Low Default Portfolios: A Proposal for Conservative Estimation of Default Probabilities

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Abstract

Low default portfolios are those for which banks have little default history, so that average observed default rates might not be reliable estimators of default probabilities (PDs). A key concern for regulators is that credit risk might be underestimated as a result of data scarcity. This paper proposes a quantitative approach to produce conservative PD estimates. Under stylised assumptions about the level of correlation between the risk factors relating to different obligors, and between the systematic risk factor in consecutive years, the estimate of portfolio-wide PD is determined by the size of the portfolio, the number of observed defaults, and the level of confidence that is placed on this empirical evidence. This central PD estimate is then used to adjust the PDs that the bank's own credit experts will have allocated to individual grades within the portfolio. The paper's original purpose was to stimulate debate within the industry on this topic. The revised paper is being issued in the hope that it may assist practitioners in this field.

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Introduction

This paper was written as a contribution to the industry debate on Low Default Portfolios in the context of Internal Ratings Based approaches to calculation of (the credit risk component of) regulatory capital requirements. It contains a proposal for a quantitative approach to conservative estimation of PD for low default portfolios. The proposals in the paper were developed in order to generate debate and discussion within the industry and among regulators, but have not been adopted as FSA policy. Specifically, the paper does not represent FSA rules or guidance. For FSA guidance on the subject of low default portfolios, readers should consult the FSA handbook, especially BIPRU Section 4[3]. The paper is being released through the FSA website in the hope that its contents will assist other analysts in dealing with this particular issue.

The paper is organised as follows. In Section 2, the issue of low default portfolios and the concerns of regulators are discussed. Section 3 comments on some previous literature. Section 4 discusses the choices needed to specify the proposed approach. Section 5 describes how the proposal could work in practice. Section 6 discusses how a regulator could position the approach within its rules, essentially as a starting point. Section 7 discusses possible criticisms of the proposal. Section 8 presents a short summary and sets out some issues that should be considered if the proposal were to be adopted for regulatory use. In the Appendix, the details of the methodology and the simulation results are set out.

Given the origins of this exercise, the paper is written in a way that suggests that it is regulators that are setting the parameters for the production of the estimates. However, to the extent that this is an approach followed within a bank without any regulatory involvement, then the role which it is envisaged that the regulator would take would instead be filled by the appropriate central risk management function.

The authors would like to thank the many colleagues in the FSA and in the industry who have commented and given advice. In particular we would like to thank Tom Wilde from Credit Suisse First Boston, who provided extensive technical commentary and who encouraged us to develop a multi-year setting for our proposal, to the other members of the FSA-industry Expert Group on low default portfolios, with whom drafts of this paper were discussed, and from the FSA Paul Sharma, who prompted the paper in the first place, Gerry Cross, Robert Hudson and Marilyn Sullivan. Any remaining errors are those of the authors.

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[3] Currently under consultation. See FSA CP06/3, Appendix 1, available from the FSA website: www.fsa.gov.uk.

1. Summary of the issue

a. Overall objective of PD estimation

Before turning to low default portfolios specifically a few comments are in order. The Basel Revised Capital Framework and the Capital Requirements Directive require firms to estimate PDs that are **long run, forward looking expected default rates** for **each grade** in **each borrower rating model**, with an appropriate **margin of conservatism**. All PD estimates need to be made with this objective in mind, regardless of whether they relate to a low default portfolio or not. We are not suggesting replacement or dilution of this objective. Accordingly, as with any approach to PD estimation, the outcome of the estimation process needs to be judged by whether or not it has delivered PDs that meet the objective.

Differing practices and liberalism over rating philosophy mean that PD estimation – and hence Pillar 1 minimum capital requirements – will vary from case to case: between banks, between different ratings systems in a single bank, and over time in a single rating system. Such differences may occur even if the underlying risk being assessed is in some sense the same. The consequence is that some judgment needs to exercised, chiefly by banks but also by regulators, when assessing the calibration of rating systems.

The extent to which a rating system is to be judged conservative is in relation to its ability to generate PDs according to the objective set out above; it may not be enough to show that PDs are conservative when judged against historical experience. This point is particularly relevant to low default portfolios.

b. LDPs not well-defined

Low default portfolios ("LDPs") are not well defined. The essence, however, is that the term refers to portfolios of credit exposures, usually to similar types of organisation or person, perhaps composed of similar facilities (although the latter is only relevant to LGD and EAD for non-Retail exposures). The portfolio is managed as a single portfolio. The number of defaults on a low default portfolio is so low that estimates of quantitative risk parameters based on historical default experience are unreliable or poor in some statistical sense. It may be simple – trivial even – to produce a historical average, but a distinguishing feature of LDPs is that such estimates may be inadequate reflections of the true risk. Portfolios that are often referred to as LDPs include portfolios of bank and sovereign exposures, exposures to large corporates, most forms of specialised lending, some other portfolios where the number of borrowers in the economy is small (housing associations, Public Private Partnerships), and some niche mortgage portfolios.

This essentially qualitative description of an LDP leaves a lot to judgment. Different people may disagree about whether a given portfolio is an LDP or not. A portfolio (of mortgages say) might be an LDP in one firm, but not in another where default data is richer.

c. Regulatory concerns

The concern on the part of regulators is chiefly one of estimation of risk parameters in an IRB context as regards portfolios that have experienced low, or even zero, default rates; regulators may be concerned that PD estimates based on simple historical averages, or judgmental considerations alone, may underestimate capital requirements for that portfolio and hence for the bank as a whole. This issue translates into concerns over the overall PD attaching to a portfolio and also to the grade-level PDs attaching to individual grades within a portfolio. [4]

A further concern is about internal portfolio management of LDPs: can firms adequately assess the risk on such portfolios and relate the risk on LDPs to the rest of a firm's business? Some firms seek to do this through a process of establishing "central tendencies" for portfolios. The concern that a regulator might reasonably have is that these central tendencies can be highly if not entirely judgmentally determined, particularly for groups of obligors whose economy-wide number is small. The role of the firms' own data in the establishment of central tendencies may not be clear.

A subsidiary concern is whether firms can adequately differentiate between borrowers in terms of risk; this is an important concern as regulator. In discussions with the industry, firms have emphasised their ability to perform this role. To the extent that credit analysis is being performed by experienced and competent staff, trained to recognise factors that could lead to a weakness in credit quality, these claims have some merit and regulators may wish to place due reliance on firms' processes here. Nevertheless, discriminative power of ratings approaches remains of high concern to regulators, being both a requirement of Basel and a foundation of good risk management. That being said, the proposal contained in this paper addresses the issue of calibration of IRB approaches for LDPs; for rank-ordering aspects, reliance needs to be placed on other approaches and techniques.

The industry feels it is important that any LDP could in principle qualify for an IRB treatment [See BBA]. It is the case that there is an IRB approach available for all portfolios and so in principle any portfolio could go on to IRB. The question is whether LDPs satisfy the IRB requirements. It is perhaps dangerous to pick on just a few of the requirements because ratings systems need to meet all the requirements not just some. But the areas where it is least clear that LDPs qualify are the following:

> "Estimates must be grounded in historical experience and empirical evidence, and not based purely on subjective or judgmental considerations." ([BCBS] Para 449)

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[4] There are also concerns over the ability of firms to estimate LGD and EAD on LDPs; the issues here are as important as estimating PD; however this paper does not go on to address LGD and EAD; the principles might be capable of being transferred to LGD and EAD estimation or else some other appropriate technique developed.

> "… a bank must add to its estimates a margin of conservatism that is related to the likely range of errors. Where methods and data are less satisfactory and the likely range of errors is larger, the margin of conservatism must be larger." ([BCBS] Para 451)

If an LDP is to qualify for IRB, then the risk estimates would need to be founded in historical default rates; in particular approaches where default rates are assigned purely judgmentally would not meet the rules – although judgment is a necessary ingredient. An approach which does not embody conservatism in the face of lack of data would also not meet the rules.

d. Data availability

Firms point out that many LDP portfolios include borrowers that are subject to external risk assessments, principally in the form of rating agency grades but external default models and estimates implied by market prices are also of relevance here. These provide a means to compare relative risk of borrowers and to assign PDs to grades. Potentially, risk estimates derived from agency grades and default models in this way could be judged to be founded in historical default rates, but not necessarily subject to conservatism. If these measures are to be used internally, firms will first need to be content that they provide appropriate measures of risk; in an IRB context, regulators would need to be persuaded as well.

For other portfolios, there may be sufficient data in the whole economy to provide reliable estimates even though an individual bank's own portfolio – which may be a small subset of all borrowers – may constitute an LDP considered in isolation. There should be ways round this issue – perhaps through data pooling. Risk estimates derived in this way could similarly be judged to be founded in historical default rates but not necessarily subject to conservatism. And their acceptability requires satisfaction with the requirements that pooling arrangements must meet under the IRB approach, most notably for comparability.

