is transformed by the matrix ***T*** to a position vector in the EME2000 frame according to [1]:

$$\boldsymbol{x}_{WGS84} = \boldsymbol{T}(\mathrm{t})\boldsymbol{x}_{\mathrm{EME2000}} = \boldsymbol{\Pi}(\mathrm{t})\boldsymbol{\Theta}(\mathrm{t})\boldsymbol{N}(\mathrm{t})\boldsymbol{P}(\mathrm{t})\,\boldsymbol{x}_{\mathrm{EME2000}} \tag{2}$$

with the following four contributions

3.3.1 Polar Motion

$$\boldsymbol{\Pi}(\mathrm{t}) = \boldsymbol{R}_{\mathrm{y}}(-\mathrm{x}_{\mathrm{p}})\boldsymbol{R}_{\mathrm{z}}(-\mathrm{y}_{\mathrm{p}}) \quad , \tag{3}$$

where x_p and y_p denote the angular coordinates of the rotation axis with respect to the adopted reference pole.

3.3.2 Sidereal Time

$$\boldsymbol{\Theta}(\mathrm{t}) = \boldsymbol{R}_{\mathrm{z}}(\Theta(\mathrm{t})) \tag{4}$$

where the true sidereal time $\Theta(t) = \overline{\Theta}(t) + \Delta\Psi\cos\varepsilon$ is expressed as the sum of the mean sidereal time $\overline{\Theta}$ and the "equation of the equinoxes". The mean sidereal time is related to the UT1 time scale which differs from Coordinated Universal Time (UTC) by up to 0.9 s.

3.3.3 Nutation

$$\boldsymbol{N}(\mathrm{t}) = \boldsymbol{R}_{\mathrm{x}}(-\varepsilon - \Delta\varepsilon)\boldsymbol{R}_{\mathrm{z}}(-\Delta\psi)\boldsymbol{R}_{\mathrm{x}}(+\varepsilon) \quad , \tag{5}$$

where ε denotes the mean obliquity of the ecliptic, $\Delta\psi$ the nutation in longitude and $\Delta\varepsilon$ the nutation in obliquity.

3.3.4 Precession

$$\boldsymbol{P}(\mathrm{t}) = \boldsymbol{R}_{\mathrm{z}}(-\mathrm{z})\boldsymbol{R}_{\mathrm{y}}(\vartheta)\boldsymbol{R}_{\mathrm{z}}(-\zeta) \tag{6}$$

with the precession angles z, ϑ, and ζ.
Characteristic values of angles related to the above described coordinate transformations are summarized in Table 3-1.

Type	Angle
Polar motion	0.3”
UTC-UT1< 1s	15”
Equation of Equinox	20”
Nutation	20”
Precession	50”/yr

Table 3-1: Typical angles involved in the transformation from WGS84 to EME2000

3.4 VELOCITY TRANSFORMATION

Since the transformation matrix $\boldsymbol{T}$ is time dependent, a velocity vector in the inertial EME2000 system is transformed according to

$$\dot{\boldsymbol{x}}_{WGS84} = \boldsymbol{T}(\mathrm{t})\dot{\boldsymbol{x}}_{\mathrm{EME2000}} + \dot{\boldsymbol{T}}(\mathrm{t})\,\boldsymbol{x}_{\mathrm{EME2000}} \tag{7}$$

to the velocity vector in the rotating WGS84 system. The time derivative of the transformation matrix may either be derived numerically from a differential quotient approximation or employing analytical approximations for the time derivative of the sidereal rotation matrix.

4. CO-MOVING ORBITAL REFERENCE FRAME

The reference frame, denoted generally as RTN (Radial, Tangential, Normal) or HCL (Height, Cross-track, Along-track), is defined by a triad of unit vectors ($\boldsymbol{e}_{\mathrm{R}}$, $\boldsymbol{e}_{\mathrm{T}}$, $\boldsymbol{e}_{\mathrm{N}}$) which may be derived from the inertial position and velocity vectors $\boldsymbol{r}$ and $\boldsymbol{v}$ of the satellite according to

$$\begin{aligned} \boldsymbol{e}_{\mathrm{R}} &= \frac{\boldsymbol{r}}{|\boldsymbol{r}|} \\ \boldsymbol{e}_{\mathrm{N}} &= \frac{\boldsymbol{r}\times\boldsymbol{v}}{|\boldsymbol{r}\times\boldsymbol{v}|} \quad . \\ \boldsymbol{e}_{\mathrm{T}} &= \boldsymbol{e}_{\mathrm{N}}\times\boldsymbol{e}_{\mathrm{R}} \end{aligned} \tag{8}$$

The RTN unit vectors may be expressed in any Earth-centered inertial reference frame.

