Assignment 1

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Exercise 2. Airpollution.

The data were obtained to determine predictors related to air pollution. We want to investigate which explanatory variables need to be included into a linear regression model with oxidant as the response variable.

```
pollution_df <- read.table("data/airpollution.txt", header=TRUE)
head(pollution_df)
```

```
##   day wind temperature humidity insolation oxidant
## 1   1   50          77       67         78      15
## 2   2   47          80       66         77      20
## 3   3   57          75       77         73      13
## 4   4   38          72       73         69      21
## 5   5   52          71       75         78      12
## 6   6   57          74       75         80      12
```

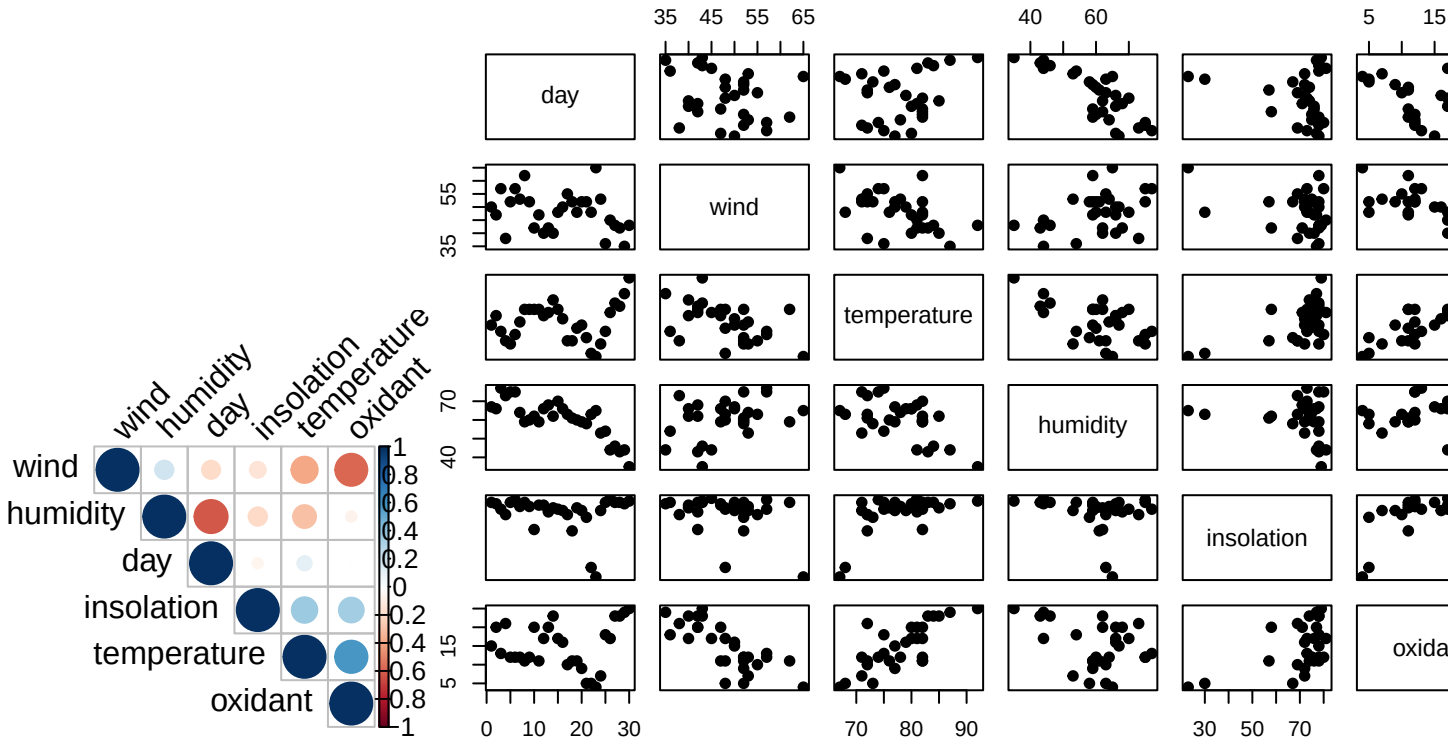
a) Make some graphical summaries of the data. Investigate the problem of potential and influence points, and the problem of collinearity.

```
# cor(pollution_df, method = c("spearman"))
if (!require("corrplot")) install.packages("corrplot")
```

```
## Loading required package: corrplot
```

```
## corrplot 0.92 loaded
```

