Контрольна робота №1

1. Розв'язати систему ер:

а) матр. мет. б) мет Гауса

$$\begin{cases} x_1 + 2x_2 - 7x_3 = -27 \\ -13x_1 - x_2 + 14x_3 = 22 \\ x_1 + x_2 + x_3 = -4 \end{cases}$$

а) Нехай

$$A = \begin{pmatrix} 1 & 2 & -7 \\ -13 & -1 & 14 \\ 1 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -27 \\ 22 \\ -4 \end{pmatrix}; \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$A \cdot X = B, \qquad X = A^{-1} \cdot B$$

$$|A| = \begin{vmatrix} 1 & 2 & -7 \\ -13 & -1 & 14 \\ 1 & 1 & 1 \end{vmatrix} = 1 \begin{vmatrix} -1 & 14 \\ 1 & 1 \end{vmatrix} -$$

$$- 2 \begin{vmatrix} -13 & 14 \\ 1 & 1 \end{vmatrix} - 7 \begin{vmatrix} -13 & -1 \\ 1 & 1 \end{vmatrix} =$$

$$= 1(-1-14) - 2(-13-14) - 7(-13+1) =$$

$$= -15 + 54 + 84 = 123$$

$|A| = 123, \neq 0, \Rightarrow$ обернена A^{-1} існує.

$$A_{11} = \begin{vmatrix} -1 & 14 \\ 1 & 1 \end{vmatrix} = -15; \quad A_{12} = -\begin{vmatrix} -13 & 14 \\ 1 & 1 \end{vmatrix} = 27;$$

$$A_{13} = \begin{vmatrix} -13 & -1 \\ 1 & 1 \end{vmatrix} = -12; \quad A_{21} = -\begin{vmatrix} 2 & -7 \\ 1 & 1 \end{vmatrix} = -9;$$

$$A_{22} = \begin{vmatrix} 1 & -7 \\ 1 & 1 \end{vmatrix} = 8; \quad A_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1;$$

$$A_{31} = \begin{vmatrix} 2 & -7 \\ -1 & 14 \end{vmatrix} = 21; \quad A_{32} = -\begin{vmatrix} 1 & -7 \\ -13 & 14 \end{vmatrix} = 77;$$

$$A_{33} = \begin{vmatrix} 1 & 2 \\ -13 & -1 \end{vmatrix} = 25$$

$$A^{-1} = \begin{pmatrix} -\frac{15}{123} & -\frac{9}{123} & \frac{21}{123} \\ \frac{27}{123} & \frac{8}{123} & \frac{77}{123} \\ -\frac{12}{123} & \frac{1}{123} & \frac{25}{123} \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{5}{41} & -\frac{3}{41} & \frac{7}{41} \\ \frac{9}{41} & \frac{8}{123} & \frac{77}{123} \\ -\frac{4}{41} & \frac{1}{123} & \frac{25}{123} \end{pmatrix}$$

$$X = A^{-1} \cdot B =$$

$$\begin{pmatrix} -\frac{5}{41} & \frac{-3}{41} & \frac{7}{41} \\ \frac{9}{41} & \frac{8}{123} & \frac{77}{123} \\ -\frac{4}{41} & \frac{1}{123} & \frac{25}{123} \end{pmatrix} \cdot \begin{pmatrix} -27 \\ 22 \\ -4 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{5}{41} \cdot (-27) - \frac{3}{41} \cdot 22 + \frac{7}{41}(-4) \\ \frac{9}{41}(-27) + \frac{8}{123} \cdot 22 + \frac{77}{123}(-4) \\ \left(-\frac{4}{41}\right)(27) + \frac{22}{123} + \frac{25}{123} \cdot (-4) \end{pmatrix} =$$

$$= \begin{pmatrix} 1 \\ -7 \\ 2 \end{pmatrix}$$

$$\begin{cases} x_1 = 1 \\ x_2 = -7 \\ x_3 = 2 \end{cases}$$

8) $\left(\begin{array}{ccc|c} 1 & 2 & -7 & -27 \\ -13 & -1 & 14 & 22 \\ 1 & 1 & 1 & -4 \end{array}\right) \underset{(L_3)-(L_1)\cdot 1}{\overset{(L_2)+(L_1)\cdot 13}{\sim}} \left(\begin{array}{ccc|c} 1 & 2 & -7 & -27 \\ 0 & 25 & -77 & -329 \\ 0 & -1 & 8 & 23 \end{array}\right)$

