Homework 3

Exercise 4.4

As a base code for this exercise the logistic.m function has been used with the ode_FE.m function. “While”-loop is used for iterating over new variable timesteps. The future of the loop is determined by the usage of the ‘if-else’-statement (controlled by the user input) with continue/break control words. Achieved result: with the timestep h of 0.078125 seconds the two curves cannot be visually distinguished and it’s becoming useless to continue decreasing the timestep (Figure 1).

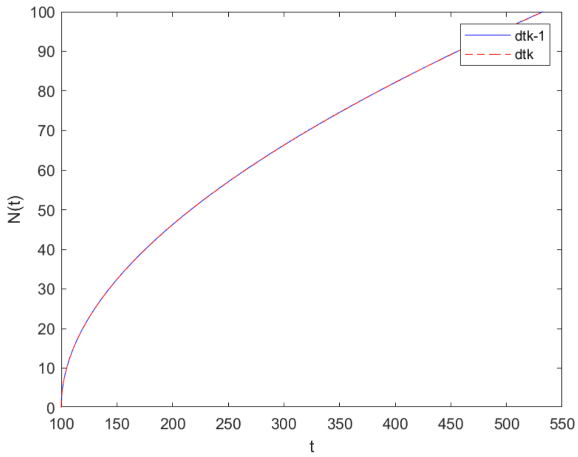

Figure 1 – previous and next logistic model numerical solutions for h = 0.078125

Exercise 4.10

The function that computes kinetic and potential energy was created in a simple way of just using the input parameters, that are equal to the results of FE and EC methods. After achieving the results, the kinetic and potential energy is calculated, and the total energy can be plotted (figures 2 and 3).

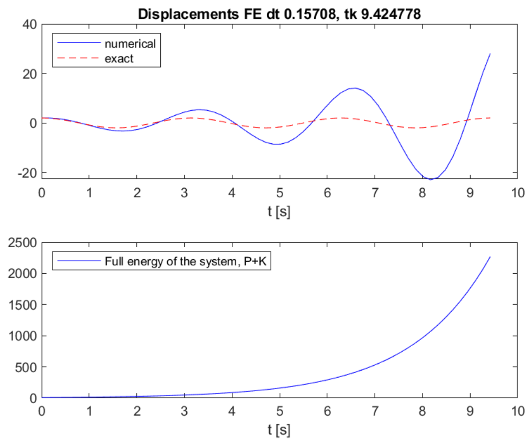

Figure 2 – numerical/exact solutions and total energy of the system, FE method

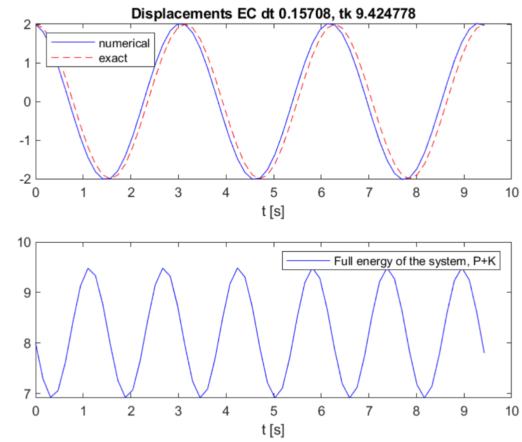

Figure 3 – numerical/exact solutions and total energy of the system, EC method

As can be seen from the plots, the FE method simply don’t create the good solution in this particular example, the total energy of the system is increasing with the increase of the oscillations. As the EC method is better for this example, the total energy of the system has the form of the numerical solution of the system and it has it peaks when the system is achieving its magnitude position.

Exercise 4.14

a) Equations for Backward Euler scheme are implemented due to those formulas:

$$u^n = \frac{\Delta t \cdot v^{n-1} + u^{n-1}}{1 + \Delta t^2 \omega^2}$$

$$v^n = \frac{-\Delta t \cdot \omega^2 \cdot u^{n-1} + v^{n-1}}{1 + \Delta t^2 \omega^2}$$

b) Formulas in section b are implemented as they are, using “for”-loop to iterate. But instead of increasing n from 1 (as we did in FE, EC methods), we are starting from 2 (because we have n-1 terms in equations) and ending with the number of discrete time points plus one additional: N_t + 1 (to iterate over all the values).

c) The plots for the different time steps are shown at Figures 4 and 5. From those it can be stated, that the small time step of dt = T/20, that was enough to solve the system with EC or FE with “magic” fix, isn’t enough in BE method. The good quality of the numerical solution is achieved at time step dt = T/2000, which will require 100 times more iterations in the loop, which is more computational power consuming. But the numerical solution in this case will even don’t have any delay to the exact one (comparing to EC or FE with “magic” fix) but will have just a little bigger peaks on a magnitude positions comparing to the exact solution.

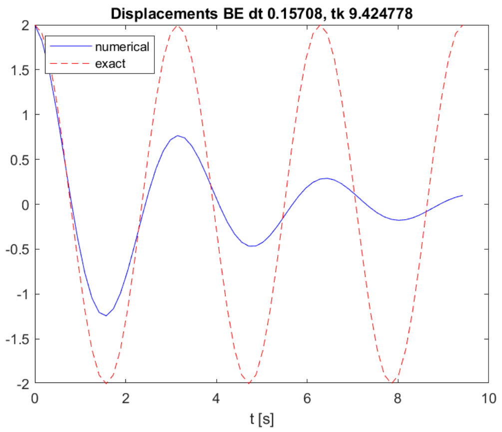

Figure 4 – numerical/exact solutions for BE method, dt = T/20

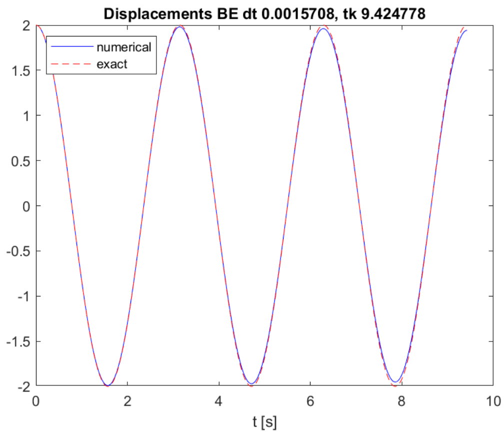

Figure 5 – numerical/exact solutions for BE method, dt = T/2000