Always silent? Endogenous central bank communication during the quiet period: a model of managing uncertainty and financial market fluctuations

Oleg Telegin

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Abstract

This paper analyzes whether the central bank needs to always adhere to the quiet period policy. The financial market model describes a multivariate trade-off in which the central bank not only focuses on the instantaneous reaction of markets to a breach of the quiet period regime but also estimates both the effects of an upcoming Board meeting and changes in market volatility. Then, it sometimes becomes more appropriate for the central bank to intervene in financial markets during the quiet period. The main drivers of this communication are the willingness to pay attention to more than just the immediate consequences of the intervention and the allocation of uncertainty for market participants between the central bank's reaction function and the uncertainty associated with the dissent during the Board meeting itself. Such communication should take a collegial approach and may contain some interesting properties, such as response asymmetry - a central bank's less willingness to tell markets negative news about its assessment of the economic conditions. Resolution of uncertainty associated with such communications may also impact the current state of the quiet period with emerging leaks, individual breaches, and unattributed informal communications.

Introduction

Many central banks adhere to some form of quiet period policy days prior to the Board of Governors meetings, during which no market-sensitive statements are allowed. This period can be called and defined in different ways. For example, the European Central Bank calls it the quiet period and defines it as the period beginning one week before and ending with the meeting. The Federal Reserve calls it the blackout period. It starts on the second Saturday before the meeting. It ends on the Thursday after the meeting, meaning that the blackout period starts ten days before and ends the day after for a normal Tuesday through Wednesday meeting. For this paper, we will use "quiet period" and "blackout period" interchangeably without implying any particular period length. We will also refer to a meeting at which the key rate decision is made as the Board of Governors meeting, although governing bodies and their meetings are also called differently for different central banks. Thus, for the same number of 8 meetings per year, the quiet period amounts to 15% of all days in a year for the ECB and 28% of all days for the Fed. And such a wide window is not uncommon - many central banks have a quiet period policy similar to the ECB. Given the increased communication activity of central banks in recent decades, such large periods with no official communications result in a highly uneven distribution of communications throughout the year. Though some communications do enter the market during the quiet period - these are unattributed communications of the "sources say" kind and individual quiet period breaches when Board of Governors members speak out on sensitive topics despite the ban. And the results of Gnan, Rieder (2023) and Ehrmann, Gnan, Rieder (2023) suggest that such communications not only move markets significantly but also represent a broadcast of dissent views that introduce noise rather than help to form a correct opinion about the regulator's policy. Therefore, the question to be answered is whether the unconditional quiet period policy is the first-best regime of central bank communication or whether there is a more flexible communication policy that yields better results.

To answer this question, our paper looks at the three-dimensional trade-off that a central bank faces while making a decision, combining both aspects currently considered by regulators and the new ones. What logic is being voiced behind the current quiet period regime? The main reason sounded is "to help prevent excessive market volatility or unnecessary speculation" (a phrase that has migrated from the seminal paper on quiet period by Ehrmann, Fratzscher (2009) directly into the logic of the ECB's official explanation). Another possible reason for the quiet period is the discussion of the forthcoming decision, which can, in particular, determine the duration of the quiet period. Thus, the Fed, which changed the start of its blackout period from 7 days before the meeting to 10 days before the meeting in 2017 (FOMC Transcripts), was guided largely by the fact that ten days before the meeting Committee participants are sent draft monetary policy alternatives and communications by officials after this date might disclose the information contained therein.

So first, the central bank focuses on the shock to the market that occurs immediately after a possible breach of the quiet period. Second, the central bank is also trying to avoid major market shocks on the press release day. This is precisely one of the basic mechanisms of our model - the central bank has to choose between a strong shock to markets at different points in time. Third, the central bank is worried about the market uncertainty regarding the Board of Governors meeting: too much uncertainty leads to increased market volatility, while too little uncertainty leads market participants to expect no surprises from the regulator, perceiving the communications as binding commitments.

That is, this paper models the actions of the central bank, which tries to prevent major market shocks, both during the quiet period and after the press release. At the same time, the regulator wants to leave itself room for maneuvering without blocking the expectations of market participants on a single option on the day of the meeting. For this purpose, the central bank separates two types of uncertainty: uncertainty about the central bank's reaction function to macroeconomic shocks and uncertainty about the Board's disagreement, which may translate into different rate decisions on the day of the meeting. For example, a central bank can remove uncertainty about its own reaction function by partially publishing the information contained in the draft of alternatives. In this case, the central bank cannot directly report the magnitude of the Board's dissent (communicating its assessment of economic conditions, formed before the start of discussions on the decision within the Board of Governors) but takes the dissent into account when deciding whether to intervene. Accordingly, the central bank makes a decision based both on its utility function and the market's expected reaction to the decision. As a result, sometimes it becomes optimal for the central bank to communicate during a quiet period, even though such communication does shake the markets considerably. On the other hand, in this case, markets do not react as strongly to the press release, better understand the reaction function of the central bank, and have a better idea of the level of dissent within the Board that is not distorted by the expression of dissenting views in the leaks, individual breaches or unattributed informal communications.

Since such a mechanism, in which the central bank intervenes verbally occasionally during a quiet period, can be quite difficult to implement, as an additional exercise, we also compare the current "never intervene" regime with the opposite "always intervene" regime. In such a scenario, the central bank can no longer communicate anything to the markets about the degree of uncertainty of the key rate decision in advance. But even the possibility of providing information about the central bank's reaction function is enough for the "always intervene" regime to lead to better results. This result starts to break down only if the central bank is much more resistant in its utility function to price shocks during the quiet period but ignores shocks resulting from the press release.

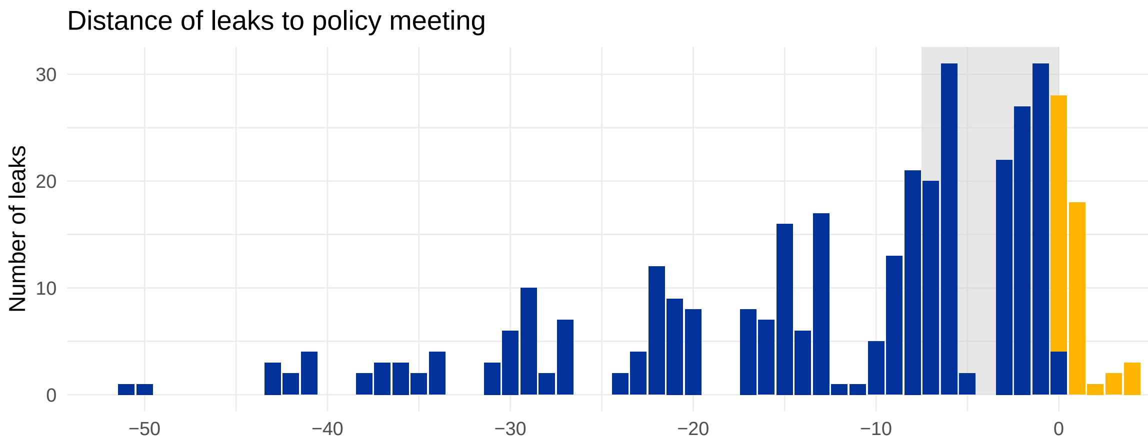
The further article is organized as follows. Section 2 discusses the paper's contribution to the literature on central bank communication, the quiet period, and pre-announcement drift. In Section 3, we formulate the model assumptions, the central bank's solution to the quiet period disruption problem, and our modeling approach. We then discuss the model results, how the presence of a multivariate trade-off between market shocks and uncertainty management affects central bank behavior, and possible methods for implementing the model's recommendations.

Literature, discussion and contribution

Our paper contributes to several literatures. First, we study the quiet period. Current research, such as Istrefi, Odendahl, and Sestieri (2022), suggests the importance of communications outside of regular monetary policy meeting days. Moreover, Bianchi, Ludvigson, and Ma (2022) showed that most of the variation in beliefs about future Fed policy doesn't occur around the FOMC announcement dates. Meanwhile, the possible desire of policymakers to prepare markets for the coming decision leads to more frequent communications before the meetings rather than after, especially before rate changes, as found in Ehrmann, Fratzscher (2007), highlighting the importance of the quiet period. According to van Dijk, Lumsdaine, and van der Wel (2016), the central bank may be quite successful in this policy, and the markets may set-up well in advance of known announcement days. Such informal communications also have limitations, Galloppo et al. (2021) inquired that the effect becomes weaker if messages start to be repeated. However, Ehrmann, Fratzscher (2009) found that the reaction of markets to news within a quiet period is strong enough to talk about an excessive shock to the markets. In addition, such news increases volatility. Despite the general idea of welfare-reducing communication, the authors in their discussion leave the possibility of not only withholding information but also mention that it can be "channeled in a specific manner". It is this idea of specific collegial communication, which aims to reveal the reaction function of the central bank but leave the uncertainty associated with board members' deliberations, that we model in this paper. At the moment, the quiet period policy means that central banks do not speak out on sensitive topics in an official way. However, there are still violations of the quiet period policy by individual Board members. Gnan, Rieder (2022) subsequently analyzed the database for all quiet period breaches. They not only confirmed and extended Ehrmann, Fratzscher (2009) findings of severe market shocks caused by quiet period violations but also found that inflation deviations and interest rate spreads of policymakers' constituencies were the main determinants of Governing Council members' violations. The influence of regional variables on the communication of policymakers is not unique to Europe; for the US, this issue has been studied by Hayo, Neuenkirch (2012).

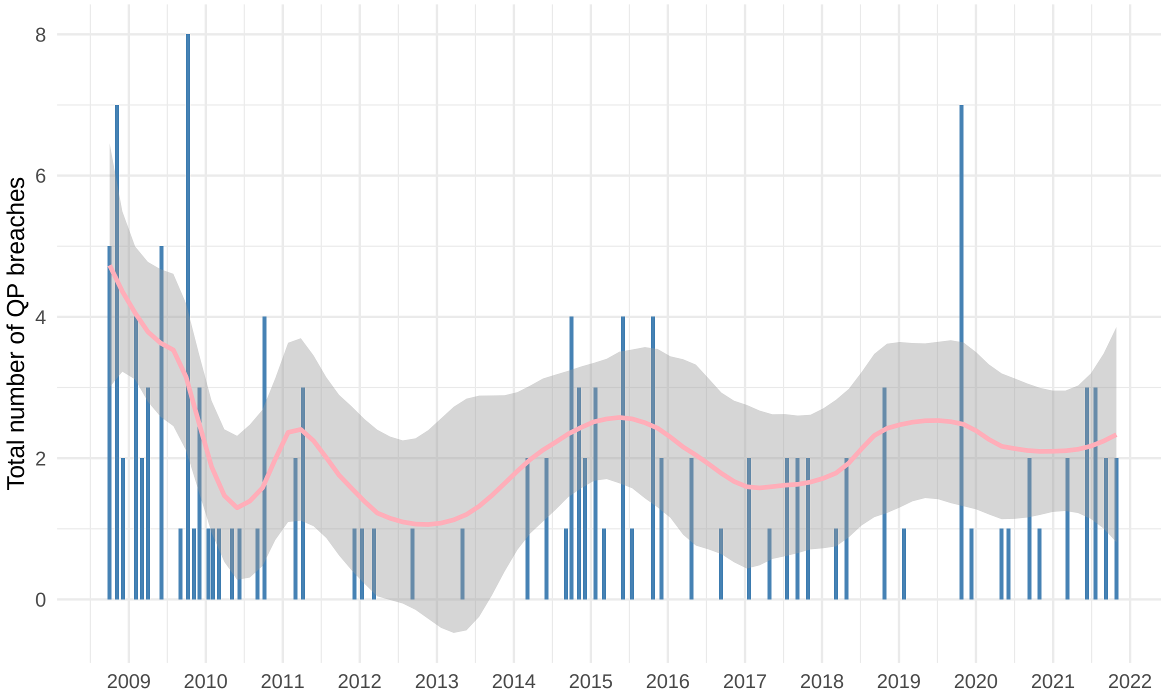
Quiet period breaches can also be carried out as anonymous unattributed communications, and Ehrmann, Gnan, and Rieder (2023) found that such communications are probably not plants, so they express dissent views. At the same time, the markets probably perceive the situation differently. This is evident from the significant reaction to such unattributed communications and the vast attention paid to so-called Fed whisperers - journalists rumored to be a regular source for Fed leaks. The simplest current example would be Nick Timiraos from the Wall Street Journal and his article The Wall Street Journal (2022) with a prediction that went against the consensus forecast of investors. Despite this fact, being released during a quiet period two days before the press release, the article was taken seriously. It influenced the expectations of markets, who saw the article as a prominent example of leakage. In the graph below from Ehrmann, Gnan, and Rieder (2023), we can see that the number of unattributed publications rises strongly shortly before the meetings (in blue) and drops immediately after the press release is published (in yellow):

Figure 1 - Time distribution of leakages



In addition, the graph below from Gnan, Rieder (2023) with the number of breaches prior to separate meetings shows that non-anonymous quiet period breaches occur regularly as well:

Figure 2 - Breaches of quiet period rules



Also, Ehrmann, Gnan, and Rieder (2023) found that attributed communication can effectively mitigate the effects of leaks, whether they are plants or an expression of dissenting views (and our paper will be agnostic as to which explanation is closer to reality). A similar idea is also explored in Vissing-Jorgensen (2020), proposing a consensus-building approach similar to that of the ECB as one way to deal with the barrage of unattributed informal communications. Hence, the issue of having official communications during the blackout period considered in our model is also a matter of counteracting the actions of certain Governors trying to pursue their agenda by adding noise to market assessments of the situation. Cacophonic communications, which can also increase due to the lack of centralized communications, can be detrimental to the welfare. Lustenberger, Rossi (2018) found that such communications result in larger macroeconomic forecast errors, Ehrmann, Fratzscher (2013) showed that insufficiently unified communication accounts for one third to one half of the market's prediction errors of FOMC policy decisions. And Vissing-Jorgensen (2019) concluded that the so-called "communication arms race" might damage both the central bank's reputation and decision-making process. A common line of thought proposed by Vissing-Jorgensen (2019) was to reduce the "lack of understanding of the Fed's decision rule." In our model, we describe exactly the communications that tell the markets Fed's reaction function while still leaving some uncertainty. However, specific suggestions of Vissing-Jorgensen (2019) were to "reduce the number of Federal Reserve districts and avoid FOMC rotation," and our model considers only the short quiet period and parses possible official collegial communications during this period.

In addition, Tillmann, Walter (2019) found that divergence between monetary authorities (the ECB and the Bundesbank, in that case) leads to higher policy uncertainty, market volatility, and higher risk premium. This result has a twofold impact on our problem. On the one hand, it confirms the intuition behind our model and the suggestion of introducing centralized communication during a quiet period. But on the other hand, this result makes a potential empirical analysis of the problem very difficult. The available quiet period communications may be perceived by markets quite differently from the counterfactuals under consideration - potential "one voice" communications aiming to reveal the central bank's assessment of the current situation.

Second, we contribute to the more general literature on central bank communication effects. The study of central bank communication problems has its origins in the rise of openness of central bank communications in the 90s and 00s, but more general Sender-Receiver communication models trace their roots back to earlier work such as Crawford, Sobel (1982). Subsequently, Morris, Shin (2002) opened up a new debate on the possible negative effects of excessive provision of public information on public welfare. After that, for the case of central bank communications, in Morris, Shin (2005), greater transparency might reduce the signal value of private sector actions. To date, the problem of central bank transparency and commitment forms a significant part of the more general problem of optimal central bank design, as can be seen in the review by Reis (2013). Also, in the modern context, according to Hahn, (2012), it is becoming easier for economic agents to obtain information independently from the central bank, which increases the importance of more open communication about the central bank's assessment of the economy. Empirically, the optimality of some intermediate level of monetary policy transparency was confirmed by Van der Cruijsen, Eijffinger, and Hoogduin (2010) for the special case of inflation persistence as a dependent variable. However, an alternative view is presented in Svensson (2005), which shows that for realistic assumptions about the parameters of the model, the result of Morris, Shin (2002) "is actually pro-transparency, not con." Another addition to this discussion was Roca (2010), in which a comparison of the beneficial effects of reduced imperfect common knowledge as a result of greater transparency and the negative effects of a potential rise in aggregate volatility concluded in favor of the steady dominance of the former. Our paper advocates an increase in transparency; however, given the existing constraint from above - the central bank cannot report to the markets precise information about the level of dissent within the Board itself and cannot take details of internal deliberations to the public.

The benefits of transparency also depend on the balance between the precision of private and public information. For example, in the set-up of Amato, Morris, and Shin (2005), when private information is very precise, high precision of public information can lead to market overreaction and drift away from fundamentals. On the contrary, the current paper investigates a period of high uncertainty and lack of precise private information, which rather points in favor of greater transparency. Another consequence of high uncertainty, as Born, Dovern, and Enders (2020) showed, is an increase in the market's reaction to the news, or, as has been shown in detail in Kurov, Stan (2018), for monetary policy the response is at least amplified in the Treasury, interest rate and foreign exchange markets. The key point of our model is that the central bank tries to manage the uncertainty, which in turn affects the risk premium demanded by investors. The strong interdependence of uncertainty associated with macro announcements and the risk premium was shown in Londono, Samadi (2023). In addition, Pflueger, Rinaldi (2022) also found that the risk premium accounts for a large part of the shock from monetary policy actions. Thus, our model contains a simple trade-off - an additional communication shock during the quiet period reduces uncertainty, changing also the market shock at the press release date.

Why do the fluctuations in financial markets that underlie the logic of the quiet period matter to the central bank? The problem of the two-way influence of monetary policy and the financial market is well researched; in particular, by examining notes of Fed's internal deliberations, Cieslak, Vissing-Jorgensen (2021) found that "Fed views the stock market as informative for policy-making," giving more weight to the stock market as one factor influencing the economy than to the stock market as a good predictor of the future direction of the economy. Theoretically, the monetary authorities' attention to financial markets can try to dampen waves of optimism and pessimism if they are not related to fundamentals as in Ifrim (2021). Similarly, in Caballero, Simsek (2022), the central bank mitigates shocks originating in the financial market itself but is willing, on the contrary, to increase the volatility of asset prices and the aggregate risk premium in case of imbalances in the real economy, using the financial market as the fastest transmission mechanism that can counter such shocks. However, this central bank's focus must have limitations. One possible negative effect of the so-called Fed put may be the potential moral hazard effects of loose policy (although Cieslak, Vissing-Jorgensen (2021) found that this effect is a concern for a relatively small fraction of policymakers). In addition, Morris, Shin (2018) deconstructed that when choosing how much weight to place on market signals and how much to other information, the central bank faces a trade-off - the more weight a market signal has, the less informative that signal becomes due to its own endogeneity. Thus, if the central bank can commit to its guidance, it significantly underweights market signals.

Another area that has received considerable attention in current research on communication effects is the Bayesian persuasion models. Kamenica, Gentzkow (2011) explored an idea, close to our model, of a Sender, sending a signal to a Receiver with the intention of influencing his actions. This paper even discusses that special case "where ω (state of the world) is a real-valued random variable, Receiver's action depends only on the expectation $\mathrm{E}[\omega]$," which makes the models even more similar. In addition, this approach is also applicable in the more special case of central bank communication. Herbert (2021) parses a set-up with heterogeneous Receivers in which the central bank sends signals about the state of the world to influence investment decisions. And the optimal strategy is to send moderating signals, calming investors in overly good/bad times, where the bias itself negatively depends on the dispersion of investor beliefs. Also, Gati (2022) investigated that when endogenous financial market's prior beliefs are included in the model, they can dampen the persuasiveness of the central bank's signal so that the central bank has to send signals that are not very precise, so that not very tight priors leave some room for communication efficiency. However, three significant differences exist between the Bayesian persuasion set-up and ours. First, the Sender, in our case, is the analytical department of the central bank, and it cannot design or select a signal to influence expectations in a certain way. It can only disclose to the public the information provided to the members of the Board of Governors. In addition, in our model, investors receive several signals about the state of the world: an initial shock unrelated to the monetary authority, the central bank's reaction to this shock, and the results of the discussion during the Board meeting. Finally, because our model operates on a shorter time horizon and models the financial market, the uncertainty aversion leads to a different risk premium. In the following work Gentzkow, Kamenica (2017) compare the results of mandatory disclosure with optional disclosure for a model in which experts send signals to Receivers about the results of experiments (whose informativeness is not costless) about the state of the world. Its result is that endogenous information will always be disclosed, and disclosure requirements thus have no impact on outcomes. The difference with our model is that the central bank cannot disclose all the information it has because it isn't able to disclose full information about the level of disagreement within the Board. From the perspective of Gati (2022), this addition of uncertainty also prevents the formation of too-tight prior beliefs and does not stifle the effectiveness of communications.

In our model, we allow the central bank's analytical department to publish its results, roughly speaking Tealbook, assuming that it describes the expected value of the decision without considering the uncertainty associated with Board members' disagreement. This premise is a simplification of the discussion started by Romer, Romer (2008), who argued that, in fact, policymakers (in that case, FOMC) do not really add value in terms of having useful information, to the forecasts of the Fed's stuff. This result was further extended by Binder, Wetzel (2018), showing that policymakers do not add value during normal times, while their forecasts can still carry useful information in worse conditions. Another view of Ellison, Sargent (2009) was that forecasts might add value, but in the sense that forecasts are worst-case scenarios used to design decisions that are robust to the misspecification of the model. But then we would, for simplicity, assume that markets can account for that and incorporate such caution into their perception of the emerging communication.

Third, we contribute to the large literature on the pre-announcement premium and, more general, studies of financial markets around major events. Starting from the seminal paper of Lucca, Moench (2015), a significant body of work examines the puzzle of large average excess returns in anticipation of monetary policy decisions. The pre-announcement drift pattern found in Lucca, Moench (2015) may undergo slight changes over time. Alam (2022) found that pre-FOMC drift is characteristic of only part of the meetings - following large key macro data releases a few days before the meeting. Lucca, Moench (2018) revisited more recent data (up to 2018) and found that drift remains, but only for meetings followed by a press conference by the Chair. In addition, sometimes, the pre-announcement premium can shift a bit within the FOMC cycle. Gu, Kurov, and Wolfe (2017) showed that if after the meeting there will be a publication of Summary of Economic Projections and a press conference, a part of the premium can be realized after the publication of the press release, due to the same mechanism of resolution of uncertainty.