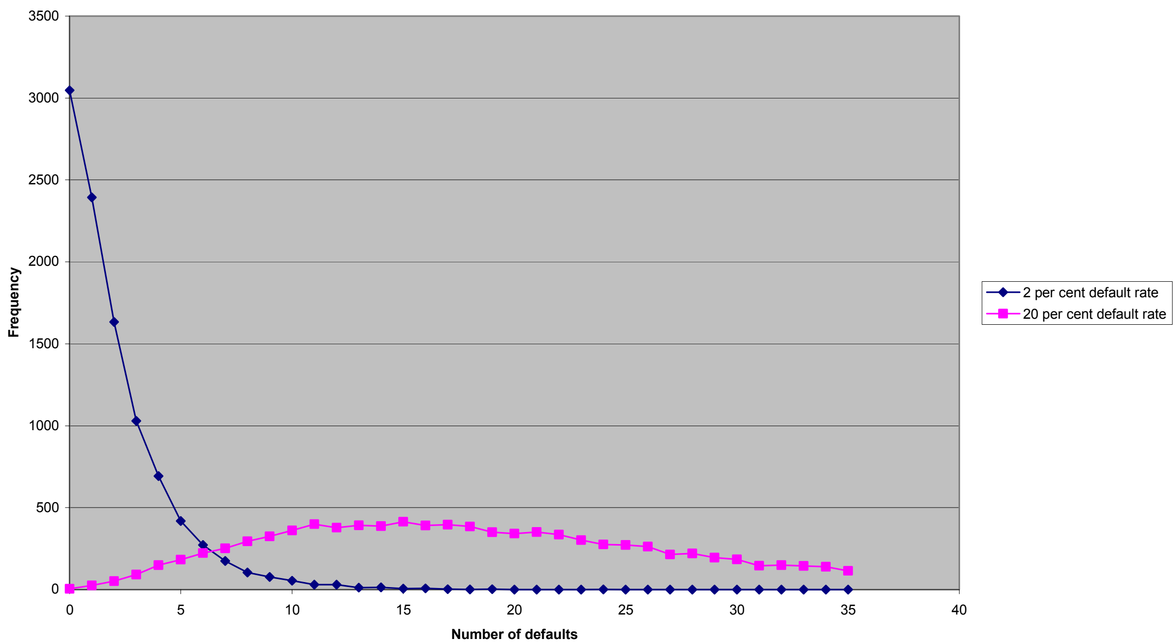
Finally there are LDP portfolios which do not benefit from external risk assessments and would remain LDPs even if a firm had all the borrowers in its portfolio. In the UK, housing associations and public-private partnership borrowers are cases in point.

Low default portfolios are not necessarily low data portfolios. A large portfolio – say a high quality mortgage portfolio – can yield a very small number of defaults on a large population. A low number of defaults needs to be seen in the light of the size of the portfolio which produces them. The larger the portfolio, the higher the credit quality can be deemed to be. Other things being equal, for a given number of defaults, a PD should be lower if the size of the portfolio is larger in terms of borrower numbers. Moreover, if a portfolio has yielded no defaults in, say, 5 years, then a PD that was estimated conservatively from this historic default experience could subsequently be reduced if the same portfolio yielded no default in a sixth year: there would be a greater quantity of evidence to support lower estimates.

e. Distribution of defaults

Low default portfolios seem to be aptly named because the issue does seem to be the actual number of defaults. This can be illustrated as follows. Chart 1 has been produced using simulation. Two hypothetical homogeneous portfolios, each of 100 obligors, and firm-to-firm asset correlation of 12 per cent were subject to 10,000 simulated outcomes. Portfolio 1 had a "true" default rate of 2 per cent, and Portfolio 2 had a "true" default rate of 20 per cent. Note that the results for Portfolio 1 are heavily skewed to the left; in particular, note the high fraction of results where no defaults were recorded. Portfolio 2 is also skewed but there is a lower chance of a very small number of defaults occurring.

Chart 1: Simulated distribution of defaults (10,000 simulations, asset correlation 12 per cent)



The graph illustrates a point about the dynamics of default rates. Some portfolios may have PDs that are well above zero. But they may still have quite a large probability of yielding no defaults in any given year. Depending on the economic environment, the observed sequence of defaults that firms see may be zero for several years in a row. With an adverse move in economic conditions, the number of defaults may then rise. The observed effect is that defaults rates in any given year will probably lie below the average. This is a consequence of the skewed nature of the distribution. In more formal statistical terms, the median lies below the mean. The challenge is to capture this risk through setting an adequately high long run average PD; if the calibration is accurate, a consequence of the skewness is that, for a majority of portfolios, the estimated PDs should usually lie above the observed default rate.

f. Materiality

As a final point, in this section, we would question the materiality to the industry of the portfolios most affected; namely those lacking both sufficient internal experience and external data and/or measures of risk on which to base default estimates. For example, few lenders have concentrations to housing associations; in the context of an IRB application, many lenders might be prepared to request partial exemption for such portfolios; i.e. leaving them on the standardised approach. This would save a lot of time and energy on the part of applicant firms and regulators. As a result, industry and regulatory effort would focus on material portfolios and quick wins.

2. Comment on other attempts to look at this question

a. Pluto and Tasche [PT]

Pluto and Tasche discuss the issue of LDPs. Their original paper of December 2004 and its revised version of April 2005 have been a stimulus to the drafting of the present paper. They analyse the issues from a statistical perspective using concepts of confidence intervals, and include an original methodology for generating grade-level PDs in a manner which it describes as the "most prudent estimation". Their paper makes use of a simple model of default, first under an assumption of independence and then under an assumption of asset correlation similar to that embodied in the Gordy model for capital requirements.

An original aspect of their paper is the proposed handling of grade-level PDs: in estimating a conservative PD for a specified grade, they propose applying their most-prudent estimation methodology, not just to data from the specific grade, but to data from all grades of inferior credit quality in addition to data from the grade in question.

The paper leaves open the question of the choice of confidence level, illustrating the methodology with a range of levels from 50 percent to 99.9 per cent: no conclusion is reached but the authors suggest that confidence intervals of less than 95 per cent appear intuitively appropriate. The authors also leave open the question of cutover from their confidence interval approach to an approach based on simple averages.

b. Schuermann and Hanson [SH]

Using credit rating histories from S&P, the authors compare PD confidence intervals obtained from various estimation techniques. Both cohort and duration estimators are employed and the intervals are built either analytically or with bootstrap. Tighter intervals are obtained with the bootstrap and the duration estimator, which points to it as a more accurate tool. However these were calculated on the basis of independent counterparties and years. The duration estimator essentially derives PDs for some grades from migration matrices and default rates in other grades. An inherent limitation is that the bootstrap will not work when no defaults are observed anywhere. The authors note that intervals invariably overlap for investment grades, that speculative grades consistently rank-order PD's monotonically across the cycle, which investment grades struggle to achieve during expansions.

c. Alan Forrest [F]

Alan Forrest of the Dunfermline Building Society has developed an analysis that has some points in common with those of Pluto and Tasche and with the present paper. He uses a straightforward model of default similar to the one in the present paper. He works in a multi-grade setting and, like Pluto and Tasche, seeks to examine the problem of estimating grade-level PDs in a default context. His approach differs from Pluto and Tasche in that he proposes a method of estimating all PDs simultaneously by working in multiple dimensions: the dimensions correspond to the grades of the rating system and each point represents a possible choice of grade-level PDs. He identifies a subset of points which are feasible (in that the grade-level PDs increase monotonically with worsening credit grades) and whose likelihood of occurring, conditional on the observed data, exceeds a preset level that depends on the confidence interval required, among other factors. This does not lead of itself to a unique outcome and Forrest proposes that the grade-level PDs chosen are those for which the regulatory capital requirement is maximised. He illustrates his method with a two-grade rating system. He emphasises the continuity between this technique and the familiar high default modelling regimes: the basic theory is common to both and the main difference in practice is that approximations can be used effectively in the high default regime.

As a comment, while the method has the attraction of dealing explicitly with grade-level PDs and of estimating them simultaneously, it is not immediately clear whether the optimisation process envisaged is practicable with several rating grades given the number of dimensions involved, nor is it clear that selecting regulatory capital as the objective function of the optimisation yields robust numerical solutions. An area that might also be investigated is how to apply the technique when the observed data is drawn from a number of different years; we think however that it would be comparatively straightforward to adapt his method to this case.

3. A possible regulatory approach

In developing a possible regulatory approach to conservative estimation of PDs for Low Default Portfolios, we have considered a number of aspects, as discussed in this section.

a. Confidence intervals

A common feature of the approaches discussed above is that of confidence intervals. This allows analysts to use some standard text book concepts in the field of parameter estimation. It is orthodox and widely accepted. We propose to make use of it.

b. Portfolio-level PDs, conditional on credit quality

The IRB approach requires firms to make grade-level estimates. On the face of it, an algorithm for estimating grade level PDs has some attraction. The Bundesbank paper [PT] addresses the issue boldly and with originality. An alternative approach is to accept that the problem is multi-dimensional in character, requiring estimation of an ordered set of PDs to a specific confidence level. On the face of it, this appears to be an intrinsically difficult statistical problem. The proposal in this paper concentrates on estimation of portfolio level PDs that then apply to grade level estimates and result in capital requirements that are conditional on credit quality. We note that modest adjustments of grade level PDs, constraining portfolio level PD to remain constant, does not have a pronounced effect on Pillar 1 capital requirements.