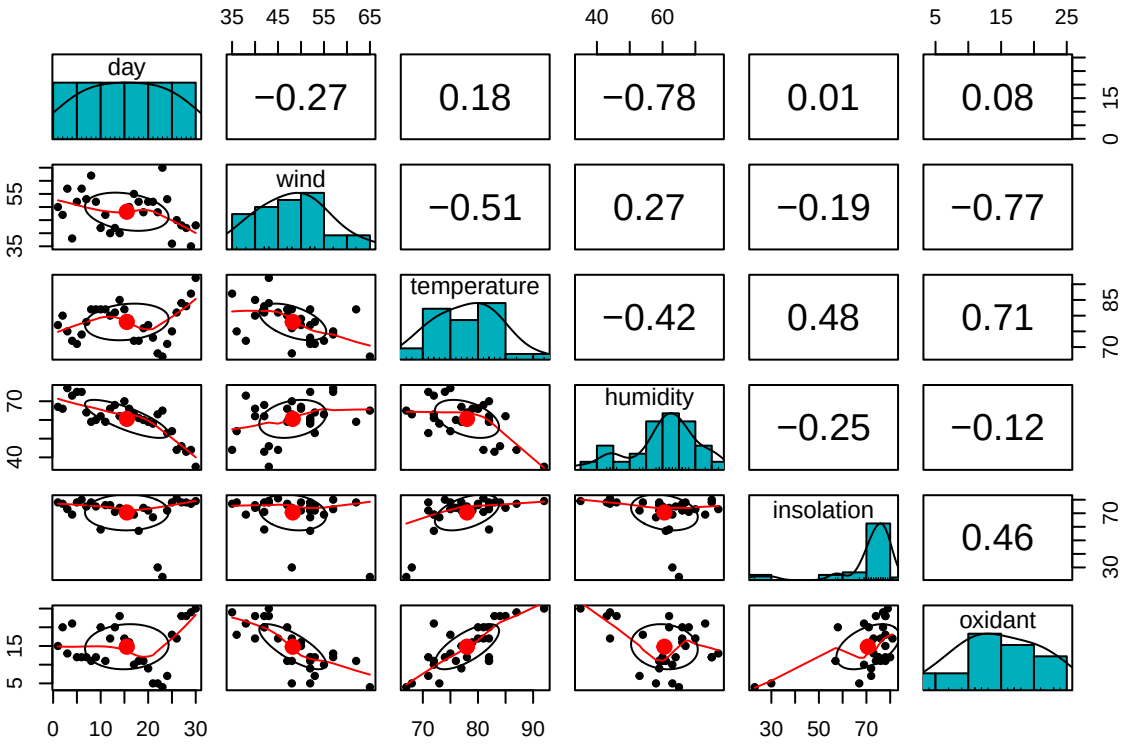
```
library(corrplot)
par(mfrow=c(2, 1))
res <- cor(pollution_df, method='kendall')
corrplot(res, type = "upper", order = "hclust",
         tl.col = "black", tl.srt = 45)
pairs(pollution_df, pch = 19)
```




```
if (!require("psych")) install.packages("psych")
```

```
## Loading required package: psych
```

```
library(psych)
pairs.panels(pollution_df,
             method = "spearman", # correlation method
             hist.col = "#00AFBB",
             density = TRUE,  # show density plots
             ellipses = TRUE # show correlation ellipses
             )
```



b) Use the added variable plot to depict the relationship between response oxidant and predictor wind. What is the meaning of the slope of fitted regression for this scatter plot?

```
if (!require("car")) install.packages("car")
```

```
## Loading required package: car
```

