CHICAGO BOOTH
The University of Chicago Booth School of Business

Time Series Analysis
Jeffrey R. Russell
Homework 2 by start of class in week 3

1. Consider the MA(1) model $y_t = \mu + \theta\varepsilon_{t-1} + \varepsilon_t$ where ε_t is white noise.
   a. What is the unconditional mean?
   b. What is the unconditional variance?
   c. Find the autocorrelations.
   d. Show that $\theta^* = \left(\frac{1}{\theta}\right)$ and $\sigma^* = \frac{1}{\sigma}$ imply the same variance and autocorrelation structure.

2. Consider the model $y_t = .5 + \left(1 + .9L + .2L^2\right)\varepsilon_t$ where $\varepsilon_t \sim iid\ N\left(0, .5^2\right)$
   a. What is the unconditional mean?
   b. What is the unconditional variance?
   c. What are the autocorrelations?
   d. What is the unconditional distribution of y_t?
   e. Is this MA model invertible? Is there an infinite AR representation? If so, find it.
   f. Is the model weakly stationary? Why or why not?

3. Consider the ARMA(1,1) model $\left(1-\beta_1 L\right) y_t = \left(1+\theta L\right)\varepsilon_t$ where ε_t is white noise.
   a. Write the the model as an MA(∞) $y_t = \sum_{j=1}^{\infty} \psi_j \varepsilon_{t-j} + \varepsilon_t$. Provide an expression for the ψ_j as a function of β_1 and θ.
   b. Find the mean, variance, and an expression for the j^{th} autocovariance.
   c. Show that the model is weakly stationary if β_1 lies inside the unit circle.
   d. Explain why the model is strongly stationary if β_1 lies inside the unit circle and the ε_t are iid Normal.

4. Consider the ARMA(p,q) model.
   a. When is the model stationary?
   b. When can the model be written as an MA(∞)?
   c. When can the model be written as an AR(∞)?

5. Use the dataset xyseris.xls for this problem. There are two series in this file.
   a. For series x, present the ACF and PACF.
   b. Use the ACF and PACF to find a good model.
   c. Estimate the model based on your conclusions from part b.
   d. Examine the correlogram (ACF) of the residuals. Does you model appear to fit the data?
   e. Repeat this exercise for series y.

6. Use the dataset TTM.xls for this problem. It contains one day of trade by trade data for the Tata Motor Limited, an NYSE listed stock. Each observation is a transaction. The time stamp is hours, minutes, 1/100's of minutes. The variable EX is the exchange on which the trade occurred. PRICE is the transaction price and SIZE is the number of shares transacted.
   a. Create the simple returns using the price variable. Present a plot.
   b. Examine the ACF and PACF. What type of model looks appropriate?
   c. Fit an MA(1) model. Does it pass the residual diagnostics?
   d. Fit an MA(2) model. Does it pass the residual diagnostic?
   e. Now fit AR(1), AR(2), and AR(3) models. Do any of these pass the residual diagnostics?
   f. Using the MA(2) model from part d, build the in sample one step ahead forecasts. Present the plot.

7. From the FRED, download seasonally adjusted monthly Consumer Price Index: Total All Items for the United States from 1980 to present. Take the log of the series and then difference it in order to create growth rates ($g_t = \ln\left(CPI_t\right) - \ln\left(CPI_{t-1}\right)$). This series corresponds to growth rates in the CPI, or inflation.

   a. Plot the series. Does it look dependent?
   b. Construct the ACF, is there significant autocorrelation in the data?
   c. Fit and present a good ARMA model for the data.
   d. Report the roots of the AR polynomial. Recall that you can "view" a model as its ARMA structure to get the roots. What do the roots tell you about the dependence? How quickly should you expect shocks to die off?

8. Let $r_t = \ln\left(p_t\right) - \ln\left(p_{t-1}\right)$ be white noise where p_t denotes the true value of an asset for the t^{th} trade of the day. p_t is not directly observed however due to market microstructure noise (i.e bid ask bounce, price discreteness). Instead, we get to observe p_t^o where $p_t^o = \eta_t p_t$. η_t is an iid multiplicative noise term that is independent of p_t. When η_t =1, the observed price is the same as the true price, but when it differs from one, the observed and true price are different. Assume that $\ln\left(\eta_t\right)$ is mean zero and finite variance so it is white noise.

   Show that the observed returns $r_t^o = \ln\left(p_t^o\right) - \ln\left(p_{t-1}^o\right)$ have a correlation structure of an MA(1) model.