This is not to suggest that relative grade-level PDs are only of minor importance in an IRB approach. For example, grade level PDs should still be accurate (or conservative) estimates of true PD. This is of particular importance if the credit quality of a portfolio were to deteriorate, either because of migration or through a change in lending practice; to the extent that the portfolio is now riskier, more capital should be held against it. Credit officers are also likely to recognise the importance of this and will also have a sense of the likely deterioration expected from grade to grade.

An ability to differentiate between levels of credit risk is one of the key requirements of an internal rating system. This is as important for LDPs as for any portfolio. This translates into proper allocation of grade-level PDs. In the same vein, it is as important to get these right as for any other portfolio. In proposing an approach based on portfolio-level average PDs, we are not implying that discriminative power and grade level PDs are being discarded or even downgraded as regulatory objectives.

Indeed, it is important for our proposal that we do not treat the portfolio as if it was composed of a single grade. A critical input to the proposal is the choice of relative PDs, as determined by the firm itself. This will allow firms to compute PDs conditional on the credit quality of the portfolio. The proposal will then deliver sensible dynamics from a regulatory perspective (lower credit quality – higher capital). Under our proposal, this can only be achieved if firms play their part in the process effectively.

c. Choice of default model

As with the concept of confidence intervals, there is a fair degree of acceptance around the pared-down Merton model, also referred to as the Vasicek model, at the heart of the revised capital framework. Under this model, defaults do not occur independently; the probability of default is driven by the distribution of future hypothetical asset values; the PD is the probability that the asset value declines to a level below some threshold. Interdependence is introduced by specifying a correlation structure between the hypothetical assets of firms: the value of each firm's assets is supposed to have a fixed correlation with a single latent variable of standard normal distribution. Under this formulation, defaults are conditionally independent given the draw of the latent variable.

The Vasicek model embodies some key features of structural firm models and commands a degree of acceptance within the industry and among regulators. While not suggesting that the model is either complete or sophisticated, it does allow us to model the key issue at stake here – the extent to which defaults at firms are non-independent and the impact of that on the distribution of default rate outcomes for given portfolios. Our proposal therefore adopts this model.

Where observations are drawn from more than one year, allowance within the model for different settings of the latent variable is made. At one extreme, it would be possible to assume that values of the latent variable are perfectly correlated from year to year; under this assumption, it would make no difference to our assessment of confidence intervals whether observations are drawn from a single year or from many years; at the other extreme, latent variables could be assumed to be independent from year to year. In practice, we have felt it desirable to model an intermediate degree of year-to-year interdependence between latent variables in successive years. The details are set out in the appendix, and require a single parameter (the year to year correlation) to be set.

The model does not deliver analytic solutions except for a few special cases such as zero defaults. As a result, the PD estimates are derived by Monte Carlo simulation, using a large number of simulations. Details are set out in the Appendix.

d. Choice of correlation

In order to specify the model, asset correlation and year-to-year correlation need to be chosen. Estimates of correlation are of questionable reliability – particularly so when we are dealing with low data problems. In order to cut to the quick, we have worked with an asset correlation assumption of 12 per cent and a year to year correlation assumption of 30 per cent. Readers will recognise the asset correlation assumption as being the lower correlation assumption in the Basel Risk-Weighted Asset formulas in Paragraph 272 of the June 2004 Framework document. It is acknowledged that the choice of these correlations is in the final analysis an arbitrary one. With further understanding of the interplay between correlation and the level of conservatism we might need to choose different correlation parameters.

Under the terms of the model, the distribution of default frequencies is driven by the pair-wise correlation between the asset values of two firms in the same portfolio; conversely, the capital calculations are driven by an assumed average asset correlation between any two firms in the economy. We might expect the asset values of two firms within the same portfolio to be more highly correlated with each other than between two firms drawn at random; for example, the asset correlation between two shipping companies might be thought to be higher than the correlation between a shipping company and a firm in a different sector. A valid implication would be that we should input an asset correlation specific to the portfolio in question. This argument has some merit and as a result it is possible that our illustrative PD estimates demonstrate too low a level of conservatism.

e. Choice of confidence interval

We find this the most awkward of the choices to make and the one where least consensus is likely to exist. We consider that the goal is to arrive at reasonably conservative estimates of average PDs. Regulators can live with the possibility that the resulting PDs are in fact underestimates of the a priori unknown and unobserved PD. For example, long run averages may prove to be underestimates and that is accepted. As consequence, we agree with Pluto and Tasche that high confidence intervals such as 95 per cent or more are not essential. We have performed calculations at both 50 per cent and 75 per cent confidence intervals and would suggest fixing on one or the other.

It should be emphasised that use of a 50 per cent or even a 75 per cent confidence interval will not of itself necessarily deliver conservative estimates. It can do so, when combined with an appropriate correlation assumption. Using a 12 per cent correlation assumption, we would note that, for a given "true" PD in the range of interest, the expected values of the confidence level PDs for both the 50th and 75th percentiles lie above the true PD. This is illustrated in the bottom graphs on page 23 in the Appendix. This suggests that the particular combination of correlation and confidence interval has some claim to produce conservative results; its ability to do so is a consequence of the skewed distribution of outcomes.

Our example has been based on a 75th percentile. We have chosen the parameters 12%, 30% and 75% (respectively asset correlation, year-to-year correlation and confidence level) for illustration only. If a bank was to use this approach they would need to justify their correlation parameter choices in terms of empirical support and by reference to the characteristics of the portfolios in question. Also they would need to assess whether 75% was an appropriate confidence level to use.

f. Definition of a low default portfolio

Definitions offered by the industry in [BBA] for example have the drawback of being judgmental and introducing a question of degree. The definitions as used by firms also appear to characterise a wide variety of portfolios as LDPs. Perhaps as a result, use of a subjective definition could lead to inconsistency or even to exaggeration of the extent of the issue.

In order to operationalise a method for regulatory use, a simple approach could be adopted. This would set a specific number of defaults as being the threshold for subjecting a portfolio to special treatment. The number would be irrespective of the total portfolio size. The level could be chosen in such a way that the magnitude of conservative adjustment to PDs resulting from the special treatment is set at around the 10 per cent to 20 per cent mark. This reflects the fact that PD estimates are subject to all kinds of noise and uncertainty, not just noise coming from sampling. We suggest that once the necessary adjustment is below the 10-20% range, then the LDP effect ceases being a significant cause of concern. The cut-off point could be refined further, but inspection of the tables in the Appendix leads us to propose a working figure of around 20 defaults.

g. Cutover from LDP to non-LDPs

There is an issue of how cutover from the treatment to be applied for exposures below the threshold and those above should be handled. We wish to avoid cliff effects and want regulatory PD to be non-decreasing as default numbers rise, other things being equal. We have considered a number of alternatives, including introducing a cap to the regulatory PD above or performing some sort of tapering approach between say 10 and 20 defaults. No proposal has been found to be ideal. For the purpose of exposition we have chosen a simple approach: below 20 defaults, the confidence interval PD is chosen; above 20 defaults, the PD chosen is set equal to the higher of the observed default rate and the confidence interval approach for 20 defaults. Details and charts are set out in the Appendix.

The drawback of the approach is that the regulatory PD curve is flat in a bounded region from 20 defaults up to a number dependent on the size of portfolio and the choice of parameters.

The advantage is that it does not obscure the behaviour of the proposal for numbers of defaults of 20 or below.

4. A proposal

In this section we outline a step by step guide to a method that firms might use in order to determine conservative PD estimates for LDPs, using the ideas from the previous section. We then illustrate it with a simple example. The steps in the proposal are as follows.

The methodology is set out in the Appendix, together with a number of tables of results from the model. Ideally we would like to demonstrate in more detail the behaviour of the chosen approach as the assumptions and parameters are varied. In particular we would like to demonstrate more clearly the behaviour of the approach as the default rate varies, as the number of obligors varies, as the correlation assumption varies, as the confidence interval varies, and so on. The goals are to articulate fully the dynamics of the chosen approach, to illustrate the drivers of conservatism, and to verify that the approach delivers sensible regulatory outcomes.

In our proposal, the regulator is envisaged as having a role to play in Steps 1 and 3. To the extent that the approach is followed within a bank without any regulatory involvement, then the role which it is envisaged that the regulator would take would instead be filled by the appropriate central risk management function.

Step 1	The regulator publishes an objective criterion for identifying a Low Default Portfolio.
Step 2	The firm identifies which of its portfolios are LDPs according to the regulator's criterion set out under Step 1.
Step 3	The regulator publishes a look-up table which gives a portfolio average PD as a function of (1) numbers of company years in the historical sample and (2) numbers of defaults observed in the historical sample. These PDs are computed by the regulator using a specified and publicly available methodology, perhaps along the lines of that described in the Appendix.
Step 4	The firm identifies its historical sample for the LDP using its own internal data. The sample should cover the full number of years specified in Basel and should also include a period of adverse credit conditions. The sample should specify the grade that each obligor was in at the start of each year of the historical period.
Step 5	The firm determines the PDs for each grade in its low default portfolio. These PDs might be derived subjectively or using an

	analytical method. The weighted average of the PDs is computed: the weight attached to each grade-level PD is proportional to the number of obligor-years present in that grade in the historical sample.
Step 6	The firm looks up the PD it is to use in order to scale its internal PDs from the regulatory table. This is termed the "look-up PD".
Step 7	The firm compares the weighted average PD with the look-up PD. If the weighted average PD is less than the look-up PD, the firm adjusts its PD upwards until the weighted average PD is equal to or above the look-up PD. This could be achieved simply through a simple scaling factor to be applied to the firm's internal grade-level PDs for the portfolio in question. As described here, the adjustment process is asymmetric: PDs can be adjusted upwards but not downwards in effect making a firm's internal PDs act as floors to the PDs resulting from the process.