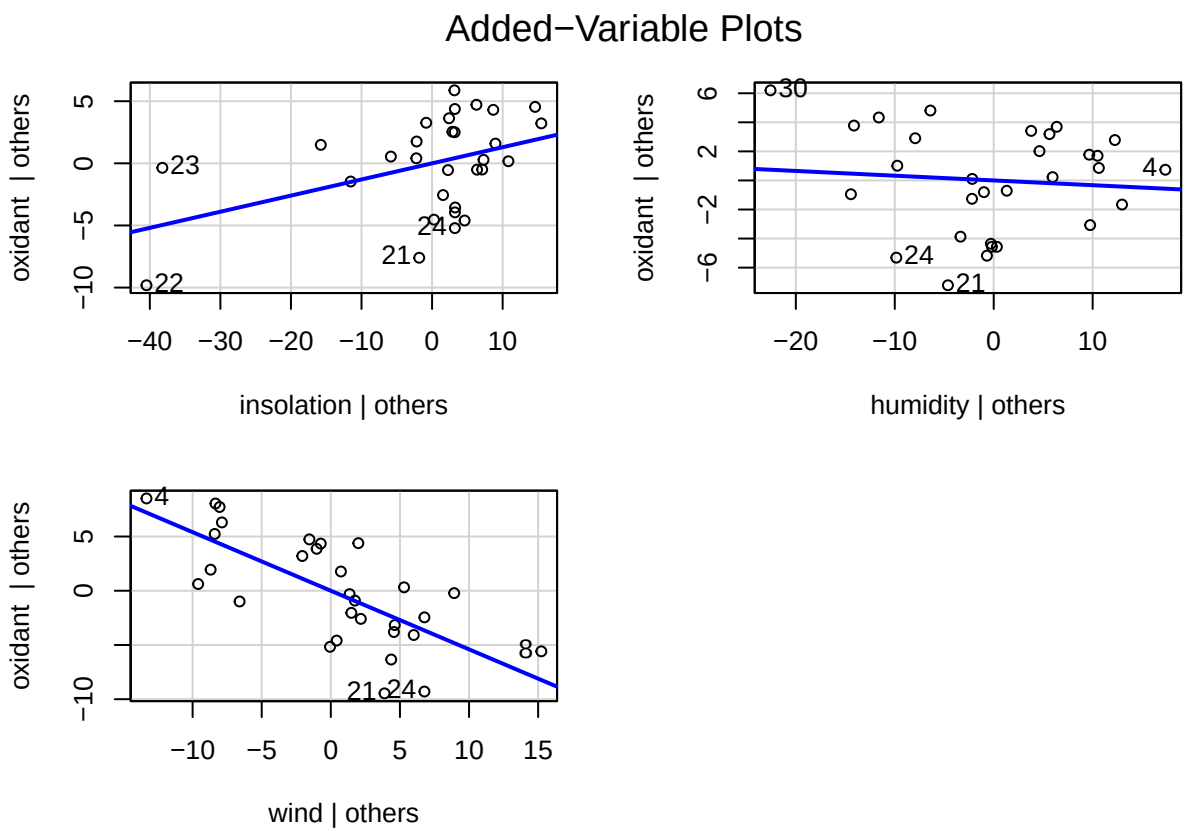
```
## Loading required package: carData
```

```
##
## Attaching package: 'car'
```

```
## The following object is masked from 'package:psych':
##
##     logit
```

```
library(car)

attach(pollution_df)
mod = lm(oxidant~insolation+humidity+wind)
par(mfrow=c(2, 1))
avPlots(mod)
```




```
summary(mod)
```

