Dynamics of Neural Systems

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Exercise Sheet 4 due Dec 14^{th} 2016

Disclaimer: *The full number of points can only be assigned if the way how the results were derived is understandable for us. Just '"seeing'" the solution and writing it down is not sufficient to obtain the full number of points!*

Please send the plots and code on the due date to the email address given above and bring your plots to the exercises. If possible, bring your laptop to show the solutions via beamer.

1 Exercise 1 Simulation of multi compartment model of passive neurite. Credits: 4

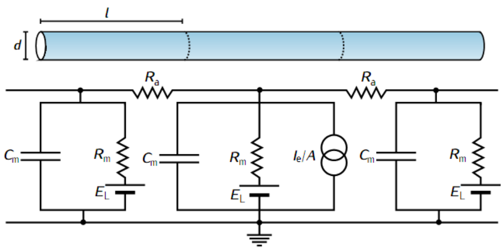

Figure 1: Multi compartment model.

Given a multi-compartment model as given in Fig. 1, assume cylindrical compartments with identical electrical properties:

- membrane capacitance $C_m = 62.8pF$,
- membrane resistance $R_m = 1.59G\Omega$,
- axial resistance $R_a = 0.0318G\Omega$, and
- $E_L = E_m = 0mV$.

(Lecture naming convention: Don't confuse with r_m and r_a or $\tilde{r}_m$ and $\tilde{r}_a$.)

An input current $I_e(j_e, t_e)$ is injected at time t_e in compartment j_e.

1.1

For a multi compartment model with an arbitrary number of compartments, derive the DEQ for the potential at compartment j, $\frac{\mathrm{d}V(j,t)}{\mathrm{d}t}$.

1.2

Approximate the derivative with the forward-Euler method and implement this model. Assume $N = 50$ as the number of compartments in the model, where the first compartment is terminated as a "sealed end" and the last compartment is terminated as a "killed end".

Hint: The boundary condition for a terminal compartment j that is sealed are $V(N-1,t) = V(N,t)$ (zero derivative of voltage / length current in last compartment), and the ones for for a killed end is $V(N,t) = 0V$, where N is the index of the last compartment.

Simulate with the step current given in Eq. (1) for a , with $j_e = 20$ and $t_e = 20ms$.

$$I_e(j,t) = \begin{cases} 0 & (t < t_e) \vee (j \neq j_e) \\ I_0 & (t_e \leq t) \wedge (j = j_e) \end{cases} \tag{1}$$

For $I_0 = 10pA$, plot the 3D plot of the potential $V(j,t)$ over both compartment and time.

1.3

Discuss the functional form of the spatially dependent stationary solution $V(j,\infty)$.

1.4

$$I_e(j,t) = \begin{cases} 0 & (t < t_e) \vee (t_s \leq t) \vee (j \neq j_e) \\ I_0 & (t_e \leq t < t_s) \wedge (j = j_e) \end{cases} \tag{2}$$

Assume the rectangle impulse input given in Eq. (2) with $I_0 = 100pA$, $j_e = 14$, $t_e = 20ms$ and $t_s = 400ms$. Simulate and 3D-plot the resulting potential $V(j,t)$.

2 Exercise 2 Simulation of single compartment Hodgkin-Huxley model of active neurite. Credits: 8

Eqs. (3)-(12) give the Hodgkin-Huxley (HH) model of an (imaginary) active neurite with input current $I_e(t)$. *(The values are given to minimize the problem of computational precision. The electrical properties (and current amplitudes) are chosen in such a way, that you can implement them by just dropping the unit (using ms as unit for* Δt*, choose, e.g.* $\Delta t = 0.025ms$*). Calculate the unit test to be sure.*
Values for a HH-model with $E_m = -65mV$ *can be found in the book by Sterratt, page 61. Beware that most values in the book are given as density over the membrane area.)*

$$C_m \frac{\mathrm{d}V}{\mathrm{d}t} = I_e(t) - \bar{g}_L(V - E_L) - \bar{g}_{\mathrm{Na}} m^3 h (V - E_{\mathrm{Na}}) - \bar{g}_{\mathrm{K}} n^4 (V - E_{\mathrm{K}}) \tag{3}$$