The resulting adjusted grade-level PDs are the outputs from this process. We envisage that these PDs would be used as a starting point from which further adjustments could made, potentially upwards or downwards, e.g. based on additional information such as external data. This is discussed further in Section 6.

Example

In this example, we assume Steps 1 and 2 have been performed and that the portfolio is caught by the LDP criterion. The example is based on a portfolio consisting of around one hundred obligors spread among seven non-default grades, A to G.

Step 3: we take the table from the Appendix corresponding to 12 per cent correlation, a 30 per cent year to year correlation and a 75 per cent confidence level. The figures needed for the example are included in Table 3 of the Appendix.

Step 4: data is available for each of the years 2000 to 2004 as set out in Tables 1 and 2. For the purposes of the example, we assume that information on the numbers of obligors in each grade and on the defaults that have occurred is available only at the start and ends of each calendar year.

Table 1: Number of obligors in each grade at start of year

Number of obligors in each grade at the start of the year							
Grade	2000	2001	2002	2003	2004	Total	
A	8	7	6	4	1	26	
B	24	25	25	24	24	122	
C	37	37	36	37	35	182	
D	23	24	25	26	25	123	
E	5	6	4	4	5	24	
F	1	3	3	2	5	14	
G	1	1	2	2	3	9	
Total	99	103	101	99	98	500	

Table 2: Defaults in each grade during calendar year

Number of defaults in each grade during calendar year							
Grade	2000	2001	2002	2003	2004	Total	
A	0	0	0	0	0	0	
B	0	0	0	0	0	0	
C	0	0	0	0	0	0	
D	0	0	0	0	0	0	
E	1	0	0	0	0	1	
F	0	1	0	0	0	1	
G	0	0	0	1	1	2	
Total	1	1	0	1	1	4	

Step 5: the firm supplies PDs for each grade; in the example, these range from 0.03% in Grade A to 30% in Grade G, as set out in Table 3.

Step 6: this is illustrated in Table 3. The average PD for the portfolio based on the grade distribution of the historical data is 1.35 per cent. Using Table 3 in the Appendix, the look-up PD corresponding to 4 defaults in 500 obligor years over 5 years is 1.69 per cent. (based on 12 per cent asset correlation, 30 per cent year-to-year correlation, and a 75 per cent confidence interval).

Step 7: since the average grade-level PD of 1.35 per cent is less than the look up PD of 1.69 per cent, an upward adjustment to the grade level PDs should be made. This can be achieved by multiplying each grade-level PD by a scale factor. The scale factor should be 1.69 /1.35 or approximately 1.25. The grade-level PDs are replaced by the scaled-up PDs.

Table 3: Firm's Chosen PDs and Weighted average PD calculation.

Calculation of weighted grade-level PD (2000-2004 data)							
Grade	Grade PD	No of obligor-years	Weight	Weight * Grade PD	*Defaults*	*Grade default rate*	Scaled-up Grade PD
A	0.03%	26	0.052	0.00%	*0*	*0.0%*	0.04%
B	0.10%	122	0.244	0.02%	*0*	*0.0%*	0.13%
C	0.30%	182	0.364	0.11%	*0*	*0.0%*	0.38%
D	1%	123	0.246	0.25%	*0*	*0.0%*	1.25%
E	3%	24	0.048	0.14%	*1*	*4.2%*	3.76%
F	10%	14	0.028	0.28%	*1*	*7.1%*	12.52%
G	30%	9	0.018	0.54%	*2*	*22.2%*	37.56%
Total		500	1.000	1.35%	*4*		

After Steps 1 to 7 have been completed, the firm would then additionally need to consider whether the PDs emerging from this process required further adjustment in either direction, perhaps because of additional information not taken account of or because there were reasons to believe that the resulting scaled-up grade-level PDs were inappropriate as forward-looking long-run PD estimates. (For the purposes of this example it is assumed that the regulator is content with the rating system's ability to discriminate amongst borrowers.) This stage is further commented on in Section 6.

Dynamic effects

When used operationally, there are two principal dynamic effects on the estimated parameters. First, the distribution of the portfolio may change over time. And second, additional observations are gained through the addition of further obligor years. Using the example already given, we look at the effect of a one-off change in credit quality and then the effect of the passage of time. In practice of course these two effects happen side by side and what is observed is the combined effect of both. Analytically, it is helpful to disentangle the two effects.

A key feature of the proposal is that a one-off deterioration in the credit quality of the portfolio evidenced by downgrades, or extending credit to new borrowers in low grades, or acquisition of new borrowers e.g. following a takeover, would result in a rise in capital. This is important because we do not want to project the historical performance of a higher quality portfolio onto a lower quality portfolio. In more technical terms, the look-up PD is a portfolio level PD that is conditioned on the portfolio distribution that gave rise to the default history observed; the forward-looking PD is conditional on the distribution of the current portfolio. A consequence of this is that improvements in credit quality would result in a reduction of the amount of capital.

First, we present some additional 2005 data relating to this portfolio. The credit quality has deteriorated as a comparison of the 2005 starting distribution with any one of the years 2000 to 2004 shows. In addition, we suppose that further defaults occur, one in each of the two grades of lowest credit quality.

Table 4: 2005 data

2005 data Grade	No of obligors at start of 2005	Defaults during 2005
A	0	0
B	9	0
C	27	0
D	31	0
E	12	0
F	11	1
G	10	1
Total	100	2

To illustrate this deterioration, we calculate the weighted average PD for the portfolio composition as it stands at the start of 2005. This is not an essential calculation to perform for the proposed method, but does provide a rough assessment of credit quality.

Table 5: Average portfolio distribution based on portfolio at start of 2005.

Calculation of weighted grade level PD (distribution at start of 2005)						
Grade	Grade PD	No of obligors	Weight	Weight * Grade PD	Scaled-up Grade PD	Weight * Scaled-up Grade PD
A	0.03%	0	0.000	0.00%	0.04%	0.00%
B	0.10%	9	0.090	0.01%	0.13%	0.01%
C	0.30%	27	0.270	0.08%	0.38%	0.10%
D	1%	31	0.310	0.31%	1.25%	0.39%
E	3%	12	0.120	0.36%	3.76%	0.45%
F	10%	11	0.110	1.10%	12.52%	1.38%
G	30%	10	0.100	3.00%	37.56%	3.76%
Total		100	1.000	4.86%		6.08%

Note that, at the start of 2005, no additional obligor years have at that point been observed. Defaults occurring prior to the start of 2005 are included among the 2004 data and have already been used to calibrate the rating system to give the scaled-up grade PDs. There is no additional data forthcoming and as a result the firm's PD grades are not adjusted further on account of the credit quality deterioration. As a result the average PD, based on the current portfolio and the scaled-up PDs, is 6.08%, considerably higher than the 1.69% look-up PD on the historic portfolio, the firm’s weighted estimate of 1.35% over 2000-04 or the 0.80% actually experienced over that period. As a result of the credit quality deterioration, the minimum Pillar 1 capital requirement would increase. The amount by which it would so depends on the relative values of the PDs supplied by the firm.

Table 6: Calculation of weighted grade level PD (2000-2005 data)

Calculation of weighted grade-level PD (2000-2005 data)							
Grade	Grade PD	No of obligor-years	Weight	Weight * Grade PD	*Defaults*	*Grade default rate*	(Non-) scaled-up Grade PD
A	0.03%	26	0.043	0.00%	*0*	*0.0%*	0.03%
B	0.10%	131	0.218	0.02%	*0*	*0.0%*	0.10%
C	0.30%	209	0.348	0.10%	*0*	*0.0%*	0.30%
D	1%	154	0.257	0.26%	*0*	*0.0%*	1%
E	3%	36	0.060	0.18%	*1*	*2.8%*	3%
F	10%	25	0.042	0.42%	*2*	*8.0%*	10%
G	30%	19	0.032	0.95%	*3*	*15.8%*	30%
Total		600	1.000	1.93%	*6*		

The historical sample now covers six calendar years and contains 600 obligor years and 6 defaults. The look-up PD in Table 3 of the Appendix corresponding to this combination is 1.89 per cent. The portfolio level average PD based on the enlarged set of historical data is 1.93 per cent. As described in Step 7 of the proposal, no upward or downward adjustment to the firm's grade level PDs would be made. A scaling factor of 1.00 is applied.

To summarise the points made in this section, an instantaneous change in credit quality will result in a change in regulatory capital; grade level PDs would not be adjusted as a result. Collection of additional data in the firm of obligor years and defaults within each grade would on the other hand justify revision of grade level PDs so as to bring them back into line with the revised conservative look-up PDs. In practice these two effects occur in parallel and it is not easy to say in advance what the net effect on capital would be.

