Time Series Analysis
Winter 2025
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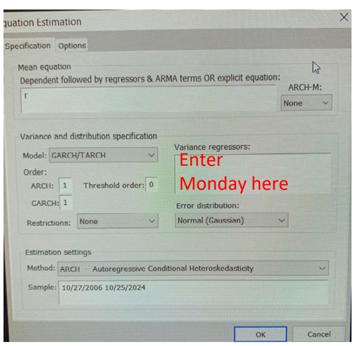
HW 5 due in class in week 7

1. Download the file VIX_SP. This file contains the S&P500 return and the VIX.

   a. Build and report a good GARCH model for the S&P500 returns. You should use squared standardized residuals to check and find a good model.

   b. Generate a "dummy variable" for Mondays using Monday=@weekday=1. This variable takes the value of 1 if Monday and zero otherwise. Estimate a GARCH model with the dummy variable on the right hand side. Notice that you don't need to use a lagged value since we know if the next day is a Monday or not at the time we forecast. Report the model. What does it say about the level of volatility on Mondays?

   In Eviews

   c. Generate a Friday dummy and re-estimate the model in part b but include the Friday dummy as well. Report the model. Is the Friday dummy significant?

   d. Now let's use the VIX series to as an additional variable to forecast the GARCH variances. The "prices" in the VIX column are annualized implied volatilities derived from near-term options on the S&P500. Convert the VIX annualized volatilities back into daily variance numbers by squaring them and then dividing by 252. Add the *lagged* daily variance VIX to the model (like you did for the Monday dummy, be sure to use a lagged value, not the contemporaneous). Report the estimated model.

e. Compare the AIC and BIC of the model in part d to the model in part a. Which model is preferred? The AIC and BIC are still valid ways to compare two different volatility models.

2. Use the dataset NASDAQ.xls. The dataset contains 5 minute continuously compounded returns for NASDAQ index from 9:30 in the morning to 2:45 in the afternoon. The returns are constructed using the difference of the logarithmic midpoint of the bid and ask prices. It also contains a variable indicating the day (1 through 32). The timestamp is in milliseconds past midnight. If you divide the timestamp by 1000 you recover seconds past midnight. Intraday returns have a deterministic component in the volatility. Volatility is high near the open and the close and relatively low in the middle of the day, a U shape or hockey stick shape over the day. This pattern repeats itself every day. You will estimate a component GARCH model for this data given by: $r_t = \sqrt{h_t}\sqrt{\phi_t} z_t$. $\phi(t) = E\left(r_t^2 \mid t\right)$ is simply the deterministic component that is purely a function of time of day. Notice that the return divided by the deterministic component results in a process that is free of any deterministic elements: $\tilde{r}_t = \dfrac{r_t}{\sqrt{\phi(t)}} = \sqrt{h_t} z_t$. Also the model says that $\tilde{r}_r$ follows a GARCH process.

    a. Let t denote the time of day variable. Choose a functional form for this time of day variable that can account for the U shape. Suggestions include a quadratic spline, a polynomial, or perhaps a piecewise linear function (I find the quadratic or cubic splines work well). You could even use a step function where volatility is assumed constant over regions of the day like every half hour. Let $\phi(t)$ denote your chosen functional form. Since $\phi(t) = E\left(r_t^2 \mid t\right)$ you can estimate parameters in $\phi(t)$ by running a regression $r_t^2 = \phi(t) + \varepsilon_t$.
    b. For a single day, plot the estimated function $\phi(t)$
    c. Next, create the series $\tilde{r}_t = \dfrac{r_t}{\sqrt{\phi(t)}} = \sqrt{h_t} z_t$. Fit a GARCH model for $\tilde{r}_t$. Report your model. This GARCH model should have a mean (unconditional variance) of about 1 and represents the fraction above or below the normal variance for a given time of day.
    d. $E\left(r_t^2 \mid F_{t-1}\right) = h_t \phi_t$. Pick one day and plot $\sqrt{\phi(t)}$ and $\sqrt{h_t}\sqrt{\phi_t}$ on the same picture. Put the time of day on the horizontal axis.