This phenomenon is not specific to U.S. securities. Thus, pre-announcement drift was recorded in the EU in the paper of Ulrich et al. (2017), and in China in the paper of Guo, Jia, and Sun (2022) (for uncertainly defined dates of announcements). In addition, a similar drift has been recorded in Hillenbrand (2021) for long-term interest rates, with the secular decline in U.S.

Treasury yields almost entirely concentrated inside the short windows around Fed meetings. Corporate bonds have seen a similar phenomenon (Abdi, Wu (2018)), and it even precedes the movements in stocks. Nor is this phenomenon exclusive to announcements by the monetary authorities. Fed news has a larger effect compared to the rest of the macro-announcements. Still, the latter can also be characterized by positive average returns realized before the announcement (Hu et al. (2022)).

There are several possible explanations for the observed drift. In our model, we start from the basic idea of uncertainty resolution, supported by empirical observation of Gao, Hu, and Zhang (2020) that the pre-announcement returns are higher for firms with high uncertainty and under conditions of higher aggregate market uncertainty. Also, Bauer, Lakdawala, and Mueller (2019) described the dynamics of uncertainty within the FOMC cycle with the resolution of uncertainty around announcements, where the level of uncertainty positively affects the magnitude of financial market reaction. The pattern of decreasing uncertainty in FOMC meetings was also described by Krieger, Mauck, and Chen (2010) for the VIX - Chicago Board Options Exchange Volatility Index (which is one of the most appropriate interpretations of the variance of future returns in our model) as an uncertainty metric. Beckmeyer, Branger, and Grünthaler (2019) showed that the main contributor to uncertainty is left-tail uncertainty, leading to more expensive insurance for bad economic states due to supply shocks in the market for crash insurance. The endogenous central bank communication considered in our model may, in particular, reduce this additional premium through the disclosure of information and reduction in the probability of large shocks on the day of the Board meeting.

But then, it is possible to go further in different directions to explain the intuition behind the resolution of uncertainty. A significant body of research has focused on explaining the puzzle through information leaks. Starting with short-term evidence of informed trading in the 30-minute news lockup period before the FOMC announcement (when government agencies provide accredited news outlets with pre-release access to information under embargo agreements) as in Bernile, Hu, and Tang (2015) and Kurov et al (2016). Similarly, as shown in Kurov, Sancetta, and Wolfe (2019), drift weakened after discontinuation of early access in the UK. However, the time period can be extended to several days before the meeting, as in Mano (2021). And evidence of markets receiving information from the Fed during the blackout period exists right up to the start of the quiet period as in Bradley et al. (2020). In our paper, we consider time periods of a few days as we try to both respond to some macroeconomic shocks at the beginning of the quiet period and smooth out both the long lack of communication within the quiet period and the sharp reaction to the press release publication. Ying (2020) explanatory theoretical mechanism includes risk compensation for the market makers who update their beliefs by observing informed trading in the period leading up to the meeting.

How exactly informed traders are informed about the upcoming decision is also an open question. Mano (2021) showed that "the Fed's informal communication with the financial sector seems to be driven by the non-voting members of the FOMC." Morse, Vissing-Jorgensen (2020) points to the role of communication between Federal Reserve governors and Federal Reserve Bank presidents. Cieslak, Morse, and Vissing-Jorgensen (2019) discussed that systematic informal communication could be in the form of both outright leaks and systematic preferential access to the Fed for some private financial institutions. The authors also discuss possible motivations for such communications applicable to the entire FOMC cycle (the paper's main result is that equity premium is earned entirely in weeks zero, two, four, and six in the FOMC cycle). For example, systematic informal communications provide flexibility in implementing more continuous policy, which also might be an argument for having some form of communication during the quiet period. Also, such communications provide "a channel for learning how the Fed's assessment of the economy compares to that of the financial sector and how markets are likely to react to a particular policy decision." Public central bank communications then remove the information asymmetry, whereby individual financial institutions are more informed. In addition, communications are a way for individual board members to drive the market perceptions toward their optimal policies, which again gives rise to cacophony communications, and thus, centralized communication might be an improvement.

Another possible explanation for the pre-announcement drift is the information acquisition by some market participants as in Ai, Bansal, and Han (2021). The main empirical fact that these models try to accommodate is that information leaks are not readily consistent with reduced realized volatility during the drift period. Another inconsistency of the information leakage hypothesis may be the inconsistency between positive drift and the results of Bradley et al. (2020), which showed that one of the causes of informal communications in the run-up to Fed meetings may be, on the contrary, information gathering via face-to-face interactions when the Fed possesses negative private information about the condition of the economy, which shouldn't end in a positive drift. The result of Bradley et al. (2020) also brings the following rationale to the disclosure of information during the quiet period. If some of the information about the state of the economy was obtained through informal communications with market participants together with two-way information exchange, then greater openness in the public disclosure of this information removes the asymmetry of information created by the Fed itself. In the information acquisition hypothesis, investor heterogeneity appears in the model. However, the empirical evidence can be explained by different variations of investor disagreement. Ai, Bansal, and Han (2021) used a set-up in which the uninformed are incentivized to bear the costs of obtaining the information and liquidate their lack of information over the informed just before the release of an important macro-announcement. An alternative option is used in Cocoma (2017), where investors are divided into those who completely trust the Fed's communication and those who completely disbelieve it.

One piece of evidence for a practical information acquisition mechanism was described in Ehrmann, Hubert (2023) - the intensity of monetary policy discussion on Twitter during the quiet period is associated with the smaller surprises on the day of the meeting. Another is Gu, Kurov (2017), showing that the pre-announcement premia in gas futures trading can be partly explained by the superior information of some of the market participants with high historical forecasting accuracy. Zhu (2021) also found that the presence of private information (which can arise from both leakage and heterogeneity in the ability of traders to process public information) causes volume dynamics to decline before announcements and emerging liquidity shocks can explain one-third of the pre-FOMC drift. An idea related to the current study - central bank communication as a cause of information acquisition before announcements - was explored in Tsukioka, Yamasaki (2020), who showed that positive news in Biege book could be one explanation for the drift, acting as information that was always public but was not processed by all investors. However, the Biege book is published before the quiet period, two weeks before the meeting. Thus, similarly, disclosure of some of the Tealbook information within the quiet period might react to a little more recent news and might act as a smoothing factor, making central bank communication more continuous. However, we remain at a more general level of modeling in this paper, without modeling investor heterogeneity or taking sides on any of the drift's explanations, a simplified version of the model we use from Hu et al. (2022) already explains a wide range of empirical facts about drift quite well.

What is the difference between the problem at hand and the pre-announcement puzzle? We do not directly model the pattern "large pre-announcement returns with small variances, followed by small post-announcement returns with large variances" from Hu et al. (2022) in our paper.

We use an approach that models the pre-announcement puzzle well, but we do not use the entire period in which uncertainty builds up and then resolves. Instead, we allow the central bank to act endogenously within this period, generating excess returns, affecting both returns and stock price volatility. We are not interested in average returns; the central bank is only trying to avoid large market shocks, in contrast to the work on pre-announcement premiums. Accordingly, the model may also be left to answer how uncertainty is resolved. Possible explanations include the information leakage hypothesis of Cieslak, Morse, and Vissing-Jorgensen (2018) and the active endogenous information acquisition hypothesis, which partially resolves the uncertainty for uninformed traders from the work of Ai, Bansal, Han (2021). In our model, the central bank is able to self-manage the reduction of this uncertainty by official communications, leaving only a fraction of the uncertainty to be shared by possible explanations for the currently observed pre-announcement puzzle.

The pre-announcement premium both in the original Lucca, Moench (2015) paper and in further Lucca, Moench (2018) concentrates in a very narrow window before the press release of about one day. Clearly, suppose the central bank does allow interventions during the quiet period. In that case, they are more likely to occur earlier, closer to the middle of the quiet period, displaying both the idea of reacting to unexpected news within the quiet period and the idea of covering the timeline more evenly with central bank communications. Thus, instead of a simple "uncertainty is generated - uncertainty is resolved in the last day before the meeting" mechanism in our model, the central bank can precede the second step with its interventions by resolving uncertainty earlier. This action generates excess returns, but at the same time, it reduces market volatility and lowers the expected magnitude of the shock at the time the key rate decision is released. In addition, the central bank separates two types of uncertainty: uncertainty about the central bank's reaction function and uncertainty about the Board's disagreement, which may translate into different monetary policy decisions on the day of the meeting. In the model, the central bank decides whether to completely eliminate the uncertainty associated with the markets' misunderstanding of its reaction function but is more cautious concerning the uncertainty about the Board of Governors' decision. By not providing precise information regarding this uncertainty, the central bank nevertheless partially reduces this uncertainty as well. On the normative side, this paper does not address the current ban on sensitive communications by individual central bankers. Still, it advocates the addition of "one voice" communications during the quiet period that can resolve some of the uncertainty generated ahead of the Board of Governors meeting while at the same time not locking the central bank into the one monetary policy decision option that markets expect following the meeting.

Model

The model description consists of several ingredients: description of shocks, assumptions, and timing of events, and the last is the financial market model with the central bank's endogenous communication.

Securities market

The model is a financial market economy with four dates, $t = 0, 1, 2, 3$. We consider a financial market with two primitive assets - a share and a bond, similar to Hu et al. (2022) model. Each unit of the bond yields a terminal sure payoff of 1 at t = 3. Each share of the stock pays a terminal risky payoff D at t = 3. However, compared to Hu et al. (2022), our model has a more complex timeline and structure of shocks, some of which are endogenous. D is given by $B = \bar{D} + \sigma\varepsilon = \bar{D} + \sigma(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$, where $\varepsilon_1 \in S_1$, $\varepsilon_2 \in S_2$, $\varepsilon_3 \in S_3$ are the market-moving news (their properties will be described in more detail below) and σ^2 shows the size of its impact on asset payoffs. The simplicity of the financial market in the model is partly justified by the ubiquity of the main mechanism of the model - building up and resolution of uncertainty, which could lead to the pre-announcement drift. We mainly consider the stock market, however, since drift is also observed in bonds (Abdi, Wu (2018)), and for interest rates (Hillenbrand (2021)) our securities market can be interpreted in slightly different ways.

Investors

Following Hu et al. (2022) there is a unit mass of identical, infinitesimal, and competitive investors who are endowed with zero units of the bond and one share of the stock. In addition, we assume that all investors have CARA utility over their terminal wealth: $U = -exp\{-\alpha W_3\}$, where $\alpha > 0$ is the risk aversion coefficient and W_3 is the wealth at t=3.

Central bank

In the model, the central bank is solving the following problem: first, it needs to determine whether to breach the blackout period regime and second, to decide on the fundamental, say the press release containing key rate and the forward guidance, on the day of the Board of Governors meeting.

What underlying communication design and central bank decision-making mechanism do we have in mind? At date 1, the market receives a news shock $\varepsilon_1 \sim \mathrm{N}(0;\delta_1)$. It can be, for example, the next release of macroeconomic statistics or any other unexpected event. It happens at the beginning of the quiet period; the central bank is in the process of preparing for the next meeting of the Board of Governors, which is why certain members of the Board of Governors have no right to speak in public. In this case, the analytical department of the bank (this entity may be called differently, depending on the regulator, but we will call it the analytical department hereafter) has to recalculate the results of the used models with the newly obtained data and prepare an analytical note describing the state of affairs in the economy by this moment, which will lie on the Board of Governors' table on the day of the meeting or during the decision process if it takes an extended period of time. The results presented in this report can be considered a starting point for discussing the meeting's decision. This assessment of the situation by the analytical department of the central bank will be our possible communication within the quiet period - a possible shock ε_2, which the central bank can communicate (or not) to the market. The form of communication can be different, for example, it is possible to publish the full analytical report, say the whole Tealbook. It is also possible to publish only partial information or even only a short message with an assessment of how the news shock influenced the regulator's assessment of the economic stance but without monetary policy alternatives included. However, the communication's nature and structure remain partly outside this paper's scope. The shock ε_2 depends on the shock ε_1. We will assume that they form a bivariate normal distribution and denote their correlation coefficient by ρ. The unconditional distribution of ε_2 is $\varepsilon_2 \sim \mathrm{N}(0;\delta_2)$, i.e., all other previous shocks are already taken into account both by the markets and the central bank. But the shock ε_1 and the reaction to it might affect ε_2. We will be agnostic about the magnitude of ρ but not about its sign. $\rho < 0$ represents the degree of countercyclicality of central bank policy. That is, in the case of a modulo large value of ρ, we get a strongly countercyclical central bank, which is more likely to respond to an unexpected positive shock ε_1 with a more restraining policy ($\varepsilon_2 < 0$) and vice versa. We end up with a conditional distribution $\varepsilon_2 \sim \mathrm{N}\left(\rho\sqrt{\frac{\delta_2}{\delta_1}}\varepsilon_1;\delta_2(1-\rho^2)\right)$. The intuitive rationale for such communication lies not only in the policymaker's previously discussed desire for more continuous communication but also in the fact that an effective communication policy should close the information deficit of markets, for which, as discussed in Byrne et al. (2023), it is also important to communicate how central banks "are assessing incoming data, and how this affects their thinking about both the current (evaluation) and future (projection) state of the economy." Also, the importance of news before FOMC meetings was highlighted in Laarits (2019) - investors may perceive the announcements themselves differently - as signals about economic conditions after positive news and as signals about the Fed's own policy stance after poor news. Thus, in communicating its assessment of the economic conditions, the Fed can close the gap between its own more accurate assessment and the uncertain assessment of investors.

At the same time, the resulting decision of the meeting may be different from the presented results of the note. The Board members use it only as a starting point for discussions. Still, as a result of the debate on the optimal monetary policy, they may make a decision that deviates from the perceived signal of the analytical note. This means there is also a third shock ε_3 in the model at date 3, resulting the discussion of the Board's members. However, we will assume that the estimation of the analytical department is unbiased, i.e., $E[\varepsilon_3] = 0$. The intuition behind this premise can be divided into two parts. The analytical department is not interested in adding known bias themselves for reputation-building reasons, since for investors to perceive the communication as an unbiased forecast, as shown in McMahon, Rholes (2023), previous forecast performance is crucial. And if, on the contrary, bias arises because of the Board filling with different views, we will assume that investors may well account for that, knowing the average preferences of the policymakers so that they will include that bias in their estimate of ε_3, and this addition is also common knowledge. An alternative view of $E[\varepsilon_3]$ is also possible, i.e., a situation in which the shock at date 3 has an expected value different from zero due to a shift in the composition of the Board of Directors either towards hawkes or doves. In such a case, another measure of uncertainty is added to the model, which has not been investigated to date. Cieslak, McMahon (2023) showed that the internal policy stance, which might also be revealed through internal FOMC deliberations, has predictive power for the risk premium. However, the predictability of this internal stance increases smoothly in the weeks after the meeting, and we do not have a clear theoretical model of the markets' expectations of this variable formed before the meeting. Therefore, for simplicity, we take the expectation of this hawks and doves imbalance to be negligible. Also, the results should not change much either if markets have relatively accurate information about such hawks/doves imbalance and can at least estimate the value of E$[\varepsilon_3]$ correctly.

Without generality restriction, we will assume the third shock to be independent of ε_1 and ε_2 and to have variance $\beta\delta_3$. The variance, in this case, consists of two summands. δ_3 is known to investors and the central bank from date 0 and represents the ex-ante expected uncertainty about the board's deliberations on the optimal decision on the meeting day. $\beta \in B$, on the other hand, is the second component of information asymmetry in our model, which becomes known to the central bank's analytical department at date 2 but remains unknown to investors. β represents the degree of confidence that the decision on the day of the rally will not deviate from the results of the analytical work presented to the members of the Board of Governors. That is, when preparing the results of analytical models, the representatives of the analytical department both see how convincing they are in favor of making a decision that does not deviate from the optimal policy results in these models and already see the internal discussions that have started, as they do not only take place on the day of the meeting but instead start from the beginning of the circulation of monetary policy alternatives. However, officials cannot communicate this level of confidence to investors. This premise is motivated by two reasons. First, such a communication would be virtually indistinguishable from a deterrent promise to make a certain decision on the day of the meeting, binding policymakers on the day of the decision. Second, such communication for β close to zero becomes closer to normative rather than positive statements. At the same time, it is carried out by persons not responsible for decision-making within the central bank. Therefore, at date 1 for both the central bank and investors $\mathrm{Var}_1[\varepsilon_3] = \mathrm{E}_1[\beta]\delta_3$. In addition, so far, we are only talking about the expected variance at date 1. In contrast, at date 2, investors see either the observance of the quiet period regime or its violation and update their opinion about the variance of ε_3. That is, for the central bank, $\mathrm{Var}_2[\varepsilon_3] = \beta\delta_3$, but it is different for markets. Let's re-define perceived ε_2 as ε_2^p. If the blackout period is violated $\varepsilon_2^p = \varepsilon_2$, and if there is no intervention $\varepsilon_2^p = 0$. Then in the case of intervention for investors $\mathrm{Var}_2[\varepsilon_3] = \mathrm{E}_2[\beta^+]\delta_3 = \mathrm{E}_2[\beta|\varepsilon_2^p \neq 0]\delta_3$ and in the case of silence $\mathrm{Var}_2[\varepsilon_3] = \mathrm{E}_2[\beta^-]\delta_3 = \mathrm{E}_2[\beta|\varepsilon_2^p = 0]\delta_3$. We have a similar difference for the expected value of shocks at dates 2 and 3; investors build their expectations based on the higher-order beliefs "we know when the central bank violates or adheres to the quiet period regime." Thus, as in Maor, Gilad, and Bloom (2013), "words are actions, and, occasionally, so is regulatory silence." For the convenience of further modeling, we relate $\delta_1 + \delta_2 + \delta_3 = 1$ so that we will essentially be interested in the ratio of variances, such as $\frac{\delta_3}{\delta_1}$, showing how much larger the uncertainty of one shock is relative to the other. It should also be noted that for the ε_2 shock we will use the conditional variance $\sigma^2_{\varepsilon_2} = \delta_2(1-\rho^2)$, since both investors and the central bank know it in advance. Normalization will not affect the modeling results since all values of δ are already known at date 0. All three shocks are normally distributed, i.e. $\varepsilon_1 \sim \mathrm{N}(0;\delta_1)$; $\varepsilon_2 \sim \mathrm{N}\left(\rho\sqrt{\frac{\delta_2}{\delta_1}}\varepsilon_1;\delta_2(1-\rho^2)\right)$; $\varepsilon_1 \sim \mathrm{N}(0;\delta_3)$

How does the central bank make a decision in this model? By observing all expectations of the markets, the central bank is guided by a quadratic utility function:

$U = -\Big(o_1\big(\widehat{\mathrm{Var}_2}[R_3] - \mathrm{Var}_2[R_3]\big)^2 + o_2\big(\mathrm{E}_2[R_2]\big)^2 + \big(\mathrm{E}_2[R_3]\big)^2\Big)$

, where R_2 and R_3 are taken given the central bank's chosen policy of adhering to the quiet period or violating it, o_1 and o_2 relative weights $(\widehat{\mathrm{Var}_2}[R_3]-\mathrm{Var}_2[R_3])^2$ and $(\mathrm{E}_2[R_2])^2 + \big(\mathrm{E}_2[R_3]\big)^2$ respectively, and $\widehat{\mathrm{Var}_2}[R_3]$ is the variance of R_3 as observed by the central bank (or what would the markets observe if they had complete information about β). That is

$U^w = -\Big(o_1\big(\widehat{\mathrm{Var}^w_2}[R_3] - \mathrm{Var}^w_2[R_3]\big)^2 + o_2\big(\mathrm{E}^w_2[R_2]\big)^2 + \big(\mathrm{E}^w_2[R_3]\big)^2\Big)$

$U^{w/o} = -\Big(o_1\big(\widehat{\mathrm{Var}^{w/o}_2}[R_3] + \mathrm{Var}^{w/o}_2[R_3]\big)^2 + o_2\big(\mathrm{E}^{w/o}_2[R_2]\big)^2 - \big(\mathrm{E}^{w/o}_2[R_3]\big)^2\Big)$

And the central bank continues to follow the policy of the blackout period if $U^{w/o} \geq U^w$, and violates it in the case of $U^{w/o} \leq U^w$.

Given the utility function used, what exactly does the central bank look at? First of all, of course, to R_2 and R_3 - the changes in the stock price at dates 2 and 3 - the very price spikes that the central bank tries to avoid. It should be noted that in our model the central bank pays attention not only to what happens immediately after the quiet period is broken - R_2, but also to the expected effect on the markets of the Board of Governors meeting itself - R_3. The parameter o_2 sets the relative weight of the shock to investors at date 2 compared to the shock at date 3. The summand responsible for the variation in our model $\widehat{\mathrm{Var}_2}[R_3] - \mathrm{Var}_2[R_3]$ is essentially the deviation of expected variance of R_3 from the level that would be observed if markets had complete information (disclosing accurate information about β, which the regulator can't do in our model). We will discuss the intuition and possible observed variables behind this parameter later in the text.

Time line

t=0: Investors and the central bank know underlying model parameters σ, δ_1, δ_2, δ_3, o_1, o_2, and the utility function, but do not yet observe shocks ε_1, ε_2, ε_3, and also β. Based on their probability distributions, investors trade the stock against the bond by submitting competitive demand functions, and the market clears at the equilibrium price P_0.

t=1: Investors and the central bank find out ε_1. Investors trade in the market again, yielding the equilibrium stock price P_1 given both ε_1 and their expectation of ε_2.
t=2: The central bank recognizes ε_2 and β. The central bank decides whether to disclose ε_2 to the market. If central bank communicates, investors recognize ε_2, but still do not know β and have only the expectation $\mathrm{E}_2[\beta^+]$. If the central bank decides not to breach the blackout period, investors do not receive information about ε_2 or β and build their expectations $\mathrm{E}_2[\varepsilon_2^-]$, and $\mathrm{E}_2[\beta^-]$. Investors trade in the market again, yielding the equilibrium stock price P_2.
t=3: Investors learn ε_3 (and ε_2 if it wasn't communicated at date 2) from the central bank meeting, dividend D is paid on the stock, and investors consume their terminal wealth.