Advantages of the proposal

The aim of this proposal is to present an approach that, if adopted by a regulator, would give firms some assurance that their approaches would embody sufficient conservatism to qualify for IRB. The hallmarks of this approach are:

- Introducing a workable definition of LDP
- Using all of the available internal data in forming estimates
- Providing a clear approach to quantification including the degree of conservatism
- Enabling firms to override or adjust risk estimates in the presence of additional relevant information.
- Addressing the chief regulatory concern of the level of capital.

- Providing a system which will result in more capital being held in the event of downgrades or deterioration in credit quality.

- Placing appropriate reliance on the skill and judgment of credit officers within regulated firms.

5. Mandatory, starting point, or fall-back

The proposal in this paper, which it is emphasised does not represent FSA Policy, needs positioning. As presented, it is an algorithm which is capable of formulaic application, after taking input from the firm in the form of the relative grade-level PDs. A firm using this approach would nevertheless be expected to use its judgment in applying the outputs of the algorithm. As such the approach would be most useful as either a starting point from which adjustments can be made or as a fall-back position to be used in the absence of other forms of data. We suggest that it could be used as a starting point from which adjustments should be made.

The approach does not make reference to any external data. It may well be that firms can improve the PD estimates by making sensible use of such data – which could justifiably reduce the regulatory PD estimate and therefore the requisite margin of conservatism. Examples of such information are external rating assessment and agency default histories and credit spread information. The approach we outline here is designed to work even where external data is available. However, if the external data is sufficiently reliable, it is arguable that it is more efficient to use that, as opposed to the Look-Up PD outlined in this paper, as the Starting Point, and the FSA's formal proposal on treatment of PD in Low Default Portfolios follows such an approach.

The approach does not explicitly take into account the time period over which the historical data is collected: As noted earlier, data should really be collected over several years. Other things being equal, this would give us greater confidence that the default experience was representative, and should therefore require a lower margin of conservatism than would data collected over just a few years. In more sophisticated variants of this approach, firms can tailor the assumptions of asset and year-to-year correlations to more accurately reflect events over the period covered.

In short, the firm should treat this estimation approach like any other: to be used sensibly and knowledgeably, factoring in the firms' own experience and judgement.

6. Possible criticisms of approach

The main criticism of this approach is that we may be trying to infer too much from too little. Low default portfolios are in many cases also portfolios where data is scarce. The suggested approach risks stretching the information contained in the data beyond what it will actually bear.

Some readers might draw the conclusion that the above approach will still result in adverse treatment of some niche portfolios such as housing associations, public-private partnership arrangements and perhaps some forms of specialised lending. If we were sure that the PDs on these portfolios were low and that asset correlations were low, then the conclusion would be right. But from a regulatory/risk management perspective, it is far from clear that the results delivered here are in appropriate. Exposures to the mentioned areas represent sectoral concentrations. As such, exposures may be subject to high degrees of interdependence between default events and a higher level of capital than that implied by the approach may be justified. Correlation may be particularly high, and observed defaults are less reliable indicators of long run averages.

The proposal will appear to penalise firms that choose to divide up their rating systems so that they cover relatively small asset types. The smaller the portfolio, the more likely any individual portfolio is to be captured by the low default portfolio criterion. The effect could be that firms group several different types of asset within a single rating grade in order to avoid this perceived penalty. There is a risk that this could lead to perverse behaviour on the part of firms. For that reason, many of the assumptions and parameter choices and the positioning of the approach as a starting point, subject to override, have been made in a way that limits the quantitative impact of the approach. Further mitigation could be made, at a cost, through firm-by-firm review, identifying instances where inappropriate groupings have been made and applying case-specific remedies.

A further criticism could be that the approach does not result in discrimination between two portfolios which happen to have the same default history, but which both the banks and their regulator might agree have differing levels of risk. The challenge here would be for the bank with the less risky portfolio to produce quantitative or other independent evidence in support of the lower level of risk.

The approach relies on internal data gathered over a number of years. We have stated above that the data history should cover an economic downturn. In practice, this may not be possible if the only available data covers a period of relatively benign economic conditions. Such a situation is not particular to LDPs; it can occur with any portfolio for which the length of data history is short. The proposal does not specifically address this issue: it would be for the firm to assess the extent to which further adjustments to the PD would be justified in view of data limitations such as these and to be able to justify its choice.

To the extent that the look-up PDs resulting from the proposal are conservative, and are higher than PDs that firms judge appropriate for their own portfolios, the approach could distort behaviour such as pricing and extension of credit. An alternative would be to allow firms to use their own internal PDs for pricing and other internal purposes; a scaling factor is then applied to scale up the internal PDs to give regulatory PDs that are consistent with the PDs emanating from the suggested proposal. This approach then appears to weaken the Use Test. Regulators need to balance two conflicting aims here: conservatism and use; it is not clear where regulators should strike this balance.

New models and new business lines may be captured by LDP definition, at least while default histories are being built up. It is a feature of this proposal that the Look-Up PDs for new portfolios will be particularly high, and it cannot be used in the absence of some experience of defaults/non-defaults.

7. Summary and conclusion

A proposal for estimating PDs for Low Default Portfolios has been put forward in this paper. The proposal is for a methodology for estimating an upper bound to portfolio level PDs, based on regulatory choices of confidence interval and correlations. It cannot be guaranteed that the resulting PDs will in all cases be conservative. The key aspect of the proposal is the linkage between grade-level PDs (specified by firms) and portfolio-level average PDs (specified by regulators). The method is simple and could be made operational with firms.

There are several caveats to the proposal and these have been noted along the way. In the minds of the authors, the main issues involved with adopting this proposal as a regulatory methodology are:

- Too little information? Where data is scarce, any given PD estimate is likely to be unreliable and open to challenge. In proposing a quantitative approach, we recognise that we are pushing at the limit of what can be inferred from the available data.

- Compatibility with the Use Test. Initial industry reaction is that the PDs resulting from this methodology will in some cases be too high for them to sustain internally. This could lead to a separation between the PDs used for internal management purposes and those used for Pillar 1 minimum capital calculations, so undermining the Use Test and weakening one of the chief disciplines of the IRB approach for the portfolios in question.

- No link to the state of the economy. The methodology does not factor in the economic conditions prevailing at the time that historical data was observed.

- Use of judgment and overrides. We have not suggested that the approach is used in a formulaic manner, but that the judgement of credit officers and risk managers are brought to bear. We have suggested some sorts of quantitative or independent information that could be used for this purpose. We have not spelled out how this layer of judgment would work in any detail; in particular regulators might consider whether standards or limitations to the exercise of judgment are warranted here. The point is that, in a practical operational context, if it was felt that downward adjustments and overrides would be made in each and every case, then it would call into question the purpose and usefulness of a quantitative-based method for addressing conservatism.

- Technical aspects. The choice of model, correlation assumptions, confidence interval and cut-over methodology are all discussed in the paper, but the choices we have made are not the only conceivable ones and it is acknowledged that other choices could reasonably be made.

8. References

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[F] Forrest, Alan (2005), *Likelihood Approaches to Low Default Portfolios,* Working Paper.

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[PT] Pluto, Katje and Dirk Tasche (2005), *Estimating Probabilities of Default for Low Default Portfolios*, Working Paper.

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Appendix

PD estimates for low default portfolios based on confidence levels

Assuming defaults occur independently, for a confidence level γ, a portfolio of size n and r observed defaults, the PD estimate as in Pluto and Tasche [PT] is the unique solution p to

$$1-\gamma=\sum_{k=0}^{r}\binom{n}{k}p^k(1-p)^{n-k}.$$

Now, modelling dependence as in the Vasicek model, we shall suppose there is a single, normally-distributed risk factor Y to which portfolio assets are correlated. We shall denote by ρ the pair-wise correlation between assets, and therefore the correlation between a given asset and the single risk factor is $\sqrt{\rho}$. Then for a given value y of Y, the right-hand side of the formula above becomes

$$\sum_{k=0}^{r}\binom{n}{k}\left(\Phi\left(\frac{\Phi^{-1}(p)+y\sqrt{\rho}}{\sqrt{1-\rho}}\right)\right)^k\left(1-\Phi\left(\frac{\Phi^{-1}(p)+y\sqrt{\rho}}{\sqrt{1-\rho}}\right)\right)^{n-k},$$

and taking the expectation across all possible values of the risk factor Y, we get the required PD estimate as the unique solution p to

$$1-\gamma=E\left[\sum_{k=0}^{r}\binom{n}{k}\left(\Phi\left(\frac{\Phi^{-1}(p)+Y\sqrt{\rho}}{\sqrt{1-\rho}}\right)\right)^k\left(1-\Phi\left(\frac{\Phi^{-1}(p)+Y\sqrt{\rho}}{\sqrt{1-\rho}}\right)\right)^{n-k}\right],$$

where Y is a standard normal variable, *c.f.* Pluto and Tasche [PT] for a proof of uniqueness of the solution in case of no observed defaults. Note that the model specified above implicitly assumes that the observation period is one year, since the binomial parameter is a conditional probability to default in one year. When there are few defaults, this is similar to a situation with several years and constant single risk factor. However this analogy breaks down when the number of defaults increases. Still for low defaults, a constant single risk factor is a conservative correlation assumption and the last part of this appendix addresses the general case when years are not fully correlated.

