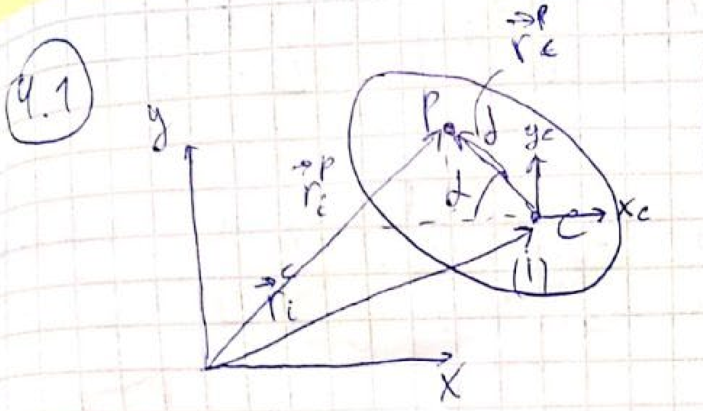
Oleg Rogov

4.1



Init:

$r_i^c = [3.3,\ 2.3]^T$

(global coords)

$\alpha = 50^\circ$

$d = 1{,}2$

Found:

r_c^P – ?

$s_i^{\prime P}$ – ?, r_i^P – ?

Solution:

$x_c^P = -d\cos\alpha = -1{,}2\cos 50^\circ$

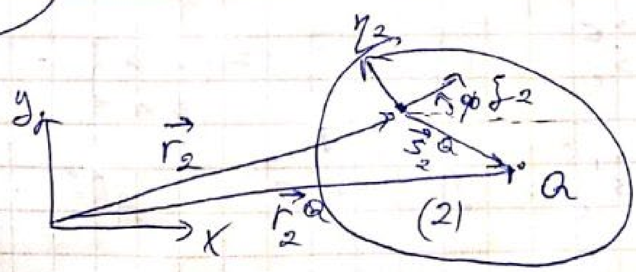
$y_c^P = d\sin\alpha = 1{,}2\sin 50^\circ$

$$\Rightarrow \vec{r}_c^{\,P} = \begin{bmatrix} -0{,}7713 \\ +0{,}9193 \end{bmatrix}$$

$$\vec{r}_i^{\,P} = \vec{r}_c^{\,P} + \vec{r}_i^{\,c} = \begin{bmatrix} -0{,}7713 \\ +0{,}9193 \end{bmatrix} + \begin{bmatrix} 3{,}3 \\ 2{,}3 \end{bmatrix} = \begin{bmatrix} 2{,}5287 \\ 3{,}2193 \end{bmatrix}$$

$s_i^{\prime P} = \vec{r}_i^{\,P}$ since no rotation in (c).

4.2



Init:

$r_2 = [5,\ 2]^T$

$\phi_2 = 30^\circ$

$r_2^Q = (5,\ 1.5)^T$.

Find:

$\vec{s}_2^{\,\prime Q}$ – ?

Solution:

$$\vec{r}_2^{\,Q} = \vec{r}_2 + A_2 \vec{s}_2^{\,\prime Q} = \vec{r}_2 + \vec{s}_2^{\,Q}$$

$$= \begin{bmatrix} 5 \\ 2 \end{bmatrix} + \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \cdot \begin{bmatrix} ? \end{bmatrix} \Rightarrow$$

$$\vec{s}_2^{\,\alpha} = \vec{r}_2^{\,\alpha} - \vec{r}_2 = \begin{bmatrix} 6 \\ 7{,}5 \end{bmatrix} - \begin{bmatrix} 5 \\ 2 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 \\ -0{,}5 \end{bmatrix}$$

$$\vec{s}_2^{\,\prime\alpha} = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix}^T \cdot \begin{bmatrix} 1 \\ -0{,}5 \end{bmatrix} = \begin{bmatrix} 1{,}1160 \\ -0{,}0670 \end{bmatrix}$$

4.4. $r_i = [3.2, \; 2.8]^T, \; \varphi_i = 80^\circ$

$s_i^{\prime A} = [-1{,}1; \; -0{,}4]^T, \; \vec{s}_i^{\,\prime B} = [1{,}9; \; 2{,}3]^T$

