Power Play: Probability of Default Predictive Ability Testing

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Probability of Default Predictive Ability Tests

Three of the most commonly used tests:

- Exact binomial
- Z-score test
- Jeffreys test.

Testing the hypothesis that the calibrated probability of default (PD) is not underestimated.

Exact binomial

$$Pr(X \geq d) = \binom{n}{d} PD^d (1 - PD)^{n-d}$$

where:

- n is the number of observations
- d is the number of defaults
- PD is the calibrated PD.

Z-score test:

$$Z_{score} = \frac{ODR - PD}{\sqrt{\frac{PD(1-PD)}{n}}}$$

where:

- ODR is the observed default rate
- PD is the calibrated PD
- n is the number of observations.

Under the assumption that the Z_{score} test statistic follows the standard normal distribution, a $p - value$ is calculated accordingly.

Jeffreys test

$$Pr(X \leq d) = \beta(q = PD, shape1 = d + 0.5, shape2 = n - d + 0.5)$$

where:

- β is the beta distribution
- PD is the calibrated PD
- d is the number of defaults
- n is the number of observations.

What if. . . ?

What if the tests lead to the opposite conclusion?

Example:

```
#number of defaults
nb = 15
#number of observations
no = 99
#calibrated pd
pdc = 0.09656014
#significance level
alpha = 0.05
```

Testing results:

```
##   Exact binomial Z-score test Jeffreys test
## 1          5.30%        3.21%         3.87%
```

Monte Carlo Simulations for Statistical Power

It helps identify which test has higher statistical power.

Steps:

1. The observed default rate is the true calibrated PD (`ODR = nb / no`)
2. Simulate random numbers from the binomial distribution with parameters `ODR` and `no`
3. Calculate the simulated `ODR` and `nb`
4. Collect the results of the tests
5. Repeat steps 2 to 4 `N` times
6. Calculate the average number of simulations for which tests reject the null hypothesis for an `alpha` significance level (a higher average indicates greater power)

Simulation results:

```
##   Exact binomial Z-score test Jeffreys test
## 1         43.10%       54.17%        54.17%
```