Discussion

Thus, the model contains 5 sources of risk that resolve over time - σ, ε_1, ε_2, ε_3, and β. At date 0, investors and the central bank learn a measure of uncertainty σ, or impact risk. At date 1, investors learn information about the initial shock to the economy ε_1 and build their expectations about the possible reaction of the central bank to this shock. At date 2, the central bank decides whether it should stick to its quiet period policy or remove the additional uncertainty associated with the central bank's reaction function for investors (by disclosing the markets ε_2). In both cases, β remains unknown to investors. However, they learn some information about β since β^+ and β^- are only truncated distributions of β. And for the central bank, this source of uncertainty is already resolved at date 2. Finally, at date 3, ε_3 - the last source of risk in the model - is revealed to investors. The intuitive idea is that inside the quiet period, the central bank tries to endogenously manage the information provided so that the resolution of uncertainty and information on the central bank's assessment of the economic situation does not cause major market shocks and investors have as accurate a picture as possible of the uncertainty of the decision to be taken during the Board meeting.

The importance of uncertainty is emphasized in $(\widehat{\mathrm{Var}_2}[R_3] - \mathrm{Var}_2[R_3])^2$ in the central bank's utility function. $\mathrm{Var}_2[R_3]$ is the uncertainty metric in our model, and as was shown in Hu et al. (2022), the most appropriate observables to describe the dynamics of $\mathrm{Var}_2[R_3]$ may be implied volatility and VIX (risk-neutral expectation of the volatility of the equity index over the next 30 days). Although VIX is a useful predictor of future realized volatility, we will not attempt to speak of our model as a model describing realized volatility of R_3 as well, since in addition to changes in expected volatility, VIX can also change due to shifts in preferences towards volatility that generate the variance risk premium (for an overview of the informational content of various uncertainty and volatility metrics, see Cascaldi-Garcia et al (2023)). Another reason is that spikes in realized volatility are tied to the publication of news; a significant cause of spikes in realized volatility is the sharp resolution of uncertainty that appears when news is published (as shown in Ai, Bansal, and Han (2021) for the FOMC press release and possible information leaks during the blackout period). Our model, in this sense, is a little close to the intertemporal mechanism in Ai, Han, and Xu (2021) - "when variations in stock market volatility are driven by information, high realized variances of past returns typically predict lower future variances." In our model, the markets will know the valuation of ε_2 sooner or later anyway, so we only choose when there is a sharp resolution of uncertainty leading to a spike in volatility, at date 2 or date 3.

Why is the utility function symmetric for both upward and downward deviations in volatility? Too high uncertainty, first, is an additional incentive for investors to mine information prior to the announcement, provoking the emergence of leaks, breaches, and unattributed communications. Secondly, if it was not partially resolved before, a sharp increase in uncertainty at the time of the press release causes a more substantial spike in volatility at date 3. On the contrary, overly low uncertainty that underestimates the degree of disagreement within the Board leads investors' expectations to become anchored on a single expected option, which can, in turn, lock in Board members, depriving them of their freedom to make a decision.

Distribution of model parameters

We have already discussed how the shocks ε_1, ε_2 and ε_3 are distributed. For the remaining variables, it is important to note that the distribution function itself does not influence the course of the model solution, so we will try to remain as agnostic as possible about the distribution parameters. We will consider ρ to be uniformly distributed on $[0; 1]$. This does not affect the solution of the model since the value of ρ becomes known to all economic agents already at date 0, but it allows us to cover all possible cases uniformly during Monte Carlo simulation.

σ^2 is exponentially distributed with location parameter λ_0 and scale parameter λ, where λ_0 and λ are known at date 0. So the support of σ^2 is $[\lambda_0, \infty)$, and $\sigma^2 - \lambda_0$ follows an exponential distribution with variance λ^2. Also, to cover more possibilities in terms of comparing uncertainty, we use uniform distributions for both parameters - $\lambda_0 \sim U[0; 1]; \lambda \sim U[0; 10]$.

For β we will use two different approaches. We duplicate the generation of Monte Carlo observations for the cases $\beta \in [0;1]$ and $\beta \in [0;2]$. The first case represents δ_3 as an upper bound on the possible uncertainty about the board meeting, while the second represents δ_3 as the expected value of this uncertainty. In both cases, we will use a uniform distribution. This does not change the course of the model solution since we always use only the first two moments of the distribution.

Empirical estimates of absolute risk aversion coefficients vary widely for many reasons, which can be seen, for example, in the work Conniffe, O'Neill (2012). We will use one value $\alpha = 0.005$ to tractabilize the results (we have too many degrees of freedom for the other variables). We use it as a realistic value, based on a review of the estimation results from Babcock, Choi, and Feinerman (1993), so we can focus on the variation of other variables in the model, especially the risk appetite of investors rather than the central bank, which we are primarily interested in.

Finally, we will be as agnostic as possible about the weights o_1 and o_2 in the central bank's utility function. Unable to use random weights with any continuous distribution, as this would interfere with the comparability of the results, we will use discrete distributions that will help us compare the results when the central bank pays strongly less, slightly less, slightly more, and strongly more attention to a certain factor in the utility function. We will use the following weights for o_1: $0,01$; $0,1$; 1; 10; 100. For o_2, on the other hand, we will use the following weights: 0; $0,01$; $0,1$; $0,5$; 1; 2; 10; 100. The reason for the different distributions is that the values of R_2 and R_3 themselves are easily comparable to each other, so we can also try to capture small nuances when their weights in the utility function are close. Whereas the deviations of the variance of $\mathrm{Var}_2[R_3]$ from $\widehat{\mathrm{Var}_2}[R_3]$ are not directly comparable to the returns, and we use a more sparse scale.

Equilibrium

In this section, we solve the model backwards. That is, at date 3, the optimization problem of generic investor looks as follows: $J_2 = \max\limits_{\theta_2} \mathrm{E}_2 \left\{ -\exp\{-\alpha[W_2 + \theta_2(D - P_2)]\} \right\} =$

$= \max\limits_{\theta_2} \mathrm{E}_2 \left\{ -\exp\{-\alpha[W_2 + \theta_2(\bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 - P_2) - \frac{1}{2}\alpha\theta_2^2 \mathrm{Var}_2[D]]\} \right\} =$

$= \max\limits_{\theta_2} \mathrm{E}_2 \left\{ -\exp\{-\alpha[W_2 + \theta_2(\bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 - P_2) - \frac{1}{2}\alpha\theta_2^2\sigma^2 \mathrm{Var}_2[\varepsilon_2 + \varepsilon_3]]\} \right\}$

, where θ_2 is the investor's demand of the risky asset at date 2.

At this stage, investors already know whether the central bank has conducted a verbal intervention, and in case of its presence $\mathrm{Var}[\varepsilon_2] = 0$. Therefore, we have 2 cases depending on the presence of the intervention. In the following, we will denote the presence of interventions by

J_2^w and the absence by $J_2^{w/o}$. With an intervention: $J_2^w = \max_{\theta_2} \mathrm{E}_2 \{ -\exp\{-\alpha[W_2 + \theta_2(\bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 - P_2) - \frac{1}{2}\alpha\theta_2^2\sigma^2 \mathrm{E}_2[\beta^+]\delta_3]\}\}$.
And without an intervention: $J_2^{w/o} = \max_{\theta_2} \mathrm{E}_2 \{ -\exp\{-\alpha[W_2 + \theta_2(\bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 - P_2) - \frac{1}{2}\alpha\theta_2^2\sigma^2 \mathrm{Var}_2[\varepsilon_2^- + \varepsilon_3]]\}\} = \max_{\theta_2} \mathrm{E}_2 \{ -\exp\{-\alpha[W_2 + \theta_2(\bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 - P_2) - \frac{1}{2}\alpha\theta_2^2\sigma^2(\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3)]\}\}$.

We can now find the equilibrium P_2 in both cases.
$\frac{\partial J_2^w}{\partial \theta_2} = \bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 - P_2 - \alpha\theta_2\sigma^2 \mathrm{E}_2[\beta^+]\delta_3$.
Then $\theta_2 = \frac{\bar{D}+\sigma\varepsilon_1+\sigma\varepsilon_2-P_2}{\alpha\sigma^2 \mathrm{E}_2[\beta^+]\delta_3}$ and with the given $\theta_2 = 1$ equilibrium $P_2 = \bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 - \alpha\sigma^2 \mathrm{E}_2[\beta^+]\delta_3$.
Where S.O.C.: $\frac{\partial^2 J_2^w}{\partial^2 \theta_2} = -\alpha\sigma^2 \mathrm{E}_2[\beta^+]\delta_3 < 0$.
In turn for the no communication scenario: $\frac{\partial J_2^{w/o}}{\partial \theta_2} = \bar{D} + \sigma\varepsilon_1 + \sigma \mathrm{E}_2[\varepsilon_2^-] - P_2 - \alpha\theta_2\sigma^2(\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3)$.
$\theta_2 = \frac{\bar{D}+\sigma\varepsilon_1+\sigma \mathrm{E}_2[\varepsilon_2^-]-P_2}{\alpha\theta_2\sigma^2(\mathrm{Var}_2[\varepsilon_2^-]+\mathrm{E}_2[\beta^-]\delta_3)}$ and with the given $\theta_2 = 1$ equilibrium price $P_2 = \bar{D} + \sigma\varepsilon_1 + \sigma \mathrm{E}_2[\varepsilon_2^-] - \alpha\sigma^2(\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3)$.
Where S.O.C.: $\frac{\partial^2 J_2^{w/o}}{\partial^2 \theta_2} = \alpha\sigma^2(\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3) < 0$.

For convenience, in the following, we will sometimes describe both cases together, denoting

$$\widetilde{\varepsilon_2} = \begin{cases} \varepsilon_2, & \text{with intervention} \\ \mathrm{E}_2[\varepsilon_2^-], & \text{without} \end{cases} \quad \text{and} \quad \widetilde{\delta_3} = \begin{cases} \mathrm{E}_2[\beta^+]\delta_3, & \text{with intervention} \\ \mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3, & \text{without.} \end{cases}$$

To describe what happens at date 1, we also need to look at what happens with the change in the information set. So for $\mathrm{Var}_2[\varepsilon_2^-]$ and $\mathrm{E}_2[\beta^-]$ at date 2, no new information occurs, the information sets at dates 1 and 2 are the same, and thus $\mathrm{Var}_1[\varepsilon_2^-] = \mathrm{Var}_2[\varepsilon_2^-]$; $\mathrm{E}_1[\beta^-] = \mathrm{E}_2[\beta^-]$. Therefore, for convenience, we will always use the notation $t = 2$ for them, even if we are talking about date 1. However, if the central bank intervenes at date 2, the market will recognize the value of ε_2 and then $\mathrm{E}_2[\beta^+] = \mathrm{E}_1[\beta^+|\varepsilon_2] \neq \mathrm{E}_1[\beta^+]$.

Now $P_2 = \bar{D} + \sigma\varepsilon_1 + \sigma\widetilde{\varepsilon_2} - \alpha\sigma^2\widetilde{\delta_3}$ and at date 1 investor's optimization problem looks as follows:
$J_2 = -\exp\{-\alpha[W_1 + \theta_1(P_2 - P_1) + \theta_2(D - P_2) - \frac{1}{2}\alpha\sigma^2\widetilde{\delta_3}]\} = -\exp\{-\alpha[W_1 + \theta_1(\bar{D} + \sigma\varepsilon_1 + \sigma\widetilde{\varepsilon_2} - \alpha\sigma^2\widetilde{\delta_3} - P_1) + \theta_2(\bar{D} + \sigma\varepsilon_1 + \sigma\widetilde{\varepsilon_2} - \bar{D} - \sigma\varepsilon_1 - \sigma\widetilde{\varepsilon_2} + \alpha\sigma^2\widetilde{\delta_3}) - \frac{1}{2}\alpha\sigma^2\widetilde{\delta_3}]\} = -\exp\{-\alpha[W_1 + \theta_1(\bar{D} + \sigma\varepsilon_1 + \sigma\widetilde{\varepsilon_2} - \alpha\sigma^2\widetilde{\delta_3} - P_1) + \frac{1}{2}\alpha\sigma^2\widetilde{\delta_3}]\}$.
$\mathrm{E}_1(J_2) = -\exp\{-\alpha[W_1 + \theta_1(\bar{D} + \sigma\varepsilon_1 + \sigma \mathrm{E}_1[\widetilde{\varepsilon_2}] - \alpha\sigma^2 \mathrm{E}_1[\widetilde{\delta_3}] - P_1) + \frac{1}{2}\alpha\sigma^2 \mathrm{E}_1[\widetilde{\delta_3}] - \frac{1}{2}\alpha \mathrm{Var}[\theta_1\sigma\widetilde{\varepsilon_2} - \alpha\sigma^2\widetilde{\delta_3}(\theta_1 - \frac{1}{2})]]\}$.
Let's redefine $\mathrm{V}_1 = \mathrm{Var}[\theta_1\sigma\widetilde{\varepsilon_2} - \alpha\sigma^2\widetilde{\delta_3}(\theta_1 - \frac{1}{2})]$.
Now F.O.C. $\frac{\partial \mathrm{E}_1[J_2]}{\partial \theta_1} = \bar{D} + \sigma\varepsilon_1 + \sigma \mathrm{E}_1[\widetilde{\varepsilon_2}] - \alpha\sigma^2 \mathrm{E}_1[\widetilde{\delta_3}] - P_1 - \frac{1}{2}\alpha \mathrm{V}_1'$.

And given $\theta_1 = 1$ equilibrium $P_1 = \bar{D} + \sigma\varepsilon_1 + \sigma \operatorname{E}_1[\widetilde{\varepsilon_2}] - \alpha\sigma^2 \operatorname{E}_1[\widetilde{\delta_3}] - \frac{1}{2}\alpha \operatorname{V}_1'$.

The problem is to express $\operatorname{E}_2[R_2]$; $\operatorname{E}_2[R_3]$; $\operatorname{Var}_2[R_3]$; $\widehat{\operatorname{Var}_2}[R_3]$ through moments of ε_2^+; ε_2^-; β^+; β^-, which we can then estimate in a Monte Carlo simulation. For this purpose, let us denote by Pr_1 the probability of intervention (estimated at date 1) identically by both market participants and the central bank (since at date 1 there is no information asymmetry yet).

Then $\operatorname{E}_1[\widetilde{\varepsilon_2}] = \operatorname{Pr}_1 \operatorname{E}_1[\varepsilon_2^+] + (1 - \operatorname{Pr}_1) \operatorname{E}_2[\varepsilon_2^-]$.

$\operatorname{E}_1[\widetilde{\delta_3}] = \operatorname{Pr}_1 \operatorname{E}_1[\beta^+]\delta_3 + (1 - \operatorname{Pr}_1)(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3)$.

$\operatorname{Var}_1[\theta_1\sigma\widetilde{\varepsilon_2} - \alpha\sigma^2\widetilde{\delta_3}(\theta_1 - \frac{1}{2})] = \operatorname{Var}_1[\theta_1\sigma\widetilde{\varepsilon_2}] + \operatorname{Var}_1[\alpha\sigma^2\widetilde{\delta_3}(\theta_1 - \frac{1}{2})] - 2\operatorname{cov}[\theta_1\sigma\widetilde{\varepsilon_2}; \alpha\sigma^2\widetilde{\delta_3}(\theta_1 - \frac{1}{2})] =$

$= \theta_1^2\sigma^2 \operatorname{Var}_1[\widetilde{\varepsilon_2}] + \alpha^2\sigma^4(\theta_1 - \frac{1}{2})^2 \operatorname{Var}_1[\widetilde{\delta_3}] - 2\operatorname{E}_1[\theta_1(\theta_1 - \frac{1}{2})\alpha\sigma^3\widetilde{\varepsilon_2}\widetilde{\delta_3}] + 2\operatorname{E}_1[\theta_1\sigma\widetilde{\varepsilon_2}]\operatorname{E}_1[\alpha\sigma^2\widetilde{\delta_3}(\theta_1 - \frac{1}{2})] =$

$= \theta_1^2\sigma^2 \operatorname{Var}_1[\widetilde{\varepsilon_2}] + \alpha^2\sigma^4(\theta_1 - \frac{1}{2})^2 \operatorname{Var}_1[\widetilde{\delta_3}] - 2\theta_1(\theta_1 - \frac{1}{2})\alpha\sigma^3 \operatorname{E}_1[\widetilde{\varepsilon_2}\widetilde{\delta_3}] + 2\theta_1(\theta_1 - \frac{1}{2})\alpha\sigma^3 \operatorname{E}_1[\widetilde{\varepsilon_2}]\operatorname{E}_1[\widetilde{\delta_3}]$.

$\frac{\partial \operatorname{V}_1}{\partial\theta_1} = 2\theta_1\sigma^2 \operatorname{Var}_1[\widetilde{\varepsilon_2}] + (2\theta_1 - 1)\alpha^2\sigma^4 \operatorname{Var}_1[\widetilde{\delta_3}] - 2(2\theta_1 - \frac{1}{2})\alpha\sigma^3 \operatorname{E}_1[\widetilde{\varepsilon_2}\widetilde{\delta_3}] + 2(2\theta_1 - \frac{1}{2})\alpha\sigma^3 \operatorname{E}_1[\widetilde{\varepsilon_2}]\operatorname{E}_1[\widetilde{\delta_3}]$.

$V_1'(\theta_1 = 1) = 2\sigma^2 \operatorname{Var}_1[\widetilde{\varepsilon_2}] + \alpha^2\sigma^4 \operatorname{Var}_1[\widetilde{\delta_3}] - 3\alpha\sigma^3 \operatorname{E}_1[\widetilde{\varepsilon_2}\widetilde{\delta_3}] + 3\alpha\sigma^3 \operatorname{E}_1[\widetilde{\varepsilon_2}]\operatorname{E}_1[\widetilde{\delta_3}]$.

Now we can express the individual summands V_1':

$\operatorname{Var}[\widetilde{\delta_3}] = \operatorname{E}_1[\widetilde{\delta_3}]^2 - \operatorname{E}_1^2[\widetilde{\delta_3}] = \operatorname{Pr}_1 \operatorname{E}_1[\beta^+]^2\delta_3^2 + (1 - \operatorname{Pr}_1)(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3)^2 - (\operatorname{Pr}_1 \operatorname{E}_1[\beta^+]\delta_3 + (1 - \operatorname{Pr}_1)(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3))^2 = \operatorname{Pr}_1(1 - \operatorname{Pr}_1)(\operatorname{E}_1[\beta^+]\delta_3 - \operatorname{Var}_2[\varepsilon_2^-] - \operatorname{E}_2[\beta^-]\delta_3)^2$.

$\operatorname{Var}_1[\widetilde{\varepsilon_2}] = \operatorname{E}_1[\widetilde{\varepsilon_2}]^2 - \operatorname{E}_1^2[\widetilde{\varepsilon_2}] = \operatorname{Pr}_1 \operatorname{E}_1[\varepsilon_2^+]^2 + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-]^2 - (\operatorname{Pr}_1 \operatorname{E}_1[\varepsilon_2^+] + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-])^2 = \operatorname{Pr}_1(1 - \operatorname{Pr}_1)(\operatorname{E}_1[\varepsilon_2^+] - \operatorname{E}_2[\varepsilon_2^-])^2$.

$\operatorname{E}_1[\widetilde{\varepsilon_2}\widetilde{\delta_3}] = \operatorname{Pr}_1 \operatorname{E}_1[\varepsilon_2^+\beta^+\delta_3] + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-(\operatorname{Var}_2[\varepsilon_2^-] + \beta^-\delta_3)] = \operatorname{Pr}_1 \delta_3 \operatorname{E}_1[\varepsilon_2^+\beta^+] + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-]\operatorname{Var}_2[\varepsilon_2^-] + (1 - \operatorname{Pr}_1)\delta_3 \operatorname{E}_2[\varepsilon_2^-\beta^-] = \operatorname{Pr}_1 \delta_3 \operatorname{E}_1[\varepsilon_2^+ \operatorname{E}_2[\beta^+]] + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-]\operatorname{Var}_2[\varepsilon_2^-] + (1 - \operatorname{Pr}_1)\delta_3 \operatorname{E}_1[\operatorname{E}_2[\varepsilon_2^-]\operatorname{E}_2[\beta^-]] = \operatorname{Pr}_1 \delta_3 \operatorname{E}_1[\varepsilon_2^+]\operatorname{E}_1[\beta^+] + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-]\operatorname{Var}_2[\varepsilon_2^-] + (1 - \operatorname{Pr}_1)\delta_3 \operatorname{E}_2[\varepsilon_2^-]\operatorname{E}_2[\beta^-]$.

Hence $V_1'(\theta_1 = 1) = 2\sigma^2 \operatorname{Pr}_1(1 - \operatorname{Pr}_1)(\operatorname{E}_1[\varepsilon_2^+] - \operatorname{E}_2[\varepsilon_2^-])^2 + \alpha^2\sigma^4 \operatorname{Pr}_1(1 - \operatorname{Pr}_1)(\operatorname{E}_1[\beta^+]\delta_3 - \operatorname{Var}_2[\varepsilon_2^-] - \operatorname{E}_2[\beta^-]\delta_3)^2 - 3\alpha\sigma^3\big(\operatorname{Pr}_1 \delta_3 \operatorname{E}_1[\varepsilon_2^+]\operatorname{E}_1[\beta^+] + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-]\operatorname{Var}_2[\varepsilon_2^-] + (1 - \operatorname{Pr}_1)\delta_3 \operatorname{E}_2[\varepsilon_2^-]\operatorname{E}_2[\beta^-]\big) + 3\alpha\sigma^3\big(\operatorname{Pr}_1 \operatorname{E}_1[\varepsilon_2^+] + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-]\big)\big(\operatorname{Pr}_1 \operatorname{E}_1[\beta^+]\delta_3 + (1 - \operatorname{Pr}_1)(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3)\big)$.

