PHYS2111 Cheat Sheet

Moisty Eyes

April 2021

1 Formula Sheet

1.1 Expectation Value

Expectation value of function $f(x)$ subject to $\Psi(x,t)$

$$\mathbb{P}(f(x)) = \int_{-\infty}^{\infty} f(x) \|\Psi(x,t)\|^2 \, \mathrm{d}x.$$

Note that the Hamiltonian operator H is

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t),$$

and the expectation value of the energy $\langle H \rangle$ is

$$\begin{aligned}\langle H \rangle &= \left\langle -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t) \right\rangle \\ &= \int_{-\infty}^{\infty} \Psi^*(x,t) \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t) \right) \Psi(x,t) \, \mathrm{d}x.\end{aligned}$$

1.2 Poisition and Momentum Operators

The position operator,

$$\hat{x} = x.$$

The momentum operator,

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}.$$

1.3 The Infinite Square Well

Time Dependent Schrodinger's Equation

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) \exp\left(-iE_n t\right).$$

with constant

$$c_n = \sqrt{\frac{2}{a}} \int_0^a \sin\left(\frac{n\pi}{a}x\right) \Psi(x,0) \, \mathrm{d}x.$$

and allowed energy

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}.$$

1.4 The Harmonic Oscillator

A harmonica oscillator has potential energy

$$V(x) = \frac{1}{2}m\omega^2 x^2.$$

With ground state $\psi_0(x)$

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left(-\frac{m\omega}{2\hbar}x^2\right).$$

The ladder operator

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{p} + m\omega x), \qquad (\omega = \sqrt{\tfrac{k}{m}})$$

and to extract the n-th state with the ladder operator

$$\psi_n(x) = A_n(\hat{a}_+)^n \psi_0(x).$$

1.5 The Free Particle

The initial condition can be expressed in the Fourier k space,

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x,0)\exp(-ikx)\,\mathrm{d}x, \qquad (k = \tfrac{\sqrt{2mE}}{\hbar})$$

we can then use this $\phi(k)$ calculated above to determine the time dependent wave equation for the free particle

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k) \exp\left(i\left(kx - \frac{\hbar k^2}{2m}t\right)\right)\mathrm{d}k.$$

Notably, the particle has a phase velocity and a group velocity,

$$v_{\text{phase}} = \frac{\omega}{k}, \qquad v_{\text{group}} = \frac{\mathrm{d}\omega}{\mathrm{d}k}. \qquad (\omega = \tfrac{\hbar k^2}{2m})$$

Also note that

$$v_{\text{group}} = v_{\text{classical}}.$$

For a free particle, note that the potential energy is zero namely $V(x,t) = 0$. Also note that we can switch from the Fourier k space and the momentum p space ($p = \hbar k$), obtaining the following

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \Psi(x,0)\exp\left(-\frac{ip}{\hbar}x\right)\mathrm{d}x,$$

and

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p)\exp\left(\frac{ip}{\hbar}x\right)\exp\left(\frac{-iEt}{\hbar}\right)\mathrm{d}p \qquad (E = \tfrac{\hbar^2 k^2}{2m})$$

1.6 The Delta-Function Potential

The delta function well has **only one** bound state (for $\alpha > 0$) namely

$$\phi(x) = \frac{\sqrt{m\alpha}}{\hbar} \exp\left(\frac{-m\alpha|x|}{\hbar}\right),$$

with **only one** allow energy

$$E = -\frac{m\alpha^2}{\hbar}.$$

The reflection coefficient R and the transmission coefficient T can be expressed as the following

$$R = \frac{1}{1+\frac{2\hbar^2 E}{m\alpha^2}}, \qquad T = \frac{1}{1+\frac{m\alpha^2}{2\hbar^2 E}}.$$

1.7 The Finite Squre Well

The finite squre well has the potential function such that (note that this is a flipped tophat function),

$$V(x) = \begin{cases} -V_0 & \text{if } -a \le x \le a, \\ 0 & \text{if } |x| > a. \end{cases}$$

TO DO LIST

- Finishing up the finite square well
- proof read
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