$(L_2):25 \sim \left(\begin{array}{ccc|c} 1 & 2 & -7 & -27 \\ 0 & 1 & -3{,}08 & -13{,}16 \\ 0 & -1 & 8 & 23 \end{array}\right) \underset{(L_3)-(L_2)}{\overset{(L_1)-(L_2)\cdot 2}{\sim}} \left(\begin{array}{ccc|c} 1 & 0 & -0{,}84 & -0{,}68 \\ 0 & 1 & -3{,}08 & -13{,}16 \\ 0 & 0 & 4{,}92 & 9{,}84 \end{array}\right)$

$(L_3):4{,}92 \sim \left(\begin{array}{ccc|c} 1 & 0 & -0{,}84 & -0{,}68 \\ 0 & 1 & -3{,}08 & -13{,}16 \\ 0 & 0 & 1 & 2 \end{array}\right) \underset{(L_2)+(L_3)\cdot 3{,}08}{\overset{(L_1)+(L_3)\cdot 0{,}84}{\sim}}$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 2 \end{array}\right);$

$$\begin{cases} x_1 = 1 \\ x_2 = -7 \\ x_3 = 2 \end{cases}$$

Перевірка

$1 + 2\cdot(-7) - 7\cdot 2 = -27$

$-13\cdot 1 - (-7) + 14\cdot 2 = 22$

$1 - 7 + 2 = -4$

2. Задано кч z_0.

1. записати z_0 в алг, тригон, пок. ф
2. зобразити z_0 на компл площині
3. зн. усі корені р-ня $z^3 - z_0 = 0$.

$$z_0 = \frac{3+5i}{4+i}$$

$$z_0 = \frac{3+5i}{4+i} \cdot \frac{(4-i)}{(4-i)} = \frac{(3+5i)(4-i)}{16-i^2} =$$

$$= \frac{1}{17}(12 + 20i - 3i - 5i^2) =$$

$$= \frac{1}{17}(17 + 17i) = 1+i$$

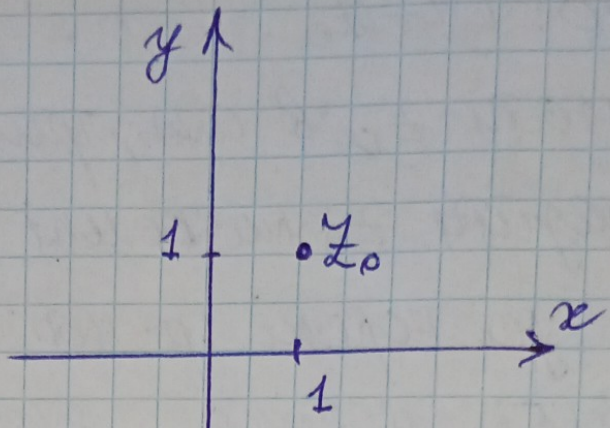
$z_0 = 1+i$ – алгебраїчна форма

$$r = \sqrt{1^2+1^2} = \sqrt{2}$$

$$tg\,\varphi = \frac{1}{1} = 1, \quad \varphi = \frac{\pi}{4}$$

$z_0 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ – тригоном. форма

$z = \sqrt{2}\, e^{i\frac{\pi}{4}}$ – показникова форма



$z_0 = 1+i$

$z^3 - \sqrt{2}\, e^{\frac{i\pi}{4}} = 0.$

$z = \sqrt[6]{2} \cdot e^{i\frac{\frac{\pi}{4}+2\pi k}{3}} = \sqrt[6]{2} \cdot e^{i \cdot \frac{\frac{\pi}{4}+2\pi k}{3}}$

