Asset Correlation Estimation in the Vasicek Model

Maximum Likelihood: Normal vs Logistic Vasicek Distribution

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The Vasicek Distribution

If a variable x has a Normal or Logistic Vasicek distribution, then x can be represented as:

$$x = F\left(\frac{F^{-1}(p) - \sqrt{\rho}z}{\sqrt{1-\rho}}\right)$$

where:

- p and ρ are the parameters of the distribution, commonly referred to as the average default rate and asset correlation, respectively;
- z represents the systemic factor drawn from the standard normal or logistic distribution; and
- F and F^{-1} denote the distribution and quantile function of the standard normal (ϕ and ϕ^{-1}) or logistic (λ and λ^{-1}) distribution, respectively.

Further, the cumulative distribution function can be defined as follows:

$$P(X \leq x) = F\left(\frac{\sqrt{1-\rho}\, F^{-1}(x) - F^{-1}(p)}{\sqrt{\rho}}\right)$$

The Asset Correlation Estimator

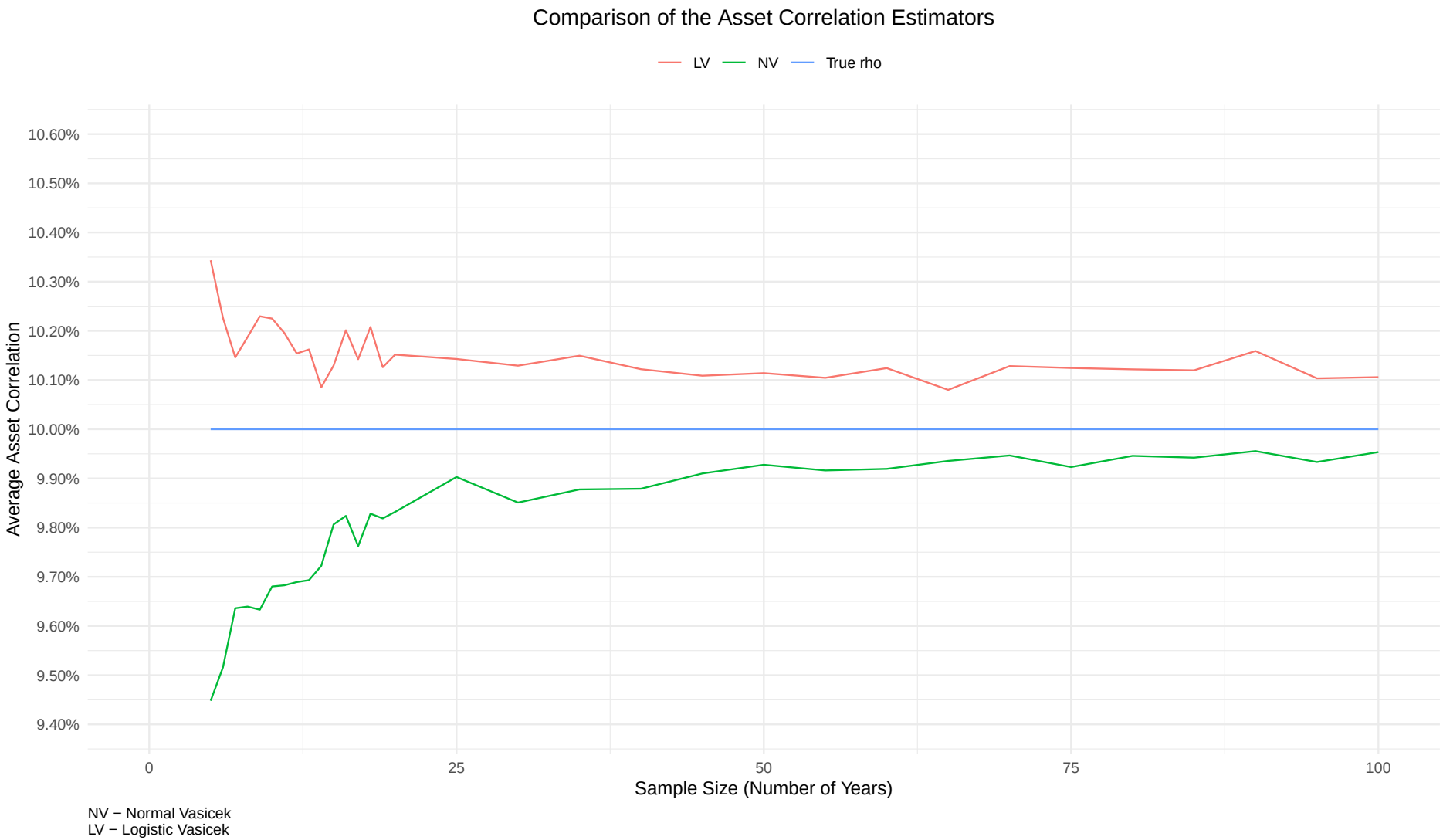
- One of the most commonly used estimation methods for fitting the parameters of the Vasicek distribution to observed data is to maximize the log-likelihood of the probability density function.
- Using the cumulative distribution function presented in slide 2, practitioners can initially derive the probability density function as a derivative of the corresponding cumulative function. Subsequently, they can estimate the asset correlation by maximizing the probability density function's log-likelihood.
- The following slides will utilize this estimation method to compare the bias of the asset correlation estimator between the Normal and Logistic Vasicek models. For both models, the parameter p is estimated as a simple average of the observed default rates.

