Impact of Pseudorandom Noise Codes on Multipath Mitigation

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BIOGRAPHY

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ABSTRACT

Multipath mitigation in GPS receiver may be based on the structure of correlators of pseudorandom sequence. Their development depends on the mathematical model of pseudorandom sequence. Two different mathematical pseudonoise code models have been given in the paper. The analysis of Gold codes that are used in satellite navigation has shown that some PRN can be described only with one of the mathematical models. These mathematical models used in computer-aided design package (CAD-package) for designing a GPS-GLONASS receiver. The developed package permits to compute statistical characteristics of correlation signals I, Q, dI, dQ, noise and multipath measurement errors, interpath and interchannel biases, biases due to technological scatter of analog path elements.

It has been shown that multipath suppression depends on GPS satellite PRN, which means that multipath suppression varies with Gold codes at the same multipath suppression technology. Such a conclusion has been confirmed by experimental results obtained with Spirent GPS simulator GSS7700

INTRODUCTION

Multipath error is one of the most powerful in the total error budget of Global Navigation Satellite Systems (GNSS) operation. There are a lot of possible solutions to mitigate multipath in the navigation receivers. They include Narrow Correlator, strobe methods, MEDLL and so on. These technologies differe by their implementation complexity, efficiency and sophistication from each other.

This paper considers signal processing in using strobe methods. Some bases of the strobe method for signal processing (patent pending) have been given in [1] and [2]. This method is intended to reduce multipath errors by the correlation of the input PRN-signal to a special reference code - a strobe sequence - consisting of pulses (strobes) with a specially designed shape. Means, which carry out the correlation, i.e., the multiplication of the signal by the strobe sequence and reference carrier phase and further integration, are called strobe correlators. Based upon the output signals of strobe correlators, discriminators of tracking systems that have the reduced multipath error can be designed.

The shape of a strobe is an instrument to reduce the multipath error in this case. In [1] and [2] the choice of a strobe shape has been done by the rule of a thumb. Varying further strobe time parameters results in the mitigation of the multipath error at an acceptable noise error. Note that the use of the strobe sequence instead of the optimal receiver's reference code always causes the increase of the noise error . In the other words, there is a interchange (interexchange) between multipath and noise errors.

Further, we will consider a strobe correlator which uses short rectangular pulses that are at PRN code changes from one state to another as a reference signal to generate dI and dQ components. Such a technology is called “Edge correlator” [4]. This suppression technology is as efficient as Narrow correlator technology and is dependent on the duration of rectangular strobes.

INPUT AND OUTPUT SIGNALS OF STROBE CORRELATOR

As an input signal of the strobe correlator the following model of the multipath signal can be used

$$c(t) = \sum_{i=0}^{L} a_i \cdot p(t-\delta_i) \cdot \exp[j(\omega_0 t + \theta_i)] + n(t)\,, \qquad (1)$$

where L is the number of reflected signals; i=0 corresponds to the line-of-sight signal while i=1,2,…L correspond to reflected signals; a_i is the amplitude of the *i-th* signal; *p(t)* is a PRN code (for example, C/A code modulated by data bits); δ_i is the time shift of the *i-th* signal related to the line-of-sight signal; ω_0 is the carrier frequency; θ_i is the phase of the *i-th* signal related to the line-of-sight signal; $n(t)$ is Gaussian noise with zero mean value. Note that adequacy of this model is confirmed by experimental results [5]. The above parameters L, a_i, δ_i, ω_0, θ_i vary over the time period due to the mutual movement of a satellite, reflected objects, and receiver's antenna.

PRN which passes through the radio frequency part (front end) can be presented as a sequence of pulses (chips)

$$p(t) = \sum_{k=-\infty}^{\infty} \mu_k \cdot [h(t-k\Delta) - h(t-(k+1)\Delta)], \qquad (2)$$

where k is integer; $\mu=\pm1$ is chip's sign; Δ is the duration of a chip that is inversely proportional to PRN clock rate; $h(t)$ is the transient process at PRN change of sign.

The correlation operation is the multiplication of $c(t)$-signal by the reference signal

$$S(t) \cdot \exp[-j\omega t] \qquad (3)$$

and integration over the time period T. The reference signal is the multiplication of the reference carrier phase $\exp[-j\omega t]$ by code $S(t)$, in our case the strobe sequence serves as such a code signal. There are two types of the strobe sequence creation which is defined by function $S(t)$:

1. creation of pulses sequences at the moment of PRN change of sign;
2. creation of pulses sequences at the moment of all PRN chip edge;

Only type 1 will be considered further.

