

# Chapter 9: Statistical Image Analysis

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January 24, 2017

# **9.1 Statistical Hypothesis Tests**

# Goal of Hypothesis Tests

- Now that we know about MR image acquisition and segmentation, we would like to **draw scientific conclusions** from our data
  - Answer questions such as:
    - Can we observe changes in the thickness of the cortex associated with learning how to juggle?
    - Which brain regions are activated when counting?

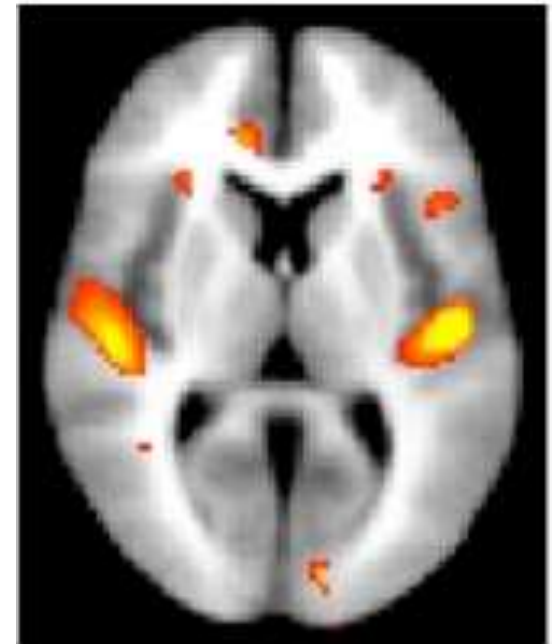
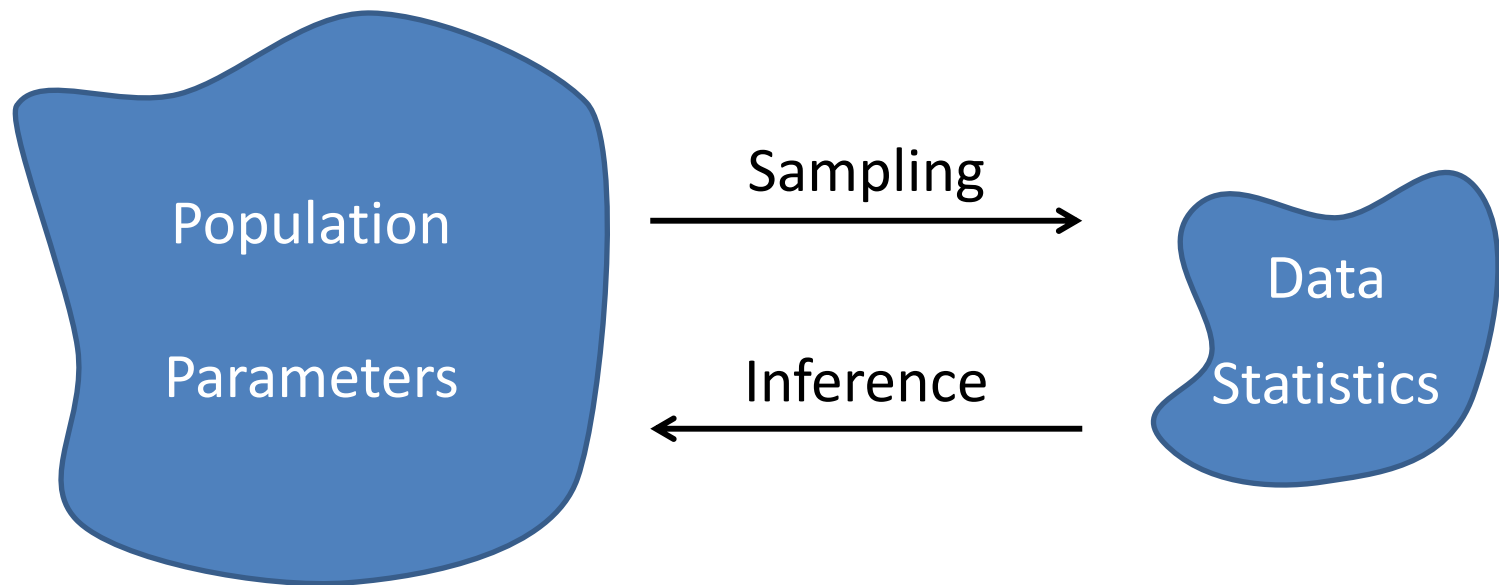


Image from Oxford brain fMRI lab

# Population vs. Sample

- **Population:** Set about which we would like to make a statement (e.g., all humans)
  - (Assumed to be) governed by some statistical distribution (e.g., Gaussian)
  - Described by certain parameters (e.g., mean and variance)
- **Sample:** Set of individuals for which we have data
  - Can compute statistics on measured data
  - Would like to make statements about the parameters *of the partially observed population* that hold with a given probability



# Hypothesis Testing: Basic Concept

- **Basic principle:** Reject a statement about the population parameters by showing that the observed data is highly unlikely if it holds
- **Null Hypothesis ( $H_0$ ):** Statement that we are trying to reject
  - The opposite of our true research hypothesis (**alternative hypothesis**  $H_A$ : “ $H_0$  is false”)
  - Often states that some factor *does not have an effect* on the data (e.g., “The changes from learning how to juggle are not visible in MRI.”)

# Type I / Type II Errors

- **Type I error ( $\alpha$ ): *Reject* a *true* null hypothesis**

- In science: reporting a false finding
- Controlled by design of statistical test
- Widely accepted level:  $\alpha=5\%$

$H_0$	True	False
Reject	Type I	Correct
Reject	Correct	Type II

- **Type II error ( $\beta$ ): *Fail to reject* a *false* null hypothesis**

- In science: Not being able to report a finding even though the alternate (research) hypothesis was correct
- “Power” of a test

# Widely Used Tests

- Widely used hypothesis tests in neuroimaging include:
  - Single-sample and two-sample t-test
  - Paired t-test
  - One- and two-sided tests
  - Analysis of variance
- I assume you are already familiar with many of them

# Definition of $p$ value

- **Definition:** The  $p$  value is the *conditional* probability of observing the computed value of the test statistic or a more extreme value *if  $H_0$  is true*
  - Often needs to be computed using software or using pre-computed lookup tables
  - Smaller  $p$ : Stronger evidence against  $H_0$
  - Obtain a test with type I error  $\alpha$  if we reject  $H_0$  if  $p < \alpha$ 
    - In neuroimaging,  $p < 0.05$  is usually taken to be “significant”



# Summary: Hypothesis Testing

- Hypothesis testing rejects a null hypothesis  $H_0$  based on the fact that the observed data is unlikely given the hypothesis.
  1. Formalize null hypothesis  $H_0$
  2. Compute test statistic from data (e.g., t score)
  3. Compute  $p$  value
    - Conditional probability of test statistic taking on the computed or a more extreme value given  $H_0$
  4. Reject  $H_0$  if  $p < \alpha$ 
    - Specify  $p$  as a measure of evidence against  $H_0$
    - $p$  does **not** provide a bound on the type I error, nor the posterior probability of  $H_0$

## **9.2 Family-Wise Error (FWE) Correction**

# Motivation: Family-Wise Errors

- **Quiz:** Suppose you have  $N=10$  coins in your wallet and throw away all for which a statistical test with type I error level  $\alpha=0.05$  rejects the null hypothesis that it is legitimate. If none of the coins is counterfeit, what is the probability of still discarding at least one of them?

$$p = 1 - (1 - 0.05)^{10} = 0.40$$

- **Lesson:** If we perform multiple tests and would like to control the rate of performing a type I error in any of them (i.e., the **family-wise error**), we have to account for the number of comparisons!
  - In brain imaging:  $N \approx 10^5$  voxels, practically certain to detect false differences when using  $\alpha=0.05$  per-voxel

# Šidàk and Bonferroni Correction

- **Šidàk correction:** An obvious way to correct for multiple comparisons is to demand a stricter  $\alpha_{\text{IND}}$

– Assuming independent tests, we obtain:

$$1 - (1 - \alpha_{\text{IND}})^N = \alpha_{\text{FW}} \text{ iff } \alpha_{\text{IND}} = 1 - \sqrt[N]{(1 - \alpha_{\text{FW}})}$$

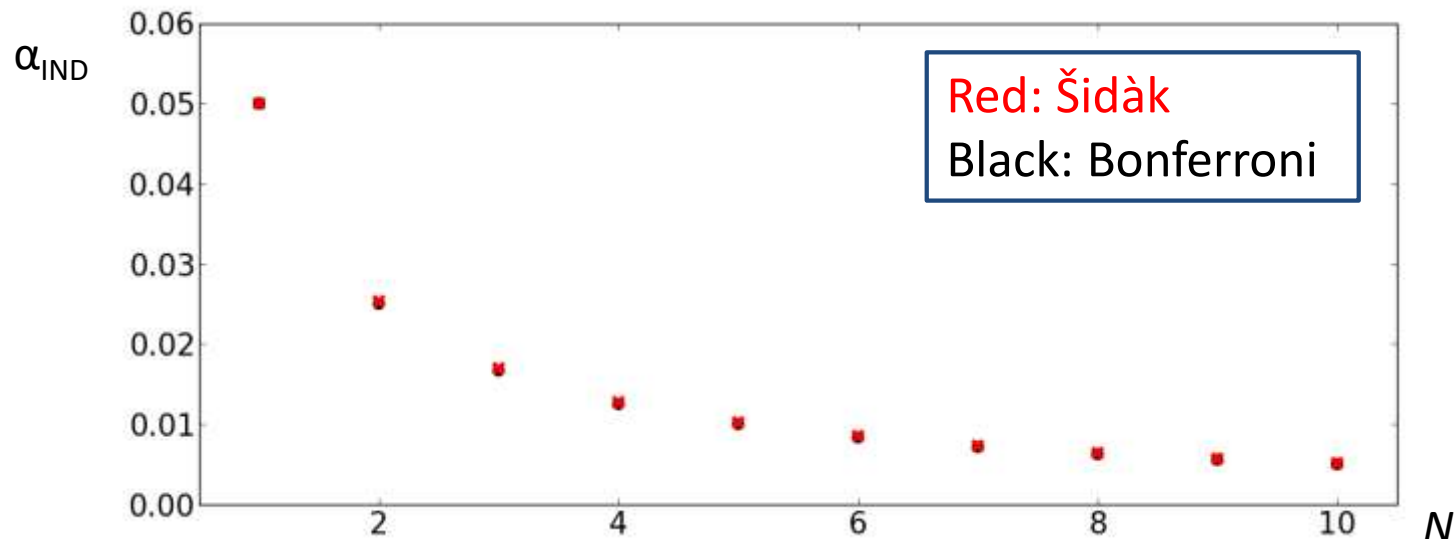
- **Bonferroni correction:** A computationally more convenient, conservative, and quite close approximation is given by:

$$\alpha_{\text{IND}} = \alpha_{\text{FW}} / N$$

# Bonferroni Correction: Proof

- Consider  $N$  hypotheses  $H_1, \dots, H_N$  with associated  $p$  values  $p_1, \dots, p_N$ . Let  $T$  be the set of  $N_T$  true hypotheses. Let  $\alpha_{\text{IND}} := \alpha_{\text{FW}}/N$ . Then,

$$\begin{aligned} FWE &= P\left(\bigcup_{i \in T} \left(p_i \leq \frac{\alpha_{\text{FW}}}{N}\right)\right) \leq \sum_{i \in T} P\left(p_i \leq \frac{\alpha_{\text{FW}}}{N}\right) \\ &= N_T \frac{\alpha_{\text{FW}}}{N} \leq \alpha_{\text{FW}} \end{aligned}$$



# Ad-hoc Correction

- In neuroimaging, tests in neighboring voxels are often strongly correlated. Thus, Šidàk and Bonferroni correction are **far too conservative** and dramatically increase type II errors.
- **Ad-hoc alternative:** Adapt threshold on test statistic until map “looks right” (e.g.,  $t > 3$ )
  - Some labs have a quasi-consensus based on their setup and experiments with predictable outcome
  - Can be useful for quick initial impression
  - More principled solution is clearly desirable to draw reproducible and reliable conclusions

# Random Field Theory

- **Random Field Theory** (RFT) is a principled way to account for spatial correlations
  - Less conservative than Bonferroni correction
- **Main result:** In 3D and for sufficiently large  $Z$  (Gaussianized t),

$$p_{FWE}(Z) \approx R \times \frac{(4 \ln 2)^{\frac{3}{2}}}{(2\pi)^2} e^{-\frac{Z^2}{2}} (Z^2 - 1)$$

with number of resolution elements (RESEL)

$$R = \frac{V}{FWHM_x \times FWHM_y \times FWHM_z}$$

where the full-width half-maximum (FWHM) refers to the hypothetical Gaussian kernel that *could have been used* to achieve observed smoothness from white noise

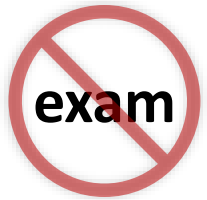
# Notes on Random Field Theory

- **Amount of smoothness** results from combined effects of intrinsic smoothness (due to physiology, PSF of acquisition) and explicit image smoothing
  - In practice, is estimated from spatial derivatives in normalized residual images
- RFT is **different** from Bonferroni correction with RESELS instead of voxels!
  - RESELS only used to make parameters in the approach more intuitive, derivation does *not* assume squared exponential decay of correlation



# Basic Idea Behind Random Field Theory

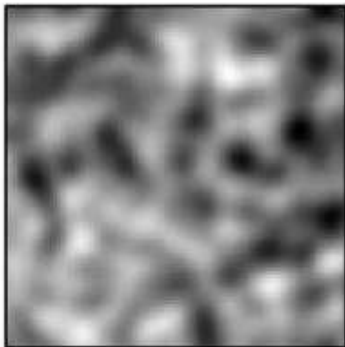
- Random Field Theory considers the **Euler characteristic**  $\chi_u$  of the excursion set  $\{\mathbf{x} \in \Omega \mid Z(\mathbf{x}) > u\}$



$$FWE = P\left(\bigcup_{i \in T} Z_i \geq u\right) = P\left(\max_i Z_i \geq u\right)$$

$$\approx P(\chi_u > 0) \approx E[\chi_u] \approx \frac{V\sqrt{|\Lambda|}}{(2\pi)^2} e^{-\frac{Z^2}{2}} (Z^2 - 1)$$

Gaussian R.F.  $Z$



Excursion set  $Z > 0.52$



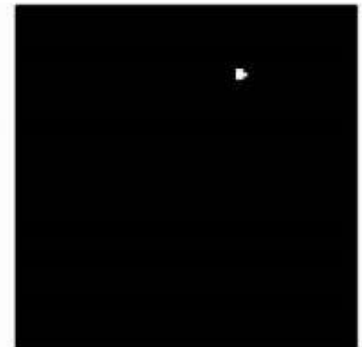
$$\chi_u = 12 - 3 = 9$$

Excursion set  $Z > 1.88$



$$\chi_u = 9 - 0 = 9$$

Excursion set  $Z > 2.75$



$$\chi_u = 1 - 0 = 1$$

# Cluster-based Testing

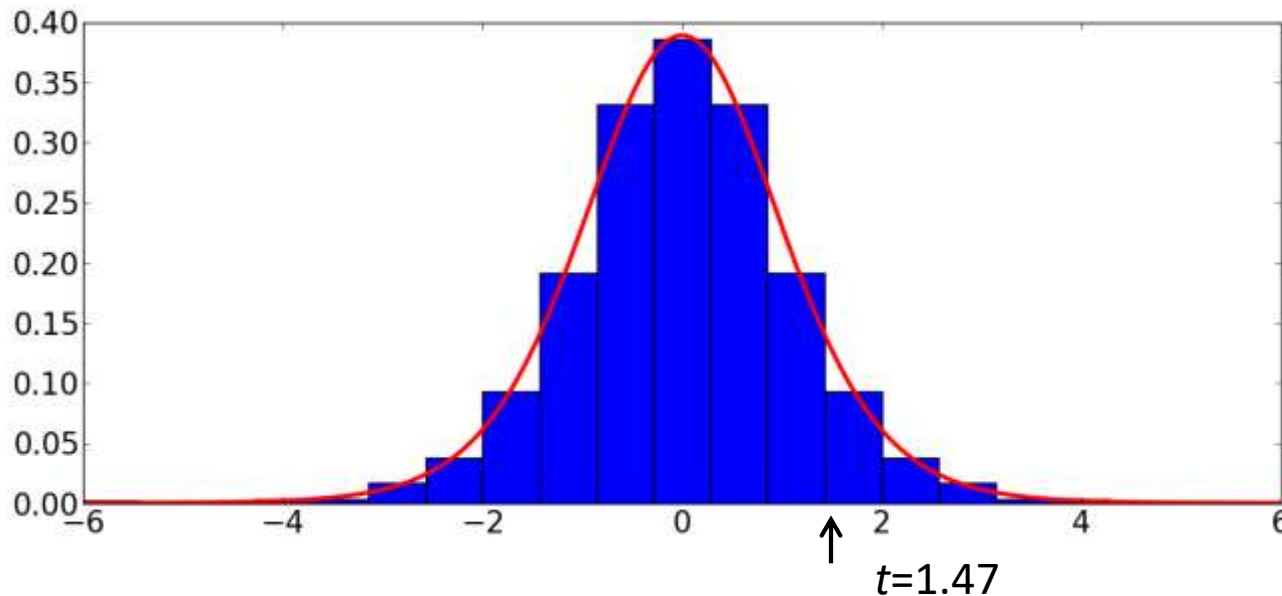
- So far, Random Field Theory still works voxel-wise
- **Alternative:** Consider size of connected regions
  - Most true effects extend over some spatial region, isolated voxels are likely to be false positives
- **Approach:**
  1. Form clusters by heuristically thresholding statistical maps at some level (e.g.,  $t > 3$ )
  2. Formal testing can be done **cluster-wise**
    - Size (number of voxels) or mass (sum of scores) serve as test statistics
    - **But:** How to determine null distributions?