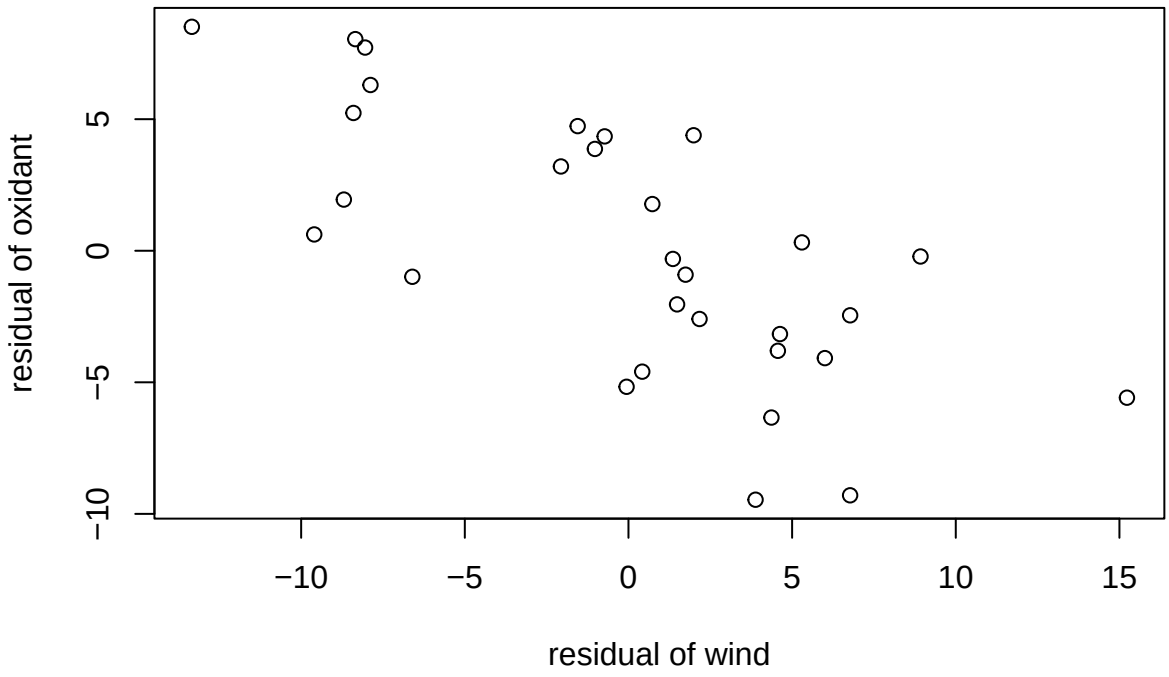
```
##
## Call:
## lm(formula = oxidant ~ insolation + humidity + wind)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.3630 -2.4212  0.5585  3.0466  5.4644
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 33.65768    7.67492   4.385  0.00017 ***
## insolation   0.12984    0.05479   2.370  0.02550 *
## humidity    -0.03266    0.07288  -0.448  0.65775
## wind        -0.54037    0.10550  -5.122 2.44e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.693 on 26 degrees of freedom
## Multiple R-squared:  0.6639, Adjusted R-squared:  0.6251
## F-statistic: 17.12 on 3 and 26 DF,  p-value: 2.423e-06
```

The slopes on the plots reflect the regression coefficients from the original miltiple regression model mod.

```
attach(pollution_df)
```

```
## The following objects are masked from pollution_df (pos = 3):
##
##     day, humidity, insolation, oxidant, temperature, wind
```

```
y = residuals(lm(oxidant~insolation+humidity))
x = residuals(lm(wind~insolation+humidity))
plot(x, y, xlab='residual of wind', ylab='residual of oxidant')
```



c) Fit a linear regression model to the data. Use both the step-up and step-down methods to find the best model. If step-up and step-down yield two different models, choose one and motivate your choice.

```
for(i in names(pollution_df)){
  if(i == 'oxidant'){next}
  # summary(lm(oxidant~i))
  print(summary(lm(paste('pollution_df$oxidant',  '~pollution_df$', i))))
}
```