$k = 0, 1, 2,$

$z_1 = \sqrt[6]{2} \cdot e^{i \cdot \frac{\pi}{12}}$

$z_2 = \sqrt[6]{2} \cdot e^{i \frac{\frac{\pi}{4}+2\pi}{3}} = \sqrt[6]{2} \cdot e^{i\frac{9\pi}{12}} = \sqrt[6]{2}\, e^{i\frac{3\pi}{4}};$

$z_3 = \sqrt[6]{2}\, e^{i \frac{\frac{\pi}{4}+4\pi}{3}} = \sqrt[6]{2}\, e^{i\frac{17\pi}{12}};$

3. $|\bar{a}| = 3, |\bar{b}| = 5, \varphi = \frac{\pi}{3}$

$\bar{c} = \alpha\bar{a} + 10\bar{b}, \bar{d} = 2\bar{a} - 17\bar{b}$

$\bar{c} \cdot \bar{d} = 0, \Rightarrow (\alpha\bar{a} + 10\bar{b})(2\bar{a} - 17\bar{b}) =$

$= 2\alpha a^2 - 170b^2 - 17\alpha|a||b|\cos\varphi + 20|b||a|\cos\varphi =$

$= 2\alpha \cdot 9 - 170 \cdot 25 - 17\alpha \cdot 3 \cdot 5\cos\frac{\pi}{3} + 20 \cdot 3 \cdot 5\cos\frac{\pi}{3} =$

$= 18\alpha - 4250 - 17 \cdot 7,5\alpha + 150 = -4100 - 109,5\alpha$

$= 0, \Rightarrow \alpha = -\frac{4100}{109,5} = -37,44$

$\bar{c} = -\frac{4100}{109,5}\bar{a} + 10\bar{b},$

$S = |\bar{c} \times \bar{d}| = \left|\left(-\frac{4100}{109,5}\bar{a} + 10\bar{b}\right)(2\bar{a} - 17\bar{b})\right| =$

$= \left|-\frac{4100}{109,5} \cdot 2\bar{a}\times\bar{a} - 170\,\bar{b}\times\bar{b} + 20\,\bar{b}\times\bar{a} + \frac{4100}{109,5} \cdot 17\bar{a}\times\bar{b}\right|$

$\bar{a} \times \bar{a} = 0, \ \bar{b} \times \bar{b} = 0$

$\bar{b} \times \bar{a} = -\bar{a} \times \bar{b}$

$= \left|-20\,\bar{a}\times\bar{b} + \frac{4100 \cdot 17}{109,5}\bar{a}\times\bar{b}\right| =$

$= \left|\left(-20 + \frac{4100 \cdot 17}{109,5}\right)|a||b|\sin\frac{\pi}{3}\right| =$

$= \left|\frac{(-20 \cdot 109,5 + 4100 \cdot 17) \cdot 3 \cdot 5 \cdot \frac{\sqrt{3}}{2}}{109,5}\right| \approx 381,5$

4. метод осн. ан. геом.

1. 4/3 т. перетину прямих

$x+2y-11=0$, $2x-y-2=0$ провести

пряму, яка перпенд. до $x+y=0$ (пр).

Визначити довжину ⊥ра, проведеного з початку к-т на шукану

пряму

2. Визначити ексцентриситет

еліпса, якщо його фокусна відстань

$=8$, а мала вісь 6.

1) Шукаємо перетин прямих

$$\begin{cases} x+2y-11=0 \\ 2x-y-2=0 \end{cases} \quad x=11-2y$$

$$2(11-2y)-y-2=0$$

$$22-4y-y-2=0$$

$$20-5y=0,\ y=4$$

$$x=11-2\cdot4=3$$

Перетин прямих

т. $M(3;4)$

$x+y=0$ і пряма, що проходить

ч/з т. M $\perp$ ні, $\Rightarrow$ $k_1 k_2 = -1$

$y=-x$, $k_1=-1$, $\Rightarrow$

$k_2 = 1$, $\Rightarrow$

$y = x+b$. $M(3;4)$; $4 = 3+b$, $b=1$

$y = x+1$ – перпендикулярна до $y=-x$.