# Permutation-based Testing

- **Permutation methods** are widely used in neuroimaging to convert test statistics into  $p$  values
  - **Null hypothesis:** The assignment of individuals to groups (e.g., healthy vs. patient) is unrelated to the value of the test statistic
  - Null distribution can be built by **permuting the group labels** w.r.t. the measurements and recomputing the test statistic for each configuration
  - $p$  value given by location of test statistic for true label assignment w.r.t. null distribution

# Illustration: Permutation-Based $p$ Value

- Example measurements for two-sample t-test:
  - $v_i^C = \{1400, 1220, 1280, 1360, 1290, 1350\}$
  - $v_i^{NC} = \{1190, 1210, 1310, 1370, 1250, 1230\}$
- The 12 measurements can be labelled in  $\binom{12}{6} = 924$  ways, leading to the following distribution of  $t$  scores:



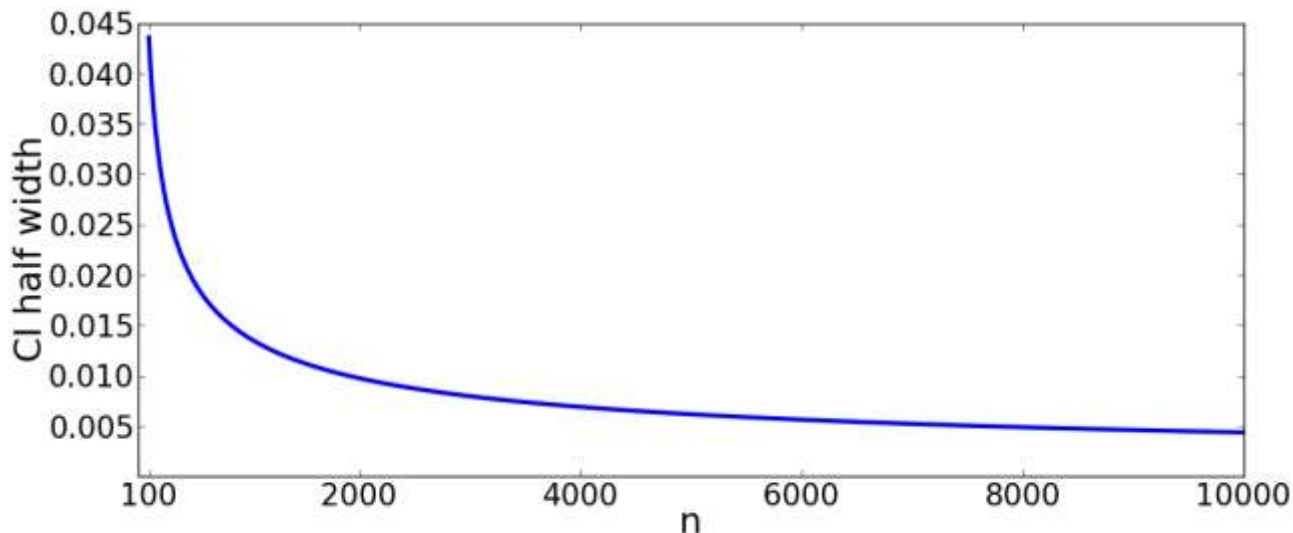
Red: Student's t-Distribution  
 $p=0.086$  (one-sided)  
Blue: Permutation-based Distribution  
 $p=0.091$

- **Note:** Based on 924 possible permutations, no  $p$  values smaller than  $1/924 \approx 0.001$  will be computed

# Required Number of Permutations

- **In practice:** With more realistic sample sizes, total number of permutations too large to enumerate, instead perform a random subsample
  - **Consequence:** Repeating the analysis of exactly the same data can lead to a different  $p$  value!
  - Randomly performing  $n$  out of a sufficiently large number of possible permutations produces the following 95% confidence interval (CI) of  $p$ :

$$p \pm 2\sqrt{p(1-p)/n}$$



# Permutation-/Cluster-Based FWE Correction

- **For each permutation** of group labels:
  - Compute per-voxel test statistic (e.g., t-Test)
  - Threshold map at some level, compute clusters
  - Store the largest / heaviest cluster in the full brain
    - This is the relevant cluster to control FWE rate!
- **For each cluster** found using the true labels:
  - Compute per-cluster “FWE-corrected”  $p$  value by comparing size / mass to the null distribution

# Reporting Cluster-Based Results

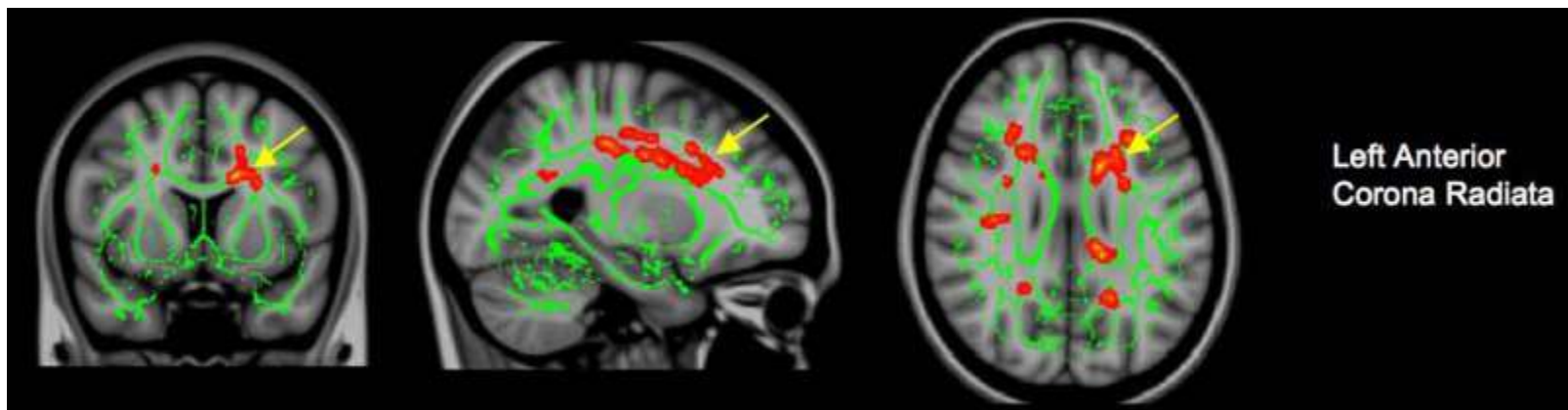
- Example: Table from [Jung et al. 2010]:

**Table 3 DTI differences between NPSLE patients and controls**

FA Controls > NPSLE

Voxels	p-value	Max-t	MNI X	MNI Y	MNI Z	Approximate white matter tract
266	0.001	3.135	30	17	18	Right superior longitudinal fasciculus
223	0.001	3.242	11	-34	25	Splenium of CC
172	0.003	3.408	-8	5	25	Body of CC
165	0.003	3.028	-26	28	12	Left anterior corona radiata
120	0.006	3.006	34	-41	28	Right superior longitudinal fasciculus

- Brain map from [Jung et al. 2010]:
  - Map  $t$  values, restricted to significant clusters



# Threshold-Free Cluster Enhancement

- **Remaining drawbacks** of cluster-based testing:
  - No principled way to select cluster-forming threshold
  - Results may change considerably based on threshold
  - A few voxels might cause clusters to split or merge
  - Pre-smoothing commonly applied, but what bandwidth?
  - Center of gravity no longer an adequate description of localization if clusters are large
- Goals of **threshold-free cluster enhancement (TFCE)**:
  - Boost belief based on agreement in spatial neighborhoods
  - Avoid having to set an arbitrary threshold
  - Still perform testing per-voxel

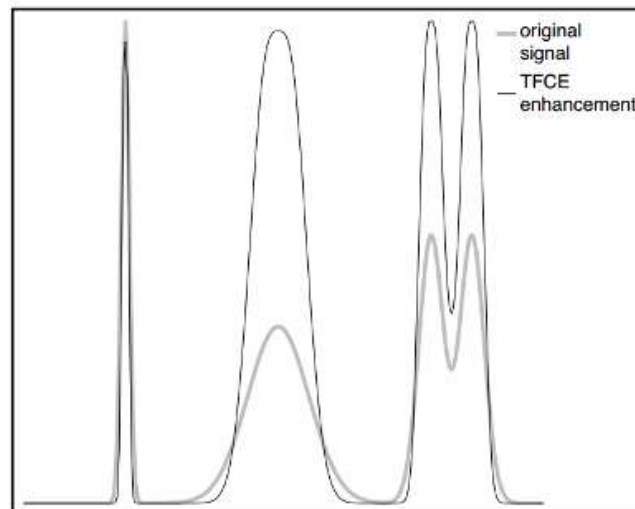
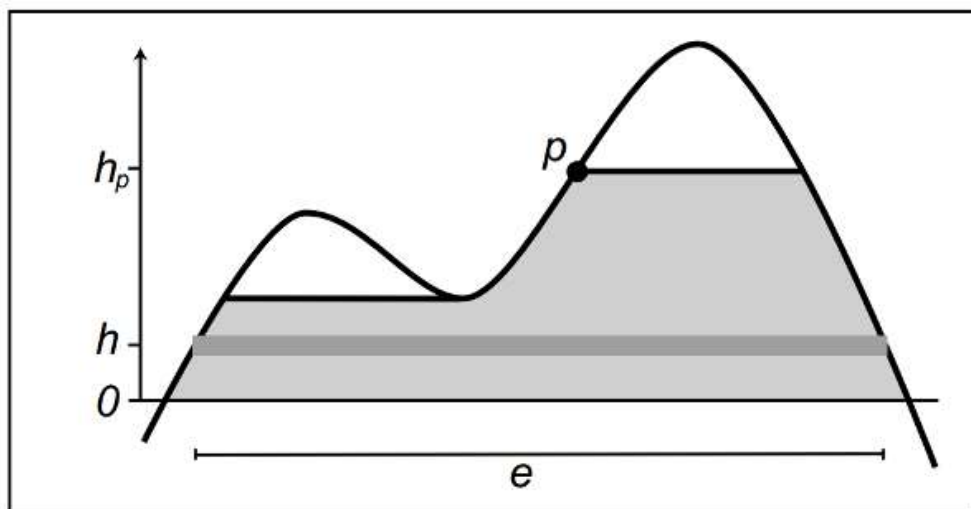


# Basic Idea of TFCE

- **“Trick” in TFCE:** Perform image transformation that boosts values (e.g.,  $t$  scores) that are surrounded by other large values

$$TFCE(p) = \int_{h=h_0}^{h_p} e(h)^E h^H dh$$

– Recommended:  $E=0.5$ ,  $H=2$



# Properties of TFCE

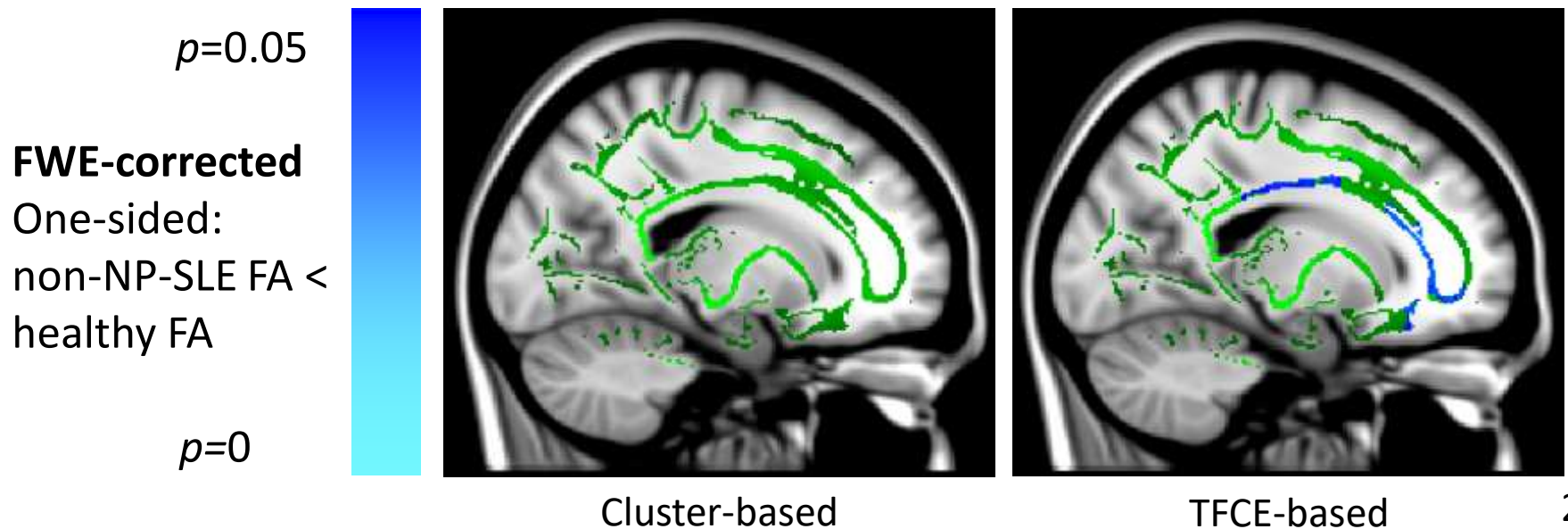
- TFCE only enhances **positive values**, sets negative ones to zero
  - Corresponds to one-sided t-Test
  - If desired, repeat test with negated t scores (account for multiple comparisons!)
- Unlike Gaussian smoothing, TFCE **preserves locations of local maxima**
- **Permutation-based testing** using TFCE:
  - Compute per-voxel test statistic (e.g., t-Test)
  - Apply TFCE to the map of t values
  - Store the largest value in the full brain
  - Testing compares per-voxel TFCE value to the null distribution

# Example: Cluster-based vs. TFCE

- *Example: Systemic Lupus Erythematosus (SLE)*
  - Autoimmune disease; would like to better understand difference between
    - NP-SLE: Neuropsychiatric variant, neural symptoms such as seizures or cognitive decline
    - non-NP-SLE: SLE without such symptoms
  - Using cluster-based analysis with cluster-forming threshold  $t > 3$ , [Jung et al. 2010] conclude that NP-SLE (but not non-NP-SLE) leads to a significant difference in brain structure
    - Might suggest that non-NP-SLE “does not affect the brain” (even though they did not test/claim this!)

# Cluster-based vs. TFCE in SLE

- Own results (based on different data):
  - Cluster-based analysis replicates Jung et al.
  - TFCE finds significant difference between healthy controls and non-NP-SLE!
    - Extent of affected regions much smaller
    - Suggests that even non-NP-SLE affects the brain, but small enough damage does not (yet) lead to neural symptoms

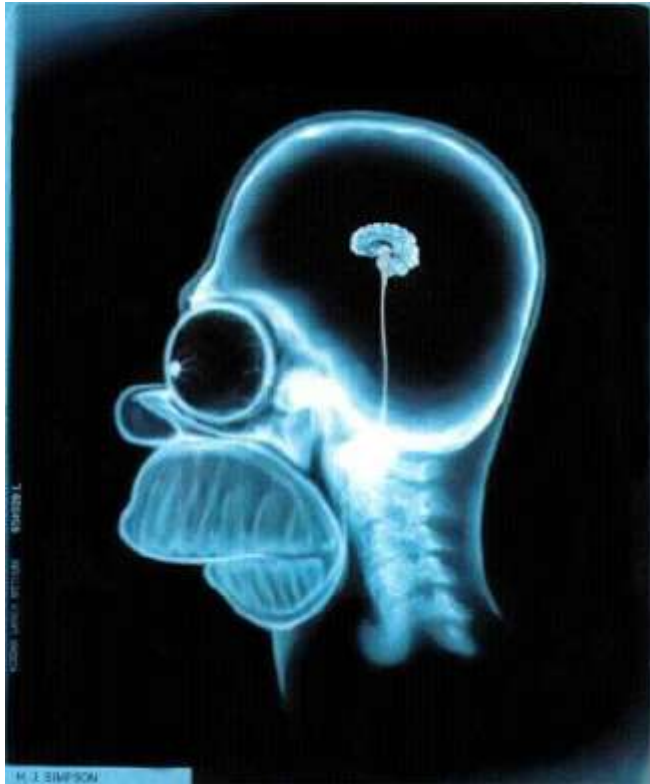


# Summary: FWE Correction

- **Type-I errors accumulate** when performing multiple hypothesis tests
- **Random Field Theory** provides a way to correct for this while accounting for smoothness
  - Less conservative than Bonferroni correction
- Another solution is to boost confidence using **information from neighboring voxels**
  - Cluster-based
    - Cluster-forming threshold, permutation-based *per-cluster* FWE corrected  $p$  value
    - Random Field Theory for clusters should be avoided
  - Threshold-Free Cluster Enhancement (TFCE)
    - Nonlinear peak enhancement, permutation-based *per-voxel* FWE corrected  $p$  value

## **9.3 Functional MRI**

# Structural vs. Functional MRI



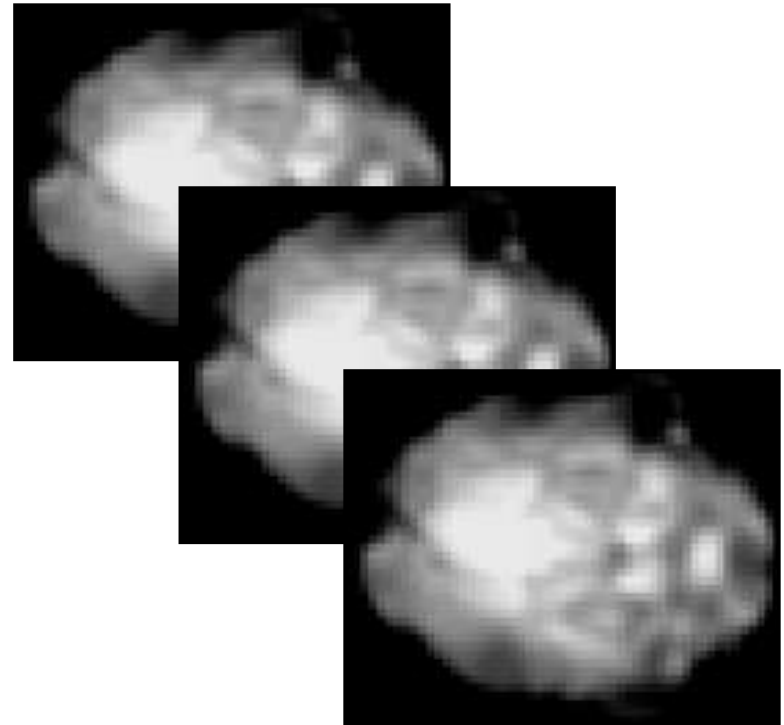
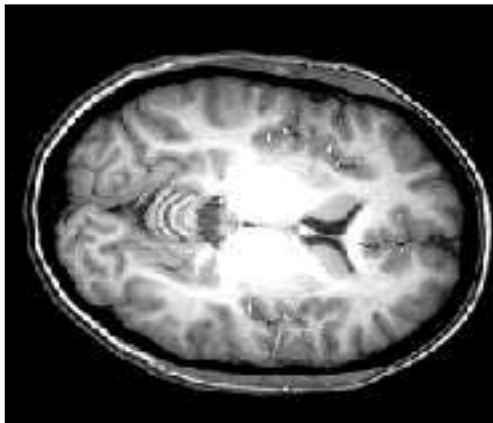
Structural MRI studies  
brain anatomy



Functional MRI studies  
brain function

# fMRI: The Raw Data

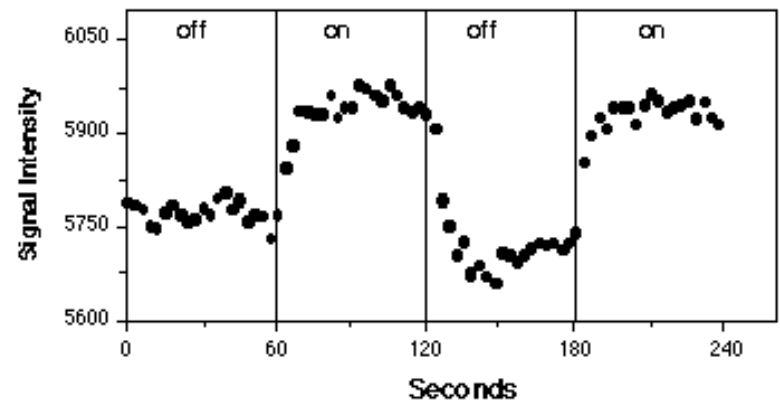
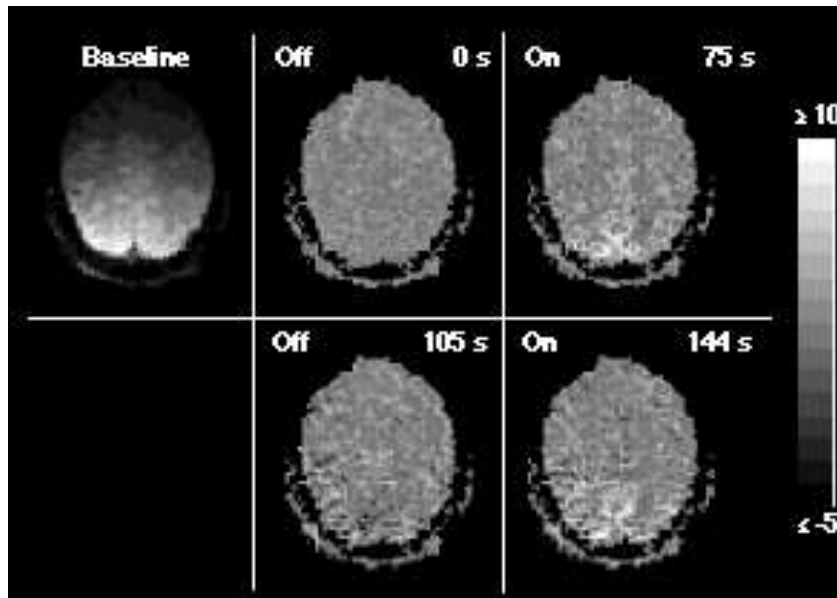
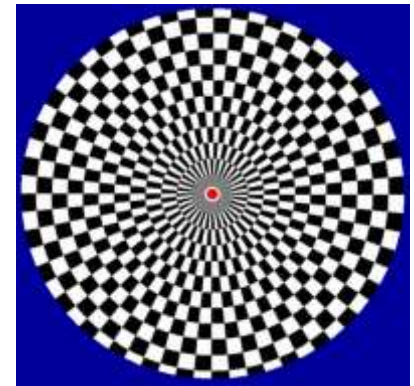
- In **structural MRI**, we take one high-resolution image (e.g.,  $1 \times 1 \times 1 \text{ mm}^3$ )
- In **functional MRI**, we repeatedly (e.g., every 2 sec) take low-resolution images (e.g.,  $3 \times 3 \times 5 \text{ mm}^3$ )





