Hosmer-Lemeshow VS Z-score Test on Portfolio Level

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Probability of Default Predictive Power Tests

- Divided into two groups:
  1. tests applicable on the rating scale level
  2. tests applicable on the rating grade or portfolio level.
- Sometimes, practitioners compare the p-value of a test at the rating scale level with the p-value of a test at the portfolio level. However, are they comparable?
- What is the testing hypothesis for each group and test?

Hosmer-Lemeshow Test - Rating Scale Level

$$HL = \sum_{g=1}^{G} \frac{(N_g PD_g - d_g)^2}{N_g PD_g (1 - PD_g)}$$

where:

- G is the number of rating grades
- N_g is the number of observations in the rating grade g
- PD_g is the calibrated PD for the rating grade g
- d_g is the number of observed defaults in the rating grade g.

Under the assumption that the HL test statistic follows the chi-square distribution with G degrees of freedom, a $p - value$ is calculated accordingly.

Testing hypothesis - the calibrated PD is true.

Z-score Test - Rating Grade or Portfolio Level

$$Z_{score} = \frac{ODR - PD}{\sqrt{\frac{PD(1-PD)}{n}}}$$

where:

- ODR is the observed default rate
- PD is the calibrated PD
- n is the number of observations.

Under the assumption that the Z_{score} test statistic follows the standard normal distribution, a $p - value$ is calculated accordingly.

The most commonly used testing hypothesis - the calibrated PD is not underestimated.

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Are they comparable?

Example:

	Rating Grade	# observations	# defaults	ODR	PD
1	RG1	47	3	0.0638	0.0307
2	RG2	95	20	0.2105	0.1161
3	RG3	68	17	0.2500	0.2907
4	RG4	53	24	0.4528	0.5514
5	RG5	37	28	0.7568	0.7648

p-values:

	Hosmer-Lemeshow test	Z-score test on portfolio level
1	2.70%	38.89%

Hosmer-Lemeshow VS Z-score on Portfolio Level cont.

Can we adjust inputs to account only for the underestimation?

Example:

	Rating Grade	# observations	# defaults	ODR	PD	PD corrected
1	RG1	47	3	0.0638	0.0307	0.0307
2	RG2	95	20	0.2105	0.1161	0.1161
3	RG3	68	17	0.2500	0.2907	0.2500
4	RG4	53	24	0.4528	0.5514	0.4528
5	RG5	37	28	0.7568	0.7648	0.7568

p-values:

	Hosmer-Lemeshow test	Z-score test on portfolio level
1	7.53%	8.58%