And then $P_1 = \bar{D} + \sigma\varepsilon_1 + \sigma \operatorname{Pr}_1 \operatorname{E}_1[\varepsilon_2^+] + \sigma(1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-] - \alpha\sigma^2\big(\operatorname{Pr}_1 \operatorname{E}_1[\beta^+]\delta_3 + (1 - \operatorname{Pr}_1)(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3)\big) - \alpha\sigma^2 \operatorname{Pr}_1(1 - \operatorname{Pr}_1)(\operatorname{E}_1[\varepsilon_2^+] - \operatorname{E}_2[\varepsilon_2^-])^2 - \frac{1}{2}\alpha^3\sigma^4 \operatorname{Pr}_1(1 - \operatorname{Pr}_1)(\operatorname{E}_1[\beta^+]\delta_3 - \operatorname{Var}_2[\varepsilon_2^-] - \operatorname{E}_2[\beta^-]\delta_3)^2 + \frac{3}{2}\alpha^2\sigma^3\big(\operatorname{Pr}_1 \delta_3 \operatorname{E}_1[\varepsilon_2^+]\operatorname{E}_1[\beta^+] + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-]\operatorname{Var}_2[\varepsilon_2^-] + (1 - \operatorname{Pr}_1)\delta_3 \operatorname{E}_2[\varepsilon_2^-]\operatorname{E}_2[\beta^-]\big) - \frac{3}{2}\alpha^2\sigma^3\big(\operatorname{Pr}_1 \operatorname{E}_1[\varepsilon_2^+] + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-]\big)\big(\operatorname{Pr}_1 \operatorname{E}_1[\beta^+]\delta_3 + (1 - \operatorname{Pr}_1)(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3)\big)$.

Thus, we got $P_1 = \bar{D} + \sigma\varepsilon_1 + \sigma \operatorname{Pr}_1 \operatorname{E}_1[\varepsilon_2^+] + \sigma(1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-] - rp_1$, where the risk premium at date 1:

$rp_1 = \alpha\sigma^2\big(\operatorname{Pr}_1 \operatorname{E}_1[\beta^+]\delta_3 + (1 - \operatorname{Pr}_1)(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3)\big) + \alpha\sigma^2 \operatorname{Pr}_1(1 - \operatorname{Pr}_1)(\operatorname{E}_1[\varepsilon_2^+] - \operatorname{E}_2[\varepsilon_2^-])^2 + \frac{1}{2}\alpha^3\sigma^4 \operatorname{Pr}_1(1 - \operatorname{Pr}_1)(\operatorname{E}_1[\beta^+]\delta_3 - \operatorname{Var}_2[\varepsilon_2^-] - \operatorname{E}_2[\beta^-]\delta_3)^2 - \frac{3}{2}\alpha^2\sigma^3\big(\operatorname{Pr}_1 \delta_3 \operatorname{E}_1[\varepsilon_2^+]\operatorname{E}_1[\beta^+] + (1 - \operatorname{Pr}_1)\operatorname{E}_2[\varepsilon_2^-]\operatorname{Var}_2[\varepsilon_2^-] +$

$(1-\Pr_1)\delta_3 \operatorname{E}_2[\varepsilon_2^-] \operatorname{E}_2[\beta^-]\big)+\frac{3}{2}\alpha^2\sigma^3\big(\Pr_1 \operatorname{E}_1[\varepsilon_2^+]+(1-\Pr_1) \operatorname{E}_2[\varepsilon_2^-]\big)\big(\Pr_1 \operatorname{E}_1[\beta^+]\delta_3+(1-\Pr_1)(\operatorname{Var}_2[\varepsilon_2^-]+\operatorname{E}_2[\beta^-]\delta_3)\big)$.

Since it is the central bank that decides whether to intervene or not, it is exactly its assessment of what will happen in the stock market that is important to us. We can calculate the components of the utility function in the absence of intervention:

$\operatorname{E}_2^{w/o}[R_3] = \operatorname{E}_2^{w/o}[P_3 - P_2] = \operatorname{E}_2^{w/o}[\bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 + \sigma\varepsilon_3 - \bar{D} - \sigma\varepsilon_1 - \sigma\widetilde{\varepsilon_2} + \alpha\sigma^2\widetilde{\delta_3}] = \operatorname{E}_2^{w/o}[\sigma\varepsilon_2 + \sigma\varepsilon_3 - \sigma\widetilde{\varepsilon_2} + \alpha\sigma^2\widetilde{\delta_3}] = \operatorname{E}_2^{w/o}[\sigma\varepsilon_2 + \sigma\varepsilon_3 - \sigma \operatorname{E}_2[\varepsilon_2^-] + \alpha\sigma^2(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3)] = \sigma\varepsilon_2 - \sigma \operatorname{E}_2[\varepsilon_2^-] + \alpha\sigma^2(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3)$.

$\operatorname{E}_2^{w/o}[R_2] = \operatorname{E}_2^{w/o}[P_2 - P_1] = \operatorname{E}_2^{w/o}[\bar{D} + \sigma\varepsilon_1 + \sigma\widetilde{\varepsilon_2} - \alpha\sigma^2\widetilde{\delta_3} - \bar{D} - \sigma\varepsilon_1 - \sigma \Pr_1 \operatorname{E}_1[\varepsilon_2^+] - \sigma(1-\Pr_1) \operatorname{E}_2[\varepsilon_2^-] + rp_1] = \operatorname{E}_2^{w/o}[\sigma\widetilde{\varepsilon_2} - \alpha\sigma^2\widetilde{\delta_3} - \sigma \Pr_1 \operatorname{E}_1[\varepsilon_2^+] - \sigma(1-\Pr_1) \operatorname{E}_2[\varepsilon_2^-] + rp_1] = \operatorname{E}_2^{w/o}[\sigma \operatorname{E}_2[\varepsilon_2^-] - \alpha\sigma^2(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3) - \sigma \Pr_1 \operatorname{E}_1[\varepsilon_2^+] - \sigma(1-\Pr_1) \operatorname{E}_2[\varepsilon_2^-] + rp_1] = -\alpha\sigma^2(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3) - \sigma \Pr_1 \operatorname{E}_1[\varepsilon_2^+] + \sigma \Pr_1 \operatorname{E}_2[\varepsilon_2^-] + rp_1$.

$\operatorname{Var}_2^{w/o}[R_3] = \operatorname{Var}_2^{w/o}[P_3 - P_2] = \operatorname{Var}_2^{w/o}[\bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 + \sigma\varepsilon_3 - \bar{D} - \sigma\varepsilon_1 - \sigma\widetilde{\varepsilon_2} + \alpha\sigma^2\widetilde{\delta_3}] = \operatorname{Var}_2^{w/o}[\sigma\varepsilon_2 + \sigma\varepsilon_3 - \sigma\widetilde{\varepsilon_2} + \alpha\sigma^2\widetilde{\delta_3}] = \operatorname{Var}_2^{w/o}[\sigma\varepsilon_2^- + \sigma\varepsilon_3 - \sigma \operatorname{E}_2[\varepsilon_2^-] + \alpha\sigma^2(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3)] = \operatorname{Var}_2^{w/o}[\sigma\varepsilon_2^- + \sigma\varepsilon_3] = \sigma^2 \operatorname{Var}_2[\varepsilon_2^-] + \sigma^2 \operatorname{E}_2[\beta^-]\delta_3$.

And $\widehat{\operatorname{Var}_2^{w/o}}[R_3] = \widehat{\operatorname{Var}_2^{w/o}}[P_3 - P_2] = \widehat{\operatorname{Var}_2^{w/o}}[\sigma\varepsilon_3] = \sigma^2\beta\delta_3$.

And in the case of an intervention:

$\operatorname{E}_2^{w}[R_3] = \operatorname{E}_2^{w}[P_3 - P_2] = \operatorname{E}_2^{w}[\bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 + \sigma\varepsilon_3 - \bar{D} - \sigma\varepsilon_1 - \sigma\widetilde{\varepsilon_2} + \alpha\sigma^2\widetilde{\delta_3}] = \operatorname{E}_2^{w}[\sigma\varepsilon_2 + \sigma\varepsilon_3 - \sigma\widetilde{\varepsilon_2} + \alpha\sigma^2\widetilde{\delta_3}] = \operatorname{E}_2^{w}[\sigma\varepsilon_2 + \sigma\varepsilon_3 - \sigma\varepsilon_2 + \alpha\sigma^2 \operatorname{E}_2[\beta^+]\delta_3] = \alpha\sigma^2 \operatorname{E}_2[\beta^+]\delta_3$.

$\operatorname{E}_2^{w}[R_2] = \operatorname{E}_2^{w}[P_2 - P_1] = \operatorname{E}_2^{w}[\bar{D} + \sigma\varepsilon_1 + \sigma\widetilde{\varepsilon_2} - \alpha\sigma^2\widetilde{\delta_3} - \bar{D} - \sigma\varepsilon_1 - \sigma \Pr_1 \operatorname{E}_1[\varepsilon_2^+] - \sigma(1-\Pr_1) \operatorname{E}_2[\varepsilon_2^-] + rp_1] = \operatorname{E}_2^{w}[\sigma\widetilde{\varepsilon_2} - \alpha\sigma^2\widetilde{\delta_3} - \sigma \Pr_1 \operatorname{E}_1[\varepsilon_2^+] - \sigma(1-\Pr_1) \operatorname{E}_2[\varepsilon_2^-] + rp_1] = \operatorname{E}_2^{w}[\sigma\varepsilon_2 - \alpha\sigma^2 \operatorname{E}_2[\beta^+]\delta_3 - \sigma \Pr_1 \operatorname{E}_1[\varepsilon_2^+] - \sigma(1-\Pr_1) \operatorname{E}_2[\varepsilon_2^-] + rp_1] = \sigma\varepsilon_2 - \alpha\sigma^2 \operatorname{E}_2[\beta^+]\delta_3 - \sigma \Pr_1 \operatorname{E}_1[\varepsilon_2^+] - \sigma(1-\Pr_1) \operatorname{E}_2[\varepsilon_2^-] + rp_1$.

$\operatorname{Var}_2^{w}[R_3] = \operatorname{Var}_2^{w}[P_3 - P_2] = \operatorname{Var}_2^{w}[\bar{D} + \sigma\varepsilon_1 + \sigma\varepsilon_2 + \sigma\varepsilon_3 - \bar{D} - \sigma\varepsilon_1 - \sigma\widetilde{\varepsilon_2} + \alpha\sigma^2\widetilde{\delta_3}] = \operatorname{Var}_2^{w}[\sigma\varepsilon_2 + \sigma\varepsilon_3 - \sigma\widetilde{\varepsilon_2} + \alpha\sigma^2\widetilde{\delta_3}] = \operatorname{Var}_2^{w}[\sigma\varepsilon_2 + \sigma\varepsilon_3 - \sigma\varepsilon_2 + \alpha\sigma^2 \operatorname{E}_2[\beta^+]\delta_3] = \operatorname{Var}_2^{w}[\sigma\varepsilon_3] = \sigma^2 \operatorname{E}_2[\beta^+]\delta_3$.

And $\widehat{\operatorname{Var}_2^{w}}[R_3]$ remains exactly the same, since $\widehat{\operatorname{Var}_2^{w}}[R_3] = \widehat{\operatorname{Var}_2^{w}}[P_3 - P_2] = \widehat{\operatorname{Var}_2^{w}}[\sigma\varepsilon_3] = \sigma^2\beta\delta_3$.

And finally we put this together into the difference of utility function $U^w - U^{w/o} = o_1\big(\widehat{\operatorname{Var}_2^{w}}[R_3] - \operatorname{Var}_2^{w}[R_3]\big)^2 + o_2\big(\operatorname{E}_2^{w}[R_2]\big)^2 + \big(\operatorname{E}_2^{w}[R_3]\big)^2 - o_1\big(\widehat{\operatorname{Var}_2^{w/o}}[R_3] - \operatorname{Var}_2^{w/o}[R_3]\big)^2 - o_2\big(\operatorname{E}_2^{w/o}[R_2]\big)^2 - \big(\operatorname{E}_2^{w/o}[R_3]\big)^2$

$U^w - U^{w/o} = o_1\big(\sigma^2\beta\delta_3 - \sigma^2 \operatorname{E}_2[\beta^+]\delta_3\big)^2 + o_2\big(\sigma\varepsilon_2 - \alpha\sigma^2 \operatorname{E}_2[\beta^+]\delta_3 - \sigma \Pr_1 \operatorname{E}_1[\varepsilon_2^+] - \sigma(1-\Pr_1) \operatorname{E}_2[\varepsilon_2^-] + rp_1\big)^2 + \big(\alpha\sigma^2 \operatorname{E}_2[\beta^+]\delta_3\big)^2 - o_1\big(\sigma^2\beta\delta_3 - \sigma^2 \operatorname{Var}_2[\varepsilon_2^-] - \sigma^2 \operatorname{E}_2[\beta^-]\delta_3\big)^2 - o_2\big(-\alpha\sigma^2(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3) - \sigma \Pr_1 \operatorname{E}_1[\varepsilon_2^+] + \sigma \Pr_1 \operatorname{E}_2[\varepsilon_2^-] + rp_1\big)^2 - \big(\sigma\varepsilon_2 - \sigma \operatorname{E}_2[\varepsilon_2^-] + \alpha\sigma^2(\operatorname{Var}_2[\varepsilon_2^-] + \operatorname{E}_2[\beta^-]\delta_3)\big)^2$.

The problem is now reduced to finding the mapping by which the central bank decides when to communicate and when not to communicate during the quiet period such that $U^{w/o} \geq U^w$ holds for those values of ε_2 and β at which it decides to follow the policy of the blackout period, and $U^{w/o} \leq U^w$ holds for those ε_2 and β at which the central bank decides to communicate.

Monte Carlo simulation

To find this solution, we will use the following algorithm. In general, we need to construct a surjective mapping $f : [S_1, S_2, \alpha, B, \theta, \sigma, \delta_1, \delta_2, \delta_3] \rightarrow \{1; 2\}$, where we have two classes - class 1 represents the decision not to communicate and class 2 represents the decision to communicate. We construct it for any parameter values α, β, θ, σ, δ_1, δ_2, δ_3 and realizations of $\varepsilon_1, \varepsilon_2$ from the corresponding distributions.

We will divide the solution into three steps:

1) Using Monte Carlo generation (see Casarin (2023) for an overview on Monte Carlo methods) for one particular set of parameters $\varepsilon_1, \alpha, \theta, \sigma, \delta_1, \delta_2, \delta_3$ for 10,000 randomly generated pairs of ε_2 and β, where $\varepsilon_2 \sim \mathrm{N}\left(\rho\sqrt{\frac{\delta_2}{\delta_1}}\varepsilon_1; \delta_2(1-\rho^2)\right)$ and $\beta \sim U[0; 2]$ we find a mapping $S_2 * B \rightarrow \{1; 2\}$; from which we get the values of the required parameters $\mathrm{E}_2[\beta^-], \mathrm{E}_2[\beta^+], \mathrm{E}_1[\varepsilon_2^+], \mathrm{E}_2[\varepsilon_2^-], \mathrm{Var}_2[\varepsilon_2^-], \mathrm{Pr}_1$. The Monte Carlo method is necessary since we do not have the closed-form solution to compare $U^w - U^{w/o}$ to zero. The iterative algorithm used to obtain this mapping is described in more detail in Appendix 1.

2) Using the obtained parameters $\mathrm{E}_2[\beta^-], \mathrm{E}_2[\beta^+], \mathrm{E}_1[\varepsilon_2^+], \mathrm{E}_1[\varepsilon_2^-], \mathrm{Var}_2[\varepsilon_2^-], \mathrm{Pr}_1$, we directly estimate the values of the utility function, by substituting parameter values into $U^w - U^{w/o}$ and get the answer to which class each point in our space $\varepsilon_2 * \beta$ belongs to.

3) We repeat steps 1 and 2, generating new parameter sets $\varepsilon_1, \alpha, \theta, \sigma, \delta_1, \delta_2, \delta_3$ from the corresponding distributions. Thus, we use steps 1 and 2 to generate essentially one observation each time in the final dataset under study. Then, the entire dataset will allow us to determine which parameters drive the central bank's decision to adhere or not to adhere to the quiet period regime. The details of the Monte Carlo algorithm are described in Appendix 1.

Results

Now we analyze the results obtained. To do this, we will answer the following questions:

1) Which scenario is preferable: "never intervene" or "endogenously intervene"? And "Never intervene" or "always intervene"?

2) How often is it optimal to break the silence?

3) For which ε_2; β do we break or follow the quiet period regime?

4) Are our responses to positive/negative shocks symmetrical?

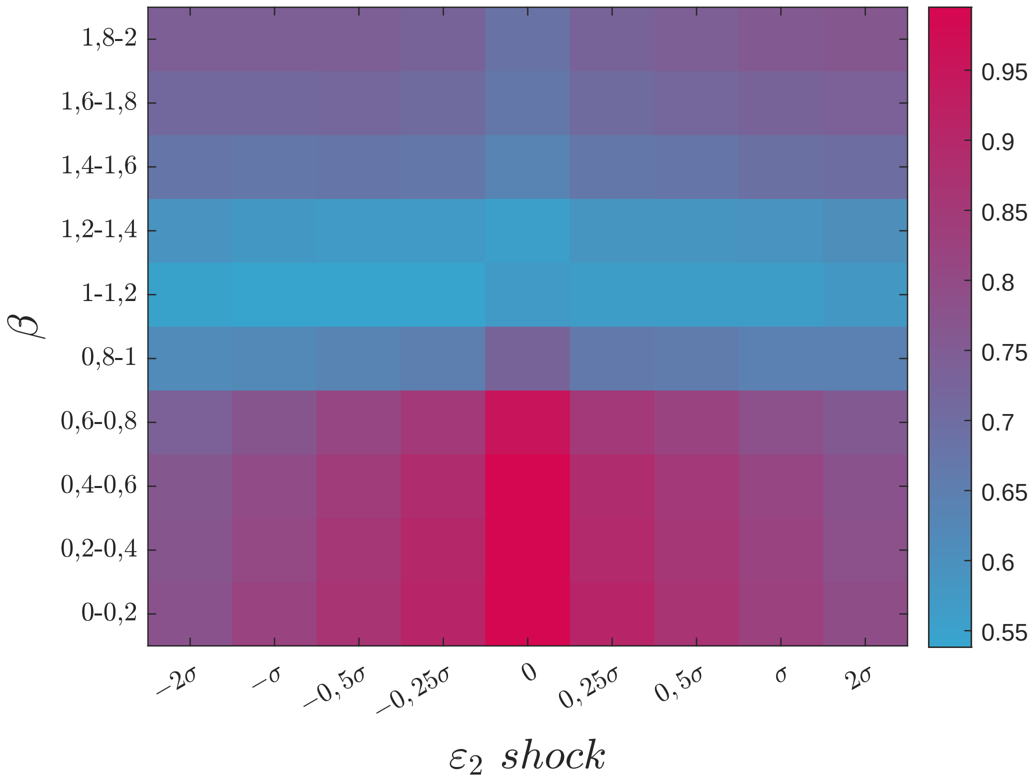
5) What factors in the model determine the answers to these questions?

Scenario comparison

Is it even necessary for the central bank to intervene during the quiet period? To answer this question, we can compare three regimes: "never intervene", "always intervene", and "endogenously intervene". Accordingly, for the first two, we assume that investors expect in advance that there will be no intervention at all or the central bank will always intervene. This means that at date 2 investors do not learn any information about β, and for them the distinction between $\mathrm{E}[\varepsilon_2^-]$ and $\mathrm{E}[\varepsilon_2^+]$ disappears. The third scenario is modeled in the previous section - the central bank determines whether to intervene depending on the values of the parameters ε_1, ε_2, α, θ, σ, δ_1, δ_2, δ_3, o_1 and o_2.

First, let's compare the utility function values for the "never intervene", "always intervene", and "endogenously intervene" scenarios:

Figure 3 - Comparison of "never intervene" and "endogenously intervene" regimes for the whole sample



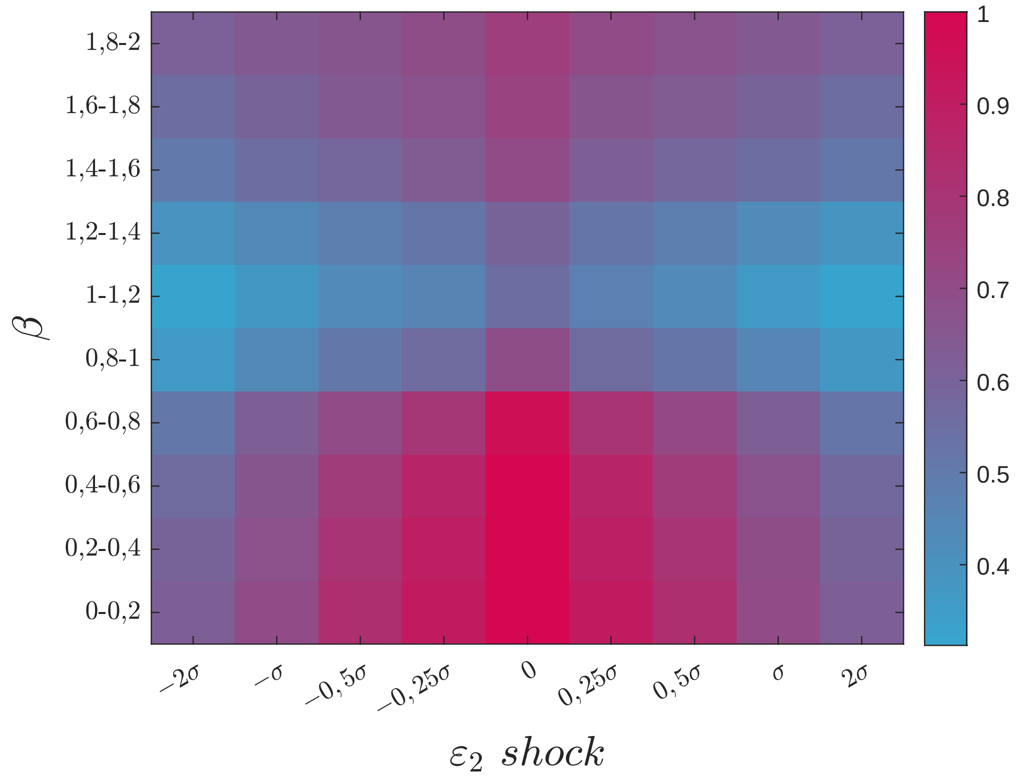
Notes: The graph indicates the probability that the value of the central bank's utility function in the "never intervene" regime is lower than in the "endogenously intervene" regime, given the distributions of ε_1, ε_3, α, β, θ, σ, δ_1, δ_2, δ_3, *and* o_1; o_2 *are distributed equally among all the values used. For* ε_2, *discrete shocks of* $-2\sigma_{\varepsilon_2}$; $-\sigma_{\varepsilon_2}$; $-0,5\sigma_{\varepsilon_2}$; $-0,25\sigma_{\varepsilon_2}$; 0; $0,25\sigma_{\varepsilon_2}$; $0,5\sigma_{\varepsilon_2}$; σ_{ε_2}; $2\sigma_{\varepsilon_2}$ *are used, while for* β *all values from 0 to 2 in steps of 0,002 are used, which are then grouped into 10 clusters in steps of 0,2.*

As you can see, it is reasonable for the central bank to stick to a regime of endogenous collegial communications during the quiet period. The situation is close enough to coin flip only for β values close to 1, but still in favor of occasional communications.