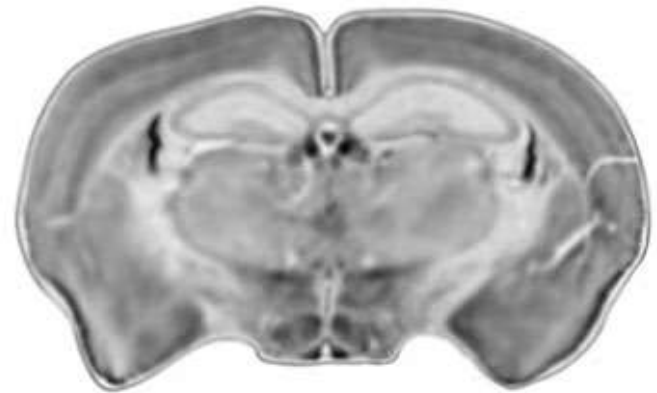
# fMRI: Activations

- In fMRI, an **activation** is measured by taking the difference between the signal for different tasks
- **Example stimulus:** Checkerboard  
OFF (60 sec) – ON (60 sec) – OFF ...
- **Example MR response:**

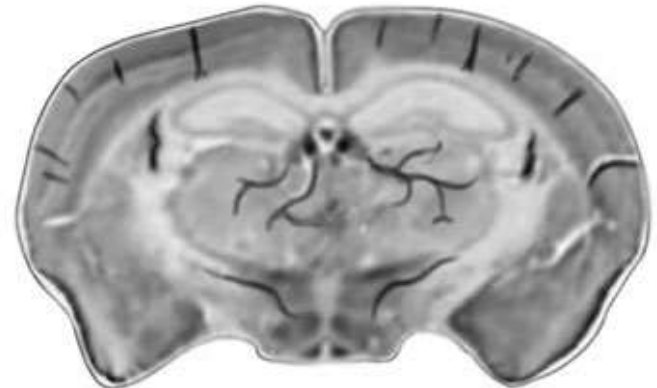


# Where Does the Signal Come From?

- **BOLD effect:** MR intensity is **B**lood **O**xygenation **L**evel **D**ependent
- **Oxygenated Blood**
  - **Diamagnetic:** (weakly) counteracts the local magnetic field
  - Effect similar to water
  - (Almost) no change in signal
- **Deoxygenated Blood**
  - **Paramagnetic:** (slightly) enhances the field
  - Decreases  $T_2^*$
  - Attenuates the MR signal

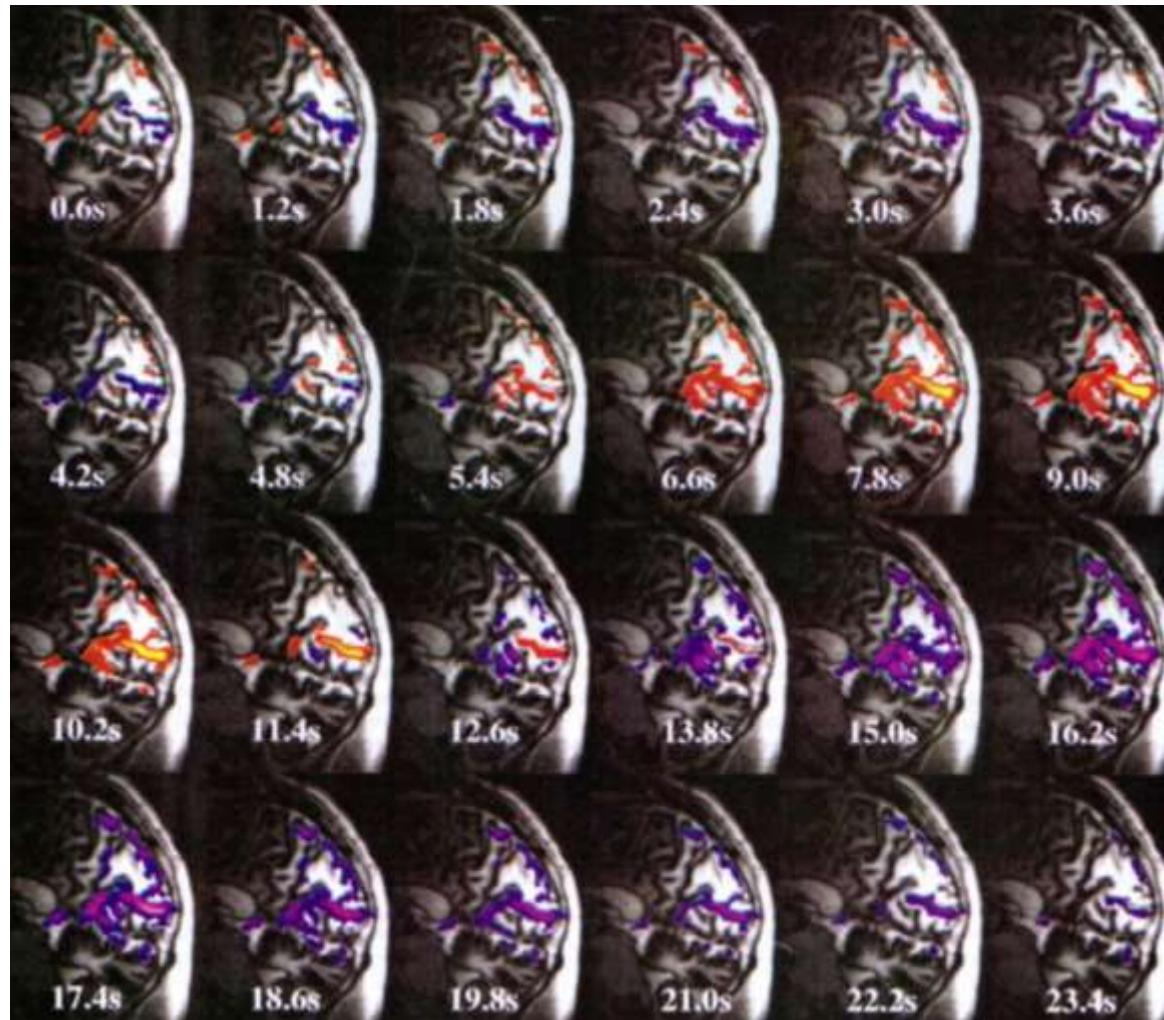


rat breathing pure oxygen



rat breathing normal air

# Temporal Complexity of BOLD Response



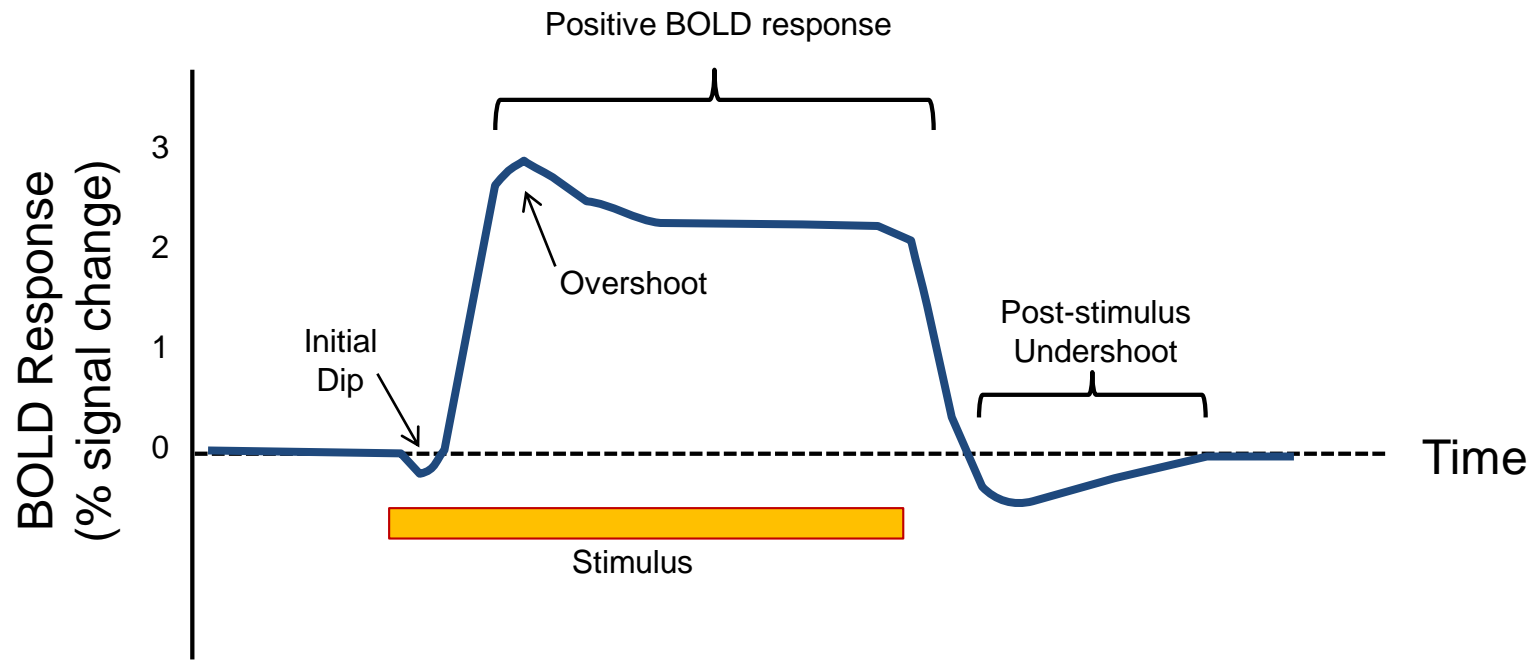
MR response to  
flashing visual  
stimulus

**Red:** Increased MR  
Signal

**Blue:** Decreased MR  
Signal

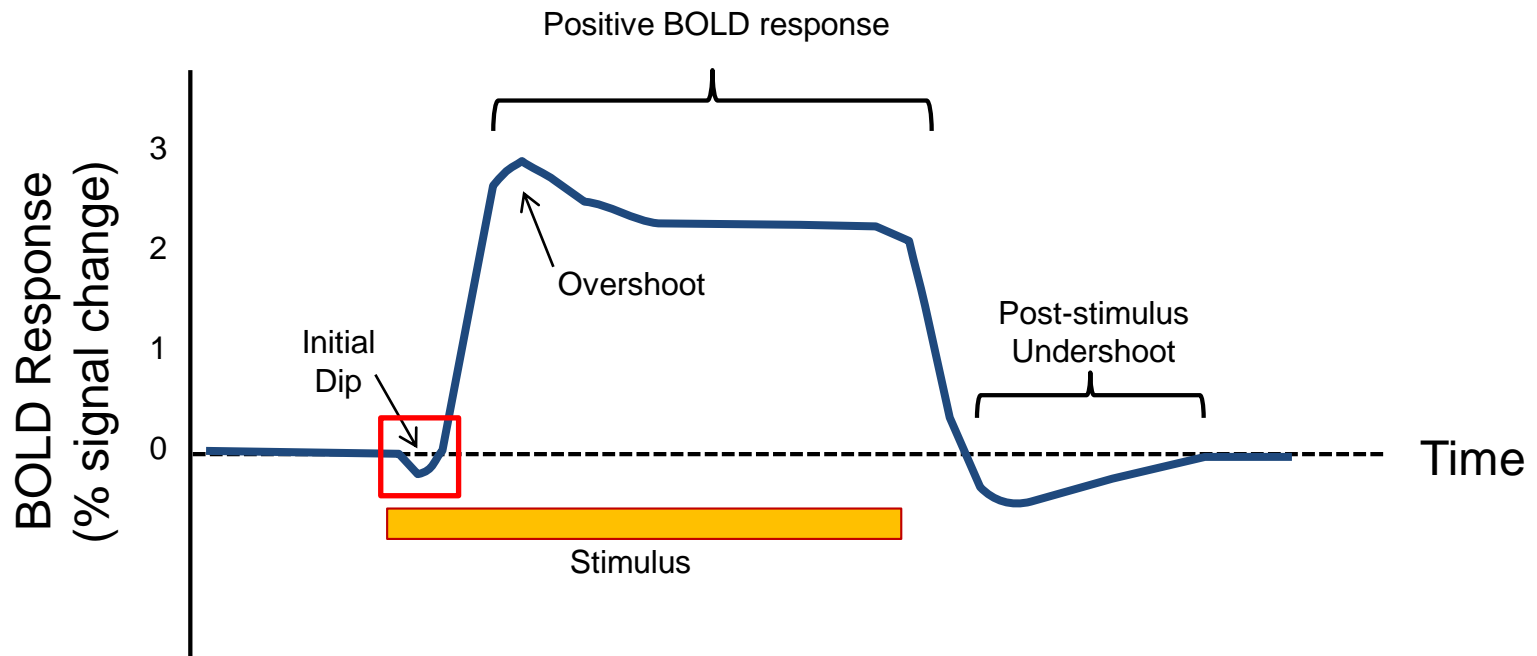
# How Does BOLD Relate to Neuronal Activity?

- Typical example of the time course of the MR signal change in response to an extended stimulus:



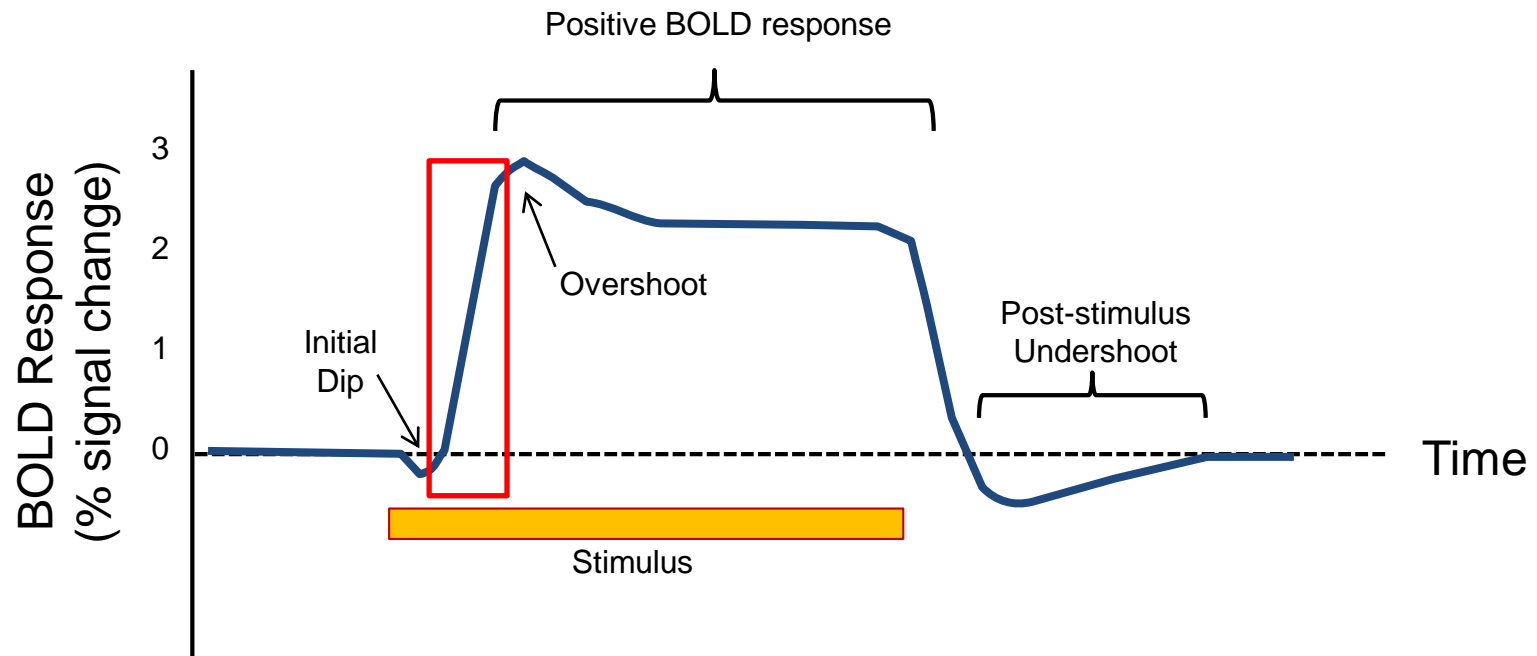
# Mechanism: Initial Dip

- Stimulus **increases neuronal activity**, which requires more oxygen and increases the amount of deoxygenated blood
  - Leads to a decrease in MR signal intensity
- **“Initial dip”** rather weak and not observed in all studies
  - If observed, often spatially more restricted than main peak



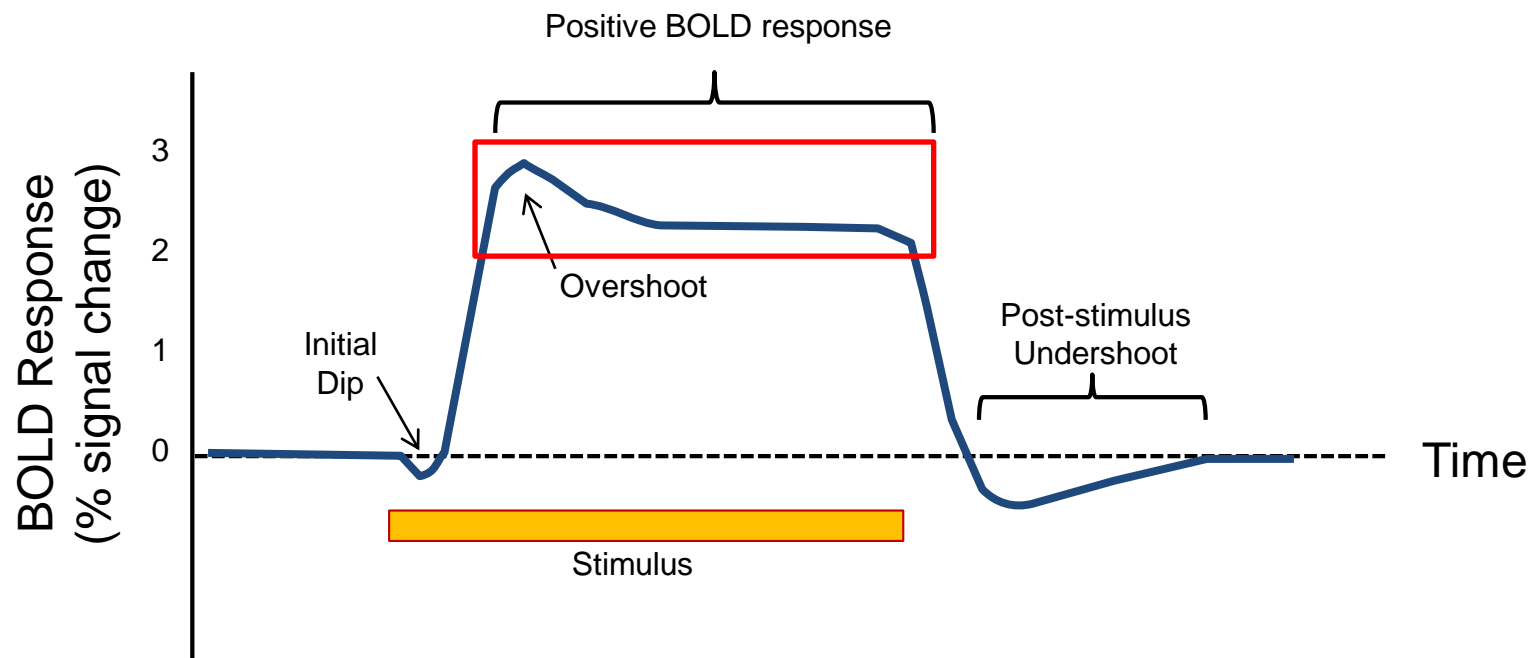
# Mechanism: Rise

- **Local blood flow increases**, providing more oxygenated blood and **overcompensating** the additional need from neural activity
  - Change in signal strength between 5% (primary sensory stimulation) and 0.1-0.5% (cognitive tasks)
  - Basis of most fMRI studies