Step-up

```
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$",
##     i))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
```

```
## -11.3373  -3.8537   0.1298   5.5403   9.1613
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)      13.68966    2.28580   5.989 1.89e-06 ***
## pollution_df$day  0.07164    0.12876   0.556    0.582
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.104 on 28 degrees of freedom
## Multiple R-squared:  0.01093,    Adjusted R-squared:  -0.02439
## F-statistic: 0.3095 on 1 and 28 DF,  p-value: 0.5824
##
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$",
##     i))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.9266 -2.5923  0.2065  2.6636  6.9077
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)        45.3171     4.8976   9.253 5.19e-10 ***
## pollution_df$wind  -0.6331     0.1005  -6.300 8.20e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.948 on 28 degrees of freedom
## Multiple R-squared:  0.5863, Adjusted R-squared:  0.5715
## F-statistic: 39.68 on 1 and 28 DF,  p-value: 8.205e-07
##
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$",
##     i))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.9400 -2.2138  0.3775  2.5550 10.9099
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -46.4292     9.9542  -4.664 6.94e-05 ***
## pollution_df$temperature   0.7850     0.1273   6.168 1.17e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 3.997 on 28 degrees of freedom
## Multiple R-squared:  0.576,  Adjusted R-squared:  0.5609
## F-statistic: 38.04 on 1 and 28 DF,  p-value: 1.167e-06
##
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$",
##     i))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.3358  -4.0749   0.8782   4.7800   8.7957
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)             27.4446     6.4368   4.264 0.000206 ***
## pollution_df$humidity  -0.2088     0.1049  -1.991 0.056317 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.745 on 28 degrees of freedom
## Multiple R-squared:  0.124,  Adjusted R-squared:  0.09273
## F-statistic: 3.964 on 1 and 28 DF,  p-value: 0.05632
##
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$",
##     i))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.9723 -4.4841 -0.3281  4.7631  8.2686
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -1.43279    5.32967  -0.269  0.79003
## pollution_df$insolation  0.22993    0.07424   3.097  0.00441 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.297 on 28 degrees of freedom
## Multiple R-squared:  0.2552, Adjusted R-squared:  0.2286
## F-statistic: 9.592 on 1 and 28 DF,  p-value: 0.004411
```

Wind variable gives maximum increase in the R^2. The variable is significant. Therefore, we can continue.

```r
for(i in names(pollution_df)){
  if(i %in% c('oxidant', 'wind')){next}
  print(summary(lm(paste('pollution_df$oxidant',  '~pollution_df$wind+pollution_df$', i))))
}
```

```
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$wind+pollution_df$",
##     i))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.4129 -2.5621  0.4498  2.3827  7.9267
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       47.84224    5.62785   8.501 4.10e-09 ***
## pollution_df$wind -0.65984    0.10489  -6.291 9.87e-07 ***
## pollution_df$day  -0.07986    0.08691  -0.919    0.366
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.959 on 27 degrees of freedom
## Multiple R-squared:  0.5989, Adjusted R-squared:  0.5691
## F-statistic: 20.15 on 2 and 27 DF,  p-value: 4.411e-06
##
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$wind+pollution_df$",
##     i))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.3939 -1.8608  0.5826  1.9461  4.9661
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -5.20334   11.11810  -0.468    0.644
## pollution_df$wind        -0.42706    0.08645  -4.940 3.58e-05 ***
## pollution_df$temperature  0.52035    0.10813   4.812 5.05e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.95 on 27 degrees of freedom
## Multiple R-squared:  0.7773, Adjusted R-squared:  0.7608
## F-statistic: 47.12 on 2 and 27 DF,  p-value: 1.563e-09
##
```

```
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$wind+pollution_df$",
##     i))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.8120 -2.2808  0.3433  3.0476  5.8757
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)           46.91570    5.68573   8.251 7.38e-09 ***
## pollution_df$wind     -0.60955    0.10971  -5.556 6.86e-06 ***
## pollution_df$humidity -0.04516    0.07866  -0.574    0.571
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.996 on 27 degrees of freedom
## Multiple R-squared:  0.5913, Adjusted R-squared:  0.561
## F-statistic: 19.53 on 2 and 27 DF,  p-value: 5.674e-06
##
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$wind+pollution_df$",
##     i))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.2119 -2.7198  0.4815  2.8733  6.2012
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)             32.32615    6.97098   4.637 8.07e-05 ***
## pollution_df$wind       -0.55639    0.09778  -5.690 4.81e-06 ***
## pollution_df$insolation  0.13161    0.05383   2.445   0.0213 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.638 on 27 degrees of freedom
## Multiple R-squared:  0.6613, Adjusted R-squared:  0.6362
## F-statistic: 26.36 on 2 and 27 DF,  p-value: 4.491e-07
```

