

# PHYS2114 Cheat Sheet

Joey Liang

June 2021

# Contents

<b>1</b>	<b>Mathematics Preliminary</b>	<b>3</b>
1.1	IMPORTANT NOTE . . . . .	3
1.2	Cartesian Coordinates . . . . .	3
1.3	Cylindrical Coordinates . . . . .	3
1.4	Spherical Coordinates . . . . .	4
1.5	Gauss' Divergence Theorem . . . . .	4
1.6	Stokes' Theorem . . . . .	5
<b>2</b>	<b>Electrostatics</b>	<b>6</b>
2.1	The Holy Trinity . . . . .	6
2.2	Electrostatic Boundary Conditions . . . . .	6
2.3	Work and Energy in Electrostatics . . . . .	7
2.4	Conductors . . . . .	7
	<b>References</b>	<b>8</b>

# 1 Mathematics Preliminary

## 1.1 IMPORTANT NOTE

The following notations are taken from Griffith's 'Introduction to Electrodynamics'.[1]

Where as the line, area and volume integral elements are the following;

- Line integral element:  $d\vec{l}$ .
  - Area integral element:  $d\vec{a}$ .
  - Volume integral element:  $d\tau$ .
- 

## 1.2 Cartesian Coordinates

The line and volume integral elements are

$$d\vec{l} = \hat{x} dx + \hat{y} dy + \hat{z} dz, \quad d\tau = dx dy dz.$$

And the following are some common operators

Gradient:  $\nabla t = \frac{\partial t}{\partial x} \hat{x} + \frac{\partial t}{\partial y} \hat{y} + \frac{\partial t}{\partial z} \hat{z},$

Divergence:  $\nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z},$

Curl:  $\nabla \times \vec{v} = \left( \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{x} + \left( \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{y} + \left( \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{z},$

Laplacian:  $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}.$

---

## 1.3 Cylindrical Coordinates

The conversion of Cartesian to Cylindrical coordinates[2] are

$$x = r \cos \phi, \quad y = r \sin \phi, \quad z = z,$$

note that  $\phi \in [0, 2\pi]$ .

The line and the volume integral elements are

$$d\vec{l} = \hat{s} ds + s \hat{\phi} d\phi + \hat{z} dz, \quad d\tau = s ds d\phi dz.$$

The following are some common vector operators

$$\text{Gradient:} \quad \nabla t = \frac{\partial t}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial t}{\partial \phi} \hat{\phi} + \frac{\partial t}{\partial z} \hat{z},$$

$$\text{Divergence:} \quad \nabla \cdot \vec{v} = \frac{1}{s} \frac{\partial}{\partial s} + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z},$$

$$\text{Curl:} \quad \nabla \times \vec{v} = \left( \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{x} + \left( \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{y} + \left( \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{z},$$

$$\text{Laplacian:} \quad \nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}.$$


---

## 1.4 Spherical Coordinates

Note that  $\theta \in [0, \pi]$  denotes the angle between z-axis and the vector of interest, and that  $\phi \in [0, 2\pi]$  denotes the angle between x-axis and the projection of the vector of interest on to the xy-plane.[2]

The conversion of Cartesian to Spherical coordinates are shown as follows

$$x = r \sin(\theta) \cos(\phi), \quad y = r \sin(\theta) \sin(\phi), \quad z = r \cos(\theta).$$

And the line and volume integral elements are

$$d\vec{l} = \hat{r} dr + r \hat{\theta} d\theta + r \sin(\theta) \hat{\phi} d\phi, \quad d\tau = r^2 \sin(\theta) dr d\theta d\phi.$$

The following are the common operators

$$\text{Gradient:} \quad \nabla t = \hat{r} \frac{\partial t}{\partial r} + \frac{\hat{\theta}}{r} \frac{\partial t}{\partial \theta} + \frac{\hat{\phi}}{r \sin(\theta)} \frac{\partial t}{\partial \phi},$$

$$\text{Divergence:} \quad \nabla \cdot \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi},$$

$$\begin{aligned} \text{Curl:} \quad \nabla \times \vec{v} = & \frac{1}{r \sin \theta} \left( \frac{\partial}{\partial \theta} \sin(\theta) v_\phi - \frac{\partial v_\theta}{\partial \phi} \right) \hat{r} \\ & + \frac{1}{r} \left( \frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} r v_\phi \right) \hat{\theta} + \frac{1}{r} \left( \frac{\partial}{\partial r} r v_\theta - \frac{\partial v_r}{\partial \theta} \right) \hat{\phi}, \end{aligned}$$

$$\text{Laplacian:} \quad \nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}.$$


---

## 1.5 Gauss' Divergence Theorem

Suppose  $V$  is a subset of  $\mathbb{R}^n$  (in the case of  $n = 3$ ,  $V$  represents a volume in three-dimensional space) which is compact and has a piecewise smooth boundary

$S$  (also indicated with  $\partial V = S$ ). If  $\vec{F}$  is a countinuously differentiable vector filed defined on a neighbourhood of  $V$ , then:

$$\iiint_V (\nabla \cdot \vec{F}) \, dV = \oint_S (\vec{F} \cdot \hat{n}) \, dS.$$