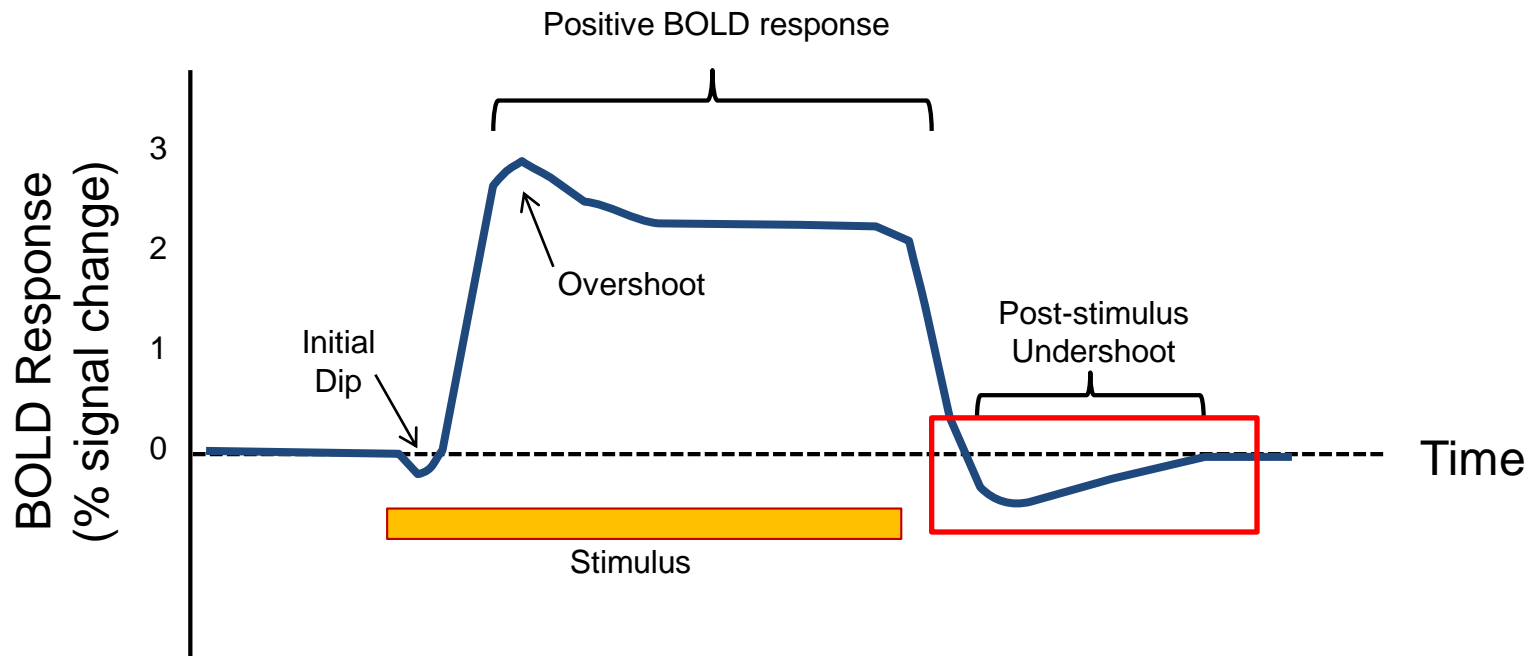
# Mechanism: Overshoot / Plateau

- While neuronal activity persists, increased oxygen uptake continues to be overcompensated
  - In “blocked” fMRI design with extended phases of stimulation, a plateau follows an initial overshoot



# Mechanism: Post-Stimulus Undershoot

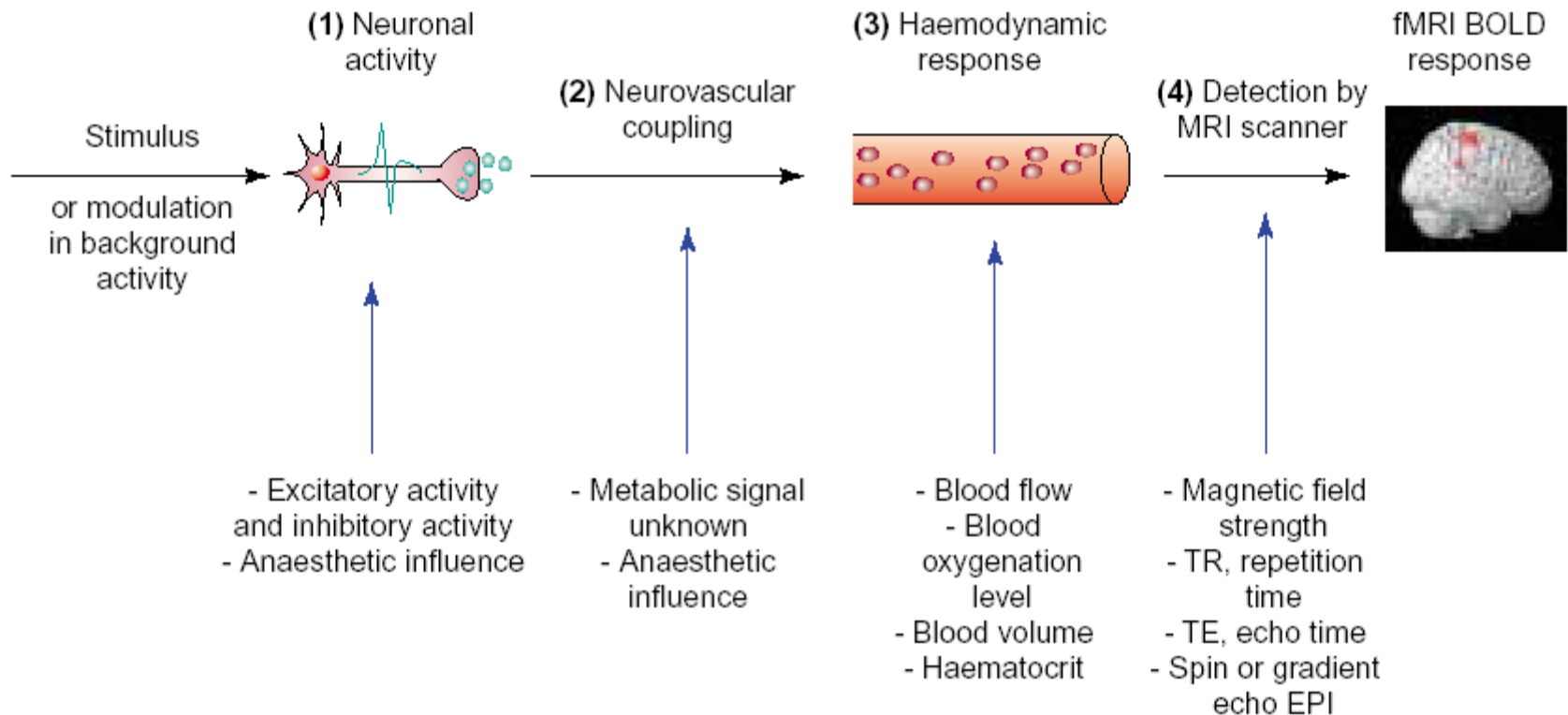
- Many studies observe a **post-stimulus undershoot** in MR intensity that often persists for tens of seconds
  - Mechanism still not fully agreed upon [van Zijl et al., 2012]
  - May indicate uncoupling of metabolic and blood flow response:
    - Blood flow returns to baseline earlier than need for oxygen
    - Much oxygen used while “cleaning up” after activity





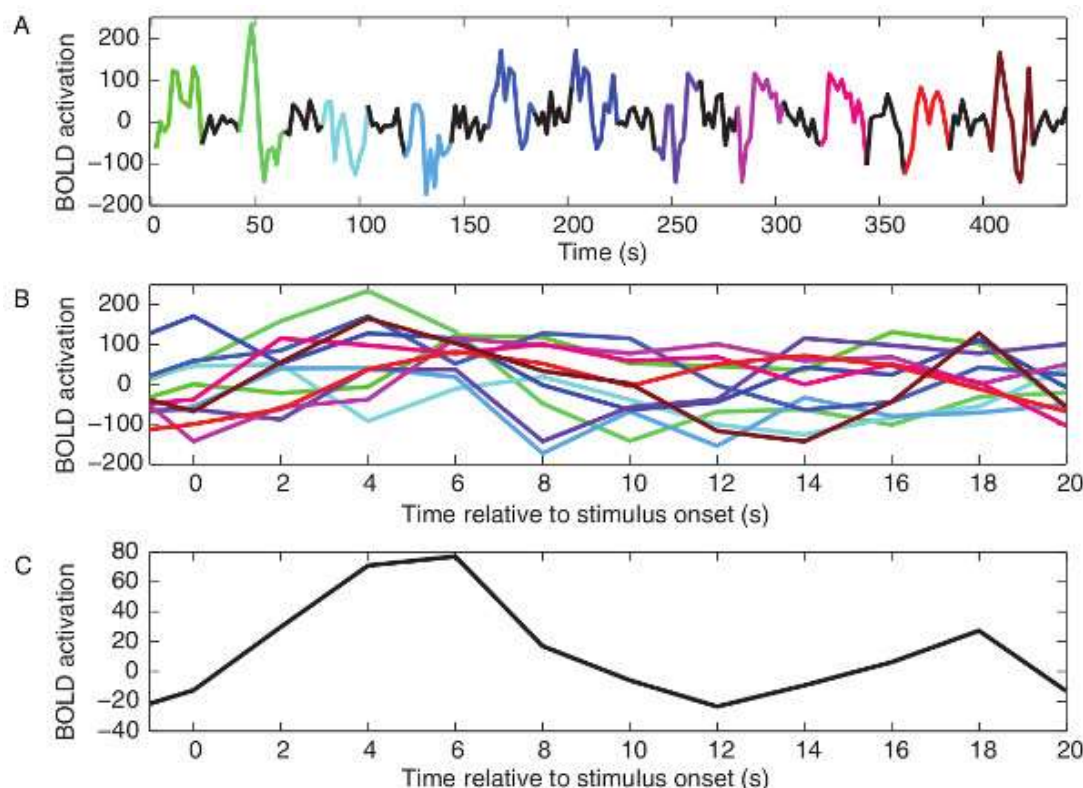
# Mechanism: Overview

- fMRI provides a rather **indirect measure** of neuronal activity

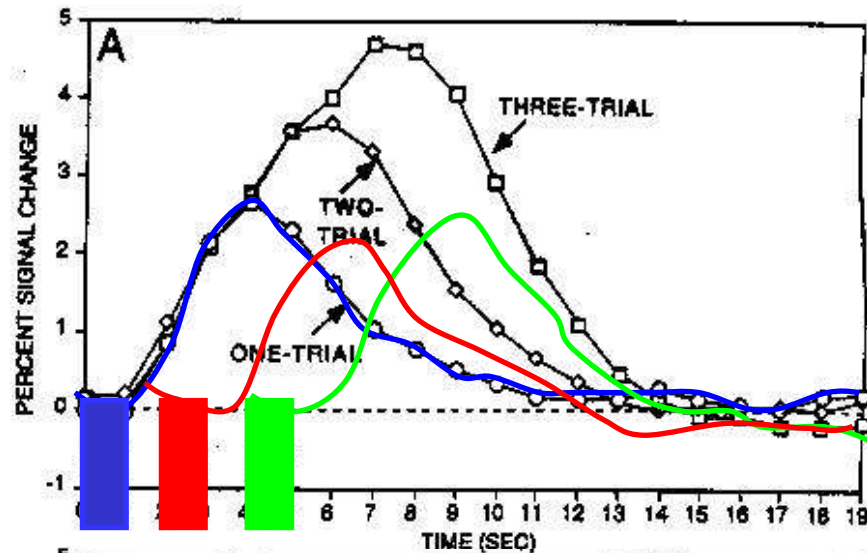


# Hemodynamic Response Function

- The **Hemodynamic Response Function (HRF)** describes the time course of MR signal change in response to a short stimulus
  - Can be estimated by repeatedly presenting a short stimulus and averaging the measurements

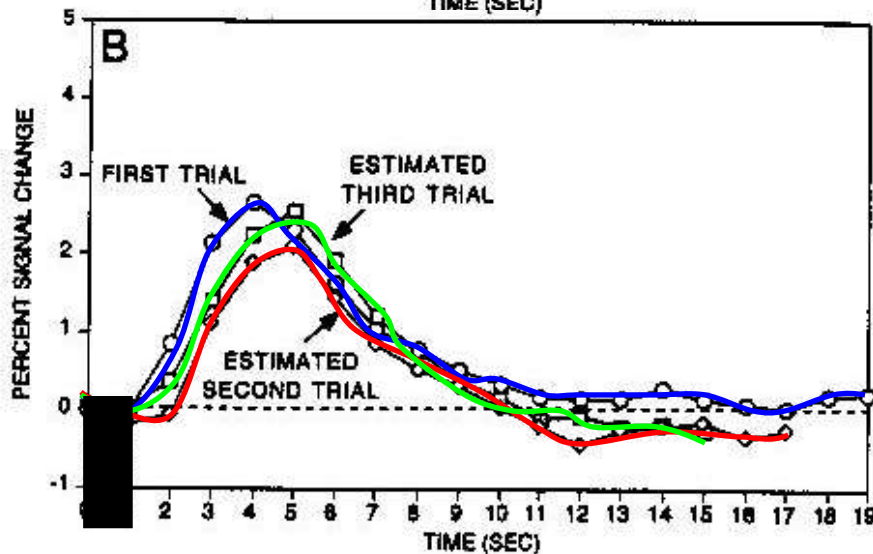


# Linearity of Hemodynamic Response



red = 2 - 1

green = 3 - 2



Sync each trial response to start of trial

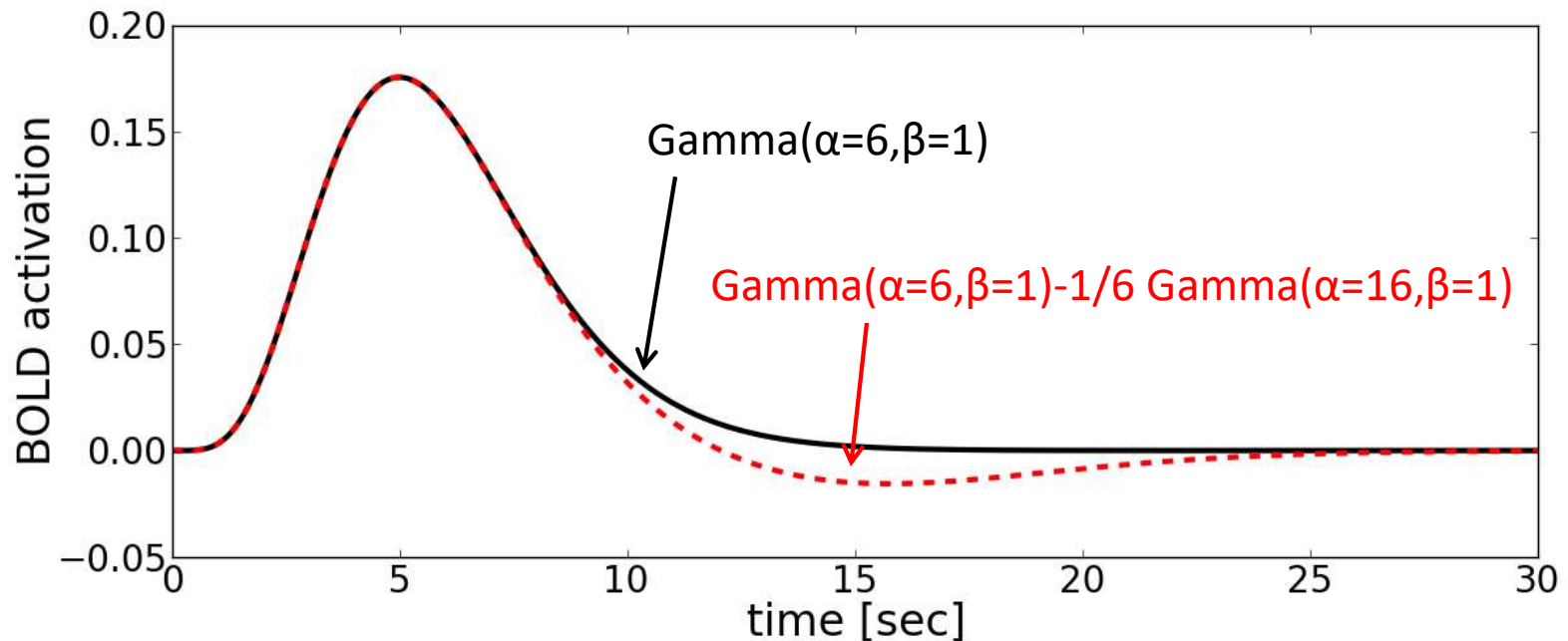
Not quite linear  
but good enough!

# Limits to Linearity of Hemodynamic Response

- Linearity is widely assumed in fMRI data analysis. Caveats of this assumption include:
  - **Spacing:** Responses to stimuli less than two seconds apart are slightly smaller than expected
  - **Duration:** Very short stimuli have a much larger response than would be expected from longer stimuli
    - Yesilyurt et al. 2008: Response to 5 ms visual stimulus half as large as to 1000 ms stimulus
- **Solution:** Quick succession of stimuli and ultra-short stimuli are rarely used in practice

# Canonical HRF

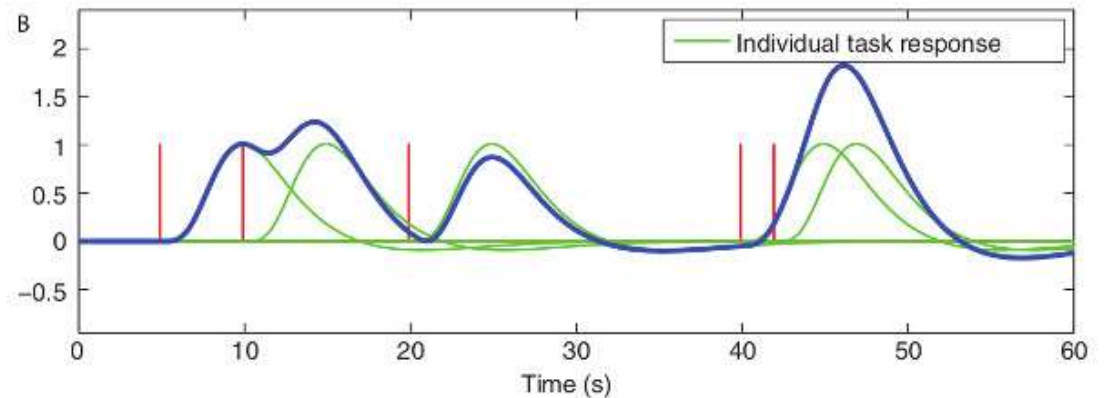
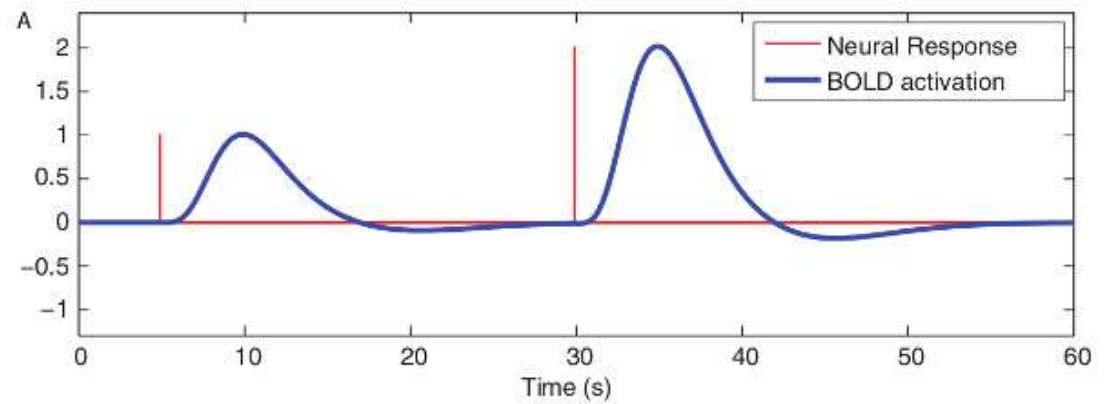
- It has been found empirically that the shape of the **Gamma distribution** (for certain parameters) closely resembles the HRF
  - Often used as a “**canonical HRF**” in analysis
  - Usually (imprecisely) called “**Gamma function**”
  - “**Double Gamma**” includes poststimulus undershoot



