The Vasicek Loss Given Default Model

Simulating the Distribution of the Parameters

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The Vasicek Loss Given Default (LGD) model is derived from the recovery equation. The recovery equation, resembling the asset return, is defined as:

$$r = \mu + \sigma\sqrt{q}z + \sigma\sqrt{(1-q)}\epsilon$$

where:

- μ is a quantity parameter (similar to parameter p from the PD model);
- σ represents the quality parameter;
- q is the sensitivity parameter (similar to parameter ρ from the PD model);
- z represents the systemic factor derived from the Vasicek PD model;
- ϵ denotes the idiosyncratic factor, independent of z, which follows a standard normal distribution.

Given that r follows a normal distribution with mean $\mu + \sigma\sqrt{q}z$ and variance $\sigma^2(1-\rho)$, model parameters can be obtained by maximizing the log-likelihood of the following probability density function:

$$f_{\mu,\sigma,q}(x) = \frac{1}{\sqrt{2\pi\sigma^2(1-q)}} e^{-\frac{(x-\mu-\sigma\sqrt{q})^2}{2\sigma^2(1-q)}}$$

Simulating the Distribution of the Parameters

Assuming specific inputs of the data-generating process, this simulation aims to derive the distributions of the parameters of the Vasicek LGD model. As the Vasicek LGD model shares the systemic factor with the Vasicek PD model, the simulation process follows these steps:

1. For the selected values of p_{sim} and ρ_{sim}, simulate T observations of the default rate from the Vasicek PD distribution (for details, see Slide 2 of the document)
2. Given the simulated default rates, calculate the parameter p_{obs} as the average of the simulated observations.
3. Given the simulated default rates and p_{obs} from steps 1 and 2, estimate the asset correlation (ρ_{obs}) using the Indirect Moment Matching method (for details, see Slide 6 of the document)
4. Based on p_{obs} and ρ_{obs}, derive the values of the systemic factor z_{obs}.

Simulating the Distribution of the Parameters cont.

5. For the selected parameters μ_{sim}, σ_{sim}, q_{sim}, and observed values of the systemic factor z_{obs}, simulate the recovery rates from the Vasicek LGD model.
6. Given the simulated recoveries, use the maximum likelihood method to estimate the parameters μ_{obs}, σ_{obs}, and q_{obs}.
7. Repeat steps 2 to 6 N times.
8. Collect the values of the estimated parameters (N), calculate their expected values, and plot their distribution.

For the described simulation design, the following data-generating inputs were selected: $p_{sim} = 0.05$, $\rho_{sim} = 0.10$, $\mu_{sim} = 0.60$, $\sigma_{sim} = 0.10$, $q_{sim} = 0.10$, $T = [10, 15, 20, 25, 30, 100]$, and $N = 10,000$.

Simulation Study Results

The following table presents the expected values of the simulated parameters:

T	mu	sigma	q
10	0.5994	0.0942	0.1451
15	0.5998	0.0966	0.1381
20	0.5997	0.0977	0.1337
25	0.6001	0.0985	0.1261
30	0.6002	0.0989	0.1237
100	0.6002	0.0996	0.1081

Simulation Study Results cont.