Temperature variable works in the same way as the previous choice. Continue.

```
for(i in names(pollution_df)){
  if(i %in% c('oxidant', 'wind', 'temperature')){next}
  print(summary(lm(paste('pollution_df$oxidant',
                         '~pollution_df$wind',
```

```
                        '+pollution_df$temperature',
                        '+pollution_df$',
                        i))))
}
```

```
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$wind",
##     "+pollution_df$temperature", "+pollution_df$", i))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.9010 -1.3477  0.1596  1.7766  3.9405
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)               -2.98987   10.94466  -0.273    0.787
## pollution_df$wind         -0.45604    0.08644  -5.276 1.63e-05 ***
## pollution_df$temperature   0.52918    0.10568   5.008 3.29e-05 ***
## pollution_df$day          -0.09711    0.06328  -1.535    0.137
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.878 on 26 degrees of freedom
## Multiple R-squared:  0.7958, Adjusted R-squared:  0.7722
## F-statistic: 33.78 on 3 and 26 DF,  p-value: 4.042e-09
##
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$wind",
##     "+pollution_df$temperature", "+pollution_df$", i))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.5887 -1.1686  0.1978  1.9004  4.1544
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)               -16.60697   13.07154  -1.270    0.215
## pollution_df$wind          -0.44620    0.08513  -5.241 1.78e-05 ***
## pollution_df$temperature    0.60190    0.11764   5.117 2.47e-05 ***
## pollution_df$humidity       0.09850    0.06316   1.559    0.131
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.874 on 26 degrees of freedom
## Multiple R-squared:  0.7964, Adjusted R-squared:  0.7729
```

```
## F-statistic: 33.89 on 3 and 26 DF,  p-value: 3.904e-09
##
##
## Call:
## lm(formula = paste("pollution_df$oxidant", "~pollution_df$wind",
##     "+pollution_df$temperature", "+pollution_df$", i))
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -6.407 -2.056  1.012  1.760  4.792
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -4.45496   11.26714  -0.395 0.695778
## pollution_df$wind        -0.42353    0.08737  -4.848 5.02e-05 ***
## pollution_df$temperature  0.47558    0.12564   3.785 0.000816 ***
## pollution_df$insolation   0.03646    0.05071   0.719 0.478636
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.976 on 26 degrees of freedom
## Multiple R-squared:  0.7816, Adjusted R-squared:  0.7565
## F-statistic: 31.02 on 3 and 26 DF,  p-value: 9.583e-09
```

Humidity has the highest R-squared increase but the variable is not significant. Therefore, we don't add it to the model. Resulting model is:

```
print(summary(lm(pollution_df$oxidant~pollution_df$wind+pollution_df$temperature)))
```

```
##
## Call:
## lm(formula = pollution_df$oxidant ~ pollution_df$wind + pollution_df$temperature)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.3939 -1.8608  0.5826  1.9461  4.9661
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -5.20334   11.11810  -0.468    0.644
## pollution_df$wind        -0.42706    0.08645  -4.940 3.58e-05 ***
## pollution_df$temperature  0.52035    0.10813   4.812 5.05e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.95 on 27 degrees of freedom
## Multiple R-squared:  0.7773, Adjusted R-squared:  0.7608
## F-statistic: 47.12 on 2 and 27 DF,  p-value: 1.563e-09
```

oxidant = -5.2 - 0.4*wind + 0.5*temperature + error, with R-squared = 0.8.

```
summary(lm(pollution_df$oxidant~ pollution_df$wind + pollution_df$temperature + pollution_df$da
```