# Convolution

- If hemodynamic response is linear, the BOLD signal is given by convolving the neural response  $f$  with the HRF  $h$

$$(h * f)(t) = \int h(\tau)f(t - \tau)d\tau$$



# Summary: Functional MRI

- **BOLD effect:** MR intensity reduced by presence of deoxygenated blood
- Amount of deoxygenated blood changes during **neuronal activity**
  - After initial dip, additional need for oxygen is overcompensated, leading to a stronger signal
  - Mechanism still not known in detail, but found to correlate mostly with local field potentials
- **Observing signal change over time** allows us to draw conclusions about brain activity
  - Tradeoff between temporal and spatial resolution

## **9.4 General Linear Model (GLM)**



# Motivation: General Linear Model

- **General Linear Model** provides a unified framework for different statistical tests
  - Provides additional flexibility that we will need for fMRI analysis
  - Standard for statistical analysis in neuroimaging
  - Expresses vector  $\mathbf{y}$  of  $N$  measurements as the product of an  $N \times (p+1)$  design matrix  $\mathbf{X}$  and a vector  $\boldsymbol{\beta}$  of  $p+1$  parameters, plus an i.i.d. Gaussian noise vector  $\boldsymbol{\varepsilon}$ :

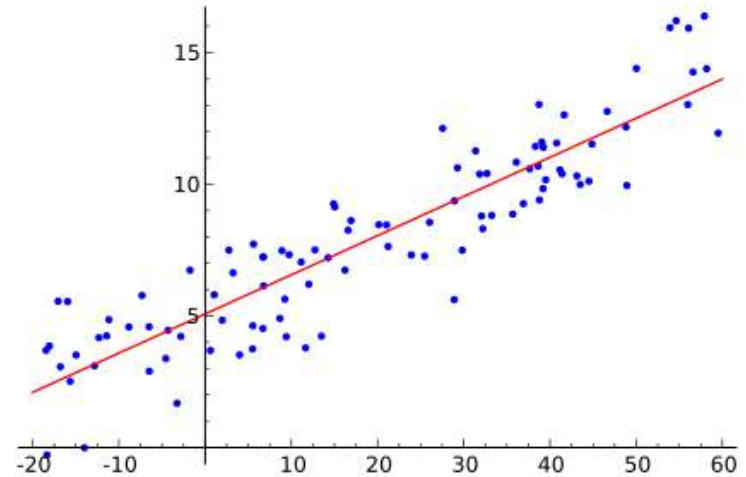
$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

# Linear Regression

- The GLM can be considered a generalization of simple linear regression:

$$\mathbf{y} = \beta_0 + \beta_1 \mathbf{x}_1 + \epsilon$$

↑      ↑  
intercept    slope



- Given data  $\{(x_{1,1}, y_1), \dots, (x_{1,N}, y_N)\}$ , the model can be written as  $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$  with  $\boldsymbol{\beta} = [\beta_0, \beta_1]^T$  and

$$\mathbf{X} = \begin{bmatrix} 1 & x_{1,1} \\ \vdots & \vdots \\ 1 & x_{1,N} \end{bmatrix}$$

# Multiple Linear Regression

- Linear regression with  $p > 1$  independent variables is called **multiple regression**:

$$y = \beta_0 + \beta_1 \mathbf{x}_1 + \cdots + \beta_p \mathbf{x}_p + \epsilon$$

- Same GLM  $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$  with longer  $\boldsymbol{\beta}$  vector
- Estimation of  $\boldsymbol{\beta}$ :
  - Since number of measurements typically exceeds  $p+1$ , we cannot simply invert  $\mathbf{X}$  to obtain  $\boldsymbol{\beta} = \mathbf{X}^{-1} \mathbf{y}$
  - Instead, multiply both sides by  $\mathbf{X}^T$  to obtain normal equations:  $\mathbf{X}^T \mathbf{y} = \mathbf{X}^T \mathbf{X} \boldsymbol{\beta}$
  - If  $\mathbf{X}$  has *full column rank*, we can solve for

$$\boldsymbol{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$$

# Two-Sample t-Test in the GLM

- *Example:* Two-sample t test (chess player vs. non-chess player) in the GLM framework:  $\boldsymbol{\beta} = [\mu^C, \mu^{NC}]^\top$

$$\begin{pmatrix} v_1^C \\ \vdots \\ v_n^C \\ v_1^{NC} \\ \vdots \\ v_n^{NC} \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{bmatrix} \boldsymbol{\beta} + \boldsymbol{\epsilon}$$

- **Null hypothesis**  $\beta_1 = \beta_2$  can be written in the general form  $\mathbf{c}\boldsymbol{\beta} = 0$  using the (row) contrast vector  $\mathbf{c} = (1 \ -1)$
- For any  $\mathbf{c}$ , the  $t$  score (with  $\nu = N - (p + 1)$ ) can be shown to be

$$t = \frac{\mathbf{c}\boldsymbol{\beta}}{\sqrt{\mathbf{c}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{c}^T\sigma^2}} \quad \text{with} \quad \sigma^2 = \frac{\boldsymbol{\epsilon}^T\boldsymbol{\epsilon}}{N-(p+1)}$$

# Regressing Out Nuisance Variables

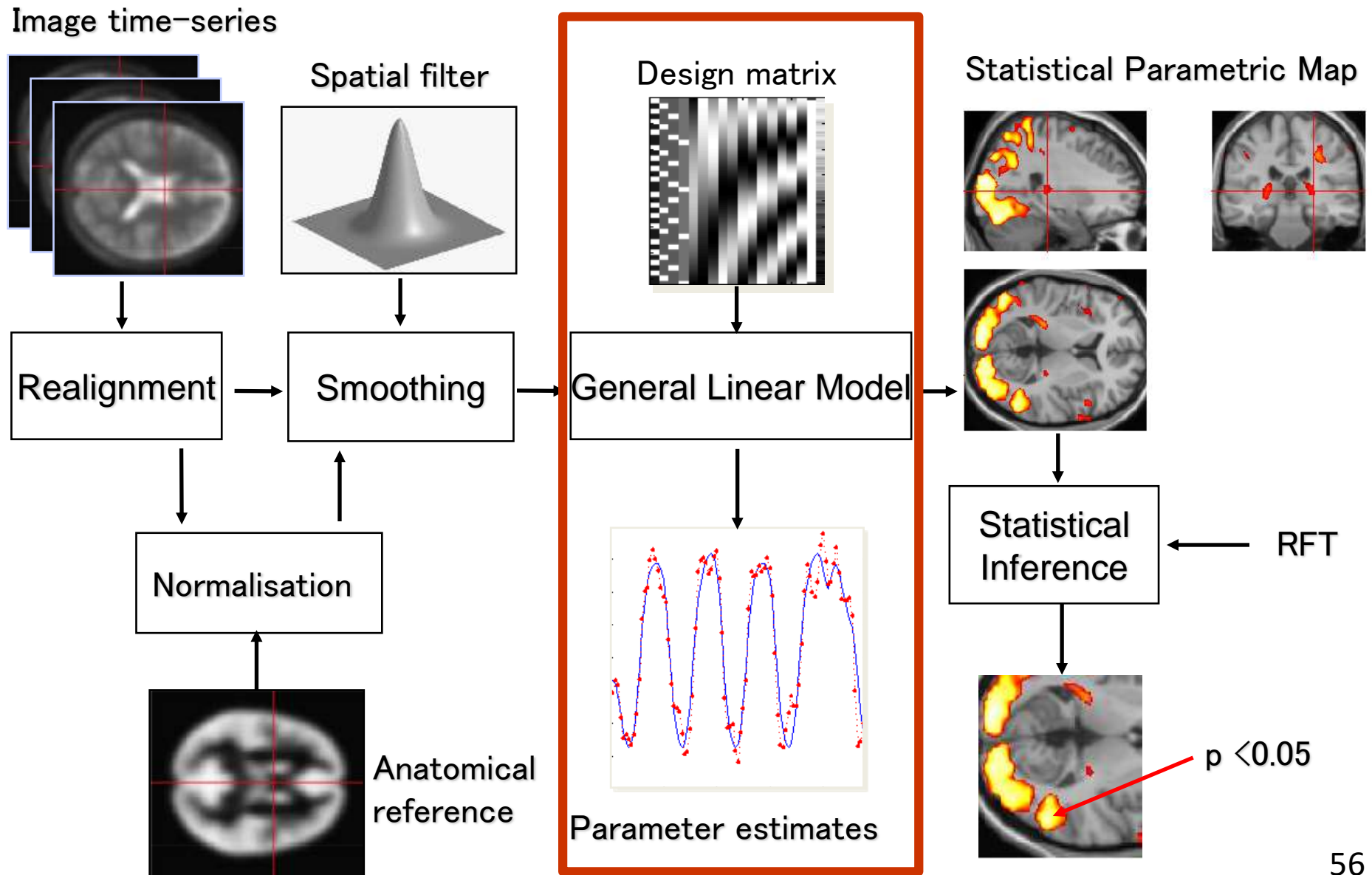
- **Additional benefit of GLM:**
  - Can insert  $\beta$ s which are ignored by the contrast to “regress out” nuisance variables of no interest
  - *Example:* Our subject pool is not age-matched. Inserting a  $\beta$  for the effect of age (corresponding to a column of ages in **X**) implies an “age-correction”
    - *Caveat:* If you model the mean of a group and include a regressor for age, your estimated mean will be at age zero, unless you mean-center the age column!
  - *Limitation:* Assumes that effect of age is linear. It’s still preferable to use matched subjects.
  - *Caveat:* In permutation-based testing, nuisance variables are *not* interchangeable under the null
    - Often regressed out separately before permutation

# Summary: General Linear Model

- The General Linear Model  $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$  unifies all statistical tests that are widely used in neuroimaging
  - Additional flexibility will be used in next section
  - Specification using design matrix  $\mathbf{X}$  and contrast vector  $\mathbf{c}$  or contrast matrix  $\mathbf{C}$
  - Can be used to compute  $t$  scores,  $F$  scores, or combined with permutation-based testing
  - Allows us to “regress out” variables of no interest

## **9.5 Statistical Analysis of fMRI Data**

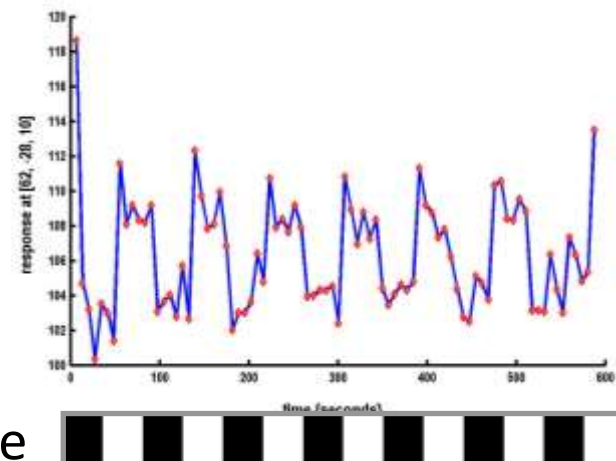
# Overview: fMRI Analysis





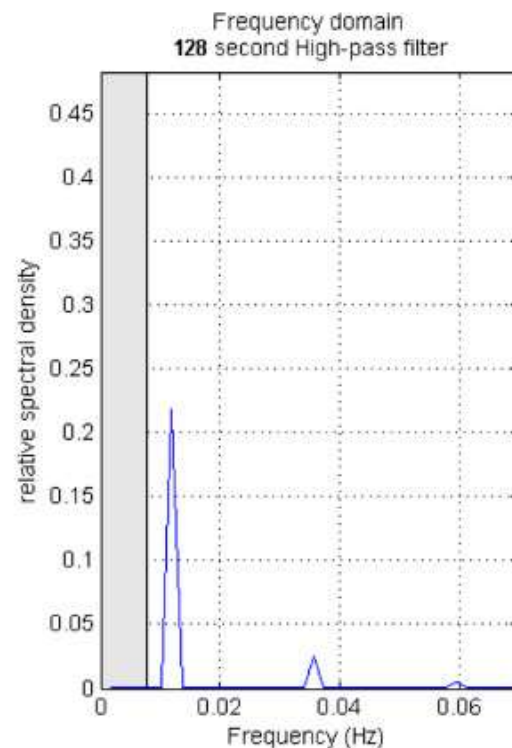
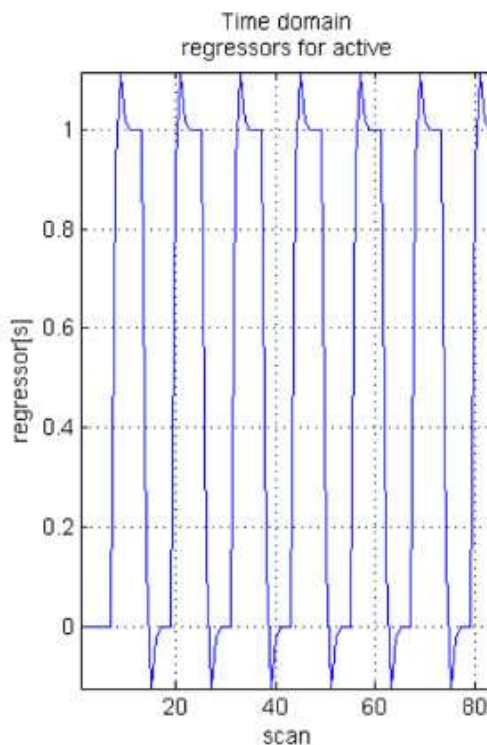
# Example: Auditory Stimulus

- **Early proof-of-concept experiment** performed at Functional Imaging Laboratory, University College London (“mother of all experiments”)
  - Full-brain EPI scan with 64x64x64 voxels (3x3x3 mm<sup>3</sup> resolution), repeated at TR=7 sec
  - Alternating between 42 sec blocks of rest and auditory stimulation (spoken words)
  - 96 acquisitions overall
    - Initial 12 discarded to reach steady state
  - Data publicly available:
    - <http://www.fil.ion.ucl.ac.uk/spm/data/auditory>



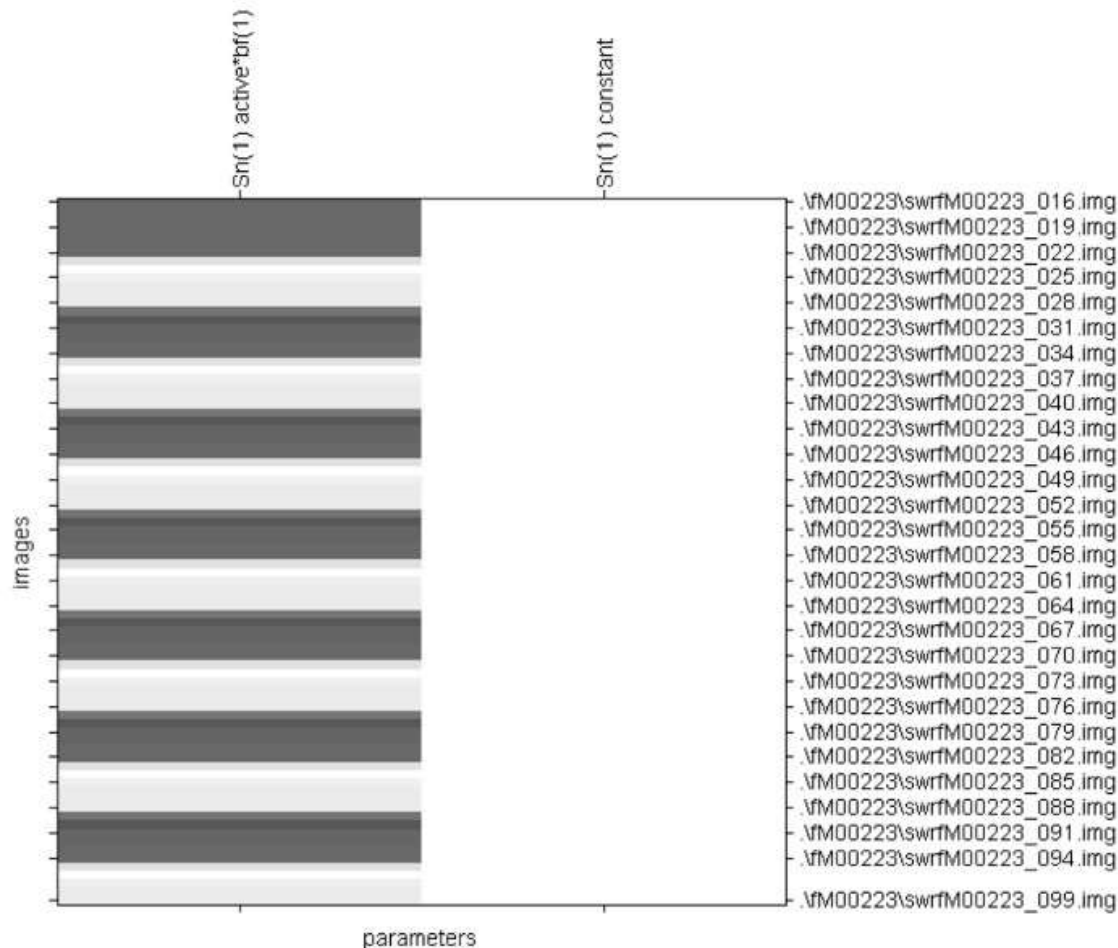
# Statistical Modeling

- In principle, we could use a mass-univariate **two-sample t-test** to compare MR images during stimulation to images at rest
- **But:** That would ignore HRF
  - Convolver stimulus with HRF predicts signal (“regressor”)
  - Use regressor as a column in General Linear Model



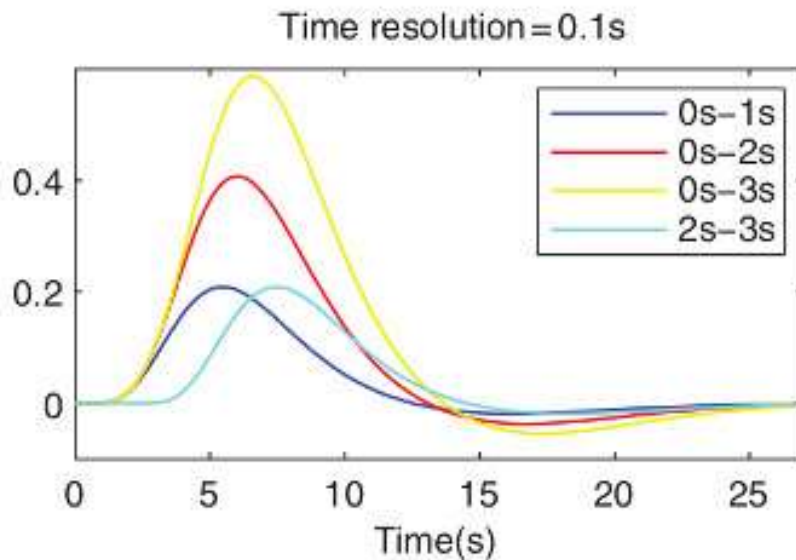
# A Typical Design Matrix

- It is common to visually inspect the design matrix for the General Linear Model:

