The Vasicek Loss Given Default Model

The Functional Form and Parameters Estimation Method

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The Vasicek Probability of Default Model

The Vasicek Probability of Default (PD) model is derived from the asset return equation represented as:

$$y = \sqrt{\rho} z + \sqrt{1-\rho}\epsilon$$

where:

- ρ is asset correlation;
- z represents the systemic factor which follows a standard normal distribution
- ϵ denotes the idiosyncratic factor which follows a standard normal distribution.

Starting from the above equation, the Vasicek PD model is defined by two parameters, p and ρ, commonly referred to as the average default rate and asset correlation, respectively.

The parameter p is usually estimated as an average of the observed default rates, while the parameter ρ can be estimated by maximizing the log-likelihood of the Vasicek probability density function:

$$f_{p,\rho}(x) = \sqrt{\frac{1-\rho}{\rho}} e^{\frac{1}{2}\left(\phi^{-1}(x)^2 - \left(\frac{\sqrt{1-\rho}\phi^{-1}(x) - \phi^{-1}(p)}{\sqrt{\rho}}\right)^2\right)}$$

Details on the Vasicek (distribution) PD model and different parameter estimation methods are available at the following link.

The Vasicek Loss Given Default Model

The Vasicek Loss Given Default (LGD) model is derived from the recovery equation. The recovery equation looks similar to the asset return and is defined as:

$$r = \mu + \sigma\sqrt{q}z + \sigma\sqrt{(1-\rho)}\epsilon$$

where:

- μ is defined as a quantity parameter (similar to the parameter p from the PD model);
- σ represents the quality parameter;
- q is the sensitivity parameter (similar to the parameter ρ from the PD model);
- z represents the systemic factor derived from the PD model;
- ϵ denotes the idiosyncratic factor independent from z which follows a standard normal distribution.

Model parameters can be obtained by maximizing the log-likelihood of the following probability density function:

$$f_{\mu,\sigma,q}(x) = \frac{1}{\sqrt{2\pi\sigma^2(1-q)}} e^{-\frac{(x-\mu-\sigma\sqrt{q})^2}{2\sigma^2(1-q)}}$$

For more information about the Vasicek LGD model, check the following paper.

Simulation Study

The dataset below is utilized solely for simulation purposes, whereas in real-world scenarios, the development of recovery rates usually tends to exhibit less volatility.

Simulation Dataset:

```
##    T     dr     rr    lgd
##    1 0.1024 0.7433 0.2567
##    2 0.1130 0.1398 0.8602
##    3 0.0546 0.2127 0.7873
##    4 0.0295 0.8631 0.1369
##    5 0.0508 0.1141 0.8859
##    6 0.0236 0.8811 0.1189
##    7 0.0350 0.6007 0.3993
##    8 0.0285 0.9723 0.0277
##    9 0.0118 0.9432 0.0568
##   10 0.0260 0.8167 0.1833
##   11 0.0298 0.2798 0.7202
##   12 0.0251 0.3129 0.6871
##   13 0.0802 0.6168 0.3832
##   14 0.0124 0.7047 0.2953
##   15 0.0272 0.6373 0.3627
##   16 0.0493 0.3651 0.6349
##   17 0.0301 0.7007 0.2993
##   18 0.1204 0.5110 0.4890
##   19 0.0093 0.8323 0.1677
##   20 0.0291 0.6406 0.3594
```

The Vasicek PD Model Parameters:

```
##        p    rho
##   0.0444 0.0965
```

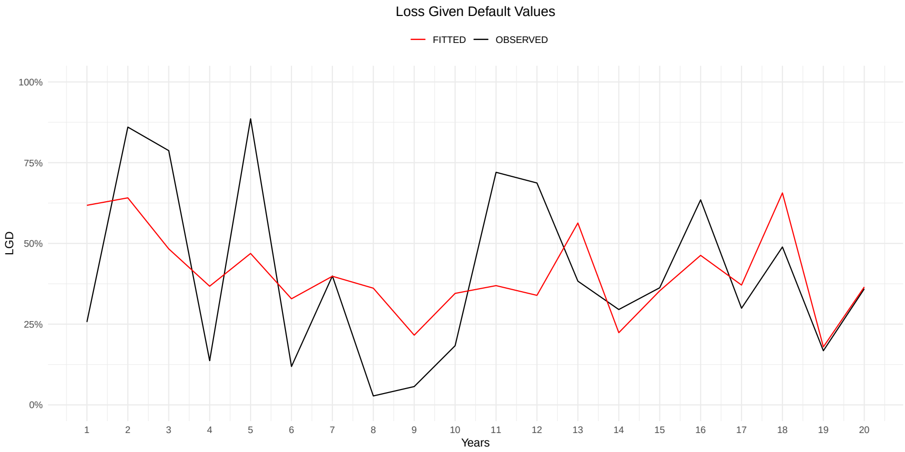
z:

```
##        1       2       3       4       5
## -1.5984 -1.7737 -0.5767  0.2998 -0.4686
##        6       7       8       9      10
##   0.5946  0.0663  0.3461  1.4488  0.4680
##       11      12      13      14      15
##   0.2862  0.5143 -1.1830  1.3904  0.4083
##       16      17      18      19      20
## -0.4241  0.2727 -1.8892  1.7238  0.3182
```

The Vasicek Recovery Model Parameters:

```
##     mu  sigma      q
## 0.5929 0.2644 0.2492
```

Simulation Study cont.



Simulation Study cont.

The following graph demonstrates the relationship between the *LGD* and *PiT PD* values under the following assumptions: $p = 0.05$, $\rho = 0.10$, $\mu = 0.60$, $\sigma = 0.30$, and $q = 0.15$.