Step-down

```
##
## Call:
## lm(formula = pollution_df$oxidant ~ pollution_df$wind + pollution_df$temperature +
##     pollution_df$day + pollution_df$humidity + pollution_df$insolation)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.6920 -1.1675  0.2582  1.8289  4.0773
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -12.04010   21.20961  -0.568  0.57553
## pollution_df$wind         -0.44749    0.09103  -4.916 5.14e-05 ***
## pollution_df$temperature   0.55714    0.15347   3.630  0.00133 **
## pollution_df$day          -0.02997    0.13995  -0.214  0.83227
## pollution_df$humidity      0.06818    0.13336   0.511  0.61384
## pollution_df$insolation    0.01822    0.05583   0.326  0.74694
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.977 on 24 degrees of freedom
## Multiple R-squared:  0.7984, Adjusted R-squared:  0.7564
## F-statistic: 19.01 on 5 and 24 DF,  p-value: 1.203e-07
```

Day has the largest p-value and the value is larger than 0.05. Removing it.

```
summary(lm(pollution_df$oxidant ~ pollution_df$wind + pollution_df$temperature + pollution_df$
```

```
##
## Call:
## lm(formula = pollution_df$oxidant ~ pollution_df$wind + pollution_df$temperature +
##     pollution_df$humidity + pollution_df$insolation)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.5861 -1.0961  0.3512  1.7570  4.0712
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -15.49370   13.50647  -1.147  0.26219
## pollution_df$wind         -0.44291    0.08678  -5.104 2.85e-05 ***
## pollution_df$temperature   0.56933    0.13977   4.073  0.00041 ***
## pollution_df$humidity      0.09292    0.06535   1.422  0.16743
```

```
## pollution_df$insolation    0.02275    0.05067   0.449  0.65728
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.92 on 25 degrees of freedom
## Multiple R-squared:  0.798,  Adjusted R-squared:  0.7657
## F-statistic: 24.69 on 4 and 25 DF,  p-value: 2.279e-08
```

Insolation is the largers from the insignificant. Removing it.

```
summary(lm(pollution_df$oxidant~ pollution_df$wind + pollution_df$temperature + pollution_df$hu
```

```
##
## Call:
## lm(formula = pollution_df$oxidant ~ pollution_df$wind + pollution_df$temperature +
##     pollution_df$humidity)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.5887 -1.1686  0.1978  1.9004  4.1544
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -16.60697   13.07154  -1.270    0.215
## pollution_df$wind         -0.44620    0.08513  -5.241 1.78e-05 ***
## pollution_df$temperature   0.60190    0.11764   5.117 2.47e-05 ***
## pollution_df$humidity      0.09850    0.06316   1.559    0.131
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.874 on 26 degrees of freedom
## Multiple R-squared:  0.7964, Adjusted R-squared:  0.7729
## F-statistic: 33.89 on 3 and 26 DF,  p-value: 3.904e-09
```

Humidity is the only insignificant. Removing.

```
summary(lm(pollution_df$oxidant~ pollution_df$wind + pollution_df$temperature))
```

```
##
## Call:
## lm(formula = pollution_df$oxidant ~ pollution_df$wind + pollution_df$temperature)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.3939 -1.8608  0.5826  1.9461  4.9661
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -5.20334   11.11810  -0.468    0.644
## pollution_df$wind        -0.42706    0.08645  -4.940 3.58e-05 ***
```

```
## pollution_df$temperature  0.52035    0.10813   4.812 5.05e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.95 on 27 degrees of freedom
## Multiple R-squared:  0.7773, Adjusted R-squared:  0.7608
## F-statistic: 47.12 on 2 and 27 DF,  p-value: 1.563e-09
```

All remaining variables are significant. Resulting model: oxidant = -5.2 - 0.4*wind* + *0.5*temperature + error, with R-squared = 0.8. The model is the same as obtained with the step-up approach.

d) Determine 95% confidence and prediction intervals for oxidant using the model you preferred in c) for wind=33, temperature=54, humidity=77 and insolation=21.

```
x1 <- pollution_df$wind
x2 <- pollution_df$temperature

mod = lm(pollution_df$oxidant ~ x1 + x2)

newxdata = data.frame(x1=33, x2=54)

predict(mod, newxdata, interval='prediction', level=0.95)
```

```
##       fit        lwr      upr
## 1 8.80281 -0.5617877 18.16741
```

```
predict(mod, newxdata, interval='confidence', level=0.95)
```

```
##       fit      lwr      upr
## 1 8.80281 1.656548 15.94907
```