Початок к-т $(0;0)$, $y-x-1=0$.

$$d = \frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}} = \frac{|-1|}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}};$$

2) $b=6$,

$c=8$

$$a^2 = b^2+c^2 = 6^2+8^2 = 10^2$$

$$e = \frac{c}{a} = \frac{8}{10} = \frac{4}{5} = 0{,}8$$

5. Піраміда $A_1A_2A_3A_4$

За доп. вект. ал:

1) кут φ між ребрами A_1A_2 і A_1A_3,

2) площу грані $A_4A_2A_1$,

3) проекцію вектора A_1A_2 на вектор $\overline{A_1A_4}$.

4) об'єм піраміди

Ан. геом:

1) р-ня ребра A_1A_4,

2) кут між ребром A_1A_2 і гранню $A_1A_2A_3$

$A_1(-3, 3, 0)$, $A_2(1, 0, 1)$

$A_3(1, 0, 3)$, $A_4(0, 3, 2)$

1) $\overline{A_1A_2} = (1-(-3), 0-3, 1-0) = (4; -3; 1)$

$\overline{A_1A_3} = (1-(-3), 0-3, 3-0) = (4; -3; 3)$

$$\cos\varphi = \frac{4\cdot4-3\cdot(-3)+1\cdot3}{\sqrt{4^2+3^2+1^2}\cdot\sqrt{4^2+3^2+3^2}} = \frac{28}{\sqrt{26\cdot34}} =$$

$$= \frac{14}{\sqrt{13\cdot17}} = \frac{14}{\sqrt{221}};$$

2) $\overline{A_1A_2} = (4; -3; 1)$

$\overline{A_1A_4} = (3; 0; 2)$

$$\overline{A_1A_2} \times \overline{A_1A_4} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 4 & -3 & 1 \\ 3 & 0 & 2 \end{vmatrix} =$$

$$= \bar{i} \begin{vmatrix} -3 & 1 \\ 0 & 2 \end{vmatrix} - \bar{j} \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} + \bar{k} \begin{vmatrix} 4 & -3 \\ 3 & 0 \end{vmatrix} =$$

$$= \bar{i}(-6) - \bar{j} \cdot 5 + \bar{k} \cdot 9$$

$$S_{A_4A_2A_1} = \frac{1}{2}\sqrt{6^2+5^2+9^2} = \frac{1}{2}\sqrt{36+25+81} =$$

$$= \frac{\sqrt{142}}{2} = \sqrt{35,5}.$$

3) $$\text{Пр}_{\overline{A_1A_4}} \overline{A_1A_2} = \frac{\overline{A_1A_2} \cdot \overline{A_1A_4}}{|\overline{A_1A_4}|} = \frac{4 \cdot 3 \quad -3 \cdot 0 + 1 \cdot 2}{\sqrt{3^2+0^2+2^2}} =$$

$$= \frac{14}{\sqrt{13}};$$

4) $\overline{A_1A_2} = (4; -3; 1)$; $\overline{A_1A_3} = (4; -3; 3)$

$\overline{A_1A_4} = (3; 0; 2)$

$$\overline{A_1A_2} \cdot \overline{A_1A_3} \cdot \overline{A_1A_4} = \begin{vmatrix} 4 & -3 & 1 \\ 4 & -3 & 3 \\ 3 & 0 & 2 \end{vmatrix} =$$

$$= 3 \begin{vmatrix} -3 & 1 \\ -3 & 3 \end{vmatrix} + 2 \begin{vmatrix} 4 & -3 \\ 4 & -3 \end{vmatrix} = 3 \cdot (-6) = -18$$