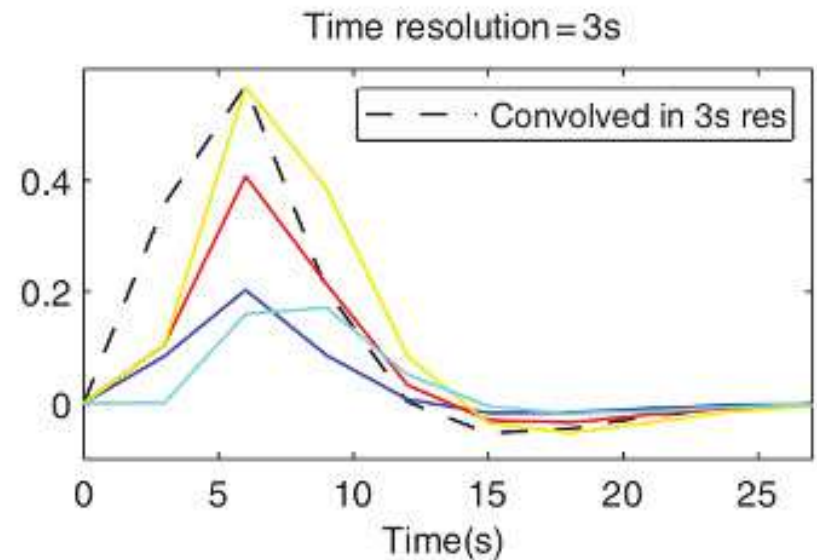


# Perform Convolution with High Resolution!

- At small temporal resolution (e.g.,  $TR=3$  s), short stimuli are undersampled ☹️
- Convolution with HRF smoothes out stimulus 😊
- But: Have to perform convolution in high temporal resolution to avoid aliasing



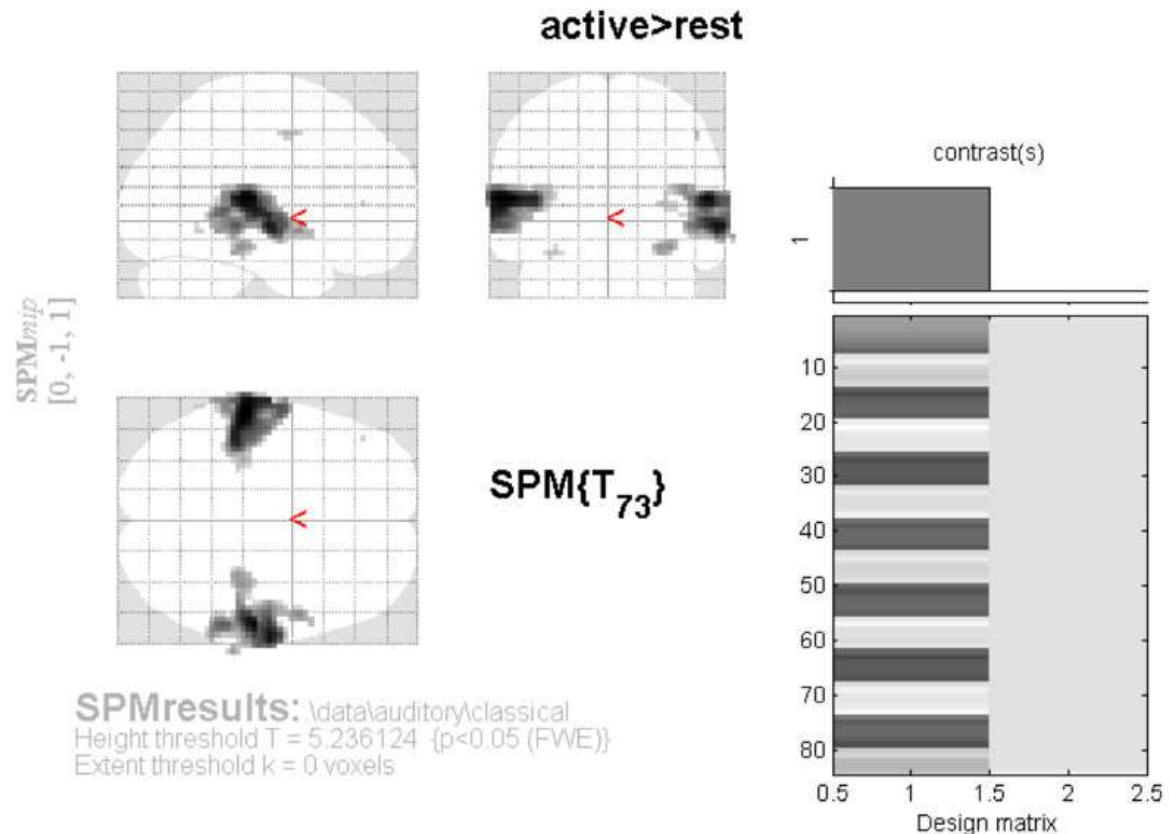
Four different stimuli  
convolved with HRF at 0.1s res



Subsampled results (color) vs.  
convolution at 3s res

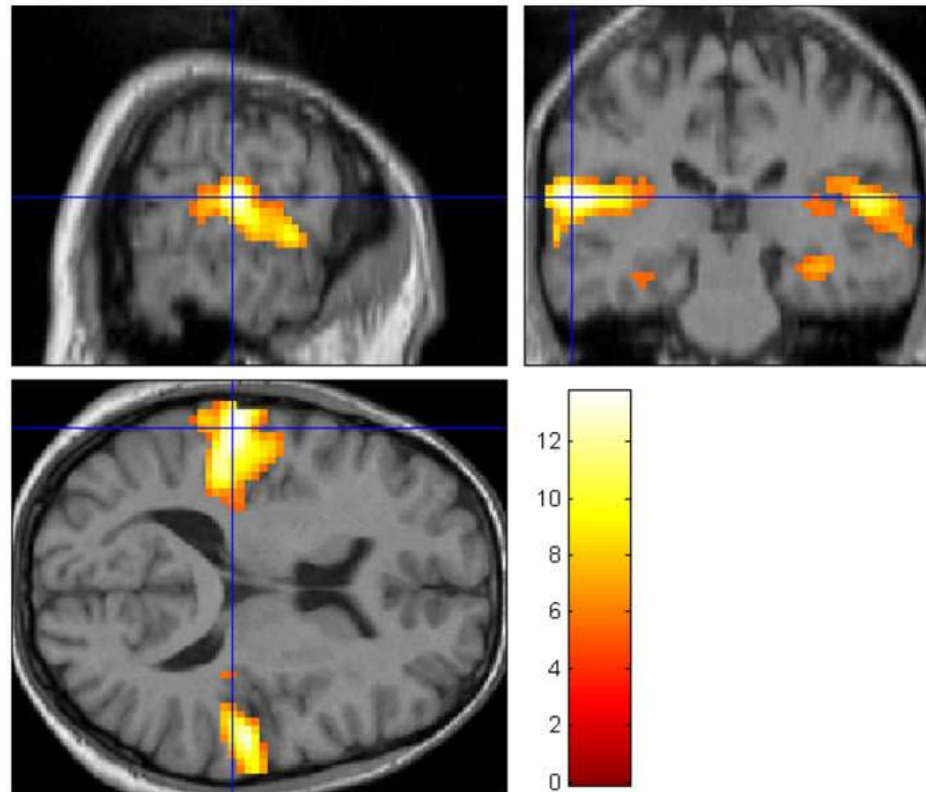
# Maximum Intensity Projection

- Perform statistical testing as in voxel-based morphometry
- **Maximum Intensity Projection:**
  - 2D map showing maximum across third dimension



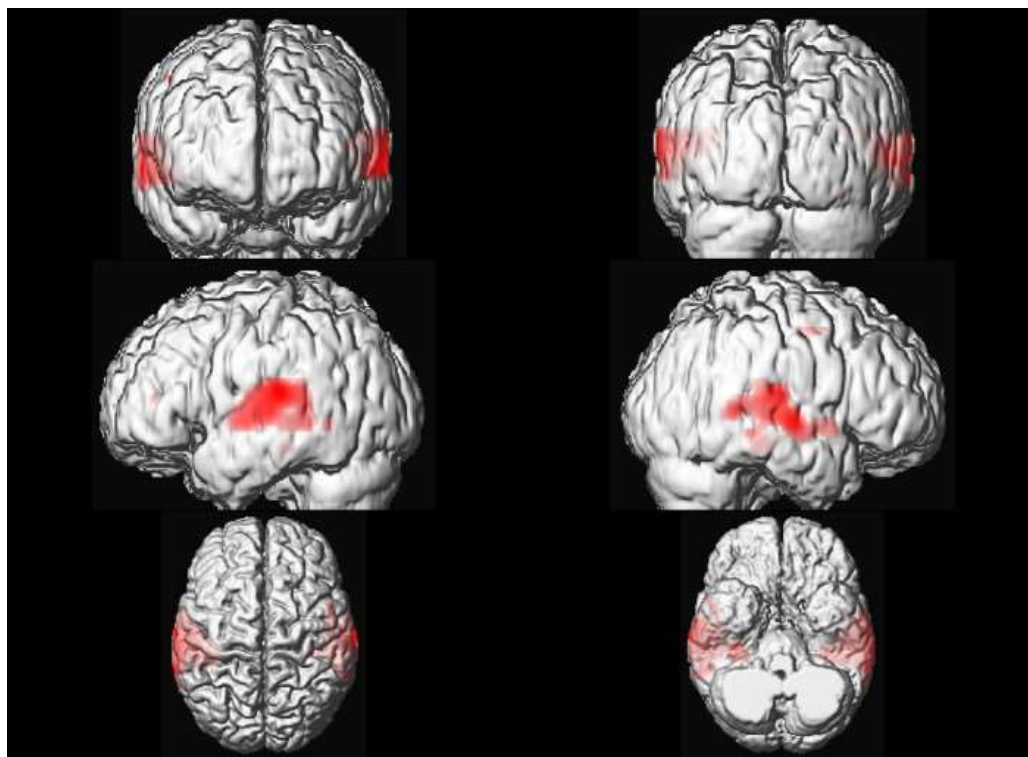
# Overlay after Coregistration

- After coregistration of functional and high-resolution structural data, activations can be overlaid



# Overlay after Normalization

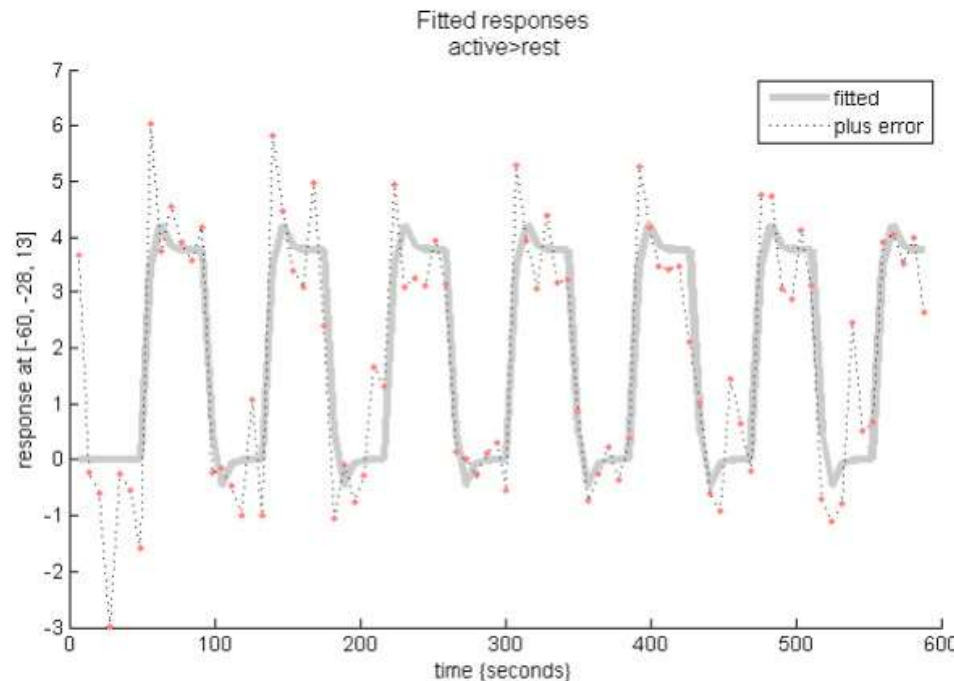
- After non-linear normalization, activations can be overlaid on a generic brain template
  - **Two-step:**  
Functional to structural scan of same subject, structural scan to template
  - Can use **generic models of gray matter surface**
    - Ignores variability in brain anatomy





# Inspecting Individual Time Series

- Selecting individual voxels (“voxel surfing”) allows us to compare their **individual time series** with the model fit
  - Check for substantial model misspecification



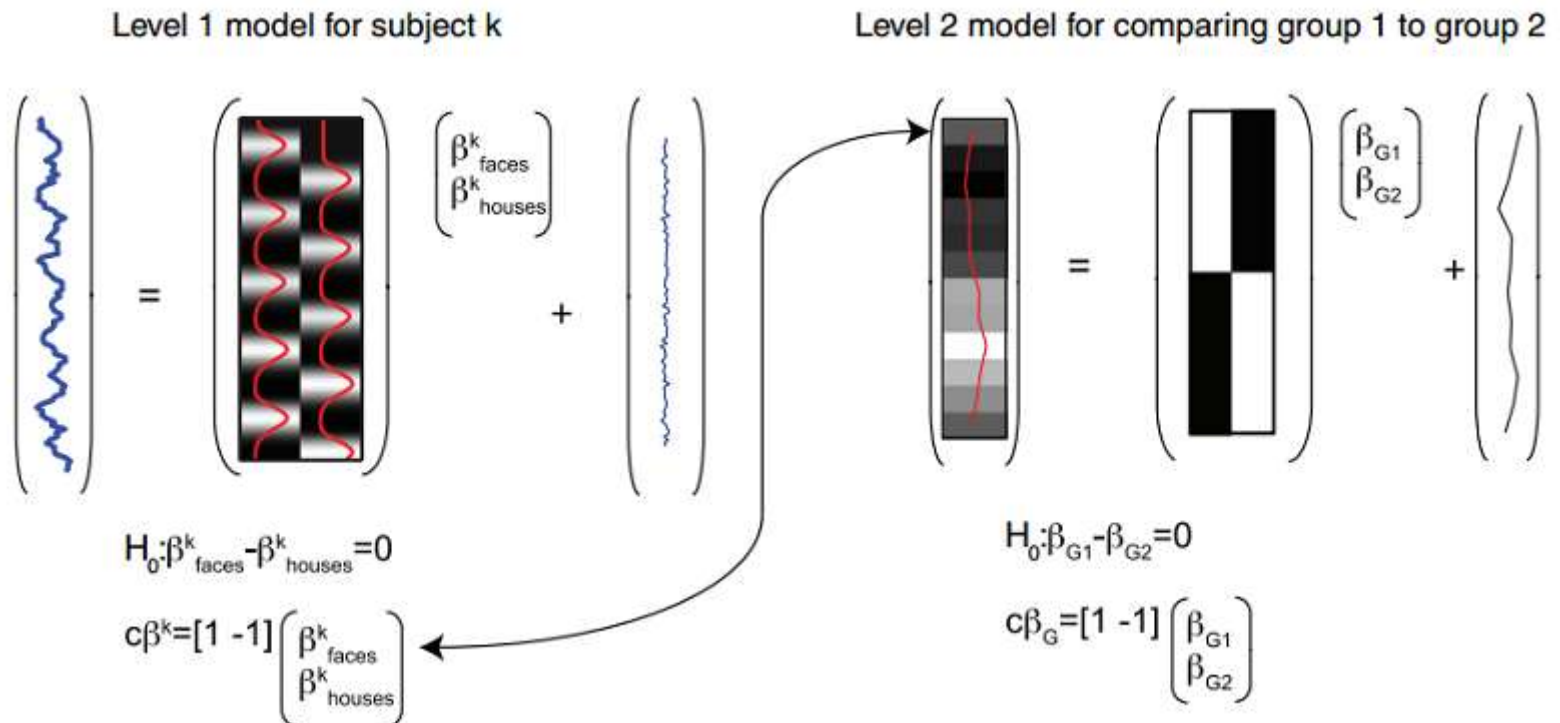


# fMRI Group Analysis

- How to combine fMRI data **from several subjects** to answer questions such as „is activation stronger in women than in men“?
- Tempting to simply normalize and temporally concatenate all scans
  - Ignores the fact that the subjects themselves are **samples from a larger population**
    - Fails to distinguish variances and degrees of freedom w.r.t. number of fMRI time steps vs. number of subjects
  - Statistical results will **not generalize** to the wider population!
  - fMRI literature calls this „fixed“ vs. „mixed“ effects

# Two-Level Analysis

- „Second-level“ models are commonly estimated in two steps, each of them using a GLM:
  1. Estimate activation per-subject
  2. Compare contrasts between groups



# Model Unequal Within-Subject Variance?

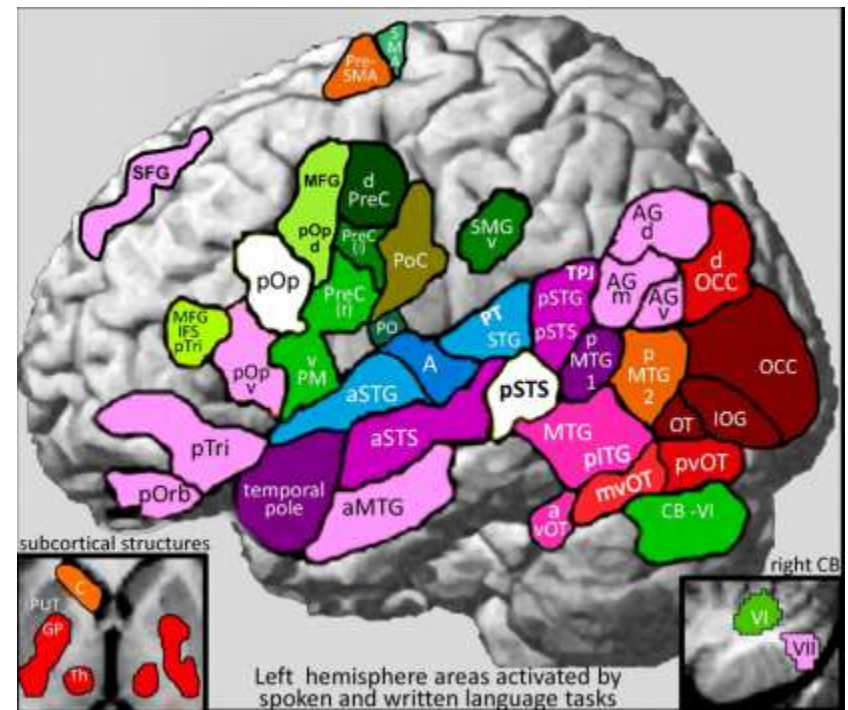
- In case variances differ between subjects, we need to perform a **weighted least squares fit** for the second-level GLM:

$$\boldsymbol{\beta} = (\mathbf{X}^T \boldsymbol{\Sigma}^{-1} \mathbf{X})^{-1} \mathbf{X}^T \boldsymbol{\Sigma}^{-1} \mathbf{y}$$

- Reduces the impact of highly uncertain subjects
- Some software packages **do not propagate** individual within-subject variances to the second level
  - Estimates of variances that have been obtained from few samples are themselves quite uncertain!

