Loss Given Default as a Function of the Default Rate

The Frye-Jacobs Loss Given Default Model

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The Frye-Jacobs LGD - Assumptions and the Functional Form

- A study by Frye and Jacobs predicts Loss Given Default (LGD) as a function of the default rate (DR).
- The LGD function relates to asymptotic DR or Point-in-Time Probability of Default (PiT PD) and Point-in-Time (PiT) LGD under certain assumptions.
- Main assumptions of the LGD function:
    - the Expected Loss (EL) and the DR (PiT PD) are co-monotonic;
    - EL and DR (PiT PD) follow a Vasicek two-parameter distribution;
    - correlation parameter (ρ) is shared between DR and EL.

The Frye-Jacobs LGD - Assumptions and the Functional Form cont.

The Frye-Jacobs LGD function has the following form:

$$cLGD = \frac{\Phi\left[\Phi^{-1}(cDR) - k\right]}{cDR}$$

with k being the Risk Index given by the following expression:

$$k = \frac{\Phi^{-1}(PD) - \Phi^{-1}(EL)}{\sqrt{1-\rho}}$$

where:

- cDR denotes conditional default rate (PiT PD);
- PD represents TTC PD;
- EL is the average loss rate; and
- ρ represents the asset correlation shared between DR and EL.

Simulation Setup

The simulation aims to generate competing predictions for the tail LGD (98^{th} percentile) utilizing the LGD function and linear regression as predictors. To give linear regression an advantage in this contest, the data-generating process is assumed to follow a linear model with known values of the intercept (a) and slope (b).
For the following assumed values: $TTC\ DR = 0.03$, $\rho = 0.10$, $\sigma^2 = 0.20$, $a = 0.50$, $b = 2.3$, and number of obligors $n = 1,000$, the simulation of a single year of data involves the following steps:

1. Simulate the value of the systemic factor z by drawing a random number from the standard normal distribution.
2. Given the $TTC\ DR$, ρ, and z from step 1, simulate the $PiT\ PD$ based on the Vasicek distribution.
3. Simulate the number of defaults D from the binomial distribution for a given n and simulated $PiT\ PD$.
4. Simulate conditional LGD $cLGD$ assuming the linear model of the form $a + b \cdot PiT\ PD$.
5. Given the $cLGD$, simulate LGD by drawing the value from the normal distribution with the mean $cLGD$ and standard deviation $\frac{\sigma}{\sqrt{D}}$.

The above process is repeated ten times, simulating a dataset for $T = 10$ years.

Simulation Setup cont.

The process described in the previous slide is simulated $N = 10,000$ times.
The LGD function and LGD linear regression model are estimated for each simulated dataset.
The elements of the LGD function are calculated as follows: parameter ρ is estimated by maximizing the log-likelihood of the Vasicek probability density function, $TTC\ DR$ is calculated as an average of the simulated T default rates, and the expected loss (EL) is calculated as an average of the product of the LGD and $PiT\ PD$.
The LGD of the linear model is calculated based on the selected values of coefficients a and b and the $TTC\ DR$.
Deriving the LGD function and LGD linear regression distributions based on the $N = 10,000$ repetitions, the final step is to compare the root mean square error of these two distributions calculated against the known 98^{th} percentile of the LGD given by the following formula:

$$LGD_{98^{th} percentile} = a + b \cdot PD_{98^{th} percentile}$$

where $PD_{98^{th} percentile}$ is defined as:

$$PD_{98^{th} percentile} = \Phi\left[\frac{\Phi^{-1}(TTC\ PD) + \sqrt{\rho} \cdot \Phi^{-1}(98^{th} percentile)}{\sqrt{1-\rho}}\right]$$

Simulation Results