The left side is a volume integral over the volume  $V$ , the right side is the surface integral over the boundary of the volume  $V$ . The closed manifold  $\partial V$  is oriented by outward-pointing normal, and  $\vec{n}$  is the outward pointing normal at each point on the boundary  $\partial V$ . ( $d\vec{S}$  may be used as a shorthand for  $\vec{n} dS$ .) In terms of the intuitive description above, the left-hand side of the equation represents the total of the sources in the volume  $V$ , and the right-hand side represents the total flow across the boundary  $S$ . [3]

---

## 1.6 Stokes' Theorem

Suppose we have a boundary  $\partial\Sigma = S$  that bounds the surface  $\Sigma$  with  $\vec{F}$  defined in  $\Sigma$ , then

$$\iint_{\Sigma} (\nabla \times \vec{F}) \cdot \vec{n} \, dS = \oint_S \vec{F} \cdot d\vec{l}.$$

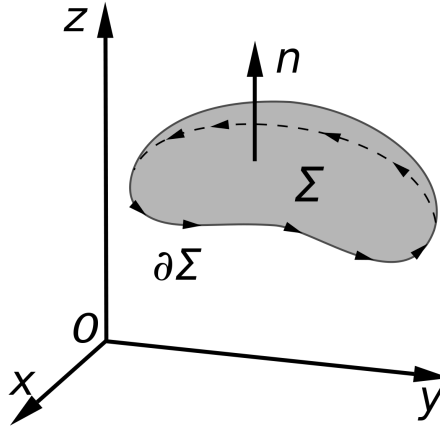


Figure 1: A visual representation for the Stokes' theorem [4]

---

## 2 Electrostatics

### 2.1 The Holy Trinity

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho}{r} d\tau \quad (2.1.1)$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \quad (2.1.2)$$

$$\mathbf{E} = -\nabla V \quad (2.1.3)$$

$$V = -\int \vec{E} \cdot d\vec{l} \quad (2.1.4)$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{r}}{r^2} \rho d\tau \quad (2.1.5)$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}; \quad \nabla \times \mathbf{E} = 0 \quad (2.1.6)$$

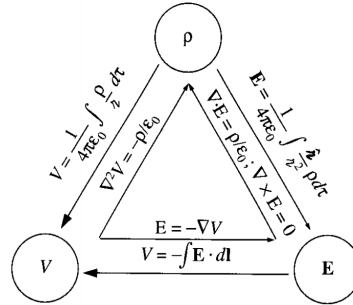


Figure 2: The Griffith's holy trinity of electrostat[1]

---

### 2.2 Electrostatic Boundary Conditions

$$\hat{n}_2 \cdot [\vec{E}_1 - \vec{E}_2] = \frac{\sigma}{\epsilon_0} \quad (2.2.1)$$

$$\hat{n}_2 \times [\vec{E}_1 - \vec{E}_2] = 0 \quad (2.2.2)$$

$$V_1 - V_2 = 0 \quad (2.2.3)$$

Note that here the "1" and "2" just refer to the different sides of the interface.

---

### 2.3 Work and Energy in Electrostatics

Three important equations for Energy in work and energy in electrostatics:

For discrete charges that are not very close together,

$$W = \frac{1}{2} \sum_{i=1}^n q_i V(\vec{r}_i). \quad (2.3.1)$$

For volume charge density  $\rho$ ,

$$W = \frac{1}{2} \int \rho V \, d\tau. \quad (2.3.2)$$

And even more simply, we can have,

$$W = \frac{\epsilon_0}{2} \int_{\mathbb{R}^3} E^2 \, d\tau. \quad (2.3.3)$$

---

### 2.4 Conductors

## References

- [1] David J Griffiths. *Introduction to electrodynamics; 3rd ed.* Upper Saddle River, NJ: Prentice-Hall, 1999. URL: <https://cds.cern.ch/record/611579>.
- [2] *Del in cylindrical and spherical coordinates*. en. Page Version ID: 1016691595. Apr. 2021. URL: [https://en.wikipedia.org/w/index.php?title=Del\\_in\\_cylindrical\\_and\\_spherical\\_coordinates&oldid=1016691595](https://en.wikipedia.org/w/index.php?title=Del_in_cylindrical_and_spherical_coordinates&oldid=1016691595) (visited on 06/03/2021).
- [3] *Divergence theorem*. en. Page Version ID: 1025269448. May 2021. URL: [https://en.wikipedia.org/w/index.php?title=Divergence\\_theorem&oldid=1025269448](https://en.wikipedia.org/w/index.php?title=Divergence_theorem&oldid=1025269448) (visited on 06/05/2021).
- [4] *Stokes' theorem*. en. Page Version ID: 1014735398. Mar. 2021. URL: [https://en.wikipedia.org/w/index.php?title=Stokes%27\\_theorem&oldid=1014735398](https://en.wikipedia.org/w/index.php?title=Stokes%27_theorem&oldid=1014735398) (visited on 06/05/2021).