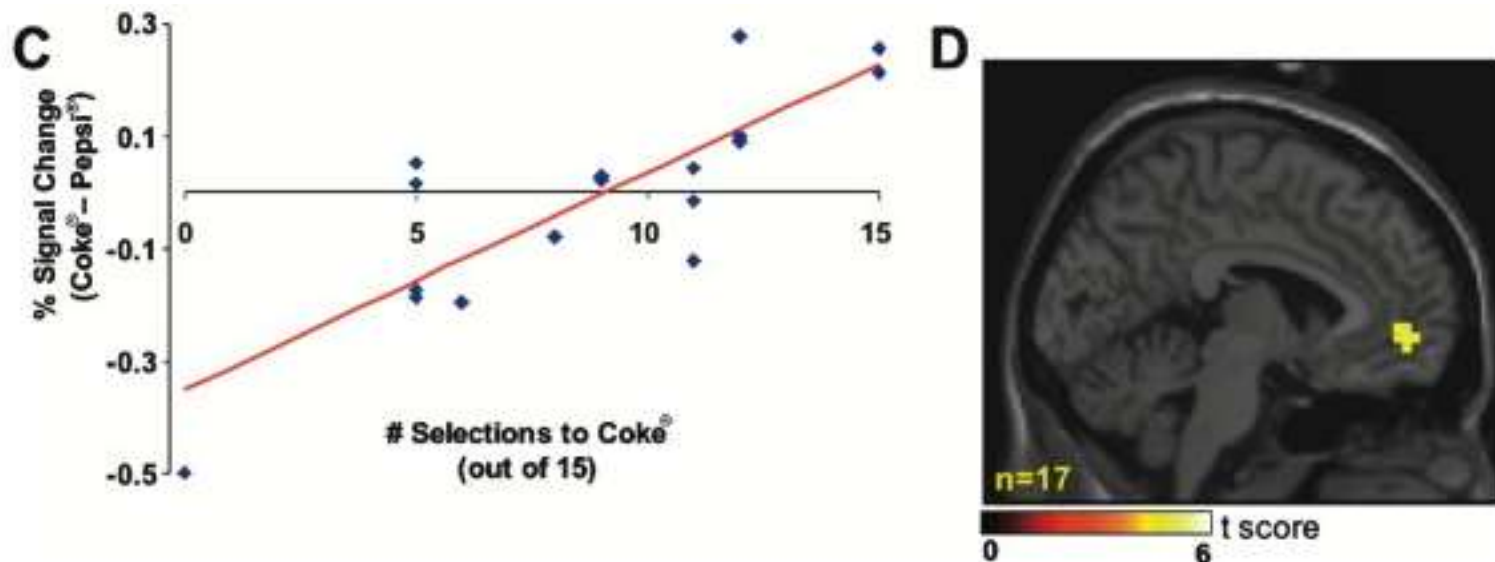
# fMRI Example: Speech

- **Price 2012** summarizes hundreds of studies that provide converging evidence on which brain regions are involved in hearing speech, producing speech, or reading
  - Reading meaningful vs. meaningless words, generating vs. repeating words, processing simple vs. complex grammar, etc.
  - Concludes that next big challenge is to understand how brain regions interact



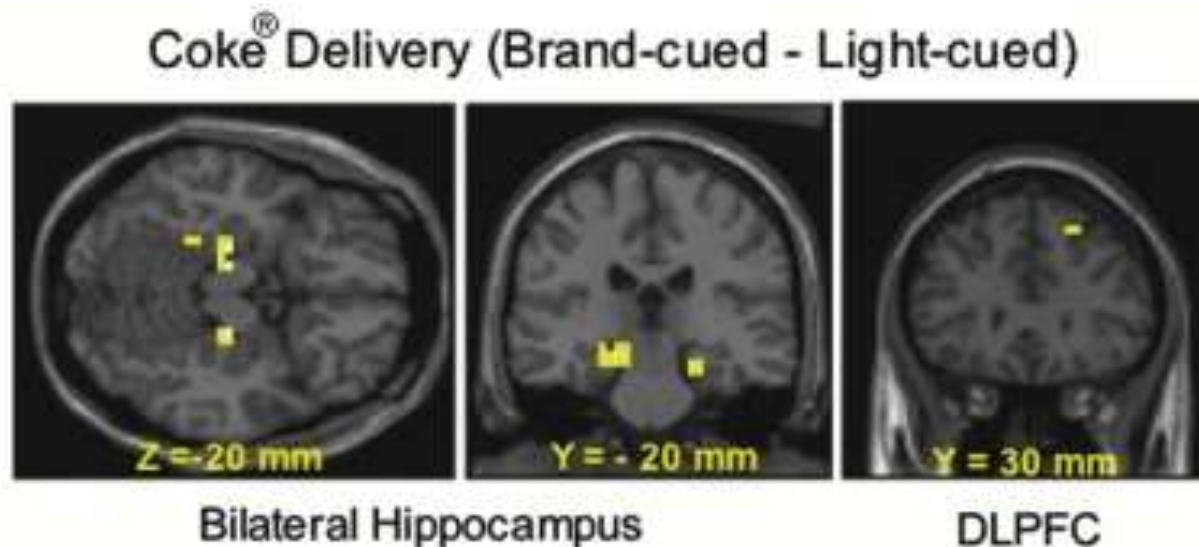
# fMRI Headline: Coke vs. Pepsi

- **McClure et al. 2004** report results on behavioral preference and fMRI response to cola with and without knowing the brand
  - People split evenly between Coke vs. Pepsi in a blind taste test
  - Prefrontal activity correlated with preference



# fMRI Headline: Coke vs. Pepsi

- Most subjects preferred labeled cup of coke over unlabeled cup of coke (despite balanced distribution during blind taste test)
- Brand-cued vs. light-cued delivery of coke led to activations in several other brain regions
  - No such effects for Pepsi



# Summary: Statistical fMRI Analysis

- After applying pre-processing...
  - Correlate observed with predicted BOLD response using the General Linear Model
  - Everything you learned about family-wise error correction applies!
  - Group analysis done in two steps to correctly account for mixed effects
- For visual presentation...
  - slice through data or present MIP
  - overlay on top of anatomical image

## **9.6 Voxel-Based Morphometry (VBM)**



# Goal of Voxel-Based Morphometry

- We just learned how to perform mass univariate tests, but how to derive images for which it makes sense to apply them?
- **Voxel-Based Morphometry** is one option
  - **Goal:** Would like to compare the size of specific brain areas between subjects
  - **But:** Segmenting specific regions of interest is difficult and limits our analysis to those (few) regions
  - **Idea:** Normalize subjects so that anatomical structures are aligned and compare regional gray matter volume *at each point of the brain*

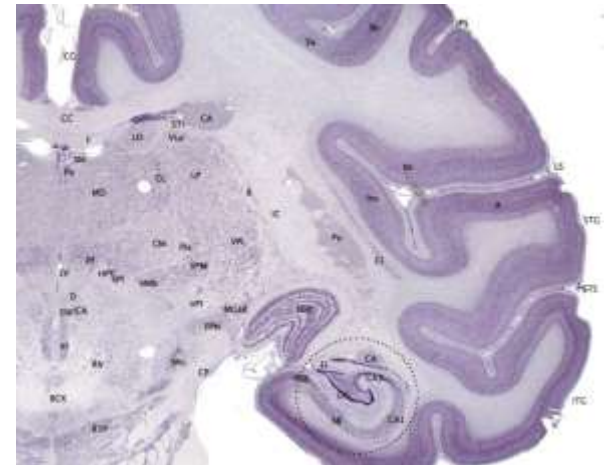
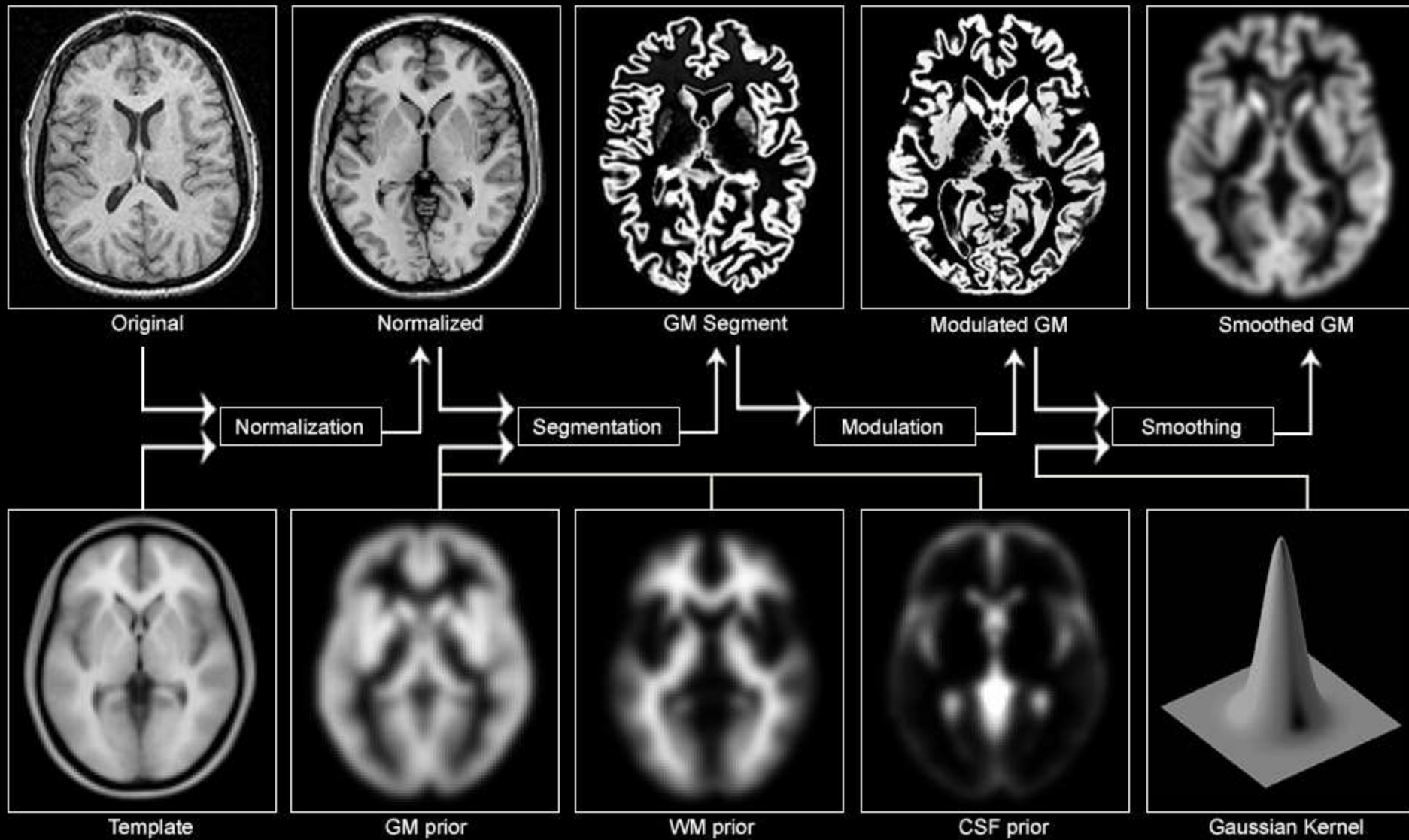


Image Source: brainmaps.org

# Voxel-Based Morphometry

## Pre-processing Overview



# Pre-Processing in VBM

## 1. Normalization

- Rigid, then affine, then non-linear registration

## 2. Segmentation

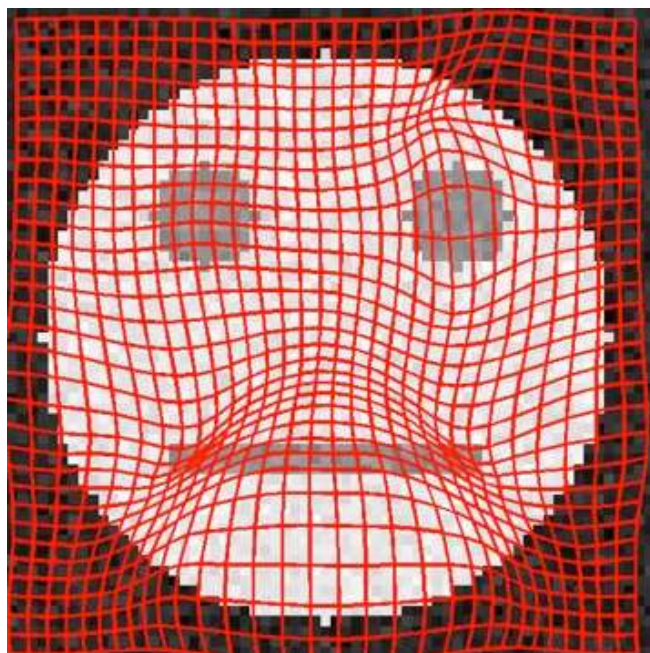
- Tissue classification into GM / WM / CSF
- MRF to deal with noise and bias fields

- **Optimized VBM:**

- Performs automated brain extraction first
- Builds a study-specific atlas
- Achieves improved gray matter alignment by segmenting first, normalizing gray matter maps, applying transformation to original image, and segmenting again (including prior maps)

# Modulation

- Desired interpretation is in terms of **gray matter volume**, but nonlinear warping distorts volumes
- **Modulation** corrects for this by multiplying normalized gray matter map with the Jacobian determinant



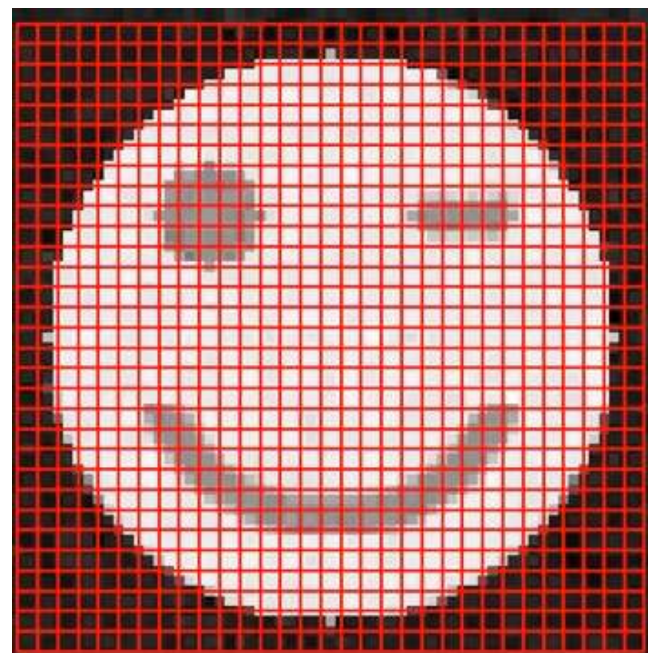
Individual Subject



Image  
warp



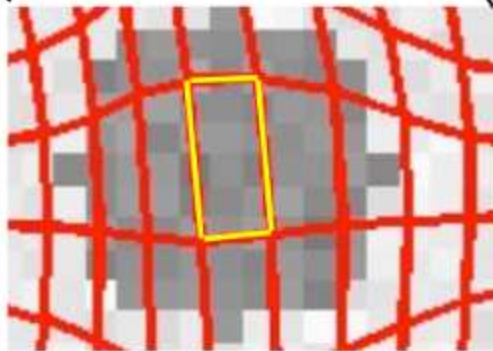
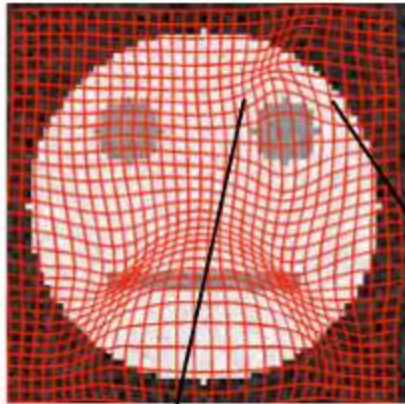
Grid  
warp



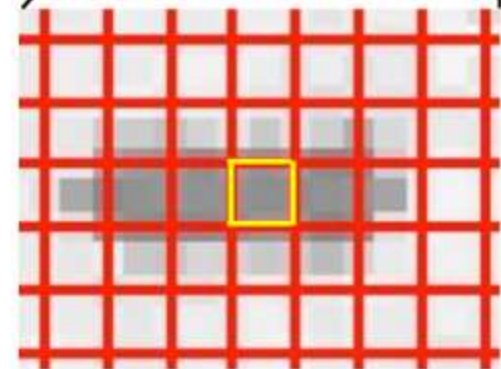
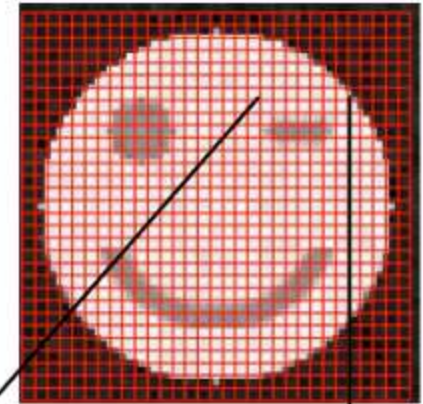
Normal Space



# Illustration: Contraction



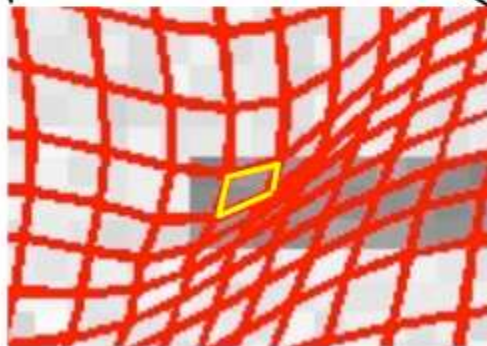
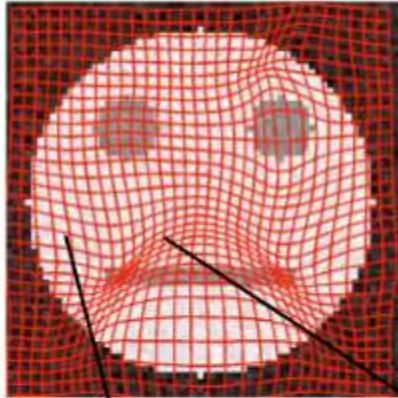
$\sim 3\text{mm}^2$  in original space



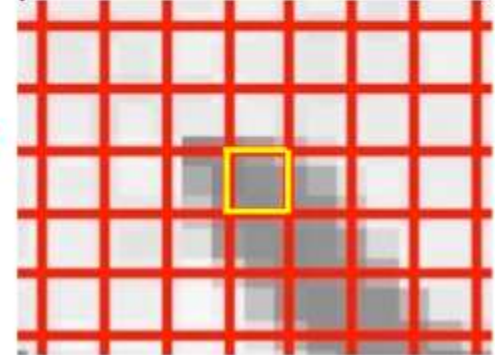
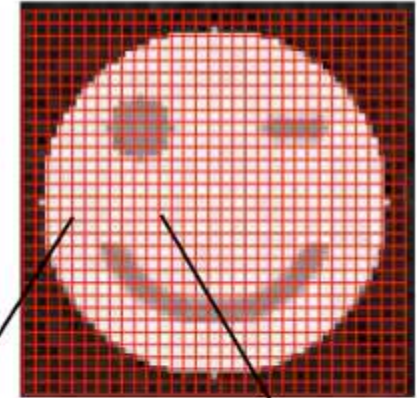
$1\text{mm}^2$  in warped space

Jacobian  $\sim 3$

# Illustration: Expansion



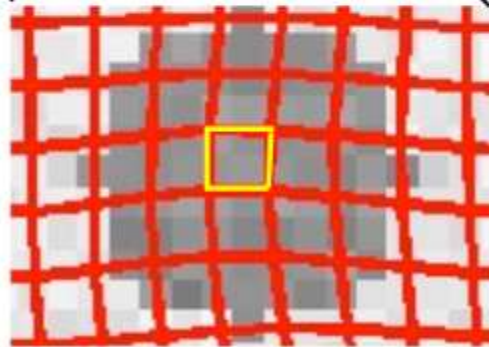
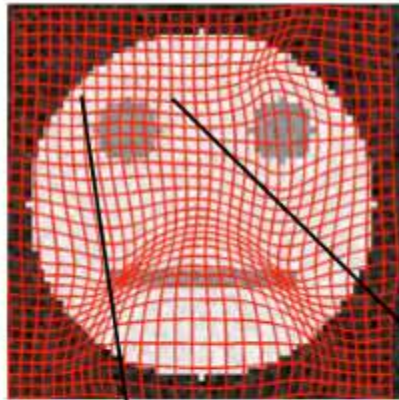
$\sim 1/3 \text{mm}^2$  in original space



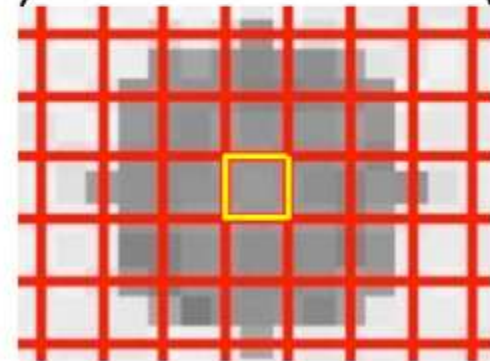
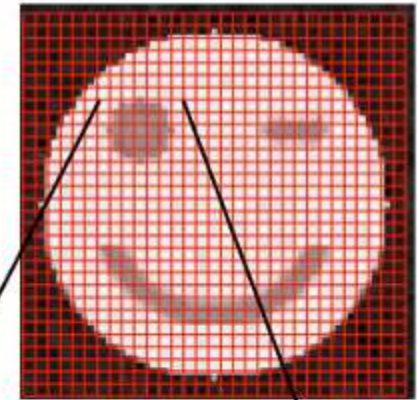
$1 \text{mm}^2$  in warped space