Estimation procedure

Simulate N independent standard normal variables (Y_i). Then find numerically the value of p such that

$$1-\gamma=\frac{1}{N}\sum_{i=1}^{N}\left[\sum_{k=0}^{r}\binom{n}{k}\left(\Phi\left(\frac{\Phi^{-1}(p)+Y_i\sqrt{\rho}}{\sqrt{1-\rho}}\right)\right)^k\left(1-\Phi\left(\frac{\Phi^{-1}(p)+Y_i\sqrt{\rho}}{\sqrt{1-\rho}}\right)\right)^{n-k}\right]. \qquad (1)$$

Results: values of look-up PDs for different correlation and confidence levels

We are using N = 1,000,000 simulated values of the single risk factor for up to 500 obligor years and 5 observed defaults, N = 100,000 for larger default rates and between 500 and 5000 obligor years, and N = 10,000 for 10,000 obligor years, owing to computation contingencies. PD figures larger than 1% have three significant digits and those smaller than 1% are rounded up to the nearest basis point. At this point we are not including time correlation.

For γ = 75%, ρ = 12%:

		n		
		100	**500**	**1000**
r	**0**	2.35%	0.60%	0.33%
	3	7.90%	2.10%	1.18%
	10	18.00%	4.90%	2.76%

For γ = 75%, ρ = 24%:

		n		
		100	**500**	**1000**
r	**0**	3.87%	1.23%	0.75%
	3	11.30%	3.70%	2.30%
	10	23.00%	7.60%	4.80%

Here are more complete tables.

Table 1: γ = 75%, ρ = 12%

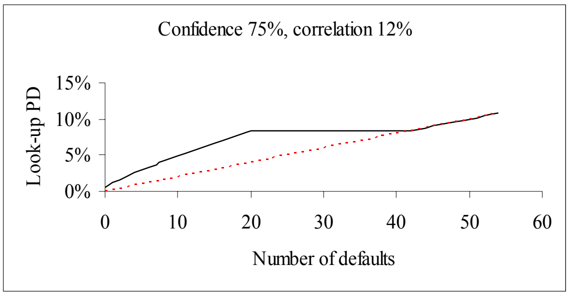
		n			
		100	**500**	**1000**	**5000**
r	**0**	2.35%	0.60%	0.33%	0.09%
	1	4.40%	1.20%	0.64%	0.17%
	2	6.21%	1.65%	0.92%	0.24%
	3	7.90%	2.10%	1.18%	0.31%
	4	9.50%	2.55%	1.45%	0.37%
	5	11.0%	2.95%	1.66%	0.44%
	6	12.5%	3.35%	1.89%	0.50%
	7	14.0%	3.75%	2.10%	0.56%
	8	15.3%	4.13%	2.35%	0.62%
	9	16.7%	4.50%	2.54%	0.67%
	10	18.0%	4.90%	2.76%	0.73%
	11	19.3%	5.25%	2.96%	0.79%
	12	20.7%	5.58%	3.20%	0.84%
	13	21.9%	5.93%	3.37%	0.90%
	14	23.1%	6.27%	3.56%	0.95%
	15	24.3%	6.63%	3.75%	1.00%
	16	25.5%	6.96%	3.96%	1.05%
	17	26.7%	7.29%	4.15%	1.10%
	18	27.8%	7.62%	4.33%	1.15%
	19	29.0%	7.95%	4.51%	1.20%
	20	30.1%	8.29%	4.69%	1.25%
	…	…	…	…	…
	80	86.0%	24.6%	14.1%	3.80%

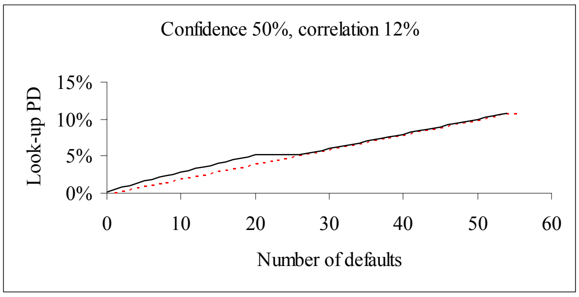
Table 2: γ = 50%, ρ = 12%

		n			
		100	**500**	**1000**	**5000**
r	**0**	1.00%	0.23%	0.13%	0.03%
	1	2.24%	0.53%	0.29%	0.07%
	2	3.45%	0.81%	0.44%	0.11%
	3	4.55%	1.08%	0.58%	0.14%
	4	5.70%	1.35%	0.73%	0.17%
	5	6.80%	1.61%	0.86%	0.21%
	6	7.88%	1.86%	1.00%	0.24%
	7	8.95%	2.11%	1.13%	0.27%
	8	10.0%	2.35%	1.27%	0.30%
	9	11.1%	2.60%	1.40%	0.34%
	10	12.1%	2.84%	1.53%	0.37%
	11	13.1%	3.08%	1.66%	0.40%
	12	14.1%	3.32%	1.79%	0.43%
	13	15.1%	3.55%	1.91%	0.46%
	14	16.1%	3.78%	2.05%	0.48%
	15	17.1%	4.05%	2.16%	0.52%
	16	18.1%	4.27%	2.28%	0.55%
	17	19.1%	4.49%	2.41%	0.57%
	18	20.1%	4.72%	2.53%	0.60%
	19	21.1%	4.94%	2.65%	0.63%
	20	22.0%	5.17%	2.78%	0.66%

Cut-off point (an example)

The PDs calculated with the confidence level as specified above are used for up to 20 observed defaults. For numbers of defaults greater than 20, in order to avoid situations when fewer defaults lead to a greater regulatory PD, for instance the latter can be chosen as the maximum of the rate corresponding to 20 defaults and the observed rate. Therefore in the case of a 50% confidence level, with 12% correlation, the look-up PDs for 20 observed defaults being 5.17% (*c.f.* Table 2 above), this value will be used for cases when 20 to 25 defaults occur, since 26 is the number from which the observed rate becomes larger than 5.17%. For more than 26 observed defaults, the observed rate is used. This gives the following charts (where n is 500).





Assessment of the level of conservatism for portfolios of varying qualities

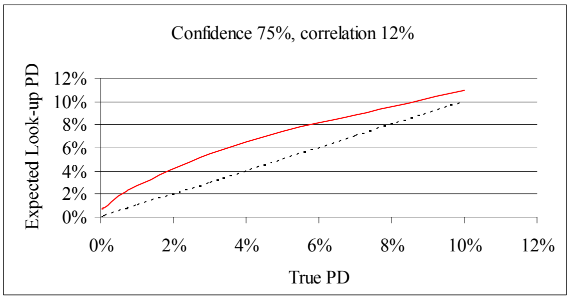
In order to get a feeling for the degree of conservatism inherent in the look-up PD, let us examine its expected output when applied to portfolios across the spectrum of credit qualities. The credit quality of a portfolio is described here by its true PD, which we do not know, but yet exists. A true PD is chosen, and the expected look-up PD for a sample of a given size with a given level of correlation is derived from Tables 1 and 2. The expected look-up PD is then compared with the true PD to check the level of conservatism. Furthermore, in order to assess the risk that the look-up PD be seriously smaller than the true PD, the probability that the former be smaller than half of the latter is also computed. This is done for several levels of the true PDs covering cases when the few observed defaults are due to the credit quality of the portfolio as well as cases when they are a lucky outcome from portfolios of worse quality. The results are displayed for a range of true PDs in the following table, for a portfolio of size 500, with asset correlation $\rho = 12\%$ and confidence level $\gamma = 75\%$.

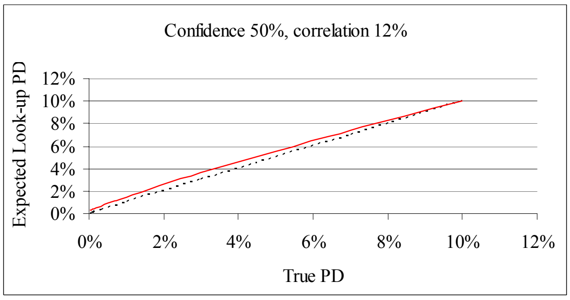
True PD	0.03%	0.05%	0.07%	0.1%	0.3%	0.5%	0.7%	1%	3%	5%	7%	10%	30%
Expected Look-up PD	0.69%	0.74%	0.79%	0.86%	1.3%	1.8%	2.2%	2.7%	5.5%	7.4%	8.8%	11%	30%
Pr (Look-up PD < True PD)	0%	0%	0%	0%	0%	0%	21%	14%	24%	24%	25%	58%	54%
*Pr (Look-up PD < 0.5*True PD)*	0%	0%	0%	0%	0%	0%	0%	0%	5.0%	4.6%	5.1%	4.4%	10%

Now for a portfolio of size 500, with confidence level 50% and correlation 12%, we get:

True PD	0.03%	0.05%	0.07%	0.1%	0.3%	0.5%	0.7%	1%	3%	5%	7%	10%	30%
Expected Look-up PD	0.27%	0.3%	0.33%	0.37%	0.65%	0.91%	1.2%	1.5%	3.7%	5.5%	7.4%	10%	30%
Pr (Look-up PD < True PD)	0%	0%	0%	0%	43%	30%	41%	42%	47%	50%	61%	58%	54%
*Pr (Look-up PD < 0.5*True PD)*	0%	0%	0%	0%	0%	30%	21%	14%	19%	18%	16%	15%	10%

The presence of zeros in the last two rows is due to the fact that the look-up tables cannot deliver look-up PDs below certain levels for these combinations of parameters.