$$v_{A_1A_2A_3A_4} = \frac{|-18|}{6} = 3$$

Д.З. 1) A_1A_4 – р-ня $\qquad A_1(-3;3;0)$

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \qquad A_4(0;3;2)$$

$$\frac{x+3}{3} = \frac{y-3}{0} = \frac{z-0}{2} \quad (A_1A_4)$$

2) φ – ? $(\widehat{A_1A_2, A_1A_2A_3})$

Визначимо $(A_1A_2A_3)$:

$$\begin{vmatrix} x+3 & y-3 & z-0 \\ 1+3 & 0-3 & 1-0 \\ 1+3 & 0-3 & 3-0 \end{vmatrix} = \begin{vmatrix} x+3 & y-3 & z \\ 4 & -3 & 1 \\ 4 & -3 & 3 \end{vmatrix} =$$

$$= (x+3)\begin{vmatrix} -3 & 1 \\ -3 & 3 \end{vmatrix} + (3-y)\begin{vmatrix} 4 & 1 \\ 4 & 3 \end{vmatrix} + z\begin{vmatrix} 4 & -3 \\ 4 & -3 \end{vmatrix} =$$

$$= (x+3)\cdot(-6) + (3-y)\cdot 8 =$$

$$= -6x - 18 + 24 - 8y = \underline{-6x - 8y + 6 = 0}$$

$$-6x - 8y + 6 = 0 \quad (A_1A_2A_3)$$

$$\Rightarrow \quad \bar{n}(-6;-8;0)$$

тепер маємо 2 вектори:

$\overline{n}(-6; -8; 0)$

то $\overline{A_1A_2} = (4; -3; 1)$

$$\cos\varphi = \frac{-6\cdot 4 - 8(-3) + 0\cdot 1}{\sqrt{6^2+8^2+0^2}\cdot\sqrt{4^2+3^2+1^2}} = \frac{0}{10\sqrt{26}} = 0$$

$$\varphi = \frac{\pi}{2}$$

Контрольна робота №2

1. 1. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)=$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}+\sqrt{x^2-1}}=$$

$$=\lim_{x\to\infty}\frac{x^2+1-(x^2-1)}{\sqrt{x^2+1}+\sqrt{x^2-1}}=$$

$$=\lim_{x\to\infty}\frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}}=$$

$$=\lim_{x\to\infty}\frac{2/x}{\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}+\sqrt{\frac{x^2}{x^2}-\frac{1}{x^2}}}=$$

$$=\lim_{x\to\infty}\frac{\frac{2}{x}\to 0}{\sqrt{1+\frac{1}{x^2}}_{\to 0}+\sqrt{1-\frac{1}{x^2}}_{\to 0}}=\frac{0}{2}=0;$$

2. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+4}-2}{\sqrt{x+9}-3}=$

$$=\lim_{x\to 0}\frac{\left(\sqrt{x+4}-2\right)\left(\sqrt{x+9}+3\right)}{\left(\sqrt{x+9}-3\right)\left(\sqrt{x+9}+3\right)}=$$

$$= \lim_{x \to 0} \frac{(\sqrt{x+9}+3)(\sqrt{x+4}-2)(\sqrt{x+4}+2)}{(x+9-9)(\sqrt{x+4}+2)} =$$

$$= \lim_{x \to 0} \frac{(\sqrt{x+9}+3)\,\cancel{x}}{\cancel{x}(\sqrt{x+4}+2)} =$$

$$= \frac{\sqrt{0+9}+3}{\sqrt{0+4}+2} = \frac{3+3}{2+2} = \frac{6}{4} = 1{,}5;$$

3. $\lim_{x \to 0} \frac{tg\, 5x}{tg\, 3x} = \left| \begin{matrix} tg\, x \sim x \\ \text{при } x \to 0 \end{matrix} \right| =$

$$= \lim_{x \to 0} \frac{5x}{3x} = \frac{5}{3};$$

4. $\lim_{x \to \infty} \left(\frac{4x+1}{4x-3}\right)^{3x+5} = \lim_{x \to \infty} \left(\frac{4x-3+4}{4x-3}\right)^{3x+5} =$

