Task 1: Approximating functions

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The thermal storage is modeled by a cylinder with length L and diameter D and it is assumed that temperature variation occurs only along the axis of the cylinder.

The temperate evolution of the solid and fluid phases is described by a system of two linear *reaction-convection-diffusion* equations:

$$\begin{aligned} \varepsilon\rho_f C_f \frac{\partial T_f}{\partial t} + \varepsilon\rho_f C_f u_f \frac{\partial T_f}{\partial x} &= \lambda_f \frac{\partial^2 T_f}{\partial x^2} - h_v(T_f - T_s) \quad x \in [0, L],\ t \in [0, T], \\ (1-\varepsilon)\rho_s C_s \frac{\partial T_s}{\partial t} &= \lambda_s \frac{\partial^2 T_s}{\partial x^2} + h_v(T_f - T_s) \quad x \in [0, L],\ t \in [0, T], \end{aligned} \tag{1}$$

with ρ being the density of the phases, C the specific heat, λ the diffusivity, ε the solid porosity, u_f the fluid velocity entering the thermal storage and h_v the heat exchange coefficient between solid and fluid. The fluid velocity is assumed to be uniform along the cylinder and varying only in time: $u_f = u$ during charging, $u = 0$ during idle and $u_f = -u$ during discharging, with u being a positive constant.

The system of equation has to be augmented with suitable initial and boundary conditions:

$$T_f(x, t=0) = T_s(x, t=0) = T_0,\ x \in [0, L] \tag{2}$$

$$\left.\frac{\partial T_s(x,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial T_s(x,t)}{\partial x}\right|_{x=L} = 0,\ t \in [0, T] \tag{3}$$

The boundary conditions for the fluid instead will be different according to the current state of the thermal storage:

- **Charging State:**

$$T_f(0, t) = T_{hot}, \qquad \left.\frac{\partial T_f(x,t)}{\partial x}\right|_{x=L} = 0, \qquad t \in [0, T] \tag{4}$$

- **Discharging State:**

$$\left.\frac{\partial T_f(x,t)}{\partial x}\right|_{x=0} = 0, \qquad T_f(L, t) = T_{cold}, \qquad t \in [0, T] \tag{5}$$

- **Idle Phase:**

$$\left.\frac{\partial T_f(x,t)}{\partial x}\right|_{x=0} = 0, \qquad \left.\frac{\partial T_f(x,t)}{\partial x}\right|_{x=L} = 0, \qquad t \in [0, T] \tag{6}$$

Objective:

To describe the temperature of the solid and the fluid respectively, as a function

of time, given a training set containing measurements.
My Approach:
The training set was noisy, so I denoised it using rolling window. I built one MLP for the fluid, and one for the solid.