

PHYS2114 Cheat Sheet

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1 Mathematics Preliminary

1.1 IMPORTANT NOTE

The following notations are taken from Griffith's 'Introduction to Electrodynamics'.[1]

Where as the line, area and volume integral elements are the following;

- Line integral element: $d\vec{l}$.
 - Area integral element: $d\vec{a}$.
 - Volume integral element: $d\tau$.
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1.2 Cartesian Coordinates

The line and volume integral elements are

$$d\vec{l} = \hat{x} dx + \hat{y} dy + \hat{z} dz, \quad d\tau = dx dy dz.$$

And the following are some common operators

Gradient: $\nabla t = \frac{\partial t}{\partial x} \hat{x} + \frac{\partial t}{\partial y} \hat{y} + \frac{\partial t}{\partial z} \hat{z},$

Divergence: $\nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z},$

Curl: $\nabla \times \vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{z},$

Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}.$

1.3 Cylindrical Coordinates

The conversion of Cartesian to Cylindrical coordinates[2] are

$$x = r \cos \phi, \quad y = r \sin \phi, \quad z = z,$$

note that $\phi \in [0, 2\pi]$.

The line and the volume integral elements are

$$d\vec{l} = \hat{s} ds + s \hat{\phi} d\phi + \hat{z} dz, \quad d\tau = s ds d\phi dz.$$

The following are some common vector operators

$$\text{Gradient:} \quad \nabla t = \frac{\partial t}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial t}{\partial \phi} \hat{\phi} + \frac{\partial t}{\partial z} \hat{z},$$

$$\text{Divergence:} \quad \nabla \cdot \vec{v} = \frac{1}{s} \frac{\partial}{\partial s} + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z},$$

$$\text{Curl:} \quad \nabla \times \vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{z},$$

$$\text{Laplacian:} \quad \nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}.$$

1.4 Spherical Coordinates

Note that $\theta \in [0, \pi]$ denotes the angle between z-axis and the vector of interest, and that $\phi \in [0, 2\pi]$ denotes the angle between x-axis and the projection of the vector of interest on to the xy-plane.[2]

The conversion of Cartesian to Spherical coordinates are shown as the follows

$$x = r \sin(\theta) \cos(\phi), \quad y = r \sin(\theta) \sin(\phi), \quad z = r \cos(\theta).$$

And the line and volume integral elements are

$$d\vec{l} = \hat{r} dr + r \hat{\theta} d\theta + r \sin(\theta) \hat{\phi} d\phi, \quad d\tau = r^2 \sin(\theta) dr d\theta d\phi.$$

The following are the common operators

$$\text{Gradient:} \quad \nabla t = \hat{r} \frac{\partial t}{\partial r} + \frac{\hat{\theta}}{r} \frac{\partial t}{\partial \theta} + \frac{\hat{\phi}}{r \sin(\theta)} \frac{\partial t}{\partial \phi},$$

$$\text{Divergence:} \quad \nabla \cdot \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi},$$

$$\begin{aligned} \text{Curl:} \quad \nabla \times \vec{v} = & \frac{1}{r \sin \theta} \left(\frac{\partial}{\partial \theta} \sin(\theta) v_\phi - \frac{\partial v_\theta}{\partial \phi} \right) \hat{r} \\ & + \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} r v_\phi \right) \hat{\theta} + \frac{1}{r} \left(\frac{\partial}{\partial r} r v_\theta - \frac{\partial v_r}{\partial \theta} \right) \hat{\phi}, \end{aligned}$$

$$\text{Laplacian:} \quad \nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}.$$

References

- [1] David J Griffiths. *Introduction to electrodynamics; 3rd ed.* Upper Saddle River, NJ: Prentice-Hall, 1999. URL: <https://cds.cern.ch/record/611579>.
- [2] *Del in cylindrical and spherical coordinates*. en. Page Version ID: 1016691595. Apr. 2021. URL: https://en.wikipedia.org/w/index.php?title=Del_in_cylindrical_and_spherical_coordinates&oldid=1016691595 (visited on 06/03/2021).