The Vasicek Loss Given Default Model

The Bias Quantification of the Sensitivity Parameter

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The Functional Form and Estimation Method of the Parameters

The Vasicek Loss Given Default (LGD) model is derived from the recovery equation. The recovery equation, resembling the asset return, is defined as:

$$r = \mu + \sigma\sqrt{q}z + \sigma\sqrt{(1-q)}\epsilon$$

where:

- μ is a quantity parameter (similar to parameter p from the PD model);
- σ represents the quality parameter;
- q is the sensitivity parameter (similar to parameter ρ from the PD model);
- z represents the systemic factor derived from the Vasicek PD model;
- ϵ denotes the idiosyncratic factor, independent of z, which follows a standard normal distribution.

Given that r follows a normal distribution with mean $\mu + \sigma\sqrt{q}z$ and variance $\sigma^2(1-\rho)$, model parameters can be obtained by maximizing the log-likelihood of the following probability density function:

$$f_{\mu,\sigma,q}(x) = \frac{1}{\sqrt{2\pi\sigma^2(1-q)}} e^{-\frac{(x-\mu-\sigma\sqrt{q})^2}{2\sigma^2(1-q)}}$$

Bias Quantification Process

The following steps outline the bias quantification process using Monte Carlo simulations:

1. Collect default and recovery rate data.
2. Select the parameters estimation method for the Vasicek PD model.
3. Based on the collected default rates and the selected method, estimate the parameters $p_{observed}$ and $\rho_{observed}$.
4. Given the values from the step 3, calculate the systemic factor $z_{observed}$
5. Based on the collected recovery data and calculated values of $z_{observed}$, estimate the parameters of the Vasicek LGD model.
6. Based on the N Monte Carlo simulations, optimize the data-generating process for q_{true} by minimizing the difference between the average value of the q_{true} distribution and the $q_{observed}$ given that the true values of parameters $\mu_{observed}$ and $\sigma_{observed}$ are equal to the estimated ones.

Note that the presented process assumes that the parameter q of the Vasicek LGD model estimated based on the observed data represent the average value of the true parameters' distribution.

Simulation Setup and Results

The following table presents the simulation inputs of the hypothetical portfolio:

```
##     T     mu  sigma q_observed
##    10 0.5994 0.0942     0.1451
##    15 0.5998 0.0966     0.1381
##    20 0.5997 0.0977     0.1337
##    25 0.6001 0.0985     0.1261
##    30 0.6002 0.0989     0.1237
##   100 0.6002 0.0996     0.1081
```

Note that the above values are directly obtained from the Vasicek models data-generating process with the value of $p = 0.05$, $\rho = 0.10$, and $q = 0.10$, while the remaining parameters μ and σ are given in the above hypothetical portfolio. Therefore, a $q_{bias\_corrected}$ closer to 0.10 indicates a better bias-corrected estimation.

After running the $(N = 10,000)$ Monte Carlo simulations for bias quantification of the asset correlation parameter, the following additional values have been obtained:

```
##     T     mu  sigma q_observed q_bias_corrected q_true
##    10 0.5994 0.0942     0.1451           0.0761    0.1
##    15 0.5998 0.0966     0.1381           0.0846    0.1
##    20 0.5997 0.0977     0.1337           0.0906    0.1
##    25 0.6001 0.0985     0.1261           0.0933    0.1
##    30 0.6002 0.0989     0.1237           0.0962    0.1
##   100 0.6002 0.0996     0.1081           0.1012    0.1
```

Simulation Setup and Results cont.