Jacobian  $\sim 1/3$

# Illustration: Volume Preservation



$\sim 1\text{mm}^2$  in original space

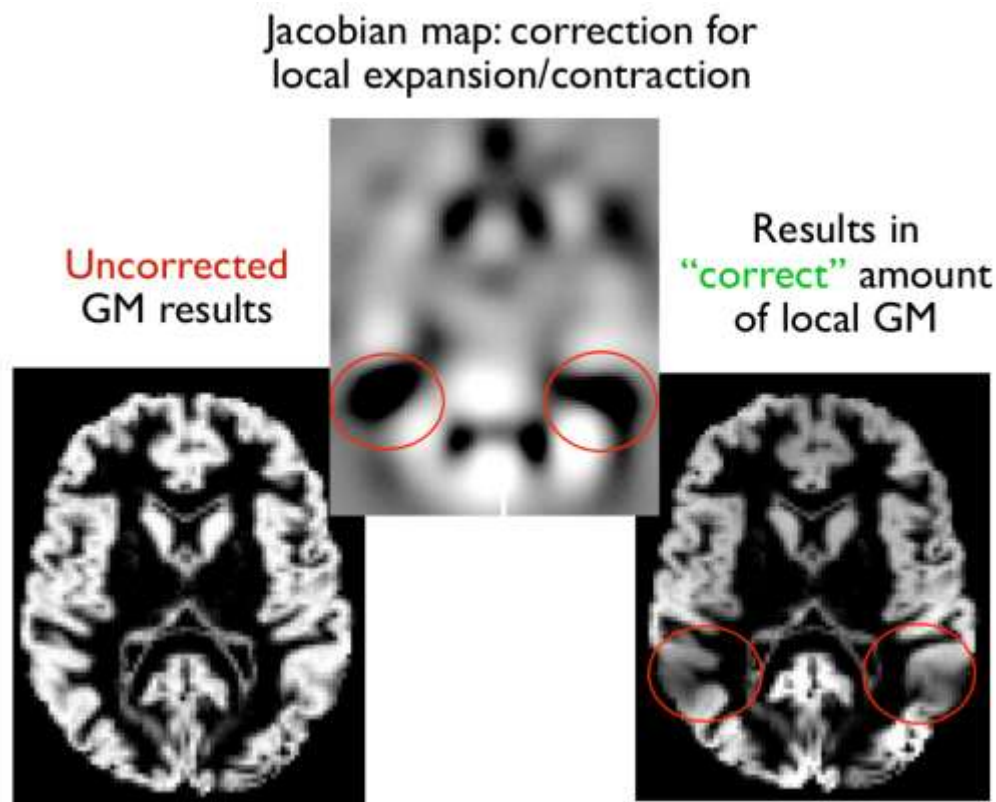


$1\text{mm}^2$  in warped space

Jacobian  $\sim 1$



# Applying the Modulation

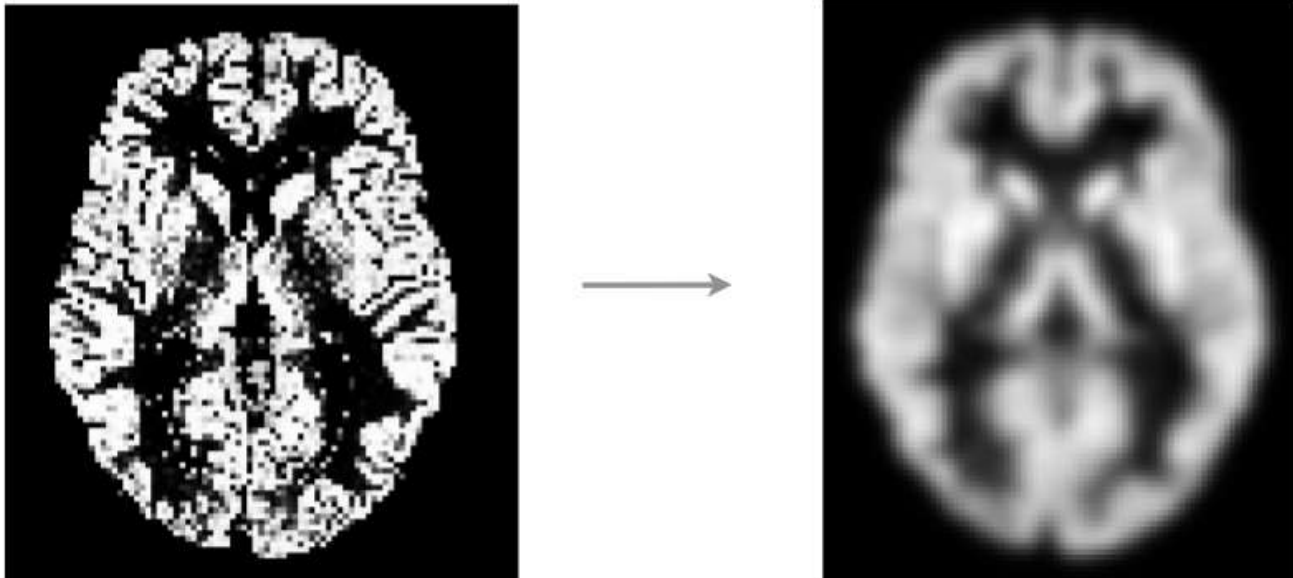


- Sometimes, modulation is omitted and results are interpreted as “gray matter density” (ratio of gray matter vs. white matter and CSF)
  - Result will depend strongly on flexibility of normalization!



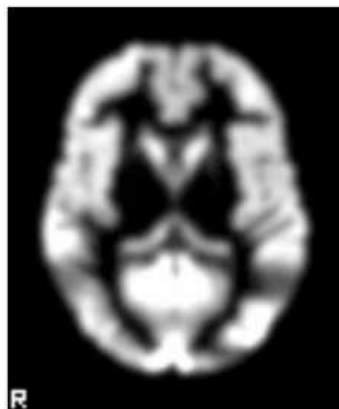
# Smoothing

- Apply some amount of **Gaussian smoothing**
  - Interpretation as “regional” (rather than strictly local) gray matter volume / density
  - Compensates for slight inaccuracies in normalization
  - Makes the data more normally distributed (central limit theorem)

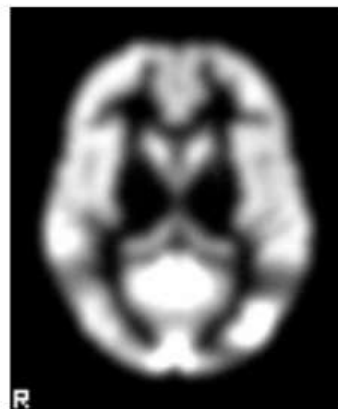
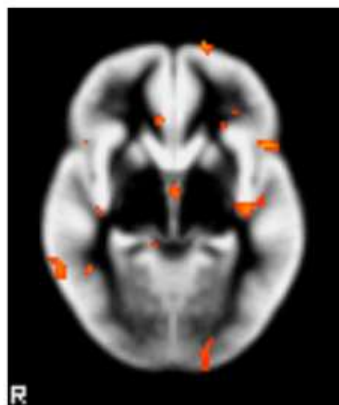


# How Much Smoothing?

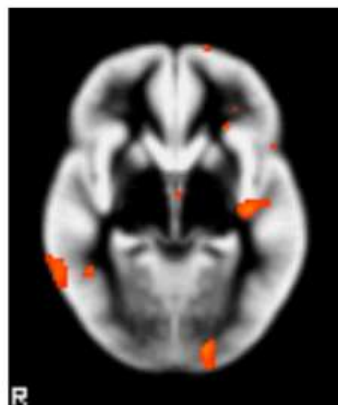
- Extent of smoothing will influence result; ideally, bandwidth should be tuned to expected size of effect



smooth=5mm

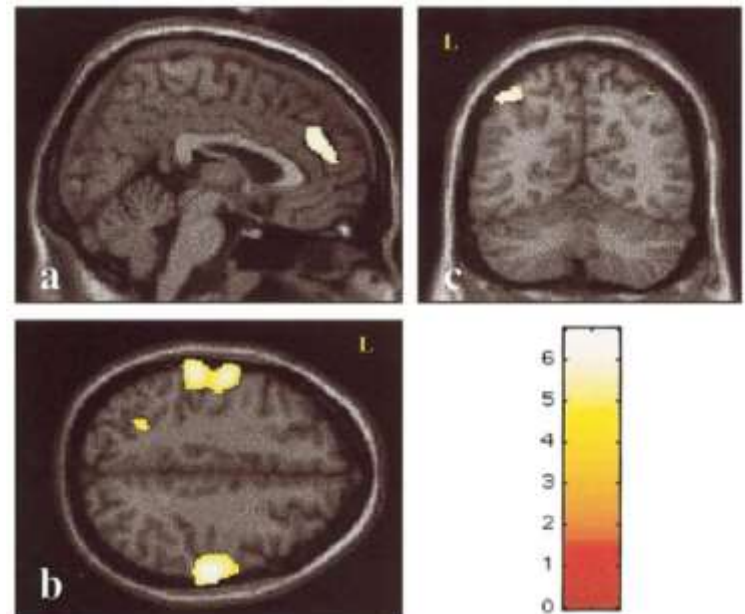
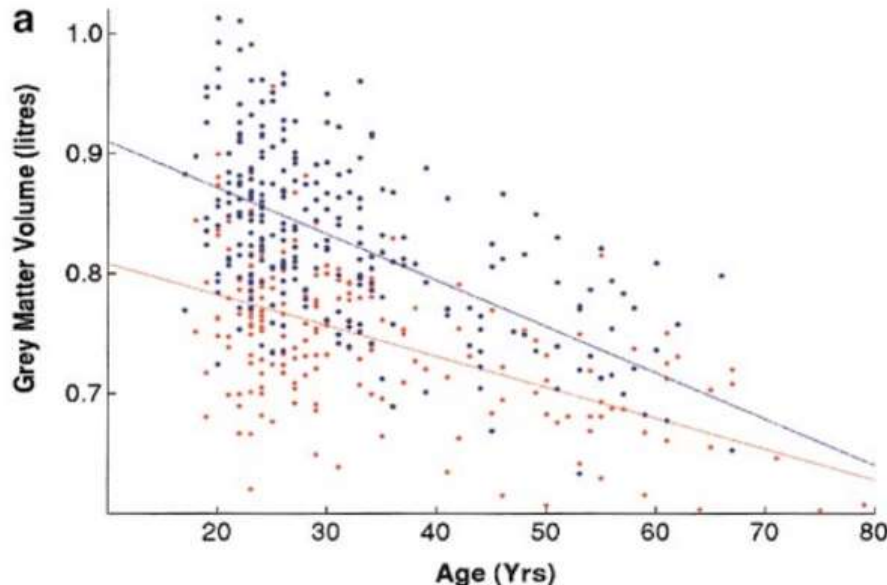


smooth=8mm



# A VBM Aging Study

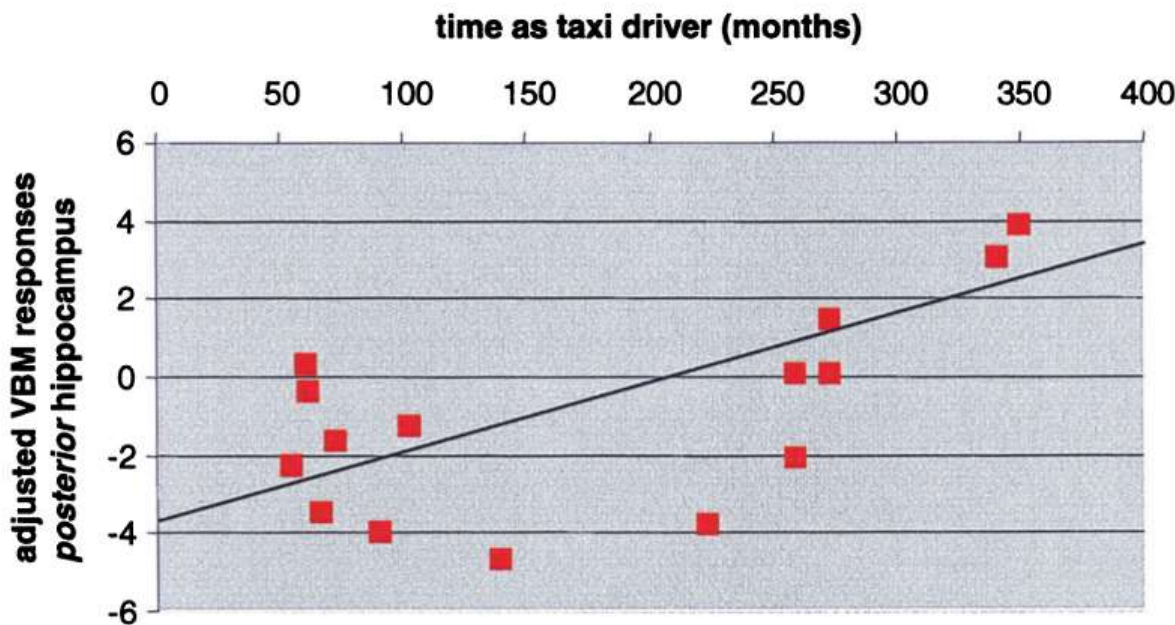
- [Good et al. 2001] used VBM to study the effect of age on global and local gray and white matter volumes in 465 healthy adults (200 female, 265 male)
  - Found global GM loss with age, accelerated in some regions, including pre-/postcentral gyrus



Accelerated loss of GM volume 83

# The VBM Taxi Driver Study

- [Maguire et al. 2000] used VBM to compare the brains of 16 taxi drivers to those of 50 gender- and age-matched non-taxi driving controls
  - Found enlarged posterior hippocampus in taxi drivers, correlated with time spent as a driver (age-corrected)



$y = -33$



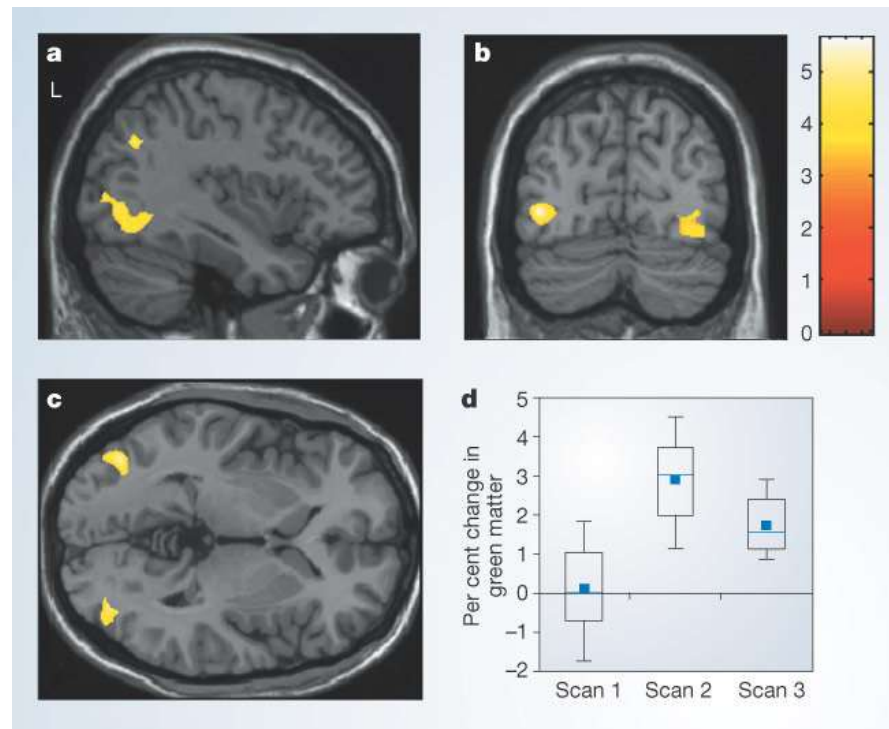
$y = -27$



$y = -20$

# A Longitudinal VBM Juggling Study

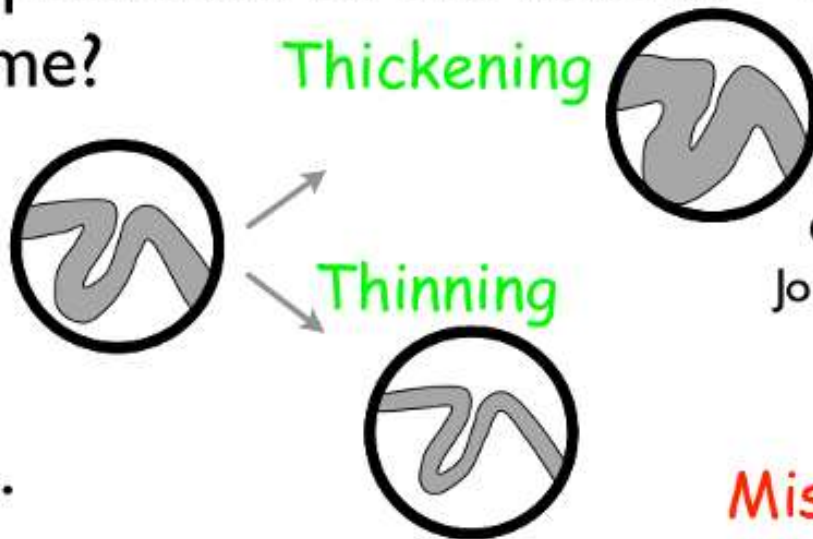
- [Draganski et al. 2004] used VBM to visualize transient structural changes in the brain associated with learning a new task (i.e., juggling)
  - Random assignment of subjects to groups, no significant differences initially
  - Changes localized in vision- and motor-related areas



# Issues with Voxel-Based Morphometry

- Controversial approach - back to the issues:

1) Interpretation of the results - real loss/increase of volume?



Courtesy of  
John Ashburner

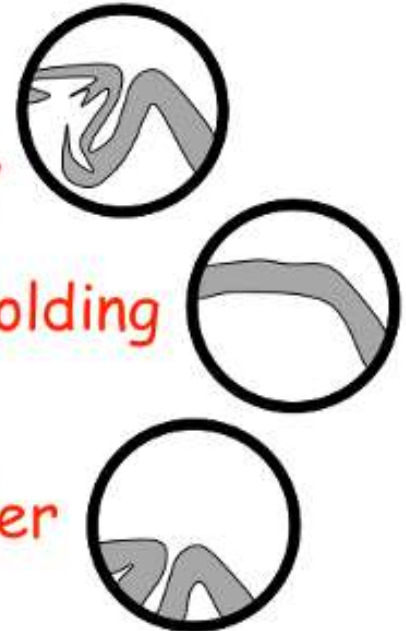
Or ...

- Difference in the contrast?
- Difference in gyrification pattern?
- Problem with registration?

Mis-classify

Folding

Mis-register



# VBM: Summary

- **Voxel-Based Morphometry (VBM)** provides a tool to study regional changes in gray or white matter volume (or “density”) based on structural MRI scans. It involves:
  - Brain Extraction
  - Normalization
  - Segmentation
  - Modulation
  - Smoothing
  - Statistical Testing
- VBM is used widely and successfully
- Points of criticism include dependence on accuracy of registration and smoothing parameters



# Further Reading

- Nicole A. Lazar: *The Statistical Analysis of Functional MRI Data*. Springer, 2008
- Russell A. Poldrack, Jeanette A. Mumford, Thomas E. Nichols: *Handbook of Functional MRI Data Analysis*. Cambridge University Press, 2011
- J. Ashburner, K.J. Friston: *Voxel-Based Morphometry – The Methods*. NeuroImage 11:805-821, 2000 [original VBM paper]
- C.D. Good et al.: *A Voxel-Based Morphometric Study of Ageing in 465 Normal Adult Human Brains*. NeuroImage 14:21-36, 2001 [optimized VBM paper]