The Model-Based Heterogeneity Testing

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Probability of Default Rating Scale

- Practitioners usually adhere to certain principles when building the Probability of Default (PD) rating scale.
- These principles lead to specific rating scale characteristics, with some being compulsory, while others may vary from one model to another and are often considered desirable but not mandatory.
- Among the compulsory characteristics of the rating scale, practitioners usually consider monotonicity and heterogeneity.

Heterogeneity Testing

- A typical way to test for heterogeneity of the rating scale is the test of two proportions on adjacent grades.
- Model-based testing offers an alternative method for conducting heterogeneity testing.
- This type of testing is based on logistic regression and the so-called nested dummy encoding of the rating grades.
- The main advantage of this type of testing is that it produces estimates that align with the most commonly used method for estimating PD models: logistic regression.
- The following slides explain the concept of nested dummy encodings, their interpretation within logistic regression, and the comparison of results between the standard heterogeneity testing method and model-based methods.

Nested Dummy Encoding in Credit Risk Modeling

- Connecting adjacent categories makes this encoding method particularly attractive in credit risk modeling.
- It is especially appealing when practitioners choose to bin numeric risk factors and seek statistically significant risk profiles between adjacent categories.
- Extending the above application, this method demonstrates its adaptability, providing opportunities to assess diverse binning methods in standard multivariate analyses.
- Another use of this method is in model-based hypothesis testing of the heterogeneity of the rating scale.

Constructing Nested Dummies

Assign a value of 0 to all categories below the chosen one and 1 to the selected category and those positioned above it.

Example

```
categories = c("A", "A", "B", "B", "C", "C", "C", "D", "D", "D")
```

```
nested_dummies
```

```
##    category_A_vs_BCD category_AB_vs_CD category_ABC_vs_D
## 1                  0                 0                 0
## 2                  0                 0                 0
## 3                  1                 0                 0
## 4                  1                 0                 0
## 5                  1                 1                 0
## 6                  1                 1                 0
## 7                  1                 1                 0
## 8                  1                 1                 1
## 9                  1                 1                 1
## 10                 1                 1                 1
```

Interpreting Nested Dummies

Binomial logistic regression with nested dummies

```
##
## Call:  glm(formula = y ~ category_A_vs_BCD + category_AB_vs_CD + category_ABC_vs_D,
##     family = "binomial", data = db)
##
## Coefficients:
##       (Intercept)  category_A_vs_BCD  category_AB_vs_CD  category_ABC_vs_D
##           -1.8554             0.2557            -0.2901             0.3589
##
## Degrees of Freedom: 999 Total (i.e. Null);  996 Residual
## Null Deviance:       855.8
## Residual Deviance: 852.6     AIC: 860.6
```

Log-odds of traget average per categorical variable

```
##         A         B         C         D
## -1.855351 -1.599634 -1.889740 -1.530876
```

Coefficient replicates

```
"(Intercept)" = lo(average["A"])
"category_A_vs_BCD" = lo(average["B"]) - lo(average["A"])
"category_AB_vs_CD" = lo(average["C"]) - lo(average["B"])
"category_ABC_vs_D" = lo(average["D"]) - lo(average["C"])
```

```
##       (Intercept) category_A_vs_BCD category_AB_vs_CD category_ABC_vs_D
##        -1.8553506         0.2557167        -0.2901060         0.3588643
```

Comparison of Two Approaches

Test of Two Proportions ($H_1 : DR_{rating_{i+1}} > DR_{rating_i}$)

```
##          rating  no  nb     dr X.squared.stat  p.val
## 1 01 (-Inf,7)  82   9 0.1098             NA     NA
## 2   02 [7,16) 349  80 0.2292         5.7839 0.0081
## 3  03 [16,26) 339 109 0.3215         7.3540 0.0033
## 4  04 [26,33)  57  19 0.3333         0.0311 0.4301
## 5  05 [33,47) 108  47 0.4352         1.6127 0.1021
## 6 06 [47,Inf)  65  36 0.5538         2.2892 0.0651
```

Model-Based Testing ($H_1 : DR_{rating_{i+1}} \neq DR_{rating_i}$)

```
##                Estimate Std. Error z value Pr(>|z|)
## (Intercept)     -2.0932     0.3533 -5.9252   0.0000
## `02 [7,16)`      0.8806     0.3755  2.3448   0.0190
## `03 [16,26)`     0.4660     0.1725  2.7019   0.0069
## `04 [26,33)`     0.0536     0.3041  0.1762   0.8601
## `05 [33,47)`     0.4324     0.3415  1.2663   0.2054
## `06 [47,Inf)`    0.4769     0.3161  1.5088   0.1314
```