Measuring Concentration Risk - A Partial Portfolio Approach

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Concentration Risk

- Concentration risk represents a significant source of risk in banking, particularly in emerging and small economies.
- Pillar 1 capital requirements don't cover concentration risk.
- Banks must autonomously estimate and set aside capital buffers to mitigate concentration risk, usually as part of Pillar 2 models.
- Inadequate recognition of concentration risk can lead to insufficient capital levels, even with apparently high capital ratios.
- A partial portfolio approach as a way to address the concentration risk and to complement the existing regulatory capital requirements (details available here).

A Partial Portfolio Approach (PPA)

Assuming a portfolio of `n` exposures with a non-granular sub-portfolio of `m` exposures (`m < n`), the PPA can be outlined in the following steps after determining the number of simulations (`B`):

1. Simulate a value for the systemic risk factor `z` from the standard normal distribution (`N[0, 1]`).
2. For each exposure in the non-granular sub-portfolio, simulate the idiosyncratic factor ϵ from the standard normal distribution (`N[0, 1]`).
3. Given the simulated systemic, idiosyncratic factor and calculated asset correlation, for each exposure in the non-granular sub-portfolio, simulate the asset return value as:

   $$y_i = \sqrt{\rho} * z + \sqrt{1 - \rho} * \epsilon_i$$

   where ρ is asset correlation, z and $epsilon_i$ systemic and idiosyncratic factor, respectively.
4. For each exposure in the non-granular sub-portfolio, simulate the default indicator as follows:

   $$I_i = y_i < N^{-1}(\overline{PD_i})$$

   where y_i is the asset return (step 3), $N^{-1}(\overline{PD_i})$ is the quantile of the standard normal variable, and $\overline{PD_i}$ is unconditional Probability of Default for exposure i.

A Partial Portfolio Approach (PPA) cont.

5. Calculate the loss for the non-granular sub-portfolio as

$$Loss^{non-granular} = \sum_{i=1}^{m} I_i * LGD_i * EAD_i$$

   where I_i is the indicator from the step 4, and LGD_i and EAD_i Loss Given Default and Exposure at Default, respectively.
6. Calculate the loss for the granular sub-portfolio as:

$$Loss^{granular} = \sum_{i=1}^{n-m} N\left[\frac{N^{-1}(\overline{PD_i}) - \rho * z}{\sqrt{1-\rho}}\right] * LGD_i * EAD_i$$

   where N and N^{-1} present standard normal cumulative distribution and quantile function.
7. Sum up losses from the non-granular and granular portfolio.
8. After repeating steps from 1 to 7 selected B times, calculate the 99.9 percentile of the loss distribution and subtract the expected loss to get the required Credit VaR.

Simulation Setup

Simulation dataset available here.

R:

```r
#lgd
lgd <- 0.30
#asset correlation
rho <- 0.05
#ead threshold
seq.ft <- c(seq(from = 0,
                to = 1.5,
                length.out = 100),
            10) / 100
#number of simulations
B <- 100000
#confidence level
cl <- 0.999
```

Python:

```python
#lgd
lgd = 0.30
#asset correlation
rho = 0.05
#ead threshold
seq_ft = np.concatenate([np.linspace(start = 0,
                                     stop = 1.5,
                                     num = 100),
                         [10]]) / 100
#number of simulations
B = 100000
#confidence level
cl = 0.999
```

R Code Extract

```
...

for   (i in 1:B) {
      #set random seed
      set.seed(i)
      #systemic factor
      z <- rnorm(n = 1)
      #idiosyncratic factor
      epsilon <- rnorm(n = nrow(db.ng))
      #asset return
      ar <- sqrt(rho)*z + sqrt(1 - rho)*epsilon
      #default indicator (non-granular portfolio)
      def.ind <- ifelse(ar < qnorm(p = db.ng$pd), 1, 0)
      #loss of the non-granular portfolio
      loss.ng <- sum(def.ind * db.ng$ead * lgd)
      #loss of the granular portfolio
      pd.cond <- pnorm(q = (qnorm(p = db.g$pd) - sqrt(rho)*z) / sqrt(1 - rho))
      loss.g <- sum(db.g$ead * pd.cond * lgd)
      #portfolio loss
      res[i] <- loss.ng + loss.g
      }

...
```

Python Code Extract

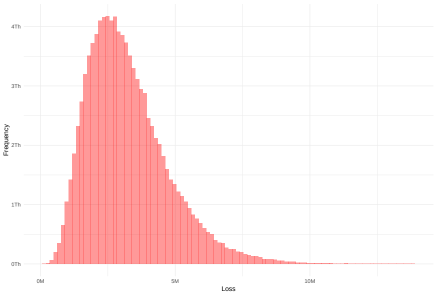
```
import numpy as np
from scipy.stats import norm
...

for i in range(B):
    #set random seed
    np.random.seed(i + 1)
    #systemic factor
    z = np.random.normal(size = 1)
    #idiosyncratic factor
    epsilon = np.random.normal(size = db_ng.shape[0])
    #asset return
    ar = np.sqrt(rho) * z + np.sqrt(1 - rho) * epsilon
    #default indicator (non-granular portfolio)
    def_ind = np.where(ar <  norm.ppf(db_ng["pd"]), 1, 0)
    #loss of the non-granular portfolio
    loss_ng = np.sum(def_ind * db_ng["ead"] * lgd)
    #loss of the granular portfolio
    pd_cond = norm.cdf((norm.ppf(db_g["pd"]) - np.sqrt(rho) * z) /
                        np.sqrt(1 - rho))
    loss_g = np.sum(db_g["ead"] * pd_cond * lgd)
    #portfolio loss
    res[i] = loss_ng + loss_g

...
```

Simulation Result Extract

Loss Distribution - Full Non-Granular Portfolio:



CVaR - Varying EAD Threshold (99.9% CL):