onsidere a transformação linear $T : \mathbb{R}^2 \to \mathbb{R}^3$ definida por $T(x_1, x_2) = (x_1 + x_2, 2x_1 + x_2, x_1 + 2x_2)$.

(a) (10 pontos) Determine a matriz da transformação T em relação às bases canônica de $\mathbb{R}^2$ e $\beta = \left\{ \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix} \right\}$ de $\mathbb{R}^3$.

(b) (5 pontos) Verifique o resultado encontrado para o vetor $u = (1, 2)$.

$$T\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}x_1+x_2\\2x_1+x_2\\x_1+2x_2\end{pmatrix} = \begin{pmatrix}1 & 1\\2 & 1\\1 & 2\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}$$

$$\beta = \begin{pmatrix}1 & 1 & 0\\1 & 0 & 1\\0 & 1 & 1\end{pmatrix}$$

$$\begin{array}{lccc} \text{canônica} & u \in U & \xrightarrow{T} & v \in V \quad v = T(u) \\ & \downarrow & & \downarrow \\ \text{bases} & x \in \mathbb{R}^2 & \longrightarrow & y \in \mathbb{R}^3 \quad y = Ax \end{array}$$

$x = [U]_\mathcal{E}$ $\quad y = [V]_\beta$

$= \mathcal{E}^{-1} U$ $\quad = \beta^{-1} V$

$$A = \begin{pmatrix}1 & 1 & 0\\1 & 0 & 1\\0 & 1 & 1\end{pmatrix} \cdot \begin{pmatrix}1 & 1\\2 & 1\\1 & 2\end{pmatrix}$$

$$A = \begin{pmatrix}1 & 0\\0 & 1\\1 & 1\end{pmatrix}$$

$y = Ax$

$\beta^{-1} \cdot V = A\ \mathcal{E}^{-1} \cdot U$

$V = \underbrace{\beta A \cdot \mathcal{E}^{-1}}_{T} \cdot U$

$T = \beta A \cdot \mathcal{E}^{-1}$

$A = \beta^{-1} \cdot T\ \mathcal{E}$

$A = \beta^{-1}\ T$

Boa prova!