5. INNOSAT REFERENCE SYSTEMS

5.1 STAR CAMERA AND ACS

The star camera employed for InnoSat is based on an input catalogue referred to the Earth mean equator and equinox of J2000 [3]. Since no particular coordinate transformations are applied for the star camera within the ACS application, the dynamics of attitude motion within the ACS are implicitly performed in the EME2000 system.

5.2 GPS-BASED NAVIGATION

The GPS-based navigation system on the InnoSat spacecraft processes raw GPS measurements. The measurement modelling involves the computation of the GPS satellite positions which is most easily performed in the WGS84 frame. Since the GPS navigation solutions also refer to the WGS84 system, this frame is, in general, a natural frame for GPS-related computations.

On the other hand, the equations of motion can most easily be implemented in an inertial frame which avoids the need for Coriolis and centrifugal accelerations. Since the EME2000 is considered as a widely applied standard frame, it has been selected as reference frame for the numerical integration of the equations of motion within InnoSat.

However, the GPS-based Navigation actually implements the EME2000* system which differs from EME2000 by neglecting polar motion and by only updating the rotation around the z-axis $\boldsymbol{\Theta}^*(t) = \mathbf{R}_z(\overline{\Theta}(t))$ where the mean sidereal time is assumed to grow linearly with GPS-time. Precession, Nutation, (UT1-UTC) correction and "equation of the equinoxes" are all set to constants.

In the framework of InnoSat, the choice of the EME2000* simplifies the ACS application however the EME2000* coordinate frame will include a small slowly changing rotation offset with respect to EME2000. The rotation offset will depend on the selected values for the tunable parameters on-board which means that the orbit estimates in EME2000* are not suitable for end-users. Instead the navigation output should be provided in the WGS84 system. This involves the back-transformation from the EME2000* to the Earth-fixed system which leaves the provided output essentially unaffected by the particular choice of the inertial reference system.

The output of the GPS based navigation shall be provided in the WGS84 frame.

5.2.1 EME2000 effect on AKE*

As the navigation output from the GPS-based navigation will be in WGS84 no error contribution from the choice of EME2000* will be propagated to the Absolute Knowledge Error (AKE).

5.2.2 EME2000 effect on APE*

The ACS will use EME2000* on-board for computation of the attitude reference. The rotation offset from EME2000 to EME2000* will thus be propagated to the attitude reference and will affect the Absolute Pointing Error (APE). The largest error contribution is due to Precession which has a linear growth of about 50” per year. Both the Nutation and Equation of Equinox have periods much longer than a year. Updating the on-board constant rotation matrix approximately once per month will maintain the total contribution to APE below 10”. The most uncertain contribution is coming from the UTC-UT1 difference which cannot be predicted.

5.3 SGP4 FRAME

As backup for the GPS-based navigation functions, mean orbital elements (TLE) will be uploaded to the spacecraft. The mean elements refer to NORAD’s SGP4 orbit propagator which allows to compute the spacecraft position with an accuracy of about 1-5 km. The resulting position and velocity coordinates refer to a specific coordinate system which requires the following transformation

$$\mathbf{x}_{EME2000} = \mathbf{P}^T(t)\mathbf{N}^T(t)\mathbf{R}_z^T(\Delta\psi\cos\varepsilon)\mathbf{R}_z^T(\omega_{\oplus}(UTC-UT1))\mathbf{x}_{SGP4} \qquad (10)$$

to the EME2000 system.

Note that the transformation from SGP4 and EME2000 only includes terms which on-board InnoSat are treated as constants. It is assumed that the resulting pointing offset can be maintained small, when compared to the accuracy of the orbit propagation, by frequently updating the SGP4 to EME2000* transformation matrix, see 5.2.2.

The output of the TLE based navigation shall be provided in the EME2000* frame.

Note: As the navigation output is only dependent on uploaded TLE’s it is not the intent that the backup navigation output shall be used by an end user. Instead if TLE based orbit navigation is needed by an end user the best suited set of TLE’s shall be propagate on ground by the end user.

6. CONCLUSION

The time and coordinate systems involved in the guidance, navigation and control functions for the InnoSat ACS have been described. Based on the InnoSat star camera, the ACS application adopts the inertial EME2000 (ICRF) system as fundamental reference system. For generality and to ease the interface with ACS, the GPS-based navigation system will perform the integration of the equations of motion in the EME2000* system which, for practical applications, can be identified with the EME2000 system.

6.1 REFERENCES

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[3] Doug Sinclair; *Star Tracker Interface Control Document,* Rev 1.26, March 17, 2016