In this case, the main driver of the outcome is the relative weight of the yield R_2 - o_2. If the central bank cares strongly about a price jump at date 2 ($o_2 > 1$), then on average, it tends to intervene less frequently, as the more significant price jumps that arise when ε_2 is communicated are now less desirable. And for the cases $o_2 = 0.5$ and $o_2 = 1$, the results are relatively trivial - the central bank should switch to the "endogenously intervene" regime; this can be seen in Appendix 6. A more meaningful case in which the "endogenously intervene" regime does not provide an advantage almost always is $o_2 = 2$. The results of the comparison of regimes in this case can be seen in the picture below:

Figure 4 - Comparison of "never intervene" and "endogenously intervene" regimes

for $o_2 = 2$

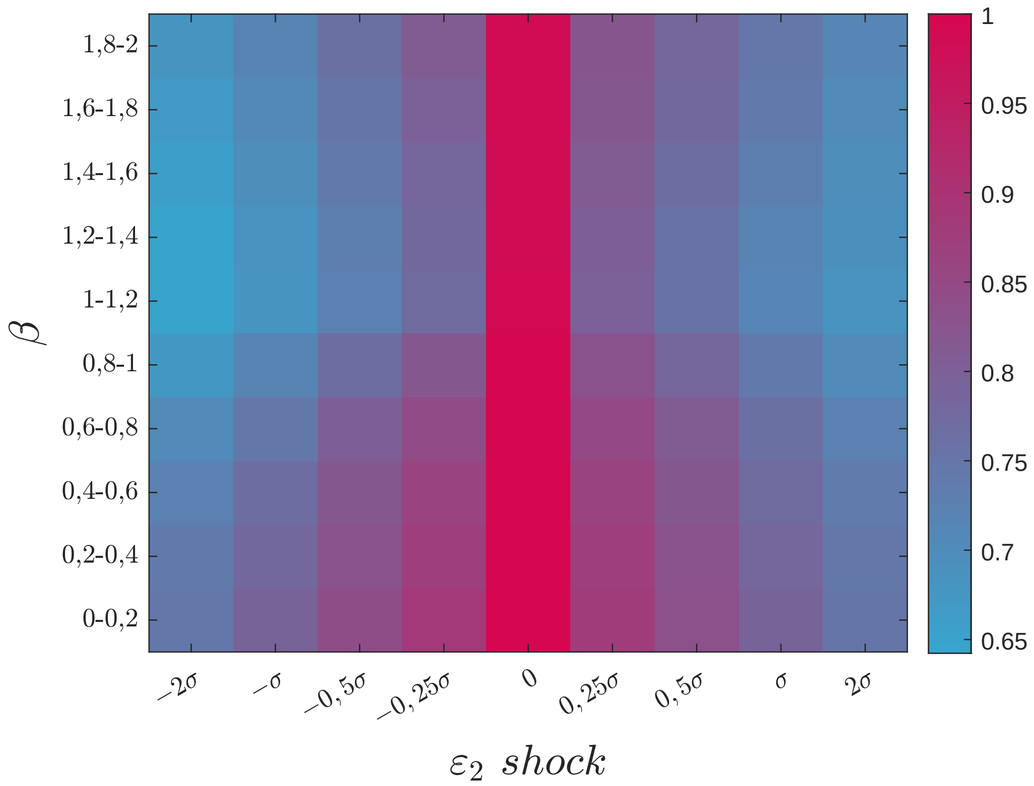


Notes: The graph indicates the probability that the value of the central bank's utility function in the "never intervene" regime is lower than in the "endogenously intervene" regime, given the distributions of ε_1, ε_3, α, β, θ, σ, δ_1, δ_2, δ_3, and o_1 is distributed equally among all the values used. For ε_2, discrete shocks of $-2\sigma_{\varepsilon_2}$; $-\sigma_{\varepsilon_2}$; $-0,5\sigma_{\varepsilon_2}$; $-0,25\sigma_{\varepsilon_2}$; 0; $0,25\sigma_{\varepsilon_2}$; $0,5\sigma_{\varepsilon_2}$; σ_{ε_2}; $2\sigma_{\varepsilon_2}$ are used, while for β all values from 0 to 2 in steps of 0,002 are used, which are then grouped into 10 clusters in steps of 0,2.

That is, even if the central bank is two times more worried about a price hike at date 2, on average, it is still reasonable for the central bank to stick to a regime of endogenous collegial communications during the quiet period. However, this result does not hold for β values close to 1 and large ε_2 shocks.

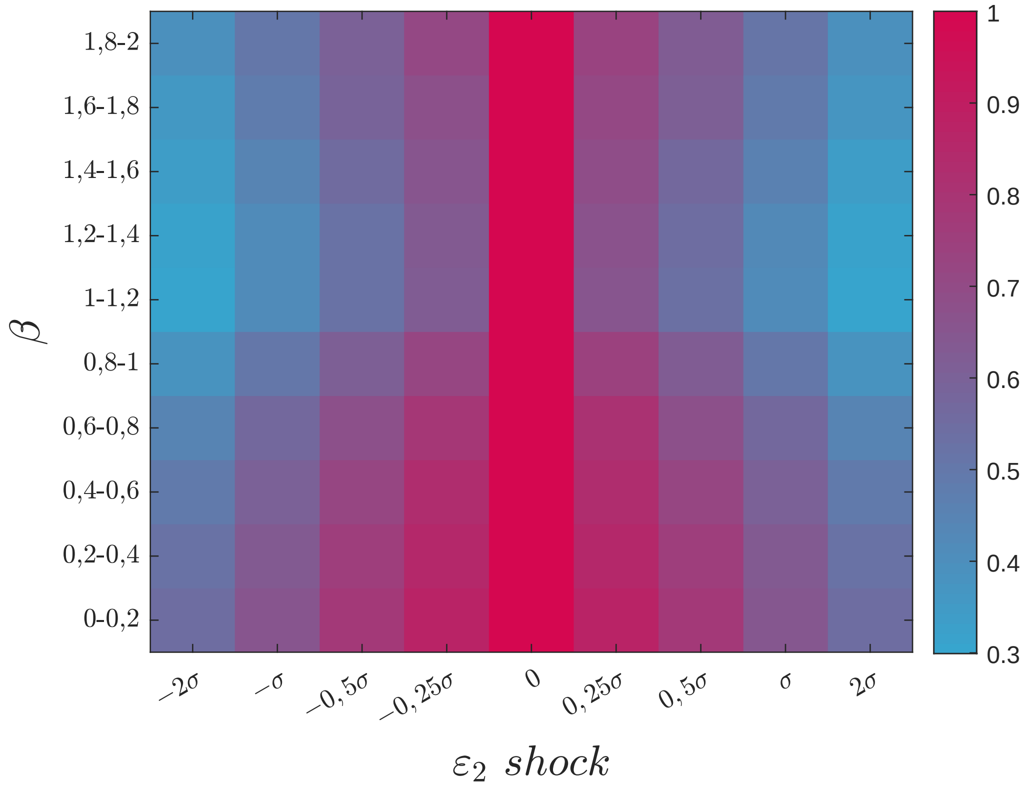
As can be seen in the graphs below, the results are similar for scenarios "never intervene" and "always intervene" for the whole sample; however, in the borderline case $o_2 = 2$, "always intervene" regime performs worse than "endogenously intervene", though still reasonably well compared to "never intervene" option.

Figure 5 - Comparison of "never intervene" and "always intervene" regimes for the whole sample



Notes: The graph indicates the probability that the value of the central bank's utility function in the "never intervene" regime is lower than in the "always intervene" regime, given the distributions of ε_1, ε_3, α, β, θ, σ, δ_1, δ_2, δ_3, and o_1; o_2 are distributed equally among all the values used. For ε_2, discrete shocks of $-2\sigma_{\varepsilon_2}$; $-\sigma_{\varepsilon_2}$; $-0,5\sigma_{\varepsilon_2}$; $-0,25\sigma_{\varepsilon_2}$; 0; $0,25\sigma_{\varepsilon_2}$; $0,5\sigma_{\varepsilon_2}$; σ_{ε_2}; $2\sigma_{\varepsilon_2}$ are used, while for β all values from 0 to 2 in steps of 0,002 are used, which are then grouped into 10 clusters in steps of 0,2.

Figure 6 - Comparison of "never intervene" and "always intervene" regimes for $o_2 = 2$



Notes: The graph indicates the probability that the value of the central bank's utility function in the "never intervene" regime is lower than in the "always intervene" regime, given the distributions of ε_1, ε_3, α, β, θ, σ, δ_1, δ_2, δ_3, and o_1 is distributed equally among all the values used. For ε_2, discrete shocks of $-2\sigma_{\varepsilon_2}$; $-\sigma_{\varepsilon_2}$; $-0,5\sigma_{\varepsilon_2}$; $-0,25\sigma_{\varepsilon_2}$; 0; $0,25\sigma_{\varepsilon_2}$; $0,5\sigma_{\varepsilon_2}$; σ_{ε_2}; $2\sigma_{\varepsilon_2}$ are used, while for β all values from 0 to 2 in steps of 0,002 are used, which are then grouped into 10 clusters in steps of 0,2.

Results comparing the communication regimes in the more trivial cases $o_2 = 0,5$ and $o_2 = 1$ can be seen in the Appendix 7.

How often is it optimal to communicate?

In the graph below, you can see how likely the central bank is to intervene at different values of the weights of the second period return R_2 and volatility of R_3. The first chart shows the average, minimum, and maximum values. And in the second, 10th, and 90th percentiles.

Figure 7

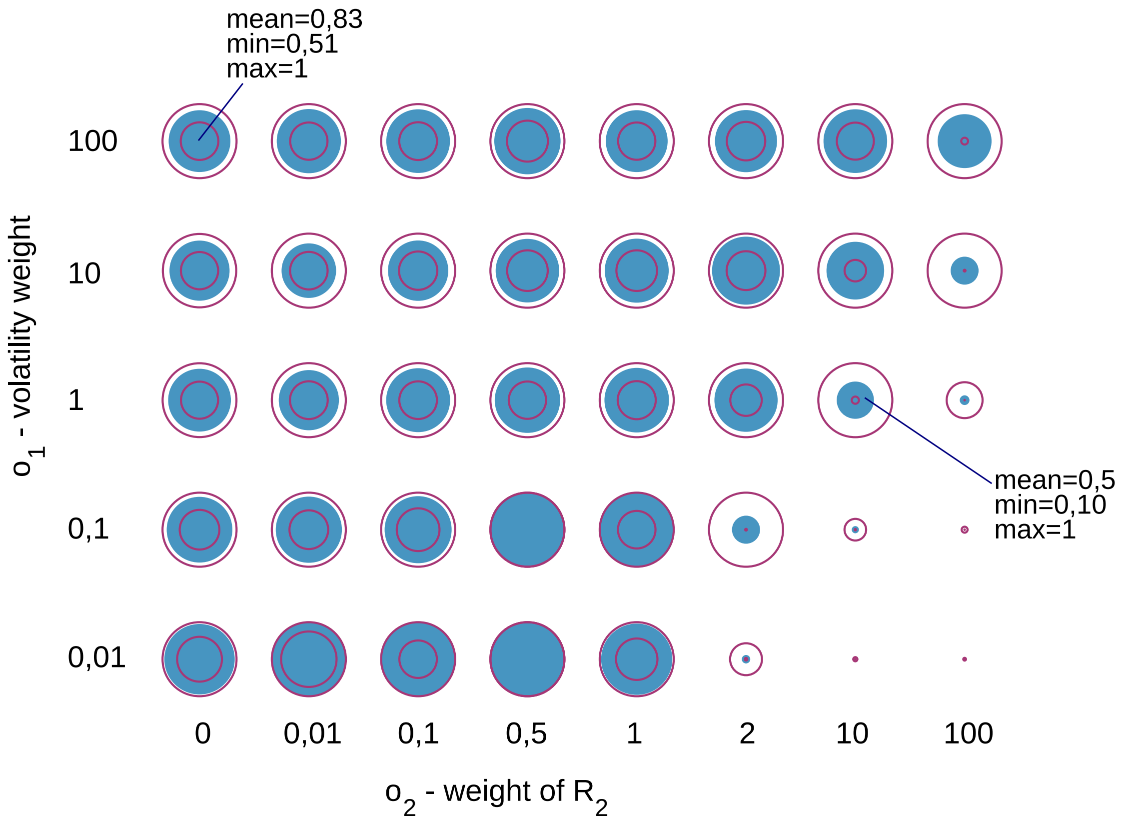
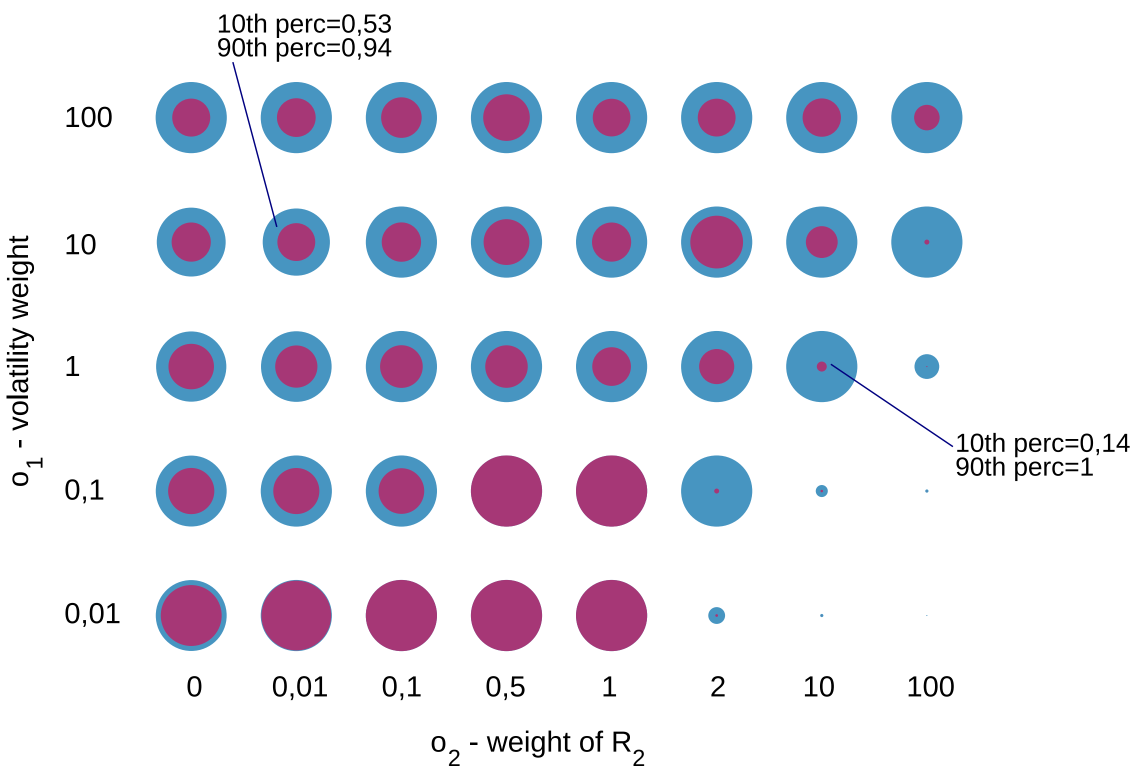

mean=0,83
min=0,51
max=1
mean=0,5
min=0,10
max=1
100
10
1
0,1
0,01
o1 - volatility weight
0
0,01
0,1
0,5
1
2
10
100
o2 - weight of R2

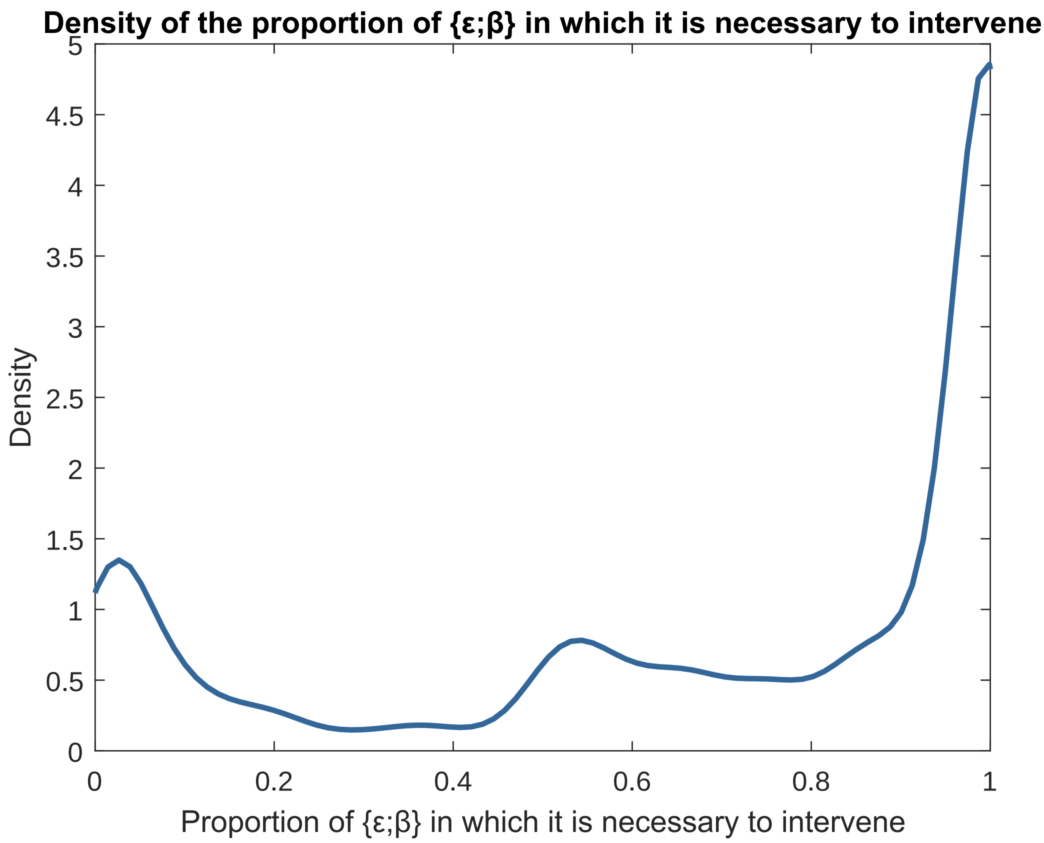
Figure 8


10th perc=0,53
90th perc=0,94
10th perc=0,14
90th perc=1
100
10
1
0,1
0,01
o1 - volatility weight
0
0,01
0,1
0,5
1
2
10
100
o2 - weight of R2

Without focusing on specific values of o_1 and o_2, the main thing that can be seen in these graphs is the high heterogeneity of the results. In some cases, the central bank should almost never intervene, while in other cases, it should intervene at any values of shock ε_2 and confidence in the future decision β.