$r_i^C = [5{,}3, \; 4.0]^T$

Find: a) $r_i^A - ?$, b) $s_i^B - ?$, c) $\vec{s}_i^{\,\prime C} - ?$

a) $$\vec{r}_i^{\,A} = \vec{r}_i + A_i \cdot s_i^{\prime A} = \begin{bmatrix} 3{,}2 \\ 2{,}8 \end{bmatrix} + A_i^{\varphi=80^\circ} \cdot \begin{bmatrix} -1{,}1 \\ -0{,}4 \end{bmatrix} =$$

$$= \begin{bmatrix} 3{,}4029 \\ 1{,}6473 \end{bmatrix}$$

b) $$\vec{s}_i^{\,B} = A_i^{\varphi=80^\circ} \cdot \vec{s}_i^{\,\prime B} = A^{\varphi=80^\circ} \cdot \begin{bmatrix} 1{,}9 \\ 2{,}3 \end{bmatrix} = \begin{bmatrix} -1{,}9351 \\ 2{,}2705 \end{bmatrix}$$

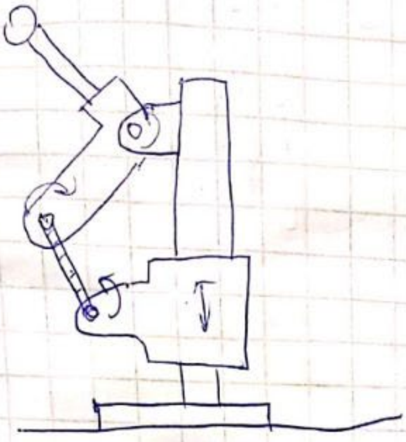
c) $$\vec{s}_i^{\,C} = \vec{r}_i^{\,C} - \vec{r}_i = \begin{bmatrix} 5{,}3 \\ 4{,}0 \end{bmatrix} - \begin{bmatrix} 3{,}2 \\ 2{,}8 \end{bmatrix} =$$

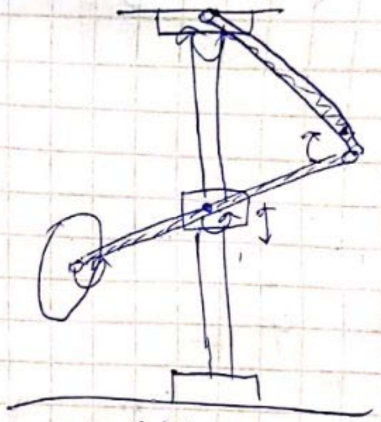
$$= \begin{bmatrix} 2{,}1 \\ 1{,}2 \end{bmatrix}, \quad \vec{s}_i^{\,\prime C} = A^{\varphi=80^\circ T} \cdot \vec{s}_i^{\,C} = A^{\varphi=80^\circ T} \cdot \begin{bmatrix} 2{,}1 \\ 1{,}2 \end{bmatrix} =$$

$$= \begin{bmatrix} 1{,}5464 \\ -1{,}8597 \end{bmatrix}$$

Сканировано с CamScanner

4.5


(a)


(b)

a) 4 bodies (links) + ground => 5 bodies => ~~5·3 = 12~~ 4 coords

b) 3 r.j => 3 · 2 = 6

1 p.j. => 1 · 2 = 2

ground => 3 constr.

total: 11 constraints

c) $m = 3(4 - 1) - 2 \cdot 4 = \underline{1}$

d) 1 independent

3 dependent

a) 5 bodies + ground => 6 bodies => ~~...~~ 5 coords

b) 4 r.j + 1 p.j. =>

10

ground => 3 constr

total: 13 constraints

c) $m = 3(5-1) - 2 \cdot 5 = 12 - 10 = \underline{2}$

d) 2 independent

3 dependent

4.6

$$\Phi_1 = x_4 + 1{,}5\cos\varphi_4 - 0{,}3\sin\varphi_4 - x_1 + 0{,}75\sin\varphi_1 = 0$$

$$\Phi_2 = y_4 + 1{,}5\sin\varphi_4 + 0{,}3\cos\varphi_4 - y_1 - 0{,}75\cos\varphi_1 = 0$$

=> J -> in Matlab.

