

# PHYS2114 Cheat Sheet

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# 1 Mathematics Preliminary

## 1.1 IMPORTANT NOTE

The following notations are taken from Griffiths' 'Introduction to Electrodynamics'.[1]

Where as the line, area and volume integral elements are the following;

- Line integral element:  $d\mathbf{l}$ .
  - Area integral element:  $d\mathbf{a}$ .
  - Volume integral element:  $d\tau$ .
- 

## 1.2 Cartesian Coordinates

The line and volume integral elements are

$$d\mathbf{l} = \hat{\mathbf{x}} dx + \hat{\mathbf{y}} dy + \hat{\mathbf{z}} dz, \quad d\tau = dx dy dz.$$

And the following are some common operators

Gradient:  $\nabla t = \frac{\partial t}{\partial x} \hat{\mathbf{x}} + \frac{\partial t}{\partial y} \hat{\mathbf{y}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}},$

Divergence:  $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z},$

Curl:  $\nabla \times \mathbf{v} = \left( \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{\mathbf{x}} + \left( \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{\mathbf{y}} + \left( \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{\mathbf{z}},$

Laplacian:  $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}.$

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## 1.3 Cylindrical Coordinates

The conversion of Cartesian to Cylindrical coordinates[2] are

$$x = r \cos \phi, \quad y = r \sin \phi, \quad z = z,$$

note that  $\phi \in [0, 2\pi]$ .

And the surface integral element with radius  $r$  constant is

$$d\mathbf{S} = r d\phi dz$$

The line and the volume integral elements are

$$d\mathbf{l} = \hat{\mathbf{s}} ds + s \hat{\phi} d\phi + \hat{\mathbf{z}} dz, \quad d\tau = s ds d\phi dz.$$

The following are some common vector operators

Gradient:  $\nabla t = \frac{\partial t}{\partial s} \hat{\mathbf{s}} + \frac{1}{s} \frac{\partial t}{\partial \phi} \hat{\phi} + \frac{\partial t}{\partial z} \hat{\mathbf{z}},$

Divergence:  $\nabla \cdot \mathbf{v} = \frac{1}{s} \frac{\partial}{\partial s} + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z},$

Curl:  $\nabla \times \mathbf{v} = \left( \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{\mathbf{x}} + \left( \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{\mathbf{y}} + \left( \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{\mathbf{z}},$

Laplacian:  $\nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}.$

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## 1.4 Spherical Coordinates

Note that  $\theta \in [0, \pi]$  denotes the angle between z-axis and the vector of interest, and that  $\phi \in [0, 2\pi]$  denotes the angle between x-axis and the projection of the vector of interest on to the xy-plane.[2]

The conversion of Cartesian to Spherical coordinates are shown as the follows

$$x = r \sin(\theta) \cos(\phi), \quad y = r \sin(\theta) \sin(\phi), \quad z = r \cos(\theta).$$

And the line and volume integral elements are

$$d\mathbf{l} = \hat{\mathbf{r}} dr + r \hat{\theta} d\theta + r \sin(\theta) \hat{\phi} d\phi, \quad d\tau = r^2 \sin(\theta) dr d\theta d\phi.$$

with the surface integral with the radius  $r$  constant is

$$d\mathbf{S} = \sin \theta d\theta d\phi.$$

The following are the common operators

Gradient:  $\nabla t = \hat{\mathbf{r}} \frac{\partial t}{\partial r} + \frac{\hat{\theta}}{r} \frac{\partial t}{\partial \theta} + \frac{\hat{\phi}}{r \sin(\theta)} \frac{\partial t}{\partial \phi},$

Divergence:  $\nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi},$

Curl:  $\nabla \times \mathbf{v} = \frac{1}{r \sin \theta} \left( \frac{\partial}{\partial \theta} \sin(\theta) v_\phi - \frac{\partial v_\theta}{\partial \phi} \right) \hat{\mathbf{r}} + \frac{1}{r} \left( \frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} r v_\phi \right) \hat{\theta} + \frac{1}{r} \left( \frac{\partial}{\partial r} r v_\theta - \frac{\partial v_r}{\partial \theta} \right) \hat{\phi},$