MATHEMATICAL MODELS OF PRN

To compute statistical parameters of signals I,Q, dI and dQ and estimate multipath error, we need to describe PRN $p(t)$ at the input of a navigation receiver, as well as function $S(t)$.

An equiprobable PRN model is often used. All the elements of pseudo-random sequence (PRS) a_n for this mathematical model are supposed to be mutually-independent and equiprobable numbers which can take values ± 1. Input antenna signal can be hence written as follows

$$S^{ant}(t) = \sum_n a_n \cdot y_0(t - n\Delta) \qquad (4)$$

where $a_n = \pm 1$ - random equiprobable numbers, $n \in (-\infty, +\infty)$, Δ - chip duration,

$$y_0(t) = 1(t) - 1(t-\Delta) \qquad (5)$$

is a unit pulse,

$$1(t) = \begin{cases} 1 \quad npu \quad t \geq 0 \\ 0 \quad npu \quad t < 0 \end{cases} \qquad (6)$$

is a unit cycle slip.

We consider the interaction of the *n-th* reference pulse at $a_n = 1$ with filter response *y(t)* to this pulse but ignore the interaction with the rest of pulses as its mathematical expectation is equal to zero.

The equiprobable model of PRN matches GLONASS signals. Note that m-sequence is used as pseudorandom noise code in GLONASS. Auto-correlation function for m-sequence can be written in the form:

$$R(\tau) = \begin{cases} 1 - |\tau| \cdot (1 + 1/N) & for \;\; |\tau| \leq \Delta \\ -1/N & for \;\; \Delta \leq |\tau| \leq (N-1) \end{cases} \qquad (7)$$

where N is the m-sequence length, Δ is the duration of a chip that is inversely proportional to PRN clock rate. In GLONASS m-sequence is generated with 9-order polynomial and hence N= 511.

Satellite navigation system GPS generates signal C/A as Gold-code sequence. Gold-codes are a sum of two m-sequences. These codes are defined by a four-valued autocorrelation. The order of the forming polynomial for C/A signal is equal 10 and correspondingly N=1023.

The auto-correlation function is four-leveled [6]:

$$R(\tau)=\begin{cases}1-|\tau|\cdot(1-x/N) & for\ |\tau|\leq\Delta \\ -1/N\ \ or & for\ \Delta<|\tau|\leq(N-1) \\ a/N\ or\ -b/N & \end{cases} \quad (8)$$

where N is PRN length equal to 1023 , Δ is the duration of a chip that is inversely proportional to PRN clock rate, x=1 or a or b ; for C/A signal a=63, b=65.

We studied Gold-codes for 32 current GPS satellite. For some GPS satellites auto-correlation functions around correlation maximums are shown in Fig.1 and Fig.2. It should be noted that within this range the auto-correlation function of a part of satellites coincides with their m-sequence auto-correlation functions, but for other satellites it does not occur. Such a fact impacts multipath suppression

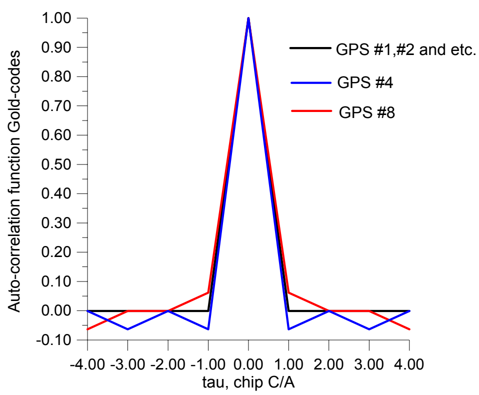

Figure 1

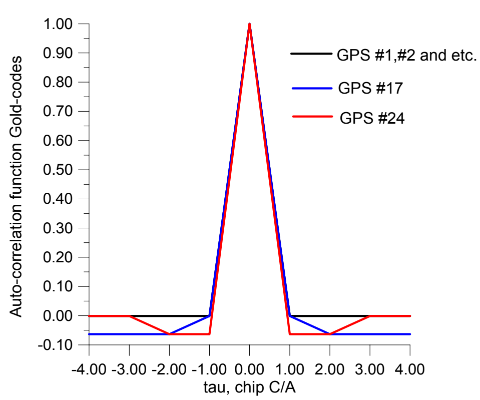

Figure 2

We developed a mathematical model for Gold-codes using a time approach. The model is based on the assumption that pulse filter (RF part navigation receiver) response decays in one-two chips.