The distribution of communication frequency $Prop^+$ for the entire sample can be seen in the graph below:

Figure 9



Typical mappings at which values of ε_2 and β central bank should communicate at specific values of $Prop^+$ can be seen in the graphs below (for convenience, we have separated cases $Prop^+ \leq 0,5$ and $Prop^+ \geq 0,5$):

Figure 10

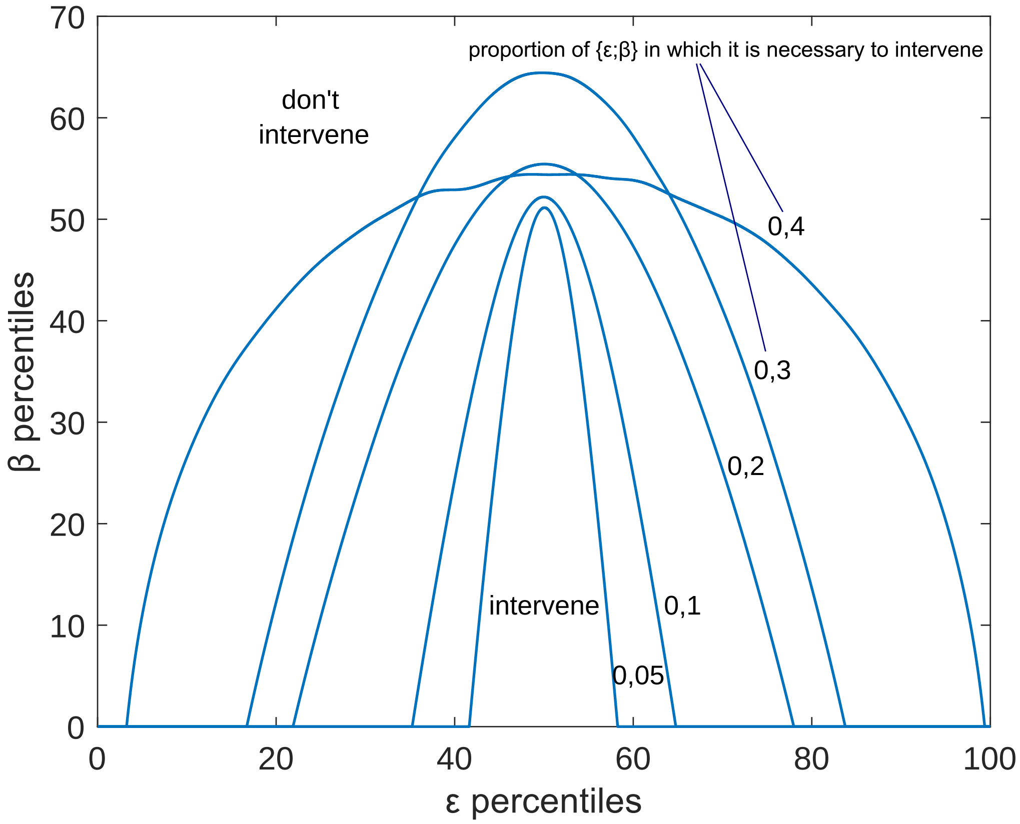
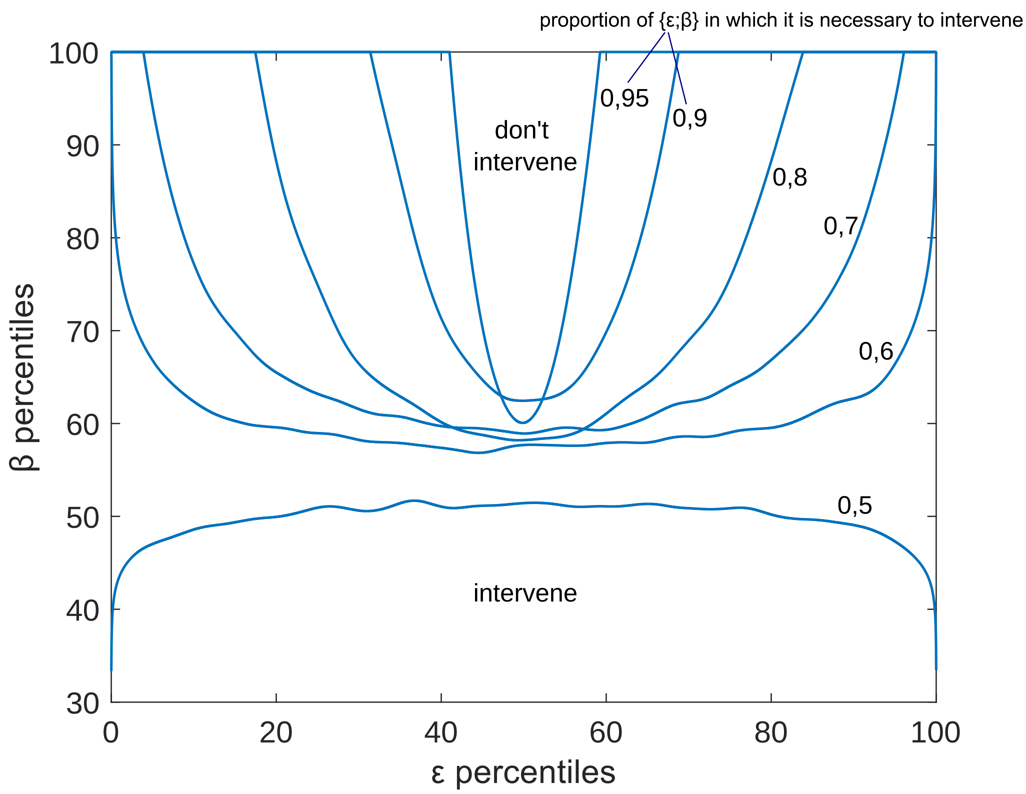

proportion of {ε;β} in which it is necessary to intervene
don't intervene
intervene
0,4
0,3
0,2
0,1
0,05
β percentiles
ε percentiles
0
10
20
30
40
50
60
70
0
20
40
60
80
100

Figure 11


proportion of {ε;β} in which it is necessary to intervene
0,95
0,9
don't intervene
0,8
0,7
0,6
0,5
intervene
β percentiles
ε percentiles
30
40
50
60
70
80
90
100
0
20
40
60
80
100

To find the determinants of central bank decisions, we first consider the most obvious candidates - weights o_1 and o_2. For o_2 - the weight of R_2 in the utility function - the intuition is fairly obvious - the more the regulator worries about what kind of spike in the financial market will occur when the hush is broken, the less willing it is to intervene by telling the market ε_2. For different values of o_1 the influence of this factor is different, the logarithm of o_2 in a regression of the form $Prop^+ = a_1 + a_2 \log_{10} o_2 + +a_3 \log_{10}^2 o_2 + const$ explains from $2,5\%$ of the variance of $Prop^+$ at $o_1 = 100$ to 74% of the variance of $Prop^+$ at $o_1 = 0,01$.

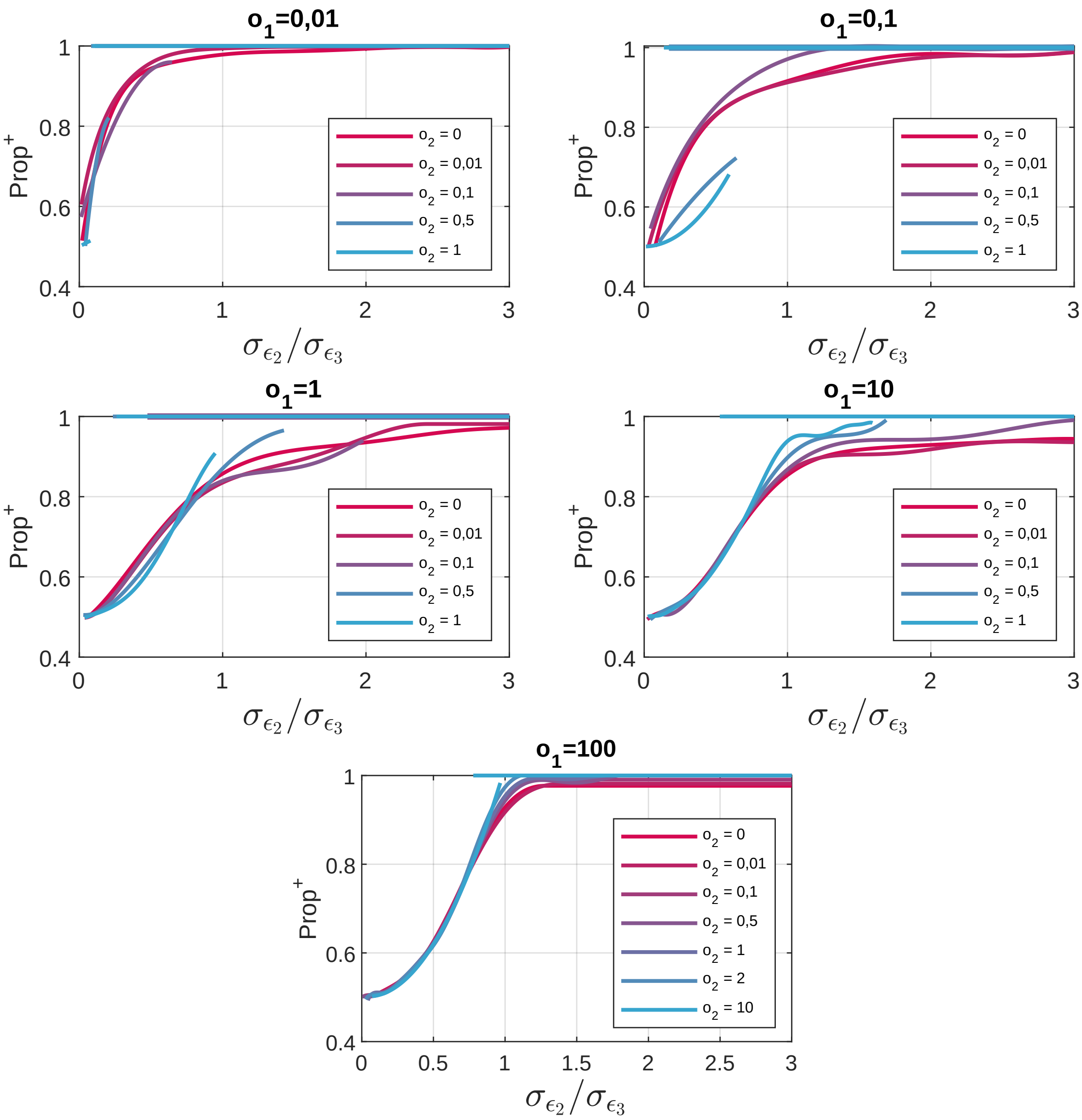
o_1 - the weight of volatility in the regulator's target function - affects $Prop^+$ differently. At $o_2 \leq 1$, the influence is negative - the more the regulator cares about volatility, the less often it intervenes with interventions. However, o_1, in this case, explains not a large part of the $Prop^+$ variance, about $10 - 15\%$ (we will discuss other factors below). But in the case of $o_2 > 1$, on the contrary, the more the regulator cares about volatility, the more often it intervenes with interventions, and the factor o_1 alone explains $50 - 70\%$ of the variance of $Prop^+$.

Let us focus on individual $(o_1; o_2)$ pairs and the determinants of $Prop^+$ within these individual cases. As we saw earlier, significant $Prop^+$ heterogeneity accompanies almost any pair of $o_1; o_2$ values. What might account for it? Intuitively speaking, in our model, the size of shocks has no effect on $Prop^+$ in our model since investors can account for it.

The main factors explaining the central bank's willingness to break the silence are δ_2 and δ_3, that is, the remaining uncertainty about the shocks (the central bank's reaction to the ε_1 shock to the economy and the discussions on the day of the meeting) after period 1. Again, uncertainty about the very first shock is already realized by the time the central bank makes its decision at date 2, so it does not directly affect the central bank's actions. For convenience in comparing the results, we normalized the sum of δ_1, δ_2 and δ_3 to one, so we need to look at the proportions of uncertainty magnitudes of different shocks.

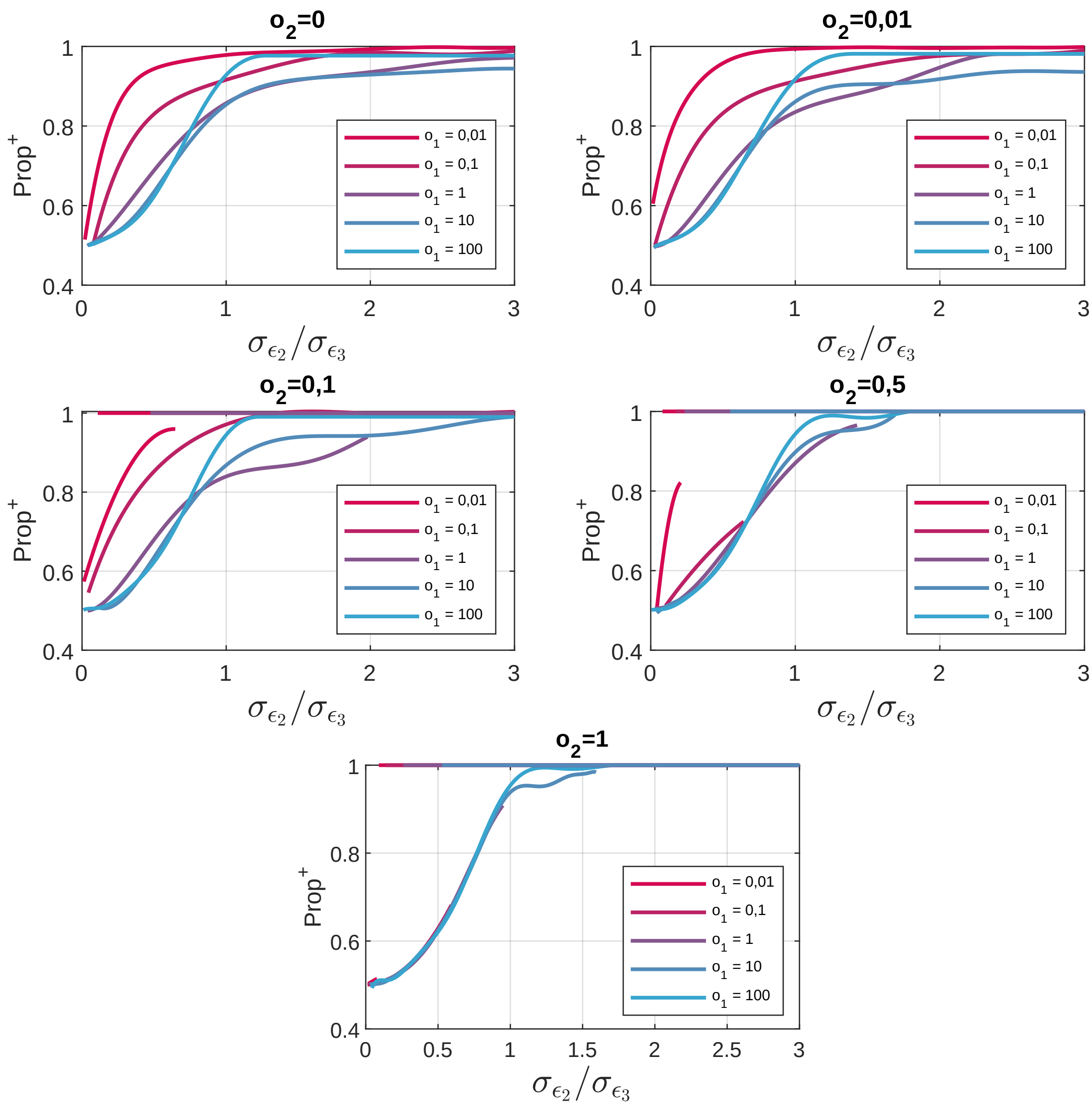
How the probability of communication within the quiet period $Prop^+$ grows with increasing ratio of the uncertainties of the second and third shocks $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3}$ for different combinations of o_1 and o_2 can be seen in the graphs below (for clarity, we group the results first by o_1 values on the first graph, then by o_2 values on the second):

Figure 12



o1=0,01
o1=0,1
o1=1
o1=10
o1=100
Prop+
σϵ2/σϵ3
o2 = 0
o2 = 0,01
o2 = 0,1
o2 = 0,5
o2 = 1
o2 = 2
o2 = 10

Figure 13



That is, it can be seen that the more significant the uncertainty about the central bank's reaction function, the more willing it is to communicate during the quiet period.

Asymmetry

For the most part, the central bank's interventions in our model are symmetric; that is, it reacts similarly to negative and positive shocks regarding whether or not to communicate. However, there is a small exception that highlights a novel mechanism. If $Prop^{+}$ is close enough to 1, i.e., the central bank intervenes very often, it becomes less willing to talk in case of negative news. The roots of the possible asymmetry lie in the existence of the pre-announcement drift itself. During the period under consideration, the stock price gradually increases due to the uncertainty risk premium demanded by investors if there are no additional shocks. But in the case where the central bank's estimate of ε_2 turns out to be negative, the mechanism is as follows - if investors know that the central bank almost always intervenes collegially during the quiet period, then in the absence of intervention, investors realize that the β is very large. And then the absence of intervention not only doesn't leave the uncertainty the same, it increases the uncertainty, meaning that at date 2 we still have a process of building up uncertainty. So investors demand a higher risk premium at date 2, which pulls the price P_2 down (see the graph below). Then, at date 3, negative ε_2 becomes known to the market, and the price P_3 decreases further. So, in the case of a negative realization of ε_2 and no central bank communication at date 2, this mechanism is just doing the central bank's job to smooth out fluctuations in the financial market. But if the central bank communicates at date 2, the markets observe ε_2, the price falls sharply and even overshoots the final price. This happens because, at high $Prop^+$, investors don't lower their β estimate very much since it is still close to the unconditional mean, and they still demand a significant risk premium, or in other words, a positive pre-announcement drift still lies ahead. And as a result, the financial market is subject to bigger swings. This is depicted in the figure below:

Figure 14

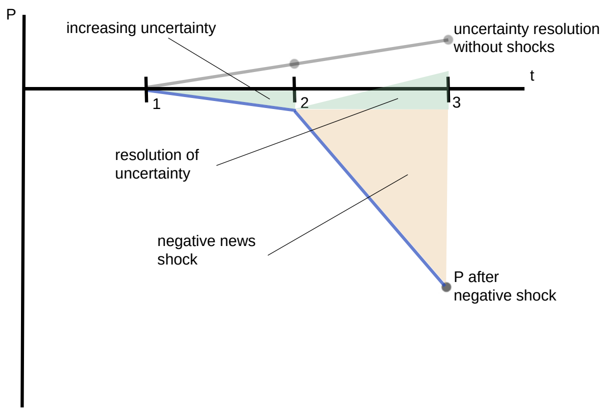


Figure 1: Quiet period

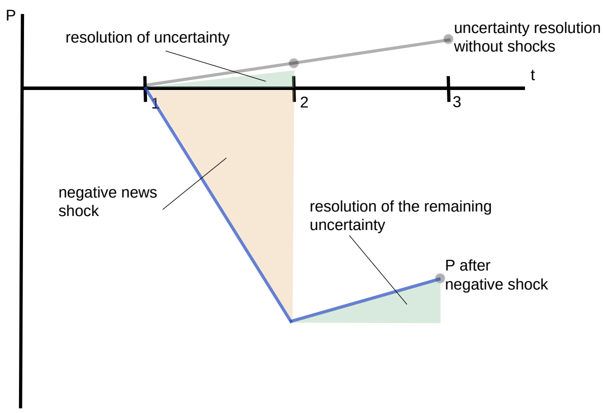


Figure 2: Communication

Empirically, analogous mechanism have been found in Kawamura et al. (2019) for the Bank of Japan, which obfuscates the reports in order not to disclose unfavorable private information. However, there may be different explanations for this behavior. Perhaps the central bank is trying to manipulate the economy in the style of dynamic inconsistency models. Our model tells a shorter-term story: the central bank observing a negative shock ε_2 may sometimes find it more profitable not to communicate information to the market because investors' realization that the central bank does not communicate only when uncertainty about the upcoming meeting is very high leads to higher uncertainty and lower stock price, forcing investors to do some of the central bank's work themselves, moving the stock price toward fundamentals even though there was no communication in this direction.

Implementability

Operating mechanisms may vary across central banks. The collegiality of the decision-making process on the day of the meeting is less important for our results. Still, the collegiality of the communication policy also differs across central banks. As described in Ehrmann, Fratzscher (2005), the Fed pursues an individualistic communication strategy, whereas the ECB and, for example, the Bank of England use a more collegial approach to communication. Our paper argues for the possible usefulness of centralized communication during the quiet period, which involves varying degrees of change in current practices across central banks. However, we suggest that these changes, albeit varying degrees, may apply to a wide range of monetary authorities. In particular, for the reason that moving to more centralized communications may mitigate the effects of cacophonic communications, as was also suggested in Vissing-Jorgensen (2019), who argued that the Fed should consider moving to more centralized communications.

Concluding remarks

When analyzing whether a central bank should always adhere to a quiet period policy, it makes sense for the regulator to look beyond the immediate consequences of such communications. Our financial market model describes a multivariate trade-off in which the central bank not only focuses on the instantaneous reaction of markets to a quiet period breaches but also evaluates both the consequences of the upcoming Board meeting and changes in market volatility. In such a case, the central bank may achieve better results with both an endogenous communication policy and a pervasive communication policy during the quiet period, provided that this communication is collegial and communicates the central bank's reaction function. Even though such a communication policy may be asymmetric and the central bank sometimes doesn't communicate negative news to the markets, it can still smooth financial market fluctuations and provide investors with more accurate information about the central bank's assessment of the economy.

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Appendix

Appendix 1 - Detailed algorithm for generating observations using the Monte Carlo method

In essence, we are trying to assign each observation in the available set to classes 1 or 2 (no communication or presence of communication within the quiet period). And for each separately taken set ε_1, α, θ, σ, δ_1, δ_2, δ_3 the problem statement is quite close to the clustering problem statement in machine learning (because for learning without a teacher the classification problem we are solving is a clustering problem). The difference is that when defining classes, we do not focus on a metric that measures the distance of a point to its class but only on the characteristics of the classes themselves. That is, if we take a finite set of points and assign a class to each point, we will get the value of the function $U^w - U^{w/o}$ for each point, and this function will not depend on the coordinates and class of the point itself, but will depend on the set of all points assigned to one and the other class. However, we can use algorithms inspired by the machine learning algorithms k-medoid and k-means. Having tested two algorithms - an analog of Partitioning Around Medoids for solving the k-medoids problem and an analog of the Hartigan-Wong algorithm for the k-means problem - it was found that the first one provides more stable convergence so that we will describe it as the main one.

