

# Chapter 5: Graphs and Networks

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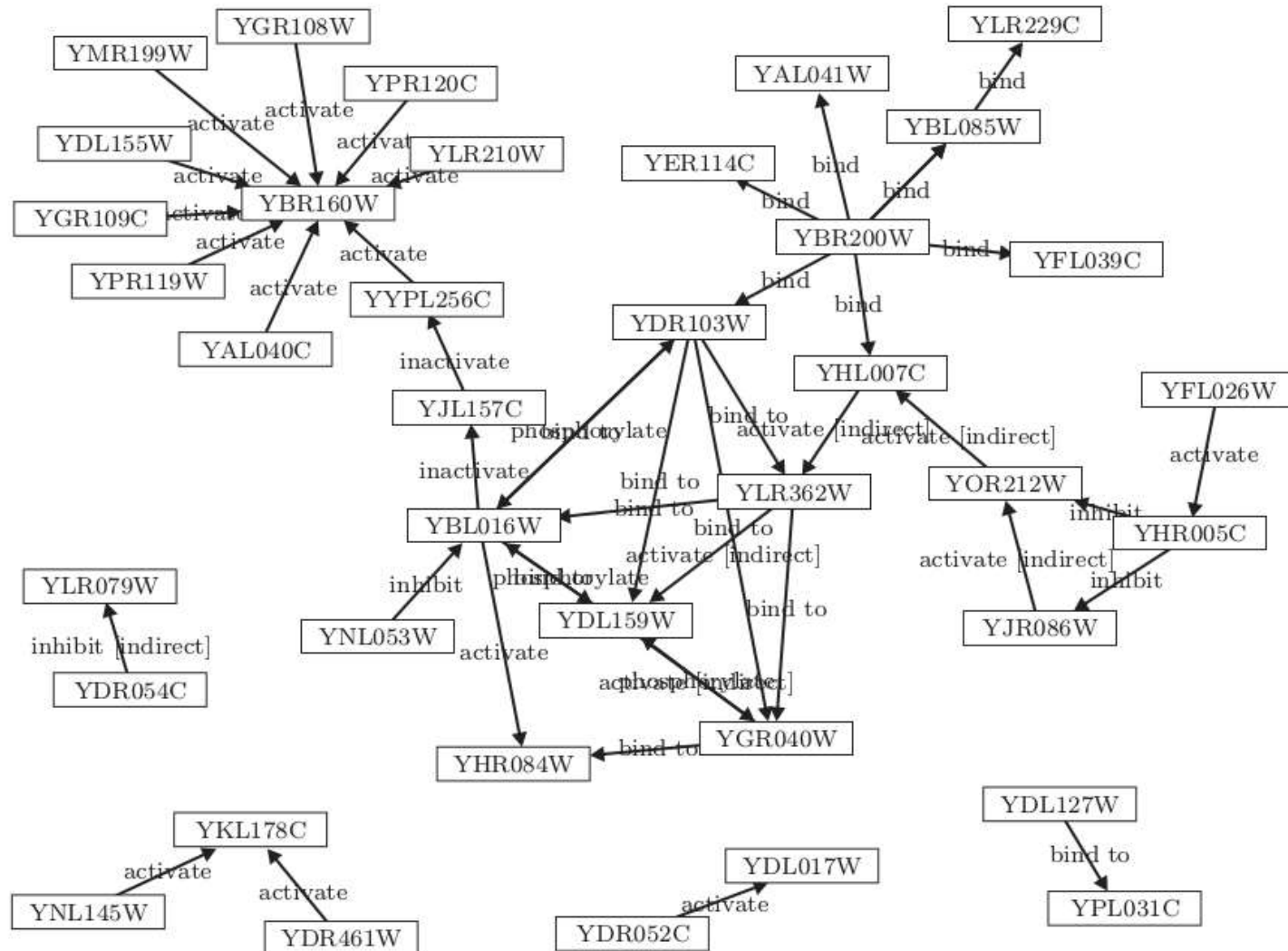
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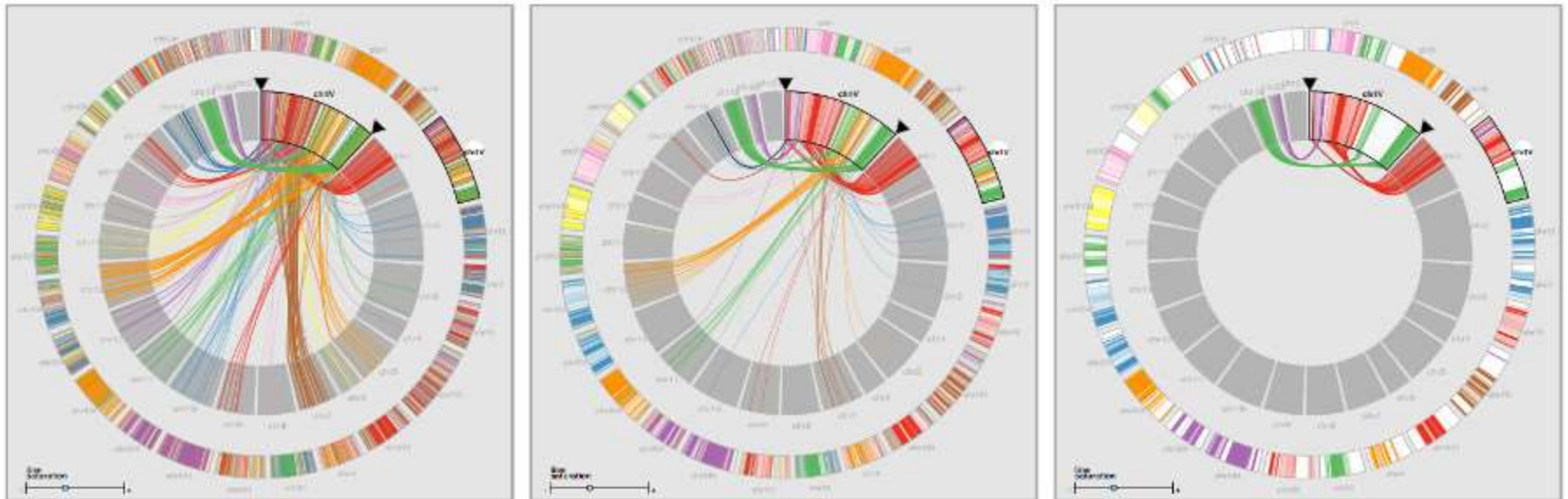
November 29, 2016