Laplacian:  $\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}.$

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## 1.5 Gauss' Divergence Theorem

Suppose  $V$  is a subset of  $\mathbb{R}^n$  (in the case of  $n = 3$ ,  $V$  represents a volume in three-dimensional space) which is compact and has a piecewise smooth boundary  $S$  (also indicated with  $\partial V = S$ ). If  $\mathbf{F}$  is a continuously differentiable vector field defined on a neighbourhood of  $V$ , then:

$$\iiint_V (\nabla \cdot \mathbf{F}) \, dV = \oiint_S (\mathbf{F} \cdot \hat{\mathbf{n}}) \, dS.$$

The left side is a volume integral over the volume  $V$ , the right side is the surface integral over the boundary of the volume  $V$ . The closed manifold  $\partial V$  is oriented by outward-pointing normal, and  $\mathbf{n}$  is the outward pointing normal at each point on the boundary  $\partial V$ . ( $d\mathbf{S}$  may be used as a shorthand for  $\mathbf{n} \, dS$ .) In terms of the intuitive description above, the left-hand side of the equation represents the total of the sources in the volume  $V$ , and the right-hand side represents the total flow across the boundary  $S$ . [3]

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## 1.6 Stokes' Theorem

Suppose we have a boundary  $\partial\Sigma = S$  that bounds the surface  $\Sigma$  with  $\mathbf{F}$  defined in  $\Sigma$ , then

$$\iint_{\Sigma} (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS = \oint_S \mathbf{F} \cdot d\mathbf{l}.$$

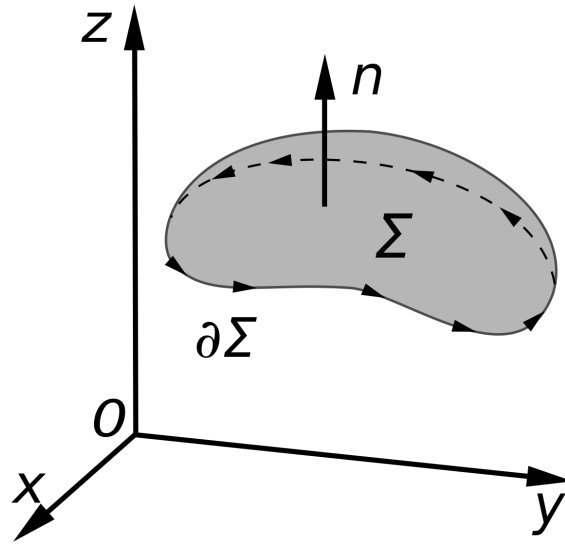


Figure 1: A visual representation for the Stokes' theorem [4]

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## 2 Electrostatics

### 2.1 The Holy Trinity

$$V = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho}{r} d\tau \quad (2.1.1)$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} \quad (2.1.2)$$

$$\mathbf{E} = -\nabla V \quad (2.1.3)$$

$$V = -\int \mathbf{E} \cdot d\mathbf{l} \quad (2.1.4)$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\hat{\mathbf{r}}}{r^2} \rho d\tau \quad (2.1.5)$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}; \quad \nabla \times \mathbf{E} = 0 \quad (2.1.6)$$

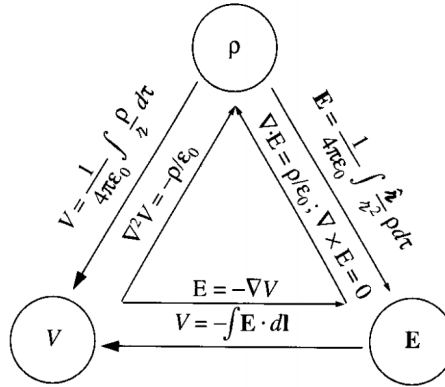


Figure 2: The Griffith's holy trinity of electrostatics[1]

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### 2.2 Electrostatic Boundary Conditions

$$\hat{\mathbf{n}}_2 \cdot [\mathbf{E}_1 - \mathbf{E}_2] = \frac{\sigma}{\epsilon_0} \quad (2.2.1)$$

$$\hat{\mathbf{n}}_2 \times [\mathbf{E}_1 - \mathbf{E}_2] = 0 \quad (2.2.2)$$

$$V_1 - V_2 = 0 \quad (2.2.3)$$

Note that here the "1" and "2" just refer to the different sides of the interface.

