PHYS2114 Cheat Sheet

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Contents

1 Mathematics Preliminary

1.1 IMPORTANT NOTE

The following notations are taken from Griffth's 'Introduction to Electromachenics'.[1]

Where as the line, area and volume integral elements are the following;

- Line integral element: $\mathrm{d}\vec{l}$.
- Area integral element: $\mathrm{d}\vec{a}$.
- Volume integral element: $\mathrm{d}\tau$.

1.2 Cartesian Coordinates

The line and volume integral elements are

$$\mathrm{d}\vec{l} = \hat{x}\,\mathrm{d}x + \hat{y}\,\mathrm{d}y + \hat{z}\,\mathrm{d}z, \qquad \mathrm{d}\tau = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.$$

And the following are some common operators

Gradient: $$\nabla t = \frac{\partial t}{\partial x}\hat{x} + \frac{\partial t}{\partial x}\hat{y} + \frac{\partial t}{\partial z}\hat{z},$$

Divergence: $$\nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z},$$

Curl: $$\nabla \times \vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z},$$

Laplacian: $$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}.$$

1.3 Cylindrical Coordinates

The conversion of Cartesian to Cylinderical coordinates[2] are

$$x = r\cos\phi, \qquad y = r\sin\phi, \qquad z = z,$$

note that $\phi \in [0, 2\pi]$.

The line and the volume integral elements are

$$\mathrm{d}\vec{l} = \hat{s}\,\mathrm{d}s + s\hat{\phi}\,\mathrm{d}\phi + \hat{z}\,\mathrm{d}z, \qquad \mathrm{d}\tau = s\,\mathrm{d}s\,\mathrm{d}\phi\,\mathrm{d}z.$$

The following are some common vector operators

Gradient: $$\nabla t = \frac{\partial t}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z},$$

Divergence: $$\nabla \cdot \vec{v} = \frac{1}{s}\frac{\partial}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z},$$

Curl: $$\nabla \times \vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z},$$

Laplacian: $$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}.$$

1.4 Spherical Coordinates

Note that $\theta \in [0, \pi]$ denotes the angle between z-axis and the vector of interest, and that $\phi \in [0, 2\pi]$ denotes the angle between x-axis and the projection of the vector of interest on to the xy-plane.[2]

The conversion of Cartesian to Spherical coordiantes are shwon as the follows

$$x = r\sin(\theta)\cos(\phi), \qquad y = r\sin(\theta)\sin(\phi), \qquad z = r\cos(\theta).$$

And the line and volume integral elements are

$$\mathrm{d}\vec{l} = \hat{r}\,\mathrm{d}r + r\hat{\theta}\,\mathrm{d}\theta + r\sin(\theta)\hat{\phi}\,\mathrm{d}\phi, \qquad \mathrm{d}\tau = r^2\sin(\theta)\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi.$$

The following are the common operators

Gradient: $$\nabla t = \hat{r}\frac{\partial t}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial t}{\partial \theta} + \frac{\hat{\phi}}{r\sin(\theta)}\frac{\partial t}{\partial \phi},$$

Divergence: $$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(v_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi},$$

Curl: $$\begin{aligned}\nabla \times \vec{v} &= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}\sin(\theta)v_\phi - \frac{\partial v_\theta}{\partial \phi}\right)\hat{r} \\ &\quad + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}rv_\phi\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}rv_\theta - \frac{\partial v_r}{\partial \theta}\right)\hat{\phi},\end{aligned}$$

Laplacian: $$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}.$$

1.5 Gauss' Divergence Theorem

Suppose V is a subset of $\mathbb{R}^n$ (in the case of $n = 3$, V represents a volume in three-dimensional space) which is compact and has a piecewise smooth boundary

S (also indicated with $\partial V = S$). If $\vec{F}$ is a countinueously differentiable vector filed defined on a neighbourhood of V, then:

$$\iiint_V \left(\nabla \cdot \vec{F}\right) \mathrm{d}V = \oiint_S \left(\vec{F} \cdot \hat{n}\right) \mathrm{d}S.$$