In both cases the probability that the look-up PD be lower than the true PD is bigger than 50% for higher true PDs. This is due to the skewness of the distribution of defaults, for a given true PD. Indeed, we know (*c.f.* Kurbat and Korablev [KK]) that the distribution of numbers of defaults is skewed to the left, and hence the observed rate will most of the time be smaller than the mean rate. Given that for lower credit qualities (higher true PDs), the observed default rate will be most of the time above the cut-off point, the result is that in these cases the look-up PD will not be used.

Level of conservatism when the credit quality is unknown: the "lucky draw" case

The analysis above tells us about the level of conservatism of the look-up PD if we have an idea about the true PD of the portfolio, but in practice this might not be the case. More precisely, we want to be prudent in case the low-default behaviour observed is a lucky realisation of a low-quality portfolio. This is why we need a quantification of the level of conservatism inherent in the look-up PD without prior knowledge on the portfolio quality. This can be achieved by noting that

$$Density(True\ PD = p \mid r\ defaults\ in\ n) \propto \Pr(r\ defaults\ in\ n \mid True\ PD = p) \times Density(True\ PD = p),$$

where "density" stands for the probability density function. Having no prior knowledge on the true PD boils down to assuming that it has a uniform distribution, whereby all values are equally likely. The expression of the first term in the right-hand side, *i.e.* the probability of getting the observed outcome given a particular value of the true PD, can be derived like the equations at the start of this appendix, taking the asset correlation into account. Finally, we get

$$Density(True\ PD = p \mid r\ defaults\ in\ n) \propto E\left[\left(\Phi\left(\frac{\Phi^{-1}(p)+Y\sqrt{\rho}}{\sqrt{1-\rho}}\right)\right)^{r}\left(1-\Phi\left(\frac{\Phi^{-1}(p)+Y\sqrt{\rho}}{\sqrt{1-\rho}}\right)\right)^{n-r}\right].$$

From this distribution, we can calculate the probability that the true PD be larger than the look-up PD, the latter being a fixed value given by the tables from the previous section, for r

defaults in *n* obligor years. For illustration purposes, let us still consider a portfolio of 500 obligor years with correlation ρ = 12%. The following table gives probabilities that the look-up PDs be smaller than the true PDs, and than half of it, for various observed default rates, without any prior knowledge on the true PD, for a confidence level of 75%.

Observed defaults	0	5	10	20
Pr (Look-up PD < True PD)	21%	23%	25%	25%
*Pr (Look-up PD < 0.5*True PD)*	8.0%	6.5%	5.1%	3.4%

Now for a confidence level of 50%, we have the following results.

Observed defaults	0	5	10	20
Pr (Look-up PD < True PD)	47%	49%	49%	50%
*Pr (Look-up PD < 0.5*True PD)*	27%	21%	19%	14%

The second rows of both tables correspond to what we might have expected from the choice of confidence levels. The third rows contain information on the risk to set PD's which are seriously too low compared to the true credit quality of the portfolio. This tells us about the overall inherent level of conservatism **if** the asset correlation **really** were 12% and **if** defaults **really** behaved as per the Vasicek model. However these are only assumptions and therefore the results in this section are approximations.

In conclusion, the best we can say is that these estimates would be conservative most of the time. However we cannot say with much certainty that a given set of estimates are conservative at each time that they are calculated. It follows that the estimated PDs may still be less than the true PDs so that capital requirements are being underestimated. The same techniques can certainly be used to give a **greater** level of comfort that the PDs being used are not being understated – by tweaking one or more of the assumptions. However, to achieve this, the PDs would usually need to be far higher than a firm's observed experience.

Evolution of the expected look-up PD as obligor years and credit quality vary

For various levels of credit quality (true PDs) and of data richness (number of obligor years), here are the expected look-up PDs for a correlation of 12% and various confidence levels. The expected look-up PD is derived in a formulaic way for each realisation of the single risk factor, and then averaged across its simulated values.

	γ = 75%	*Number of obligor years*			
	ρ = 12%	**100**	**500**	**1000**	**5000**
True PD	**0.03%**	2.4%	0.69%	0.42%	0.19%
	0.1%	2.6%	0.86%	0.61%	0.37%
	0.3%	3.0%	1.3%	1.1%	0.7%
	1%	4.2%	2.7%	2.3%	1.4%
	3%	7.6%	5.5%	4.4%	3.2%
	10%	17%	11%	10%	10%
	30%	34%	30%	30%	30%

	γ = 50%	*Number of obligor years*			
	ρ = 12%	**100**	**500**	**1000**	**5000**
True PD	**0.03%**	1.0%	0.27%	0.18%	0.08%
	0.1%	1.1%	0.37%	0.28%	0.18%
	0.3%	1.4%	0.65%	0.55%	0.39%
	1%	2.3%	1.5%	1.4%	1.1%
	3%	4.5%	3.7%	3.2%	3.0%
	10%	12%	10%	10%	10%
	30%	30%	30%	30%	30%

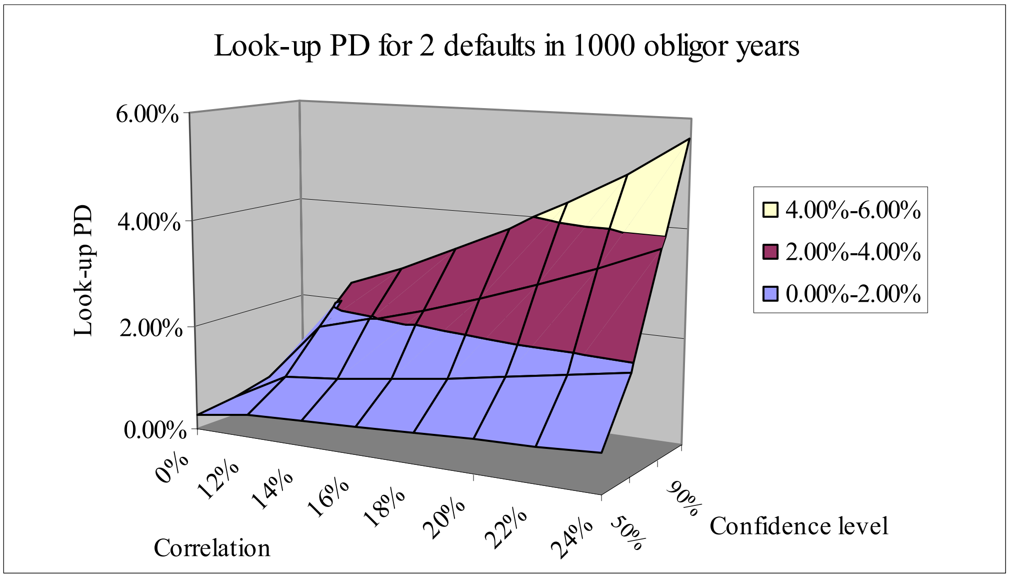
Evolution of the look-up PD as confidence level and correlation vary

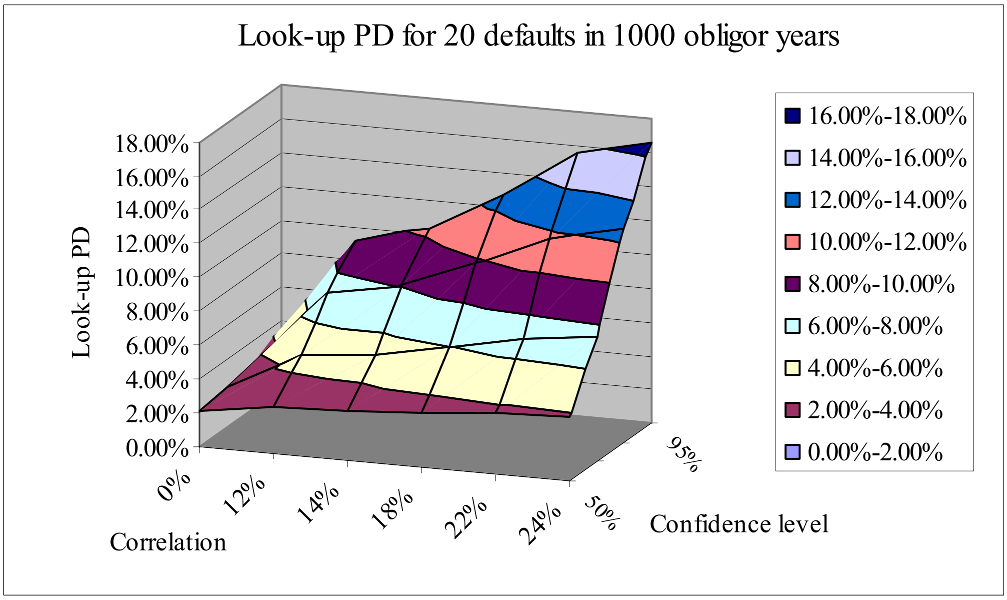
For various levels of correlation and confidence, here are the look-up PDs for an observed configuration of two, respectively 20 defaults in 1000 obligor years. The figures are numerical solutions to Equation (1) for the appropriate levels of confidence and correlation.

	n = 1000	*Confidence level*			
	r = 2	**50%**	**75%**	**90%**	**95%**
Correlation	**0%**	0.27%	0.39%	0.53%	0.63%
	12%	0.44%	0.92%	1.70%	2.36%
	14%	0.48%	1.04%	1.96%	2.77%
	16%	0.52%	1.17%	2.25%	3.25%
	18%	0.57%	1.31%	2.59%	3.76%
	20%	0.62%	1.47%	2.96%	4.32%
	22%	0.68%	1.65%	3.36%	4.93%
	24%	0.74%	1.84%	3.80%	5.62%

	n = 1000	*Confidence level*			
	r = 20	**50%**	**75%**	**90%**	**95%**
Correlation	**0.00**	2.07%	2.38%	2.70%	2.89%
	0.12	2.78%	4.69%	7.25%	9.20%
	0.14	2.91%	5.09%	8.05%	10.35%
	0.18	3.22%	5.95%	9.77%	12.70%
	0.22	3.57%	6.90%	11.65%	15.57%
	0.24	3.75%	7.45%	12.67%	16.68%

The figures below illustrate these two configurations.