## 2.3 Work and Energy in Electrostatics

Three important equations for Energy in work and energy in electro statics:  
For discrete charges that are not very closed together,

$$W = \frac{1}{2} \sum_{i=1}^n q_i V(\mathbf{r}_i). \quad (2.3.1)$$

For volume charge density  $\rho$ ,

$$W = \frac{1}{2} \iiint_V \rho V \, d\tau. \quad (2.3.2)$$

And even more simply, we can have,

$$W = \frac{\epsilon_0}{2} \iiint_{\mathbb{R}^3} E^2 \, d\tau. \quad (2.3.3)$$

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## 2.4 Conductors

There are **five** fundamental properties of a conductors,

1.  $E = 0$  inside a conductor.
2.  $\rho = 0$  inside a conductor.
3. Any net charge resides on the surface.
4. A conductor is an equipotential.
5.  $E$  is perpendicular to the surface, just outside a conductor.

Now, let's consider the force per unit area  $\mathbf{f}$  on any charges that rests on the surface of the conductor, with references to page 102 in the book,<sup>1</sup> we know that

$$\mathbf{f} = \sigma \mathbf{E}_{\text{average}} = \frac{1}{2} \sigma (\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}}). \quad (2.4.1)$$

In particular, that for conductors, where we know  $\mathbf{E}_{\text{below}} = 0$  (equipotential inside the conductor), we obtain

$$\mathbf{f} = \frac{1}{2\epsilon_0} \sigma^2 \hat{\mathbf{n}} \quad (2.4.2)$$

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## 2.5 Capacitors

Note that the one thing we need to know about capacitance is that it is nothing but a “proportionality” between the stored charge and the potential difference;

$$Q = CV.$$

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<sup>1</sup>‘The book’ is exclusively used in this note to ‘Introduction to Electromagnetism’ by David J. Griffiths

## 3 Special Techniques

### 3.1 Uniqueness Theorems

**First uniqueness theorem:** The solution to Laplace's equation in some volume  $\mathcal{V}$  is uniquely determined if  $V$  is specified on the boundary surface  $S$ .

**Second uniqueness theorem:** In a volume  $\mathcal{V}$  surrounded by conductors and containing a specified charge density  $\rho$ , the electric field is uniquely determined if the *total charge* on each conductor is given. (The region as a whole can be bounded by another conductor, or else unbounded.)[1]

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### 3.2 Separation of Variables

#### 3.2.1 Cylindrical Coordinate

You can find the full derivation [here](#). For those who just care about the result (of potential  $V$ ) without  $\hat{\mathbf{z}}$  dependence

$$V(s, \phi) = a_0 + b_0 \ln s + \sum_{k=1}^{\infty} [s^k (a_k \cos k\phi + b_k \sin k\phi) + s^{-k} (c_k \cos k\phi + d_k \sin k\phi)] . \quad (3.2.1)$$

#### 3.2.2 Spherical Coordinate

Once again, you can find the full derivation [here](#). And the general result for the potential  $V(s, \theta, \phi)$  is

$$V(s, \theta, \phi) = \sum_{l=0}^{\infty} \sum_l^{m=-l} \left( A_{lm} r^l + \frac{B_{lm}}{r^{l+1}} \right) P_l^m(\cos \theta) \exp(im\phi) \quad (3.2.2)$$

---

## 4 Electric Field in Matter

### 4.1 Induced Dipoles

Suppose we have an uniform electric field  $\mathbf{E}$ , and an atom (dipole) is in this field. This atom will now possess some dipole moment. The dipole moment is **most of the times** approximately proportional to the field.

$$\mathbf{p} = \alpha \mathbf{E}. \quad (4.1.1)$$

---

### 4.2 Bound Charges

Let's suppose we have a blob of polarised material, with dipole moment  $\mathbf{p} = \mathbf{P}(\mathbf{r}') d\tau'$  in each volume element. The total potential is