The left side is a volume intgral over the volume V, the right side is the surface integral over the boundary of the volume V. The closed manifold ∂V is oriented by outward-pointing normal, and $\vec{n}$ is the outward pointing normal at each point on the boundary ∂V. ($\mathrm{d}\vec{S}$ may be used as a shorthand for $\vec{n}\,\mathrm{d}S$.) In terms of the intuitive description above, the left-hand side of the equation represents the total of the sources in the volume V, and the right-hand side represents the total flow across the boundary S.[3]

1.6 Stokes' Theorem

Suppose we have a boundary $\partial\Sigma = S$ that bounds the surface Σ with $\vec{F}$ defined in Σ, then

$$\iint_\Sigma (\nabla \times \vec{F}) \cdot \vec{n}\,\mathrm{d}S = \oint_S \vec{F} \cdot \mathrm{d}\vec{l}.$$

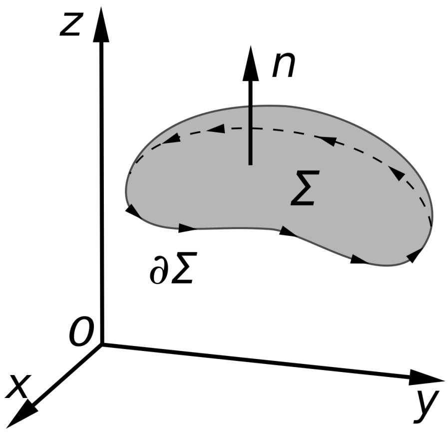

Figure 1: A visual representation for the Stokes' theorem[4]

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2 Electrostatics

2.1 The Holy Trinity

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{\mathscr{r}}\,\mathrm{d}\tau \tag{2.1.1}$$

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \tag{2.1.2}$$

$$E = -\nabla V \tag{2.1.3}$$

$$V = -\int \vec{E}\cdot \mathrm{d}\vec{l} \tag{2.1.4}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\rho\,\mathrm{d}\tau \tag{2.1.5}$$

$$\nabla\cdot E = \frac{\rho}{\varepsilon_0};\quad \nabla\times E = 0 \tag{2.1.6}$$

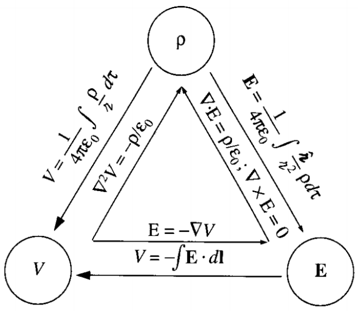

Figure 2: The Griffith's holy trinity of electrostat[1]

2.2 Electrostatic Boundary Conditions

$$\hat{n}_2\cdot[\vec{E}_1 - \vec{E}_2] = \frac{\sigma}{\varepsilon_0} \tag{2.2.1}$$

$$\hat{n}_2\times[\vec{E}_1 - \vec{E}_2] = 0 \tag{2.2.2}$$

$$V_1 - V_2 = 0 \tag{2.2.3}$$

Note that here the "1" and "2" just refer to the different sides of the interface.

References

[1] David J Griffiths. *Introduction to electrodynamics; 3rd ed.* Upper Saddle River, NJ: Prentice-Hall, 1999. URL: https://cds.cern.ch/record/611579.

[2] *Del in cylindrical and spherical coordinates.* en. Page Version ID: 1016691595. Apr. 2021. URL: https://en.wikipedia.org/w/index.php?title=Del_in_cylindrical_and_spherical_coordinates&oldid=1016691595 (visited on 06/03/2021).

[3] *Divergence theorem.* en. Page Version ID: 1025269448. May 2021. URL: https://en.wikipedia.org/w/index.php?title=Divergence_theorem&oldid=1025269448 (visited on 06/05/2021).

[4] *Stokes' theorem.* en. Page Version ID: 1014735398. Mar. 2021. URL: https://en.wikipedia.org/w/index.php?title=Stokes%27_theorem&oldid=1014735398 (visited on 06/05/2021).