This model is based on the interaction of the *n-th* element of the reference signal with *n*-th, with *(n-1)*-th and with *(n-2)*-th input signal element. There are possible cases of positive shifts - transition (-1, +1) and non-transition (+1, +1), as well as inversions to them: negative shifts - transition (+1, -1) and non-transition (-1, -1). The model considers different combination of four adjacent code elements. We calculated differences Δn_{n-2}^{1} (when there was a transition) between the sum of pulses $a_{n-2}=+1$ at the positive transition and $a_{n-2}=-1$ at the negative transition and the sum of pulses $a_{n-2}=-1$ at the positive transition and $a_{n-2}=+1$ at the inverse one (one should calculate the difference between the two sums, as well as difference Δn_{n-2}^{0} – when there was non-transition). The same calculations have been made for the *(n+1)-th* pulse: Δn_{n+1}^{1} - when there is transition, Δn_{n+1}^{0} - non-transition case. The analysis showed that from 32 Gold-codes the characteristics of 17 codes coincide with the characteristics of m-sequence, that is $n_1 \approx n_0$; $\Delta n_{n-2}^{1}=\Delta n_{n-2}^{0}=\Delta n_{n+1}^{1}=\Delta n_{n+1}^{0}=0$. PRN codes are 1; 2; 3; 5; 9; 11; 12; 13; 14; 20; 23; 25; 26; 27; 29; 31; 32. As to other codes, we summarized the data in Table 1.

Table 1

№ PRN	n_1	n_0	Δn_{n-2}^{1}	Δn_{n-2}^{0}	Δn_{n+1}^{1}	Δn_{n+1}^{0}
4	512	511	-32	-33	+32	-33
6	512	511	-32	-33	+32	-33
7	480	543	-32	+31	+32	+31
8	544	479	+64	0	-64	0
10	512	511	+32	+31	-32	+31
15	480	543	-32	+31	+32	+31
16	512	511	-32	-33	+32	-33
17	480	543	-32	+31	+32	+31
18	512	511	+32	+31	-32	+31
19	512	511	-32	-33	+32	-33
21	480	543	-32	+31	+32	+31
22	544	479	+32	-33	-32	-33
24	480	543	-64	0	+64	0
28	512	511	-32	-33	+32	-33
30	512	511	+32	+31	-32	+31

3. COMPUTATION OF MULTIPATH ERROR

The new PRN model for GPS allowed to modify CAD-package [7] for the navigation receivers design. Using the package one can estimate measurement errors of different types (noise, multipath, etc) taking into account the predetermined input data.

The input data are as follows:

1) Parameters of the received signal
   a. carrier frequency of the signal;
   b. signal code (GPS or GLONASS; C/A or P-code);
   c. carrier-to-noise power density ratio C/N_0;
   d. maximal Doppler frequency shift;
   e. power of the reflected signal (its relative amplitude).

2) RF-part parameters
   a. quartz oscillator frequency;
   b. local oscillator frequencies.
   c. complex frequency characteristics (CFC) of filtering elements

3) ADC parameters
   a. sampling rate $f_s = 1 / T_s$;
   b. variant of signal quantization.

Output parameters:

1) Statistical characteristics of correlation signals I, Q, dI, dQ

The computation of these parameters is carried out with the help of numerical methods rather than simulating approach, that is not so time-consuming.

2) phase and code noise errors

3) spurious harmonics due to low-level approximation of reference oscillations in ASIC.

4) interpath and interchannel biases

5) phase and code multipath errors

The calculation procedure is given in [1], but we modified it considering our time approach.

Fig.3 and Fig.4 presents the envelops of multipath errors for different GPS satellites which were obtained with the help of CAD-package. The results showed that for PRN (given in Table 1), there is multipath error at delay between reflected and direct signal more than 300 m.