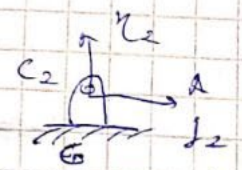
```
syms x1 x4 phi4 phi1 y1 y4 ...
    dx1 dx4 dphi4 dphi1 dy1 dy4 ...
    ddx1 ddx4 ddphi4 ddphi1 ddy1 ddy4

F1 = x4 + 1.6*cos(phi4) - 0.3*sin(phi4) - x1 + 0.75*sin(phi1);
F2 = y4 + 1.6*sin(phi4) + 0.3*cos(phi4) - y1 - 0.75*cos(phi1);
J = jacobian([F1, F2], [x1,y1,phi1,x4,y4,phi4]);
J1 = [-1,  0, (3*cos(phi1))/4, 1, 0, - (3*cos(phi4))/10 - (8*sin(phi4))/5];
J2 = [ 0, -1, (3*sin(phi1))/4, 0, 1,   (8*cos(phi4))/5 - (3*sin(phi4))/10];
dF = J * [dx1; dy1; dphi1; dx4; dy4; dphi4]; % F_dot = J*q_dot
ddF = J * [ddx1; ddy1; ddphi1; ddx4; ddy4; ddphi4] ...
 + jacobian([J1, J2], [x1,y1,phi1,x4,y4,phi4]) * [dx1; dy1; dphi1; dx4; dy4;
dphi4]; % F_2dot = J_dot*q_dot + J*q_2dot
```

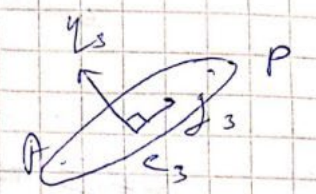
$$\dot{x} = J\dot{q} \Rightarrow J \cdot \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\varphi}_1 \\ \dot{x}_4 \\ \dot{y}_4 \\ \dot{\varphi}_4 \end{bmatrix}$$

$$\Rightarrow \dot{\Phi} =$$

$$\ddot{x} = \dot{J}\dot{q} + J\ddot{q}$$

(4.8)





$$\eta_2^A = 0 \qquad \xi_2^A = r$$

$$\cancel{\Phi_1 = r_1 + s_1^A - r_j - s_j^A = 0}$$

$$\cancel{\Phi_2 = x_j}$$

$$\Phi_2 = r_2 + s_2^A - r_3 - s_3^A = 0$$

$$\Phi_3 = (x_g^G - \xi_3^P)(y_j^G - y_g^P) - (y_g^G - y_3^P)(x_g^G - x_j^P) = 0.$$

(4.9). $s_1' = [1, 2, -0.5]^T$, $s_2' = [-0.3, 0.8]^T$.

$$q = [x_1, y_1, \varphi_1, x_2, y_2, \varphi_2]^T$$

a) if $\varphi_1 = 30°$, $\varphi_2 = 45°$: Φ_q for

$$\Phi = s_1^T s_2 \quad ?$$

Solution: $s_1 = A_1 s_1' = \begin{bmatrix} -0,87 & -0,5 \\ 0,5 & 0,87 \end{bmatrix} \begin{bmatrix} 1,2 \\ -0,5 \end{bmatrix}$

$= \begin{bmatrix} 1,2892 \\ 0,1670 \end{bmatrix}$

$s_2 = A_2 s_2' = A^{\varphi = 45^\circ} \cdot \begin{bmatrix} -0,3 \\ 0,8 \end{bmatrix} = \begin{bmatrix} -0,7778 \\ 0,3536 \end{bmatrix}$

$\varphi^0 = s_1^T s_2 = \begin{bmatrix} 1,2892 & 0,1670 \end{bmatrix} \cdot \begin{bmatrix} -0,7778 \\ 0,3536 \end{bmatrix} =$

$= -0,9437.$

6) if $x_1 = 6.2$, $y_1 = 1$, $\varphi_1 = 30^\circ$, $x_2 = -1,9$, $y_2 = 2,3$, $\varphi_2 = 45^\circ$: $\varphi^0 = \vec{s}_1 \times d$, $d =$

$= [x_2 - x_1, y_2 - y_1]^T = [-1,9 - 6,2, 1,3]^T =$

$= [-8,1, 1,3]^T.$

Solution:

$s_1 = [6,2; 1].$

$\Rightarrow \cancel{s_1 \times d =} \begin{bmatrix} \cancel{6,2} & \cancel{-8,1} \\ \cancel{1} & \cancel{1,3} \\ 0 & 0 \end{bmatrix}.$

$\varphi^0_1 = (s_{1x} d_y - s_{1y} d_x) = 1,29 \cdot 1,3 - 0,97 \cdot (-8,1$

$= 1,58 + 1,38 = 2,94.$