The Binomial Tests for PD Model Validation

The Independent and Correlated Binomial Distributions

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The Binomial Test for PD Model Validation

- The binomial test is a frequently employed method for validating Probability of Default (PD) estimates reported for each rating category of internal rating systems.
- The binomial test works under the null hypothesis that the PD of a rating is not underestimated.
- The most common implementation of the binomial test relies on the assumption that default events in the rating category under consideration are independent.
- How can practitioners adjust the standard binomial test to account for correlated defaults?
- How do the results of independent and correlated binomial tests compare?

The Binomial Test - Independent Defaults

The most commonly used version of the binomial test relies on the assumption that default events in the rating category under consideration are independent.

Given a confidence level q, the null hypothesis is rejected if the number of defaulters k in this rating category is greater than or equal to a critical value $k*$, which is defined as:

$$k^* = min\left\{ k \mid \sum_{i=k}^{n} \binom{n}{i} PD^i (1 - PD)^{n-i} \leq 1 - q \right\}$$

where:

- n is the number of debtors;
- PD is the probability of default in the rating category.

The Binomial Test - Correlated Defaults

Applying the binomial test under the assumption of correlated defaults complicates the mathematical framework.

Two methods are presented to analyze the impact of default correlation on the critical value of the number of defaulters.

The first method involves exact numerical calculation through numerical integration:

$$P(D_n \leq k) = \int_{-\infty}^{\infty} \sum_{i=0}^{k} \binom{n}{i} \left[\Phi\left(\frac{\Phi^{-1}(PD) - \sqrt{\rho}\, x}{\sqrt{1-\rho}} \right) \right]^{i} \left[1 - \Phi\left(\frac{\Phi^{-1}(PD) - \sqrt{\rho}\, x}{\sqrt{1-\rho}} \right) \right]^{n-i} \phi(x)\, dx$$

The Binomial Test - Correlated Defaults cont.

The second method is an analytical approximation defined as follows:

$$P(D_n \leq k) = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\binom{k}{n} - PD}{\sqrt{\rho}}\right)$$

where:

- D_n is the observed number of defaults;
- n is the total number of debtors in the rating category;
- PD is the probability of default in the rating category;
- ρ represents the asset correlation;
- $\phi(x)$ denotes the standard normal density;
- Φ denotes the distribution of the standard normal distribution.

Simulation Results

Correlated Defaults

Example 1:

pd	n	rho	q	exact k	appox. k
0.01	1000	0.00	0.99	19	11
0.01	1000	0.05	0.99	35	32
0.01	1000	0.10	0.99	49	47
0.01	1000	0.15	0.99	63	62
0.01	1000	0.20	0.99	77	76

Example 2:

pd	n	rho	q	exact k	appox. k
0.05	1000	0.00	0.99	68	51
0.05	1000	0.05	0.99	128	125
0.05	1000	0.10	0.99	172	169
0.05	1000	0.15	0.99	212	210
0.05	1000	0.20	0.99	252	250

Independent Defaults

pd	n	q	k
0.01	1000	0.99	19
0.05	1000	0.99	68

Simulation Results cont.