In more detail, we will follow the following iterative algorithm:
1a) Generate a random set of parameters ε_1, α, θ, σ, δ_1, δ_2, δ_3 from the corresponding distributions.
1b) Generate 10,000 points $(\varepsilon_2;\ \beta)$ according to the available distributions ε_2 and β.
1c) Starting with an arbitrary distribution of points into classes $[1;2]$, we iteratively change the classes of the points as follows:
- For the available classes, we will count $\mathrm{E}_2[\beta^-]$, $\mathrm{E}_2[\beta^+]$, $\mathrm{E}_1[\varepsilon_2^+]$, $\mathrm{E}_2[\varepsilon_2^-]$, $\mathrm{Var}_2[\varepsilon_2^-]$, Pr_1 and the values of the function $U^w - U^{w/o}$ for each point. Recall that a point must belong to class 1 (we do not break silence) if $U^w - U^{w/o} > 0$. Accordingly, we obtain some number of points with the wrong class. We will call the error metric in this case

$$Err = \begin{cases} 0, & \text{if } U^w - U^{w/o} > 0 \text{ and } class = 1 \\ 0, & \text{if } U^w - U^{w/o} < 0 \text{ and } class = 2 \\ |U^w - U^{w/o}|, & else \end{cases}$$

Now we find $\mathrm{argmax}_{\varepsilon_2;\beta}\, Err$ and change the class of this point. Thus, we find the point with the most erroneous value of the $U^w - U^{w/o}$ function and rerelate it to another class. The class itself becomes correct, however, we now have to recalculate the values of $\mathrm{E}_2[\beta^-]$, $\mathrm{E}_2[\beta^+]$, $\mathrm{E}_1[\varepsilon_2^+]$, $\mathrm{E}_2[\varepsilon_2^-]$, $\mathrm{Var}_2[\varepsilon_2^-]$, Pr_1
1d) Repeat the previous step, finding the next point with the largest error value, until no more points are incorrectly assigned to their class, i.e., $\sum_1^{10000} Err = 0$. The cases when the algorithm does not converge to zero will be described below.
2) The second step of the algorithm is not computationally complex (we only need to calculate the values of the utility function $U^w - U^{w/o}$ and compare them to zero), so we can use a denser grid. Let's partition our space $\varepsilon_2 * \beta : [-\infty; \infty] * [0;1]$ by a $10000 * 10000$ grid and calculate the value of the utility function for each node in the grid. The grid nodes are distributed uniformly at β and according to the 10,000th percentile at ε_2. Using the values $\mathrm{E}_2[\beta^-]$, $\mathrm{E}_2[\beta^+]$, $\mathrm{E}_1[\varepsilon_2^+]$, $\mathrm{E}_2[\varepsilon_2^-]$, $\mathrm{Var}_2[\varepsilon_2^-]$, Pr_1, obtained in the first step, we estimate $U^w - U^{w/o}$ for each point and assign the point to the corresponding class.

Empirically, the algorithm converges fairly quickly ($\leq 8,000$ iterations) to a stable separation of all points into two classes, with no erroneously assigned classes $\sum\limits_{i=1}^{10000} Err_i = 0$. In rare cases, the algorithm does not converge quickly, which is often because k-means-like algorithms can find local extrema rather than global extrema. In this case, the algorithm may fall into a loop that circularly changes the class of one or more points without reaching $\sum_1^{10000} Err = 0$. For such cases, we set several constraints that stop the first step of the algorithm. First, if the loop contains a result of 1 erroneously defined point at some step, we stop the algorithm after ten repetitions. Second, we set an upper limit of 10,000 iterations, after which we stop the algorithm as soon as it reaches a local minimum inside the loop and use the obtained results for the second step of the algorithm. Third, suppose this loop at each step contains the same number of erroneous point classes or the number of erroneously defined point classes > 30 (i.e., $> 0.3\%$ of all points). In that case, we stop the algorithm after 11,000 iterations and proceed to the second step. These conditions constitute a vulnerability for the accuracy of our algorithm. However, the very presence of the second step is a check of the correctness of the algorithm, as we will discuss below. Loops that lead to the number of iterations greater than 10,000 occur in slightly less than 4% of cases; in exactly 2/3 of these cases the number of incorrectly defined classes is less than or equal to 10 ($\leq 0.1\%$ points), in 1.08% cases the number of errors is between 10 and 30 (from 0.1% to 0.3% points) and in 0.25% cases the number of errors is greater than 30 ($> 0.3\%$ points).

In this case, the presence of the second step allows us to check the accuracy of the algorithm, i.e., to check whether it generates the same class partitioning as in the first step. Using the obtained $\mathrm{E}_2[\beta^-]$, $\mathrm{E}_2[\beta^+]$, $\mathrm{E}_1[\varepsilon_2^+]$, $\mathrm{E}_2[\varepsilon_2^-]$, $\mathrm{Var}_2[\varepsilon_2^-]$, Pr_1 directly assign points to classes by comparing $U^w - U^{w/o}$ to zero. Then we check how close the values of these parameters (again $\mathrm{E}_2[\beta^-], \mathrm{E}_2[\beta^+], \mathrm{E}_1[\varepsilon_2^+], \mathrm{E}_2[\varepsilon_2^-], \mathrm{Var}_2[\varepsilon_2^-]$ and Pr_1) estimated in the first step are to those obtained in the second step.

Appendix 2 - Second-order condition for V_1'

S.O.C: $\frac{\partial^2 \mathrm{E}_1[J_2]}{\partial^2\theta_1} = -\frac{1}{2}\alpha\frac{\partial^2 V_1}{\partial^2\theta_1} = -\frac{1}{2}\alpha\Big[2\sigma^2 \mathrm{Pr}_1(1-\mathrm{Pr}_1)(\mathrm{E}_1[\varepsilon_2^+] - \mathrm{E}_2[\varepsilon_2^-])^2 + 2\alpha^2\sigma^4 \mathrm{Pr}_1(1-\mathrm{Pr}_1)(\mathrm{E}_1[\beta^+]\delta_3 - \mathrm{Var}_2[\varepsilon_2^-] - \mathrm{E}_2[\beta^-]\delta_3)^2 - 4\alpha\sigma^3\big(\mathrm{Pr}_1\,\delta_3\,\mathrm{E}_1[\varepsilon_2^+\beta^+] + (1-\mathrm{Pr}_1)\,\mathrm{E}_2[\varepsilon_2^-]\,\mathrm{Var}_2[\varepsilon_2^-] + (1-\mathrm{Pr}_1)\delta_3\,\mathrm{E}_2[\varepsilon_2^-\beta^-]\big) + 4\alpha\sigma^3\big(\mathrm{Pr}_1\,\mathrm{E}_1[\varepsilon_2^+] + (1-\mathrm{Pr}_1)\,\mathrm{E}_2[\varepsilon_2^-]\big)\big(\mathrm{Pr}_1\,\mathrm{E}_1[\beta^+]\delta_3 + (1-\mathrm{Pr}_1)(\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3)\big)\Big] = -\frac{1}{2}\alpha\Big[2\sigma^2 \mathrm{Pr}_1(1-\mathrm{Pr}_1)(\mathrm{E}_1[\varepsilon_2^+]-\mathrm{E}_2[\varepsilon_2^-])^2 + 2\alpha^2\sigma^4 \mathrm{Pr}_1(1-\mathrm{Pr}_1)(\mathrm{E}_1[\beta^+]\delta_3 - \mathrm{Var}_2[\varepsilon_2^-] - \mathrm{E}_2[\beta^-]\delta_3)^2 - 4\alpha\sigma^3\big(\mathrm{Pr}_1\,\delta_3\,\mathrm{E}_1[\varepsilon_2^+\,\mathrm{E}_2[\beta^+]] + (1-\mathrm{Pr}_1)\,\mathrm{E}_2[\varepsilon_2^-]\,\mathrm{Var}_2[\varepsilon_2^-] + (1-\mathrm{Pr}_1)\delta_3\,\mathrm{E}_1[\mathrm{E}_2[\varepsilon_2^-]\,\mathrm{E}_2[\beta^-]]\big) + 4\alpha\sigma^3\big(\mathrm{Pr}_1\,\mathrm{E}_1[\varepsilon_2^+] + (1-\mathrm{Pr}_1)\,\mathrm{E}_2[\varepsilon_2^-]\big)\big(\mathrm{Pr}_1\,\mathrm{E}_1[\beta^+]\delta_3 + (1-\mathrm{Pr}_1)(\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3)\big)\Big] = -\frac{1}{2}\alpha\Big[2\sigma^2 \mathrm{Pr}_1(1-\mathrm{Pr}_1)(\mathrm{E}_1[\varepsilon_2^+]-\mathrm{E}_2[\varepsilon_2^-])^2 + 2\alpha^2\sigma^4 \mathrm{Pr}_1(1-\mathrm{Pr}_1)(\mathrm{E}_1[\beta^+]\delta_3 - \mathrm{Var}_2[\varepsilon_2^-] - \mathrm{E}_2[\beta^-]\delta_3)^2 - 4\alpha\sigma^3\big(\mathrm{Pr}_1\,\delta_3\,\mathrm{E}_1[\varepsilon_2^+]\,\mathrm{E}_1[\beta^+] + (1-\mathrm{Pr}_1)\,\mathrm{E}_2[\varepsilon_2^-]\,\mathrm{Var}_2[\varepsilon_2^-] + (1-\mathrm{Pr}_1)\delta_3\,\mathrm{E}_1[\varepsilon_2^-]\,\mathrm{E}_1[\beta^-]\big) + 4\alpha\sigma^3\big(\mathrm{Pr}_1\,\mathrm{E}_1[\varepsilon_2^+] + (1-\mathrm{Pr}_1)\,\mathrm{E}_2[\varepsilon_2^-]\big)\big(\mathrm{Pr}_1\,\mathrm{E}_1[\beta^+]\delta_3 + (1-\mathrm{Pr}_1)(\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3)\big)\Big] = -\frac{1}{2}\alpha\Big[2\sigma^2 \mathrm{Pr}_1(1-\mathrm{Pr}_1)(\mathrm{E}_1[\varepsilon_2^+] - \mathrm{E}_2[\varepsilon_2^-])^2 + 2\alpha^2\sigma^4 \mathrm{Pr}_1(1-\mathrm{Pr}_1)(\mathrm{E}_1[\beta^+]\delta_3 - \mathrm{Var}_2[\varepsilon_2^-] - \mathrm{E}_2[\beta^-]\delta_3)^2 + 4\alpha\sigma^3\Big(-\mathrm{Pr}_1\,\delta_3\,\mathrm{E}_1[\varepsilon_2^+]\,\mathrm{E}_1[\beta^+] - (1-\mathrm{Pr}_1)\,\mathrm{E}_2[\varepsilon_2^-]\,\mathrm{Var}_2[\varepsilon_2^-] - (1-\mathrm{Pr}_1)\delta_3\,\mathrm{E}_2[\varepsilon_2^-]\,\mathrm{E}_2[\beta^-] + \mathrm{Pr}_1^2\,\mathrm{E}_1[\varepsilon_2^+]\,\mathrm{E}_1[\beta^+]\delta_3 + \mathrm{Pr}_1(1-\mathrm{Pr}_1)\,\mathrm{E}_1[\varepsilon_2^+](\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3) + \mathrm{Pr}_1(1-\mathrm{Pr}_1)\,\mathrm{E}_2[\varepsilon_2^-]\,\mathrm{E}_1[\beta^+]\delta_3 + (1-\mathrm{Pr}_1)^2\,\mathrm{E}_2[\varepsilon_2^-](\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3)\Big)\Big] = -\frac{1}{2}\alpha\Big[2\sigma^2 \mathrm{Pr}_1(1-$

$\Pr_1)(\mathrm{E}_1[\varepsilon_2^+] - \mathrm{E}_2[\varepsilon_2^-])^2 + 2\alpha^2\sigma^4 \Pr_1(1-\Pr_1)(\mathrm{E}_1[\beta^+]\delta_3 - \mathrm{Var}_2[\varepsilon_2^-] - \mathrm{E}_2[\beta^-]\delta_3)^2 + 4\alpha\sigma^3 \Big(-\delta_3 \mathrm{E}_1[\varepsilon_2^+]\mathrm{E}_1[\beta^+]\Pr_1(1-\Pr_1) - \mathrm{E}_2[\varepsilon_2^-](\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3)\Pr_1(1-\Pr_1) + \Pr_1(1-\Pr_1)\mathrm{E}_1[\varepsilon_2^+](\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3) + \Pr_1(1-\Pr_1)\mathrm{E}_2[\varepsilon_2^-]\mathrm{E}_1[\beta^+]\delta_3\Big)\Big] = -\frac{1}{2}\alpha\Big[2\sigma^2\Pr_1(1-\Pr_1)(\mathrm{E}_1[\varepsilon_2^+] - \mathrm{E}_2[\varepsilon_2^-])^2 + 2\alpha^2\sigma^4\Pr_1(1-\Pr_1)(\mathrm{E}_1[\beta^+]\delta_3 - \mathrm{Var}_2[\varepsilon_2^-] - \mathrm{E}_2[\beta^-]\delta_3)^2 + 4\alpha\sigma^3\Pr_1(1-\Pr_1)\Big(\mathrm{E}_1[\varepsilon_2^+](-\delta_3\mathrm{E}_1[\beta^+] + \mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3) - \mathrm{E}_2[\varepsilon_2^-](\mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3 - \mathrm{E}_1[\beta^+]\delta_3)\Big)\Big] = -\frac{1}{2}\alpha\Big[2\sigma^2\Pr_1(1-\Pr_1)(\mathrm{E}_1[\varepsilon_2^+] - \mathrm{E}_2[\varepsilon_2^-])^2 + 2\alpha^2\sigma^4\Pr_1(1-\Pr_1)(\mathrm{E}_1[\beta^+]\delta_3 - \mathrm{Var}_2[\varepsilon_2^-] - \mathrm{E}_2[\beta^-]\delta_3)^2 + 4\alpha\sigma^3\Pr_1(1-\Pr_1)(\mathrm{E}_1[\varepsilon_2^+] - \mathrm{E}_2[\varepsilon_2^-])(-\delta_3\mathrm{E}_1[\beta^+] + \mathrm{Var}_2[\varepsilon_2^-] + \mathrm{E}_2[\beta^-]\delta_3)\Big] = -\alpha\Pr_1(1-\Pr_1)\Big[\sigma(\mathrm{E}_1[\varepsilon_2^+] - \mathrm{E}_2[\varepsilon_2^-]) - \alpha\sigma^2(\mathrm{E}_1[\beta^+]\delta_3 - \mathrm{Var}_2[\varepsilon_2^-] - \mathrm{E}_2[\beta^-]\delta_3)\Big]^2 < 0$

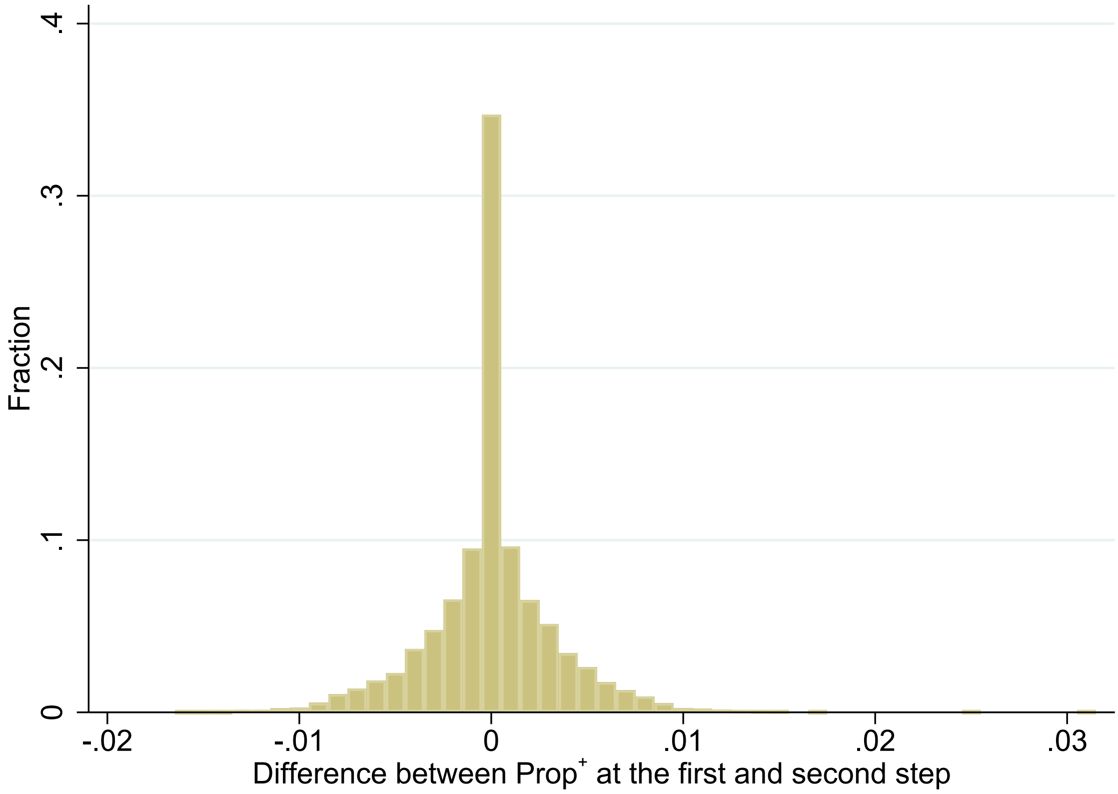
Appendix 3 - Robustness checks

We use several different ways to check the robustness of the results. First, we will use a comparison of the results of the first and second steps as a metric of the quality of the algorithm used. That is, how accurately the points ε_2^+; ε_2^-; β^+; β^- estimated by the iterative algorithm in the first step from the resulting mapping $S_2 * B \to \{1; 2\}$ when substituted into the utility function reproduce that mapping for us. We will not compare individual points and decisions on the first and second steps to intervene or not to intervene because in the first step we work with 10,000 randomly distributed points due to computational complexity, and in the second step we build a fixed grid of 100,000,000 points. But we can compare, first, the $Prop^+$ values, and second, the $\mathrm{E}_2[\varepsilon_2^+]$; $\mathrm{E}_2[\varepsilon_2^-]$; $\mathrm{E}_2[\beta_2^+]$; $\mathrm{E}_2[\beta_2^-]$ moments. The absolute difference in the values of the indicators is given in the table below; for $Prop^+$ and β, we use the values themselves, and for ε_2 we use the percentiles of the distribution to make the observations comparable between each other. In addition, for observations with $Prop^+ > 0,99$, i.e., when we have only isolated cases requiring central bank intervention, the algorithm still performs accurately in terms of the deviations of $Prop^+$ and $\mathrm{E}_2[\varepsilon_2]$ in the first and second steps of the algorithm. Still, it underperforms in terms of $\mathrm{E}_2[\beta_2]$ deviations. This is because the first step of the algorithm works with more sparse subsets of the space $\{\varepsilon_2; \beta\}$, so leaving only a few points out of 10,000 in the first step, in the second step, this is transformed into a region for a small interval ε_2 (it can be of slightly different shape, as we discussed earlier). These few points in the first step in terms of the value of the exponent $E[\beta^-]$ do not specify this region precisely enough, so $E[\beta^-]$ can vary quite a bit. Therefore, we separately report the deviation values for all observations and observations with $Prop^+ < 0,99$.

Also consider the distribution of the difference $Prop^+$ at the first and second steps - $Prop_1^+ - Prop_2^+$.

Figure 15

Moments	Mean	Std. deviation
$Prop^+$	0,0022	0,0028
$\mathrm{E}_2[\varepsilon_2^+]$	0,33	0,30
$\mathrm{E}_2[\varepsilon_2^-]$	0,74	3,09
$\mathrm{E}_2[\beta_2^+]$	0,0085	0,0157
$\mathrm{E}_2[\beta_2^-]$	0,0048	0,0063
$\mathrm{E}_2[\beta_2^+]all$	0,0072	0,0133
$\mathrm{E}_2[\beta_2^-]all$	0,0643	0,1709



It can be seen that the difference is very close to 0 and does not have huge outliers.

Another way to check the robustness of the results would be to use different distributions for β. We will use two alternative scenarios. In one of them, β will also be uniformly distributed, but on an interval from 0 to 1, reflecting the idea that δ_3 sets a maximum for the variance of R_3 and β shows how much this uncertainty is reduced by the analytical work done. The second scenario used is the assumption that β is truncated normally with a mean of 1 and a standard deviation of 0,3, enclosed between 0 and 2. That is, $\beta \sim \mathrm{N}(1; 0, 09)$. The standard deviation exponent is chosen so that almost the entire distribution density is centered between 0 and 2, even without boundaries. The comparison with our baseline scenario can be either quantitative or qualitative. Qualitatively

We also use increasing the number of generated pairs (ε_2; β) for one observation as one of the robustness checks. For better accuracy when moving to the second step of the algorithm, we repeat the class search procedure for 10,000 generated points 5 times, then estimating $\mathrm{E}_2[\beta^-], \mathrm{E}_2[\beta^+], \mathrm{E}_1[\varepsilon_2^+], \mathrm{E}_2[\varepsilon_2^-], \mathrm{Var}_2[\varepsilon_2^-], \mathrm{Pr}_1$ for all available 50,000 points. The reason for this simplification of the algorithm (we do not take just 50,000 points for the iterative class change procedure) is the computational complexity, which increases nonlinearly with the number of points, making it extremely computationally expensive to estimate for 50,000 points at once directly.