$$= \lim_{x \to \infty} \left(1 + \frac{4}{4x-3}\right)^{3x+5} = \lim_{x \to \infty} \left(1 + \frac{1}{x - 3/4}\right)^{3x+5} =$$

$$= \lim_{x \to \infty} \left(1 + \frac{1}{x-3/4}\right)^{(x-3/4)\left(\frac{1}{x-3/4}\right)(3x+5)} =$$

$$= e^{\lim_{x \to \infty} \frac{3x+5}{x-3/4}} = e^{3};$$

2. $y'(x)$ - ?

1) $y = \left(\frac{x^2+3}{x^3}\right) - \sin(\sqrt{x})$;

$$y' = \frac{(x^2+3)'x^3 - (x^2+3)(x^3)'}{x^6} - (\sin\sqrt{x})' = \frac{2x\cdot x^3 - (x^2+3)3x^2}{x^6} - \cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}} = \frac{2x^4 - 3x^4 - 9x^2}{x^6} - \frac{\cos\sqrt{x}}{2\sqrt{x}} = \frac{2x^2 - 3x^2 - 9}{x^4} - \frac{\cos\sqrt{x}}{2\sqrt{x}} =$$

$$= \frac{-x^2-9}{x^4} - \frac{\cos\sqrt{x}}{2\sqrt{x}};$$

2) $\begin{cases} y = \sqrt[3]{t^2+2} \\ x = \sqrt{t-1} \end{cases}$

$$y'_x = \frac{\left(\sqrt[3]{t^2+2}\right)'_t}{\left(\sqrt{t-1}\right)'_t} = \frac{\frac{1}{3}(t^2+2)^{-\frac{2}{3}}\cdot 2t}{(t-1)^{-1/2}\cdot\frac{1}{2}} =$$

$$= \frac{4}{3}\,\frac{\sqrt{(t-1)}\cdot t}{\sqrt[3]{(t^2+2)^2}};$$

3) $x^2+y^2-xy=x+y$

$2x+2yy'-y-xy'=1+y'$

$2x-y-1=xy'+y'-2yy'$

$2x-y-1=y'(x+1-2y)$

$$y'=\frac{2x-y-1}{x+1-2y}$$

4) $y=x^x$

$y'=?$ $\ln y = x\ln x$

$(\ln y)'=(x\ln x)'$

$$\frac{y'}{y}=\ln x+\frac{x}{x}$$

$$\frac{y'}{y}=\ln x+1$$

$y'=y(\ln x+1)$

$y'=x^x(\ln x+1)$

5) $$y=\frac{e^{\sin x}}{\operatorname{tg}\sqrt{x}}$$

$$y'=\frac{(e^{\sin x})'\operatorname{tg}\sqrt{x}-e^{\sin x}\cdot(\operatorname{tg}\sqrt{x})'}{\operatorname{tg}^2\sqrt{x}}=$$

$$= \frac{e^{\sin x} \cdot \cos x \, tg\sqrt{x} - e^{\sin x} \cdot \frac{1}{\cos^2\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}}{tg^2\sqrt{x}} =$$

$$= \frac{e^{\sin x}(\cos x \cdot \cos^2\sqrt{x} \cdot 2\sqrt{x} \cdot tg\sqrt{x} - 1)}{tg^2\sqrt{x} \cdot \cos^2\sqrt{x} \cdot 2\sqrt{x}};$$

