The Logistic Vasicek Distribution

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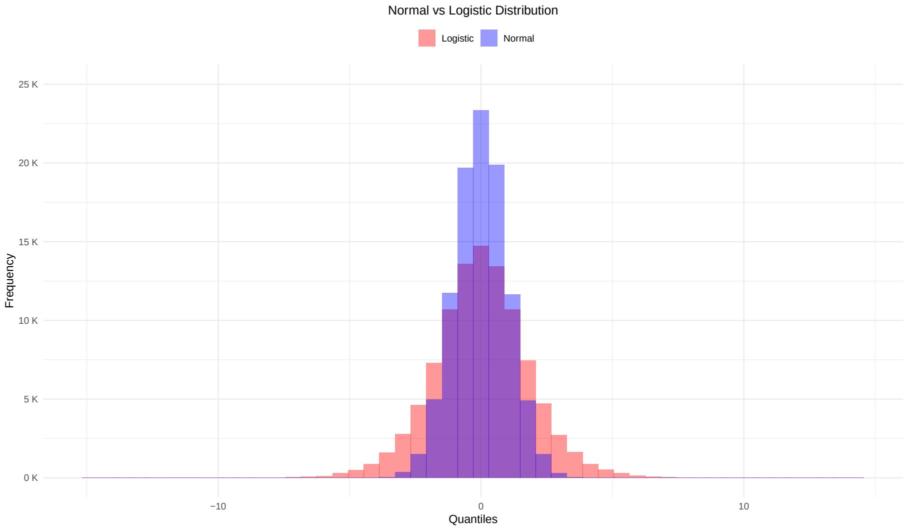
Adjusting the Vasicek Model Using the Logistic Distribution

- Witzany suggests replacing the standard normal distribution function with the logistic distribution in the Vasicek model, which could be a natural approach for calculating regulatory capital requirements, aligning with the typical use of logistic regression in Probability of Default (PD) modeling.
- In addition to calculating regulatory capital requirements, the Logistic Vasicek model could be explored for other applications, such as developing IFRS9 ECL challenger models or measuring concentration risk. These areas also often rely heavily on the Vasicek model.
- Another interesting area of investigation would be to examine the bias in estimating the asset correlation parameter under normal and logistic distributions in the Vasicek model for smaller sample sizes.

Normal vs Logistic Distribution



The Logistic Vasicek Distribution

The Logistic Vasicek distribution is derived similarly to the classical Vasicek distribution from the asset return formula:

$$y_i = \sqrt{\rho} z + \sqrt{1-\rho}\epsilon_i$$

with the key difference being that the systemic (z) and idiosyncratic (ϵ) factors are assumed to come from a logistic distribution rather than a normal distribution.

A technical disadvantage of this new assumption is that the sum of two independent logistically distributed variables is not necessarily another logistically distributed variable. However, this approximation can be considered valid for asset correlation (ρ) values up to 30%. For a more detailed comparison, refer to slide 6.

The Logistic Vasicek Distribution cont.

The Logistic Vasicek distribution is still a two-parameter ($0 < p < 1$ and $0 < \rho < 1$) continuous distribution on the range 0 to 1. If a variable x has a Logistic Vasicek distribution, then x can be represented as:

$$x = \lambda\left(\frac{\lambda^{-1}(p) - \sqrt{\rho}z}{\sqrt{1-\rho}}\right)$$

where:

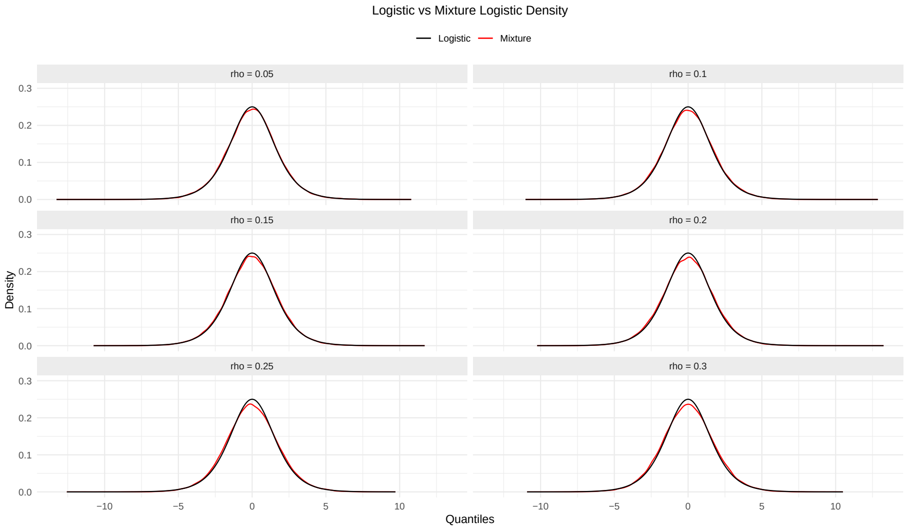
- p and ρ are the parameters of the distribution, commonly referred to as the average default rate and asset correlation, respectively;
- z represents the systemic factor drawn from the logistic distribution with mean 0 and standard deviation of 1; and
- λ and λ^{-1} denote the distribution and quantile function of the logistic distribution with mean 0 and standard deviation of 1, respectively.

The cumulative distribution function is defined as follows:

$$F_{p,\rho}(x) = \lambda\left(\frac{\sqrt{1-\rho}\,\lambda^{-1}(x) - \lambda^{-1}(p)}{\sqrt{\rho}}\right)$$

where the probability density function is calculated as the first derivative of the aforementioned cumulative probability function.

Logistic vs Mixture-Logistic Distribution



Normal vs Logistic Cumulative Distribution of the Vasicek-Distributed Variable