Adjustment of look-up PDs for multi-year observations

The method outlined at the start of this appendix did not account for the lower correlation between default events farther from each other. This section aims at adjusting look-up PDs for such decaying temporal correlation. The approach presented in Pluto and Tasche [PT] is adopted and the 'single risk factor' model used, which inspired the Basel II IRB formulae. For an observation period of T years with n initial obligors, the change $V_{i,t}$ in asset value of Obligor i in Year t is modelled as

$$V_{i,t} = \sqrt{\rho} S_t + \sqrt{1-\rho} X_{i,t}$$

where S_t is the systematic risk factor common to all obligors, $X_{i,t}$ the idiosyncratic factor, and ρ the pair-wise correlation between assets of different obligors in any given year. All idiosyncratic factors $X_{i,t}$ are assumed to be independent standard normal variables, also independent of the systematic factor. Now S_t can vary across the years and its consecutive values are modelled as multivariate standard normal variables with correlation matrix

$$\begin{pmatrix} 1 & \theta & \cdots & \theta^{T-1} \\ \theta & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \theta \\ \theta^{T-1} & \dots & \theta & 1 \end{pmatrix}$$

so that the correlation between Years i and j is $\theta^{|i-j|}$. The year-to-year correlation θ is chosen strictly smaller than one, so that years farther apart be less correlated.

Adjusted estimation procedure for the look-up PDs

Now, given a realisation $(S_1,...,S_T)$ of the systematic factor, if we denote by p the PD of an obligor, so that $\Phi^{-1}(p)$ is the default threshold of the yearly change in asset value $V_{i,t}$, then the probability that this obligor defaults in any given year within the observation period is

$$\pi(S_1,\ldots,S_T) = 1 - \prod_{t=1}^{T}\left(1 - \Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\rho}S_t}{\sqrt{1-\rho}}\right)\right)$$

and therefore the likelihood of observing no more than r defaults out of n initial obligors during the whole period is

$$\sum_{k=0}^{r}\binom{n}{k}E\left[(\pi(S_1,\ldots,S_T))^k(1-\pi(S_1,\ldots,S_T))^{n-k}\right].$$

For a given level of confidence γ, equating the latter expression with $1-\gamma$ will give the desired value of p. Simulations of the systematic risk factor consist of independent draws of the T-year sequence $S_1,...,S_T$.

Results of the adjustment

Model inputs are set as follows.

- Confidence level: γ = 75%
- Asset correlation: ρ = 12%
- Year-to-year correlation: θ = 30%

The matrices describing correlations between different years, which result from the choice of value for θ are presented below for each length of observation period.

For T = 5 years:

$$\begin{pmatrix} 100\% & 30\% & 9\% & 2.7\% & 0.81\% \\ 30\% & 100\% & 30\% & 9\% & 2.7\% \\ 9\% & 30\% & 100\% & 30\% & 9\% \\ 2.7\% & 9\% & 30\% & 100\% & 30\% \\ 0.81\% & 2.7\% & 9\% & 30\% & 100\% \end{pmatrix}$$

For T = 6 years:

$$\begin{pmatrix} 100\% & 30\% & 9\% & 2.7\% & 0.81\% & 0.24\% \\ 30\% & 100\% & 30\% & 9\% & 2.7\% & 0.81\% \\ 9\% & 30\% & 100\% & 30\% & 9\% & 2.7\% \\ 2.7\% & 9\% & 30\% & 100\% & 30\% & 9\% \\ 0.81\% & 2.7\% & 9\% & 30\% & 100\% & 30\% \\ 0.24\% & 0.81\% & 2.7\% & 9\% & 30\% & 100\% \end{pmatrix}$$

Model outputs are the PD estimates corresponding to various observed defaults and portfolio sizes. They are illustrated in the following look-up tables. Observation periods of five and six years are chosen in order to highlight the reduction in look-up PDs gained from an extra year of data. For the sake of comparison, in Table 1 at the beginning of the appendix, the column of look-up PDs for 500 obligor-years can be related to a situation when T = 5 years, n = 100 obligors and θ = 100% (since the single risk factor underpinning these estimates is implicitly assumed to be constant over the years). The adjusted look-up PDs for such a configuration are presented in the first column of Table 3. The effect of the adjustment can also be observed by comparing the third column of Table 1 (for 1,000 obligor years) with the second column of Table 3, where the 1,000 obligor-years consist of 200 obligors across five years.

Table 3: T = 5 years

Years		**5**	**5**	**5**
n		**100**	**200**	**500**
Obligor-years		**500**	**1000**	**2500**
r	**0**	0.37%	0.20%	0.09%
	1	0.72%	0.38%	0.17%
	2	1.06%	0.56%	0.24%
	3	1.37%	0.72%	0.32%
	4	1.69%	0.89%	0.38%
	5	2.00%	1.05%	0.45%
	6	2.30%	1.21%	0.52%
	7	2.61%	1.37%	0.59%
	8	2.91%	1.52%	0.66%
	9	3.21%	1.68%	0.72%
	10	3.50%	1.83%	0.79%
	11	3.81%	1.98%	0.85%
	12	4.10%	2.14%	0.92%
	13	4.40%	2.29%	0.98%
	14	4.70%	2.44%	1.05%
	15	5.00%	2.59%	1.11%
	16	5.30%	2.74%	1.17%
	17	5.60%	2.89%	1.24%
	18	5.91%	3.04%	1.30%
	19	6.21%	3.19%	1.36%
	20	6.51%	3.34%	1.42%
	…	…	…	…
	80	32.6%	12.9%	5.02%

For T = 6 years:

Years		**6**	**6**	**6**
n		**100**	**200**	**500**
Obligor-years		**600**	**1200**	**3000**
r	**0**	0.30%	0.16%	0.07%
	1	0.59%	0.31%	0.13%
	2	0.86%	0.45%	0.19%
	3	1.12%	0.59%	0.25%
	4	1.38%	0.72%	0.31%
	5	1.63%	0.85%	0.37%
	6	1.89%	0.98%	0.42%
	7	2.13%	1.11%	0.48%
	8	2.38%	1.24%	0.53%
	9	2.63%	1.37%	0.59%
	10	2.88%	1.49%	0.64%
	11	3.13%	1.62%	0.69%
	12	3.37%	1.74%	0.74%
	13	3.62%	1.87%	0.80%
	14	3.87%	1.99%	0.85%
	15	4.12%	2.12%	0.90%
	16	4.37%	2.24%	0.95%
	17	4.63%	2.36%	1.00%
	18	4.87%	2.49%	1.05%
	19	5.13%	2.61%	1.10%
	20	5.38%	2.73%	1.15%
	…	…	…	…
	80	28.0%	10.7%	4.14%

For large numbers of defaults (80 in these tables), despite having accounted for time correlation decay, the look-up PD can be larger than Table 1 might suggest. This is because the underlying assumptions differ, as mentioned in the introduction of this appendix. Table 1 was derived in a simplistic framework of nT obligors having one opportunity to default, whereas Table 3 is derived in a framework of n obligors having T opportunities to default. The latter method will start to give higher PDs than the former when the number of observed defaults gets closer to the portfolio size. This is a realistic feature. Indeed the earlier approach could in principle allow for more defaults than there are obligors in the portfolio! This is not an issue given that we focus on low default situations, where both methods coincide once the single risk factor is kept constant.

Notes

Even though the look-up PD presented in this appendix has a confidence level attached to it, this is just one of the various parameters determining its value. Since choices of correlation levels (both asset and year-to-year correlations) and of models (Vasicek, Pluto and Tasche [PT]) are based on judgement and thereby arbitrary, it is actually difficult to know exactly how conservative the final output really is. The only certainty is that the look-up PD will respond to changes in inputs in a sensible manner. The advantage of the framework presented here is that the judgemental parts of the process are very precisely delimited, and can thereby be easily singled out and assessed.

The PD estimates presented here take into account the number of years over which the observation window stretches. However they do not reflect either the position of this window in the economic cycle, or how scattered defaults were within the window. These two limitations should be the objects of further adjustments by the bank's own credit experts to ensure that the final estimates are forward-looking and representative of a long run.