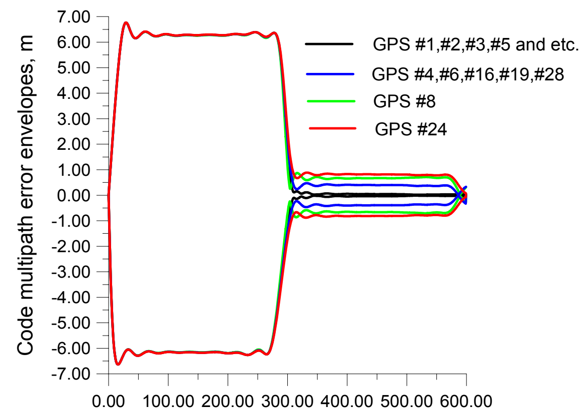

Figure 3

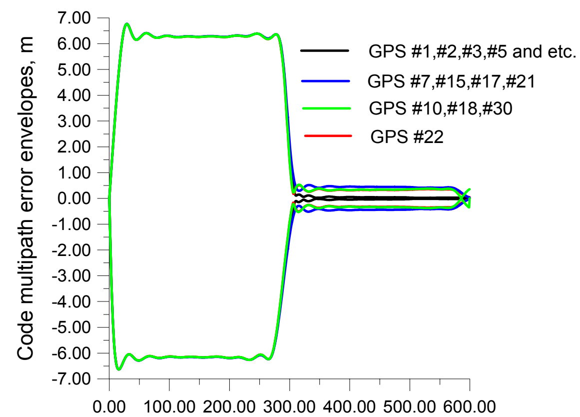

Figure 4

EXPERIMENTAL STUDY ON MULTIPATH ERRORS

We have studied multipath errors in the receivers of different manufacturers. To make these experiments Spirent GSS7700 simulator of GPS signals was used. The simulator allows both direct and reflected signals from satellites to be generated. The test results were in good correlation with model calculations. Fig5. and Fig6. shows the combined multipath errors and noise errors for the receiver of one of the companies. According to the results, the signal from satellite #8 has multipath error at the delay of the reflected signal exceeding 300 m. In Fig7. you can see test data for Legacy receiver, which had switched off special option of multipath suppression technology. This figure presents also the calculated multipath envelope. It should be noted that experimental and computed results match well.

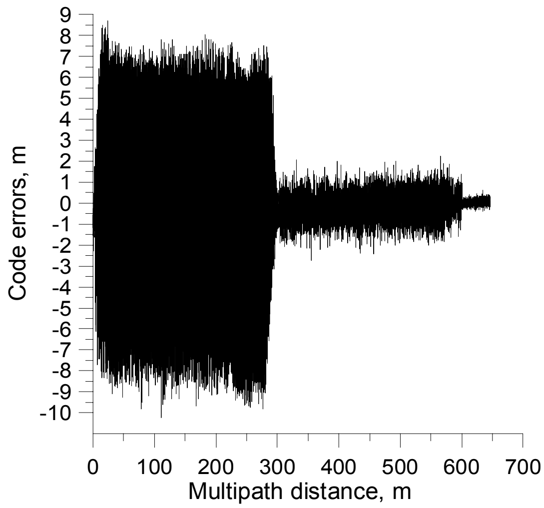

Figure 5

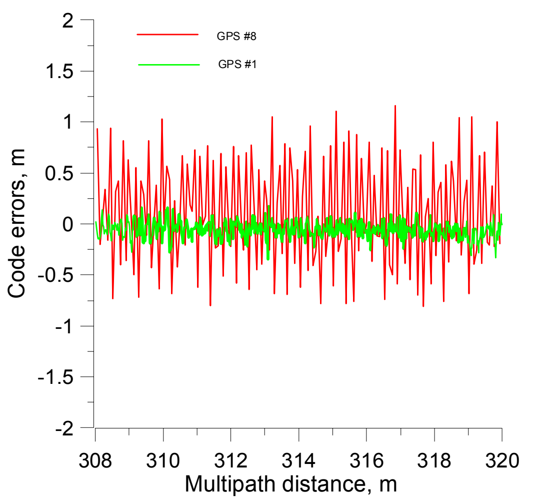

Figure 6

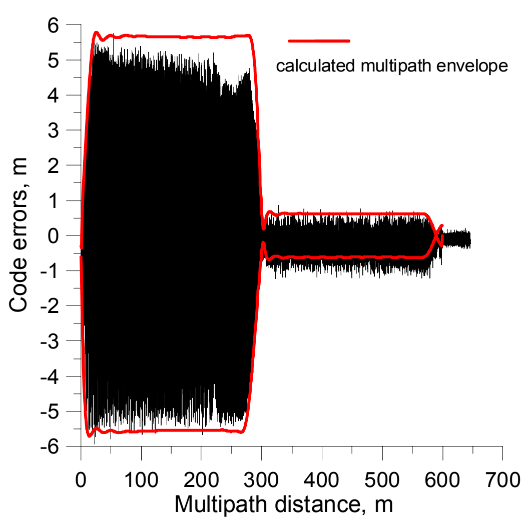

Figure 7

CONCLUSION

Different Gold-codes of different GPS satellites affect multipath errors differently. The extent of the influence depends on the technology of multipath mitigation used. The Gold-codes affect is different when one uses different technology of multipath mitigation. Example, some multipath mitigating C/A code tracking methods can induce meter-level biases of C/A code measurements [8].

CAD-package which takes into account peculiarities of different Gold codes was developed. It is used to design modern navigation receivers. The results of this package operation were tested experimentally.

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