3. Р-ня нормалі і р-ня дотичної до кривої y т. з $x_0 = 2$

$$y = \frac{x^3+2}{x^3-2};$$

$$y - y_0 = y'(x_0)(x - x_0)$$

$$y' = \frac{3x^2(x^3-2) - (x^3+2) \cdot 3x^2}{(x^3-2)^2} =$$

$$= \frac{\cancel{3x^5} - 6x^2 - \cancel{3x^5} - 6x^2}{(x^3-2)^2} =$$

$$= \frac{-12x^2}{(x^3-2)^2};$$

$$y'|_{x_0=2} = \frac{-12 \cdot 2^2}{(2^3-2)^2} = \frac{-48}{6^2} = \frac{-48}{36} =$$

$$= -4/3;$$

$$y - y_0 = \left(-\frac{4}{3}\right)(x - x_0)$$

$$y - \frac{2^3+2}{2^3-2} = -\frac{4}{3}(x-2)$$

$$y - \frac{10}{6} = -\frac{4}{3}(x-2)$$

$$y = \frac{5}{3} - \frac{4}{3}x + \frac{8}{3}$$

$$y = -\frac{4}{3}x + \frac{13}{3}$$ — рівняння дотичної

$$y - y_0 = \frac{-1}{y'(x_0)}(x - x_0)$$

$$y - \frac{5}{3} = -\frac{1}{(-4/3)}(x-2)$$

$$y = \frac{5}{3} + \frac{3}{4}(x-2)$$

$$y = \frac{5}{3} + \frac{3}{4}x - \frac{6}{4}$$

$$y = \frac{3}{4}x + \frac{5}{3} - \frac{3}{2} = \frac{3}{4}x + \frac{1}{6}$$

$$y = \frac{3}{4}x + \frac{1}{6}$$ рівняння нормалі

4. 1) $\int\left(\frac{5}{x}-6e^{-4x}+6\cdot5^{3x}\right)dx=$

$=5\ln x-6\frac{e^{-4x}}{-4}+\frac{6\cdot5^{3x}}{3\ln5}+C=$

$=5\ln x+1{,}5e^{-4x}+2\cdot\frac{5^{3x}}{\ln5}+C;$

2) $\int\frac{1}{4x^2-24x+32}dx=$

$=\frac{1}{4}\int\frac{1}{x^2-6x+8}dx=$

$x^2-6x+8=0$

$D=6^2-4\cdot8=4$

$x_{1,2}=\frac{6\pm2}{2};\ x_1=4;\ x_2=2$

$x^2-6x+8=(x-4)(x-2)$

$=\frac{1}{4}\int\frac{1}{(x-4)(x-2)}dx$

$\frac{1}{(x-4)(x-2)}=\frac{A}{x-4}+\frac{B}{x-2}$

$A(x-2)+B(x-4)=1$

$x(A+B)-2A-4B=1$

$$\begin{array}{c|l} x & A+B=0 \\ x^0 & -2A-4B=1 \end{array}$$

$$A=-B$$

$$-(-2B)-4B=1$$

$$-2B=1, \quad B=-\frac{1}{2}$$

$$A=\frac{1}{2}$$

$$\frac{1}{(x-4)(x-2)}=\frac{1}{2(x-4)}-\frac{1}{2(x-2)};$$

$$=\frac{1}{4}\int\left(\frac{1}{2(x-4)}-\frac{1}{2(x-2)}\right)dx=$$

$$=\frac{1}{8}\ln(x-4)-\frac{1}{8}\ln(x-2)+C;$$

3) $\int \cos^8 4x \sin^5 4x\,dx=$

$$=\int \cos^8 4x\cdot(1-\cos^2 4x)^2 \sin 4x\,dx=$$

$$=\left|\begin{array}{l}\cos 4x=u, \\ -4\sin 4x\,dx=du\end{array}\right|=$$

$$= \int u^8 \cdot (1-u^2)^2 \cdot \frac{du}{-4} =$$

$$= -\frac{1}{4} \int u^8 (1 + u^4 - 2u^2)\, du =$$

$$= -\frac{1}{4} \int (u^8 + u^{12} - 2u^{10})\, du =$$

$$= -\frac{1}{4} \left(\frac{u^9}{9} + \frac{u^{13}}{13} - 2\frac{u^{11}}{11} \right) + C =$$

$$= -\frac{1}{36} \cos^9 4x - \frac{1}{52} \cos^{13} 4x +$$

$$+ \frac{2}{44} \cos^{11} 4x + C \;;$$