Simulation Setup

1. Select the inputs for simulating random data from the Vasicek distribution: sample size ($T = 5$), unconditional PD ($pd = 0.05$), asset correlation ($\rho = 0.10$), time-dependent systemic factor (z) from the standard normal and logistic distribution with an autoregression coefficient ($\theta = 0.30$).
2. Using the simulated data, estimate the asset correlation by maximizing the Log-Likelihood of the Vasicek Probability Density Function, considering the different underlying distributions of the Vasicek model (normal and logistic).
3. Store the results and repeat step 2 a total of N times ($N = 10,000$).
4. Calculate the average asset correlation for each estimation model, then compare these averages to the true ρ.
5. Change the sample size T from step 1, and repeat steps 2 through 4.

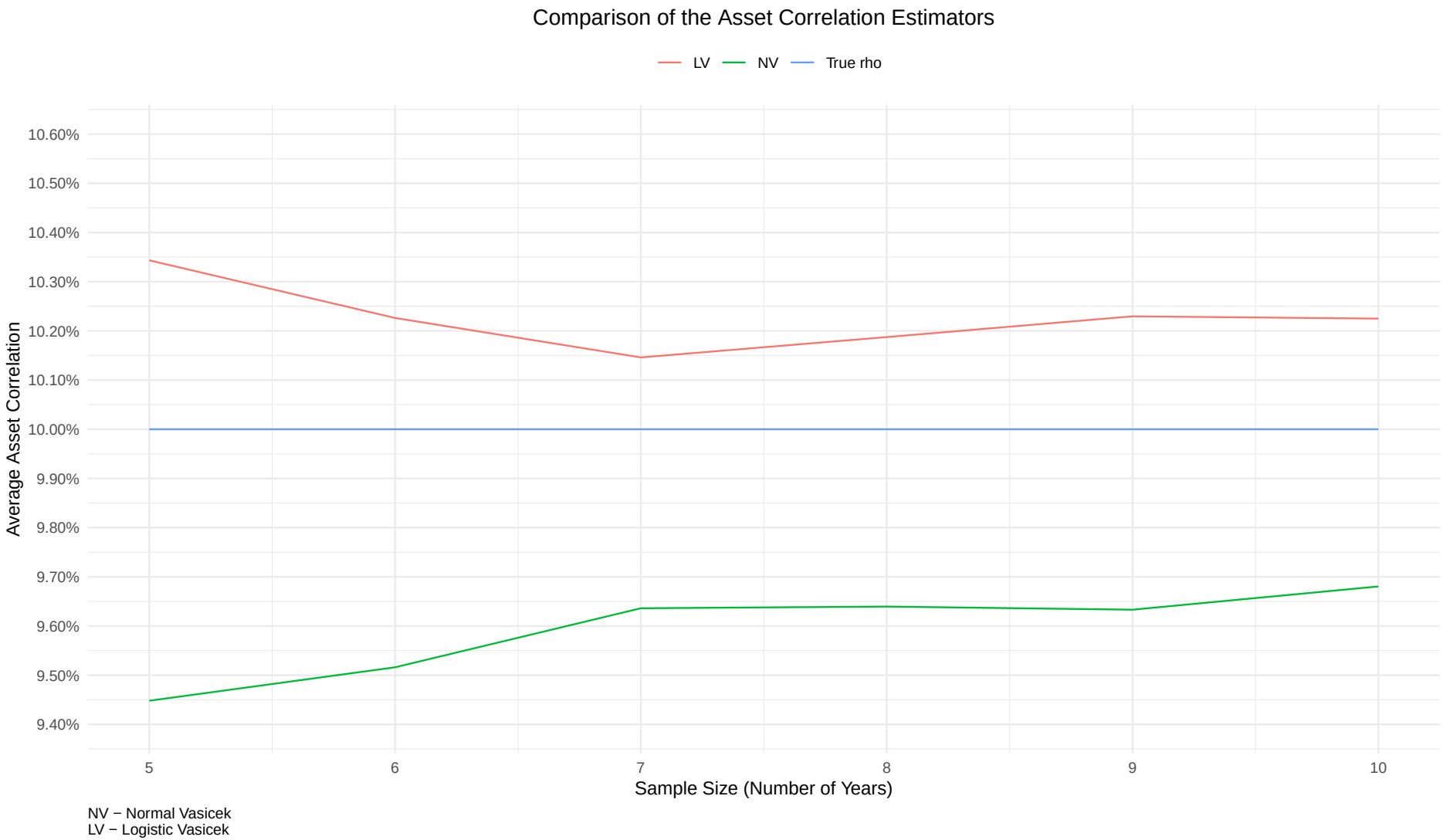
Practitioners are encouraged to test and simulate the bias of the estimators with other parameters of the both Vasicek-distributed variable.

Simulation Results



NV − Normal Vasicek
LV − Logistic Vasicek

Simulation Results cont.



NV − Normal Vasicek
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Simulation Results cont.