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \iiint_{\mathcal{V}} \frac{\hat{\mathbf{z}} \cdot \mathbf{P}(\mathbf{r}')}{r'^2} d\tau'.$$



And after some maths, we then obtain

$$V = \frac{1}{4\pi\epsilon_0} \oint_S \frac{1}{r} \mathbf{P} \cdot d\mathbf{a}' - \frac{1}{4\pi\epsilon_0} \iiint_V \frac{1}{r} (\nabla \cdot \mathbf{P}) d\tau'. \quad (4.2.1)$$

Notice that

$$\underbrace{\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}}_{\text{Surface bound charge}}, \quad \text{and} \quad \underbrace{\rho_b = -\nabla \cdot \mathbf{P}}_{\text{Volume bound charge}}.$$

Those are called bound charges (surface bound charges & volume bound charge), we can just find them to get the voltage of the polarised dielectric blob instead of calculating the massive integral.

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## 4.3 The Electric Displacement

### 4.3.1 Gauss's Law in the Presence of Dielectrics

Blah blah blah there is something called the “free charge” because we don't live in a perfect world. So the total charge density is now

$$\rho = \rho_b + \rho_f.$$

and Gauss' law reads

$$\begin{aligned} \epsilon_0 \nabla \cdot \mathbf{E} &= \rho = \rho_b + \rho_f = -\nabla \cdot \mathbf{P} + \rho_f; \\ \Rightarrow \quad \rho_f &= \nabla \cdot (\epsilon_0 \mathbf{E} + \mathbf{P}); \end{aligned}$$

$$\Rightarrow \quad \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}; \quad (4.3.1)$$

$$\Rightarrow \quad \nabla \cdot \mathbf{D} = \rho_f; \quad (4.3.2)$$

$$\Rightarrow \quad \oint_S \mathbf{D} \cdot d\mathbf{a} = Q_{f\text{enc}}. \quad (4.3.3)$$

*Trick:* In most of cases,  $\mathbf{D}$  is determined exclusively by the free charge if we are dealing with symmetrical systems.

### 4.3.2 Boundary Conditions

$$D_{\text{above}}^\perp - D_{\text{below}}^\perp = \sigma_f; \quad (4.3.4)$$

$$\mathbf{D}_{\text{above}}^\parallel - \mathbf{D}_{\text{below}}^\parallel = \mathbf{P}_{\text{above}}^\parallel - \mathbf{P}_{\text{below}}^\parallel; \quad (4.3.5)$$

$$E_{\text{above}}^\perp - E_{\text{below}}^\perp = \frac{\sigma}{\epsilon_0}; \quad (4.3.6)$$

$$\mathbf{E}_{\text{above}}^\parallel - \mathbf{E}_{\text{below}}^\parallel = 0. \quad (4.3.7)$$


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## 4.4 Linear Dielectrics

Iff a space that is entirely filled with a homogeneous linear dielectric, then

$$\nabla \cdot \mathbf{D} = \rho_f \quad \text{and} \quad \nabla \times \mathbf{D} = 0.$$

And, also

$$\mathbf{D} = \varepsilon_0 \mathbf{E}_{\text{vac}}$$

where  $\mathbf{E}_{\text{vac}}$  is the field the same free charge would produce in the absence of dielectric.

So, we can conclude that

$$\mathbf{E} = \frac{1}{\varepsilon} \mathbf{D} = \frac{1}{\varepsilon_r} \mathbf{E}_{\text{vac}}. \quad (4.4.1)$$

Now, just for the sake of a very simple example, if we have a free charge  $q$  that is embedded in a large dielectric, the field it produces is

$$\mathbf{E} = \frac{1}{4\pi\varepsilon} \frac{q}{r^2} \hat{\mathbf{r}}. \quad (4.4.2)$$

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## References

- [1] David J Griffiths. *Introduction to electrodynamics; 3rd ed.* Upper Saddle River, NJ: Prentice-Hall, 1999. URL: <https://cds.cern.ch/record/611579>.
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