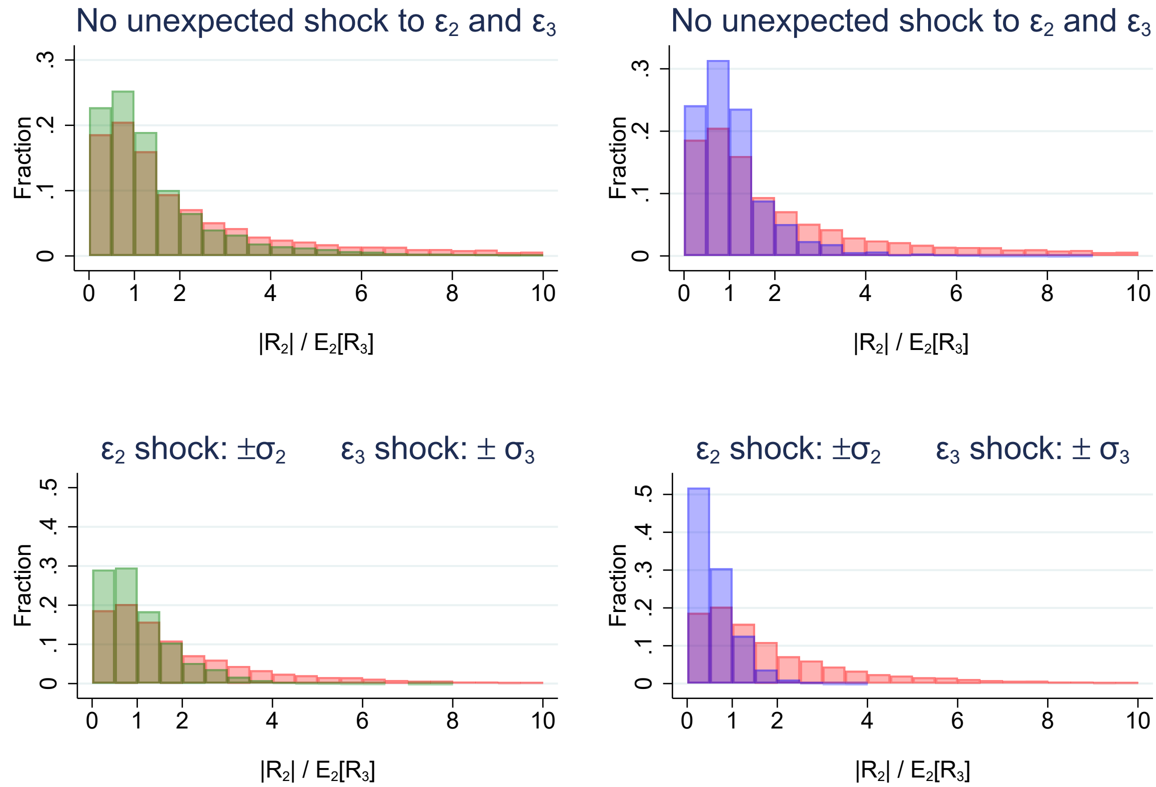
Appendix 4 - Comparison of returns

What does the central bank see as the immediate consequences of a breach of a quiet period in our model? A significant market reaction immediately after a hush is one of the main motivations for central banks to make decisions in real life, so let's look at what the model says about this. To do this, let's compare the values of the yields R_2 and R_3 themselves, using the measure $|R_2|/\mathrm{E}_2[R_3]$ - how many times larger a shock can be observed in the market at the time of a blackout period breach compared to expectations from the day of the rally. We are only interested in cases when the central bank decides to intervene. For convenience, we will consider $\beta = 1$, the mean value for β, and different shocks for ε_2 equal to 0; $-\sigma_{\varepsilon_2}$; σ_{ε_2}. That is, for no unexpected shock ε_2 and $\varepsilon_2 = \mathrm{E}_1[\varepsilon_2] = \rho\sqrt{\frac{\delta_2}{\delta_1}}\varepsilon_1$, we also use $\varepsilon_3 = \mathrm{E}_1[\varepsilon_3] = 0$. But for shock $\varepsilon_2 = \mathrm{E}_1[\varepsilon_2] \pm \sigma_{\varepsilon_2} = \rho\sqrt{\frac{\delta_2}{\delta_1}}\varepsilon_1 \pm \sqrt{\delta_2(1-\rho^2)}$ we will already use $|R_2|/|R_3|$ and instead of waiting for R_3 we will use a similarly sized shock for ε_3: $\varepsilon_3 = \mathrm{E}_1[\varepsilon_3] \pm \sigma_{\varepsilon_3} = 0 \pm \sqrt{\delta 3}$. We do this in order not to bias the results towards increasing the weight of R_2: if we take a shock of size σ for R_2, then in the case of a zero shock to R_3 the ratio $|R_2|/\mathrm{E}_2[R_3]$ will, of course, almost always be greater than one, so we use shocks of size σ for both returns as "shocks of equal surprise." At the same time, the shocks' signs do not significantly impact the result of the analysis, so that we will use a common sample for shocks of size $\pm\sigma$ for both dates.

For no unexpected shocks ε_2 and ε_3, the median value of $|R_2|/\mathrm{E}_2[R_3]$ is 1,55 and only for 35% of observations $|R_2| < \mathrm{E}_2[R_3]$, i.e., markets move more powerfully in most cases when there is a breach of a blackout period policy than after the press release following the Board meeting. That said, if we look at the restricted sample when $o_2 > 1$ or the sample for different o_1, the results do not change much, i.e., the central bank's target function does not significantly impact the results. However, the comparison of returns in dates 2 and 3 is undoubtedly affected by the ratio of the standard deviations of the shocks ε_2 and ε_3. For clarity, we consider the density of the distribution of $|R_2|/\mathrm{E}_2[R_3]$ and $|R_2|/|R_3|$ in three cases - for the whole sample; in the case where $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 1$, i.e. the uncertainty of the Board of Governors meeting is believed by investors to be larger than the uncertainty of the central bank reaction function, and for $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 0,5$, i.e. the uncertainty of the Board of Governors meeting is believed by investors to be significantly larger:

- unrestricted sample, - $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 1$. - $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 0,5$.

Figure 16



The density is plotted from 0 to 10 for clarity, where for no unexpected shock, 10 corresponds to the 88th percentile of observations for the unrestricted sample and the 99th percentile for samples with $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 1$ *and* $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 0,5$*; for shocks* $\pm\sigma$ *10 corresponds to the 96th percentile of observations for unrestricted sampling and for* $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 1$ *and* $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 0,5$ *the maximum value in the sample is less than 10.*

For no unexpected shocks, the median value of $|R_2|/\,\mathrm{E}_2[R_3]$ decreases from 1,55 to 1,04 for $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 1$ and to 0,93 for $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 0,5$. For shocks of $\pm\sigma$, the median value of $|R_2|/|R_3|$ decreases from 1,41 for the whole sample to 0,84 at $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 1$ and to 0,49 at $\sigma_{\varepsilon_2}/\sigma_{\varepsilon_3} < 0,5$.

In Gnan, Rieder (2023) when comparing the effects of the quiet period breaches and the reaction to the ECB's press release statements from the Altavilla et al. (2019), the effect of breaches was between 50 and 80 percent of the effects of press releases. Board leaks are not quite equivalent to the interventions considered in this paper, but they are the closest such events. We find that even with higher Board meeting uncertainty relative to the central bank's target function, the returns at date 2 are quite large, consistent with these empirical findings.

Moreover, the weight of R_2 in the central bank's target function does not strongly affect the ratios of $|R_2|/\,\mathrm{E}_2[R_3]$ and $|R_2|/|R_3|$. That is, even if the central bank pays close attention to the magnitude of the shock at date 2, it should still occasionally break the silence even though the stock price will undergo a large jump at date 2. In addition, the model generates a R_2 return larger in absolute value than the R_3 return

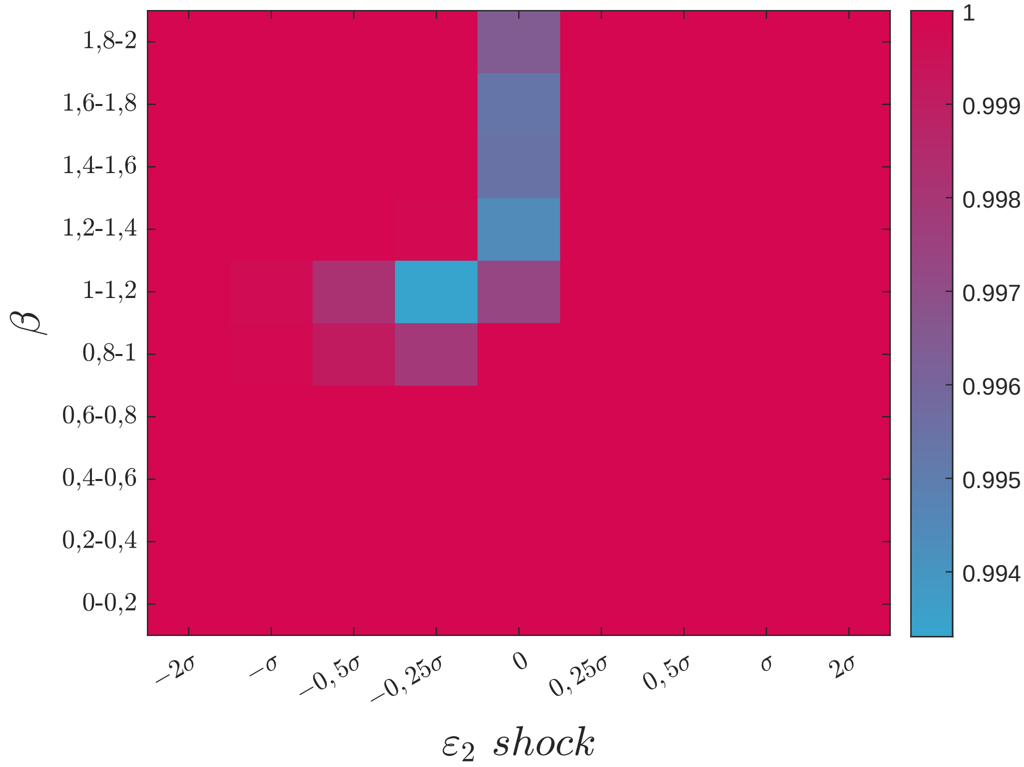
Appendix 5 - Case of insignificant volatility

What happens if we further reduce the weight of variation $(\sigma^2\beta\delta_3 - \sigma^2 \mathrm{E}_2[\beta^+]\delta_3)^2$ in the central bank's target function, e.g., use $o_1 = 0.001$ or $o_1 = 0.0001$? In that case, the model degenerates to a trivial model in which $Prop^+ \approx 0$ for $o_2 > 1$ and $Prop^+ \approx 1$ for $o_2 \leq 1$. That is, if the central bank is completely unimportant to any considerations of variation as a result of the board meeting, then all it has to do is compare the weights of the yields R_2 and R_3 and decide either to always inform the market at date 2 or never to do so. This is understandable because breaking the quiet period regime generates, as found earlier, a significant jump in stock price R_2 even compared to the effect of the rally itself. In contrast, the absolute value of R_2 will be smaller if the blackout period regime is preserved. In particular, the currently adopted quiet period policy fits the case $o_1 \approx 0$; $o_2 > 1$ quite well, but these factors are largely left out of the public discussion of the blackout period regime.

Appendix 6 - Comparison of "never intervene" and "always intervene" regimes for $o_2 = 0,5$ and $o_2 = 1$

In the $o_2 = 1$ case (i.e., where the central bank is equally averse to large stock market jumps at dates 2 and 3), overall, the comparison is clearly in favor of the "always intervene" policy, as can be seen in the figure below. The central bank almost always increases the value of the utility function by switching to an intervention policy, and even for particular betas and huge negative shocks, the benefits of intervention are outweighed at least 66% of the time. The asymmetry, in this case, is an artifact of the risk premium required by investors: a large negative shock, if not communicated to the market at date 2, will just be partially offset by the required risk premium, leading to a not so large price jump at date 3. That is P_2 - the stock price at date 2 in the case of complete absence of intervention will be lower because of the large required risk premium (since it will include all the risks of ε_2, ε_3 and β), and in the case of a negative realization of ε_2 it just might be close enough to P_3.

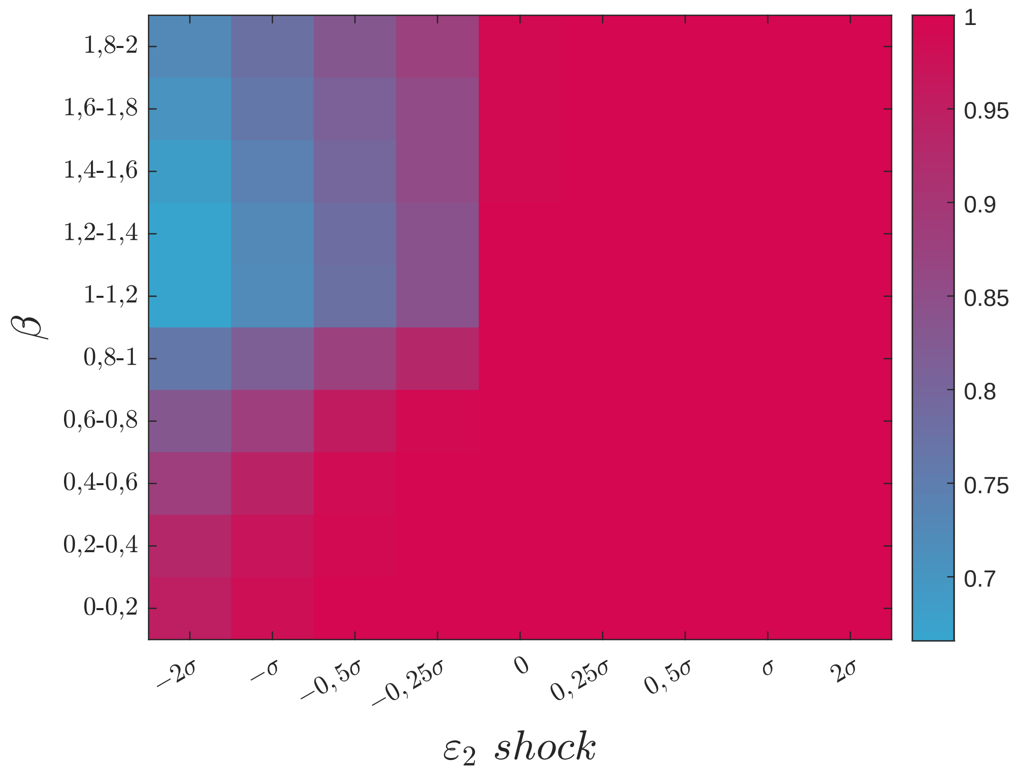
Figure 17 - Comparison of "never intervene" and "always intervene" regimes for $o_2 = 1$



Notes: The graph indicates the probability that the value of the central bank's utility function in the "never intervene" regime is lower than in the "always intervene" regime, given the distributions of ε_1, ε_3, α, β, θ, σ, δ_1, δ_2, δ_3, and o_1 is distributed equally among all the values used. For ε_2, discrete shocks of $-2\sigma_{\varepsilon_2}$; $-\sigma_{\varepsilon_2}$; $-0,5\sigma_{\varepsilon_2}$; $-0,25\sigma_{\varepsilon_2}$; 0; $0,25\sigma_{\varepsilon_2}$; $0,5\sigma_{\varepsilon_2}$; σ_{ε_2}; $2\sigma_{\varepsilon_2}$ are used, while for β all values from 0 to 2 in steps of 0,002 are used, which are then grouped into 10 clusters in steps of 0,2.

The case of $o_2 = 0,5$ (i.e., where the central bank pays a little more attention to price spikes on the day of the BoE meeting) is already fairly trivial, as seen in the figure below. The central bank will always be in a better position if it nearly always intervenes.

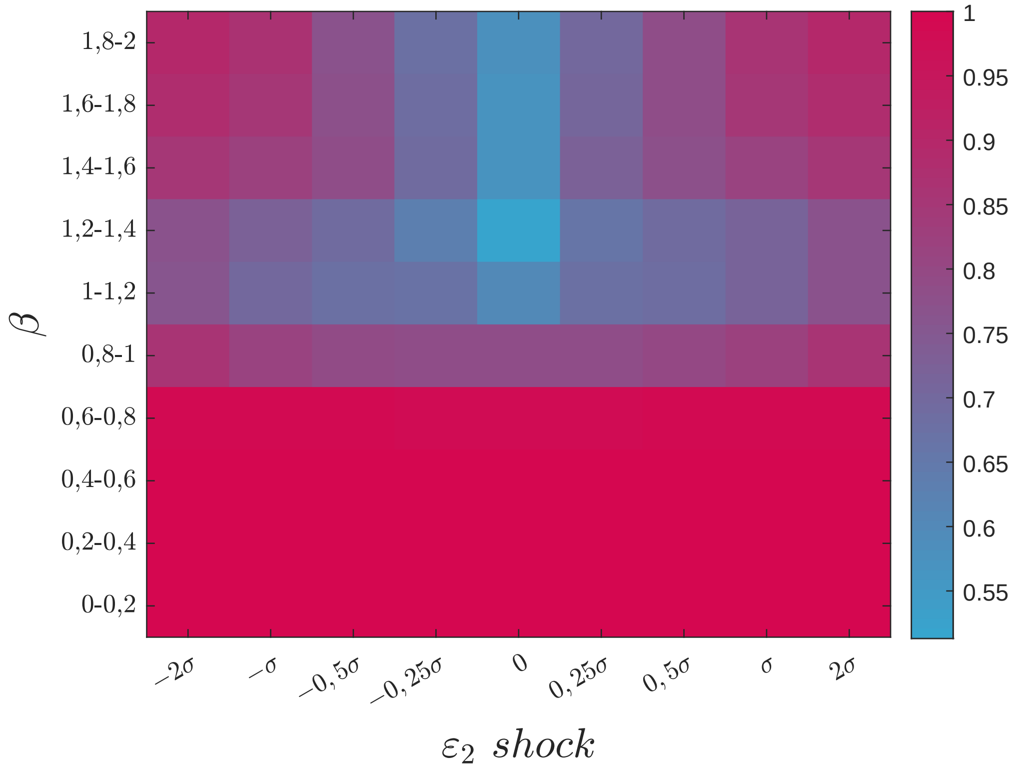
Figure 18 - Comparison of "never intervene" and "always intervene" regimes for $o_2 = 0,5$



Notes: The graph indicates the probability that the value of the central bank's utility function in the "never intervene" regime is lower than in the "always intervene" regime, given the distributions of ε_1, ε_3, α, β, θ, σ, δ_1, δ_2, δ_3, and o_1 is distributed equally among all the values used. For ε_2, discrete shocks of $-2\sigma_{\varepsilon_2}$; $-\sigma_{\varepsilon_2}$; $-0,5\sigma_{\varepsilon_2}$; $-0,25\sigma_{\varepsilon_2}$; 0; $0,25\sigma_{\varepsilon_2}$; $0,5\sigma_{\varepsilon_2}$; σ_{ε_2}; $2\sigma_{\varepsilon_2}$ are used, while for β all values from 0 to 2 in steps of 0,002 are used, which are then grouped into 10 clusters in steps of 0,2.

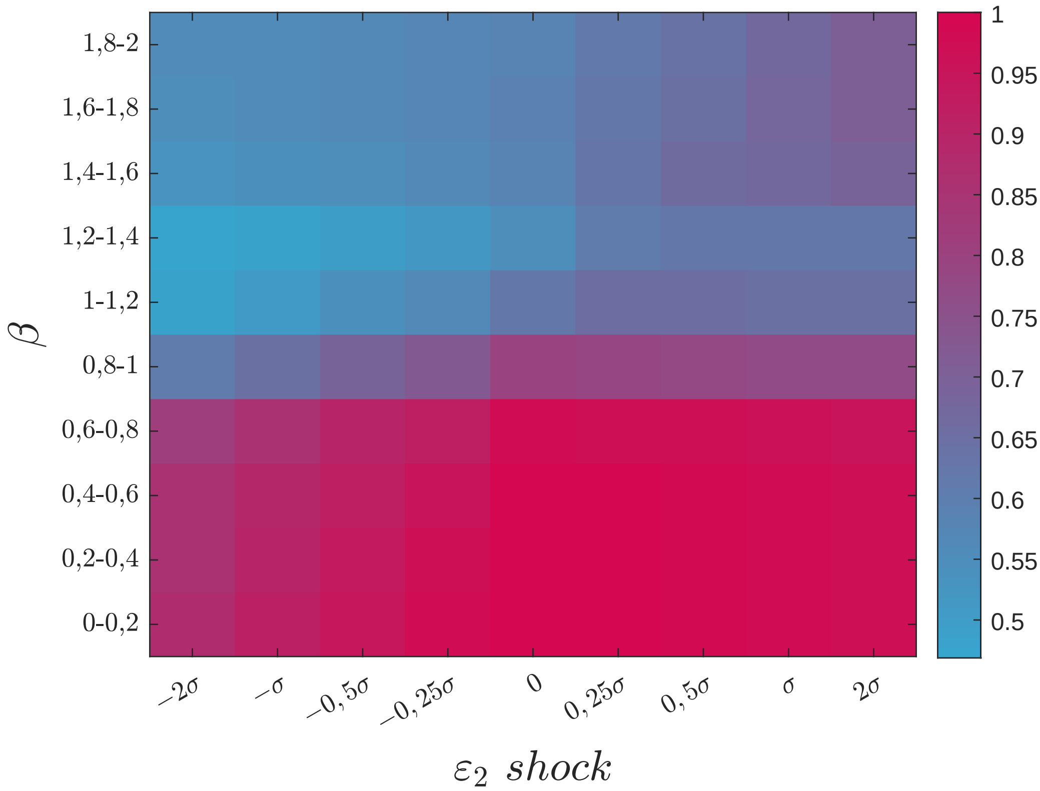
Appendix 7 - Comparison of "never intervene" and "endogenously intervene" cases for $o_2 = 0,5$ and $o_2 = 1$

Figure 19 - Comparison of "never intervene" and "endogenously intervene" regimes for $o_2 = 0,5$



Notes: The graph indicates the probability that the value of the central bank's utility function in the "never intervene" regime is lower than in the "endogenously intervene" regime, given the distributions of ε_1, ε_3, α, β, θ, σ, δ_1, δ_2, δ_3, and o_1 is distributed equally among all the values used. For ε_2, discrete shocks of $-2\sigma_{\varepsilon_2}$; $-\sigma_{\varepsilon_2}$; $-0,5\sigma_{\varepsilon_2}$; $-0,25\sigma_{\varepsilon_2}$; 0; $0,25\sigma_{\varepsilon_2}$; $0,5\sigma_{\varepsilon_2}$; σ_{ε_2}; $2\sigma_{\varepsilon_2}$ are used, while for β all values from 0 to 2 in steps of 0,002 are used, which are then grouped into 10 clusters in steps of 0,2.

Figure 20 - Comparison of "never intervene" and "endogenously intervene" regimes for $o_2 = 1$



Notes: The graph indicates the probability that the value of the central bank's utility function in the "never intervene" regime is lower than in the "endogenously intervene" regime, given the distributions of ε_1, ε_3, α, β, θ, σ, δ_1, δ_2, δ_3, and o_1 is distributed equally among all the values used. For ε_2, discrete shocks of $-2\sigma_{\varepsilon_2}$; $-\sigma_{\varepsilon_2}$; $-0,5\sigma_{\varepsilon_2}$; $-0,25\sigma_{\varepsilon_2}$; 0; $0,25\sigma_{\varepsilon_2}$; $0,5\sigma_{\varepsilon_2}$; σ_{ε_2}; $2\sigma_{\varepsilon_2}$ are used, while for β all values from 0 to 2 in steps of 0,002 are used, which are then grouped into 10 clusters in steps of 0,2.