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \alpha_m(1-m) - \beta_m m \tag{4}$$

$$\alpha_m = \begin{cases} 0.1 \frac{V-25}{1-\exp\left(-\frac{V-25}{10}\right)} & V \neq 25mV \\ 1 & V = 25mV \end{cases} \tag{5}$$

$$\beta_m = 4\exp\left(-\frac{V}{18}\right) \tag{6}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \alpha_h(1-h) - \beta_h h \tag{7}$$

$$\alpha_h = 0.07\exp\left(-\frac{V}{20}\right) \tag{8}$$

$$\beta_h = \frac{1}{1+\exp\left(-\frac{V-30}{10}\right)} \tag{9}$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \alpha_n(1-n) - \beta_n n \tag{10}$$

$$\alpha_n = \begin{cases} 0.01 \frac{V-10}{1-\exp\left(-\frac{V-10}{10}\right)} & V \neq 10mV \\ 0.1 & V = 10mV \end{cases} \tag{11}$$

$$\beta_n = 0.125\exp\left(-\frac{V}{80}\right) \tag{12}$$

The electric properties for this neurite are:

- $C_m = 1\mu F$, membrane capacitance.
- $E_{\mathrm{Na}} = 115mV$, sodium equilibrium potential.
- $E_{\mathrm{K}} = -12mV$, potassium equilibrium potential.
- $E_{\mathrm{L}} = 10.6mV$, leak equilibrium potential.
- $V(0) = 0mV$, (starting) membrane resting potential.
- $\bar{g}_{\mathrm{Na}} = 120mS$, maximum conductance for sodium channel.

- $\bar{g}_{\mathrm{K}} = 36mS$, maximum conductance for potassium channel.
- $\bar{g}_{\mathrm{L}} = 0.3mS$, maximum leak conductance.

At $t = 0ms$, the model is in equilibrium with $E_m = V(0) = 0mV$, i.e. there is no membrane potential change and therefore no change in channel activation.

As additional (debugging) help, the characteristics for the m,h,n gating probabilities at steady state are given in Fig. 2:

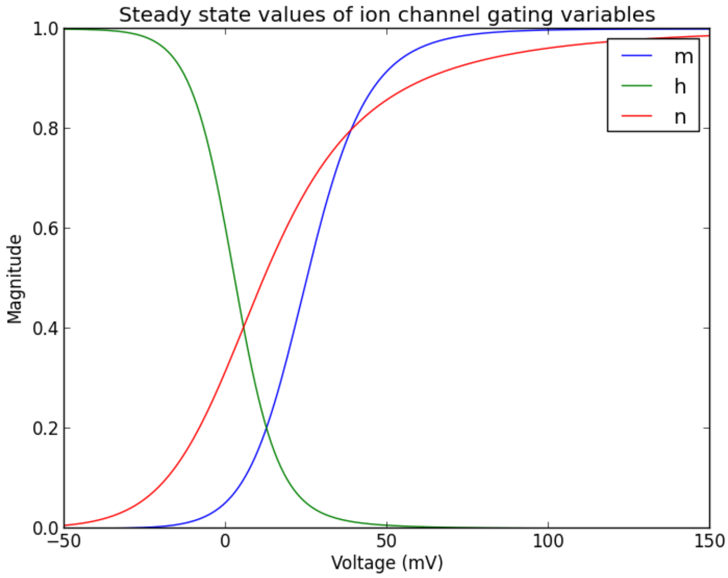

Figure 2: m,h,n characteristics in stationary state.

2.1

Given the Hodgkin-Huxley (HH) model of an active neurite with input current $I_e(t)$, Eqs. (3)-(12), assume a single compartment model, i.e. no axial current, and approximate all DEQs.

2.2

Implement the approximated HH model and plot $V(t)$ for the input current given in Eq. (2) with $t_e = 50ms$, $t_s = 300ms$ for different amplitudes $I_0 = 0\mu A$, $I_0 = 3\mu A$, $I_0 = 6\mu A$and $I_0 = 8\mu A$.

2.3

Simulate with different amplitudes of input current: from $I_0 = 0\mu A$ to $I_0 = 20\mu A$ in steps of e.g. $\Delta I = 0.5\mu A$. If you want, you can increase the timeframe for the simulation, e.g. $t_s = 800ms$, for better resolution of the firing rate. Plot the firing rate in the stationary state (i.e. ~ 10ms after stimulus onset) as a function of the input current $I_e(t)$. *Hint: Calculate the firing rate, e.g. by counting the number of times a certain threshold is crossed in one direction only, e.g. rising, and divide by the time-frame you looked at.*