# Protein-Protein Interactions in Yeast



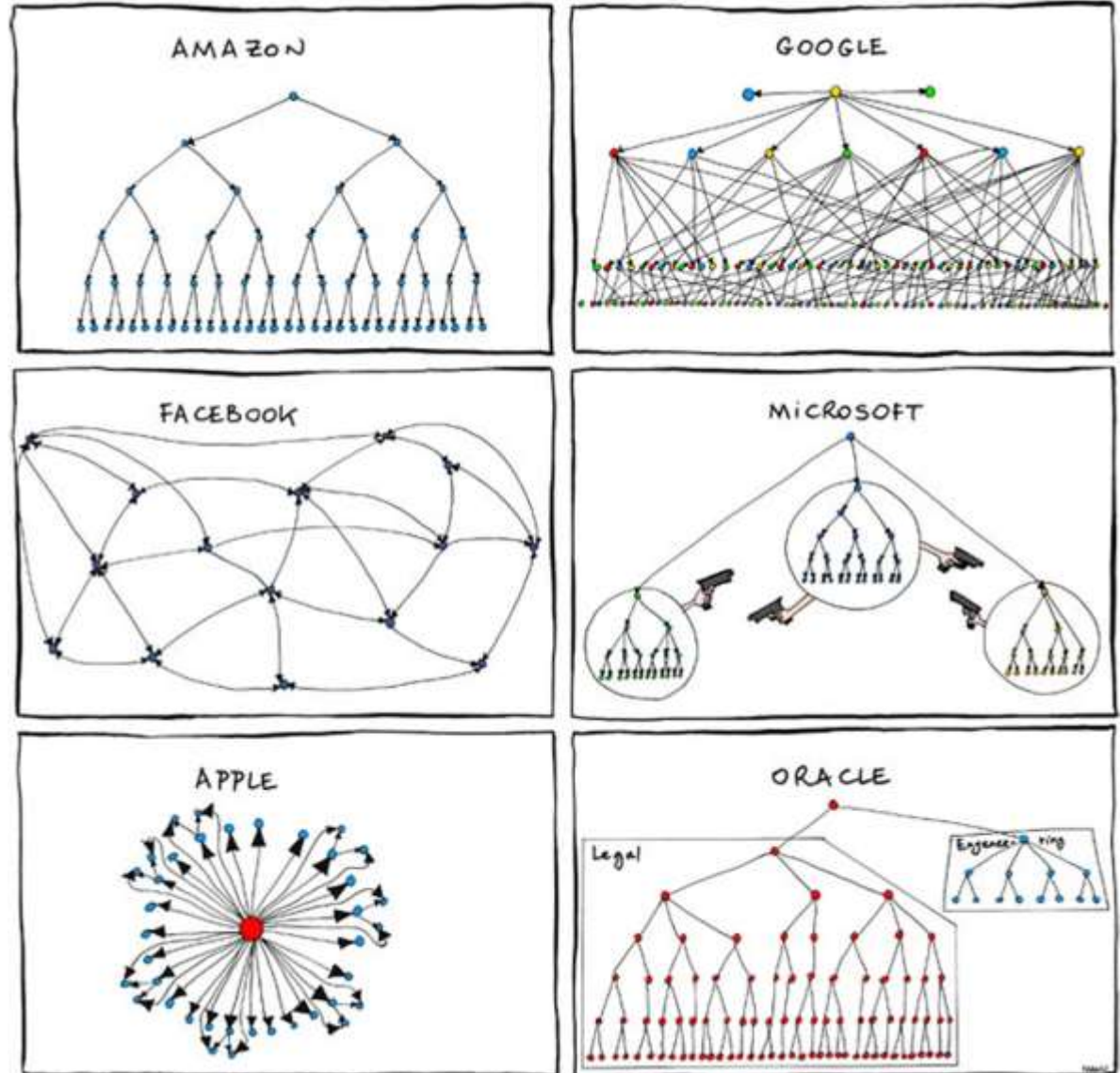
# Comparative Genomics

- [Meyer et al. 2009]: Synteny (conservation of genomic features) between genomes



# Organization Charts

- By Manu Cornet







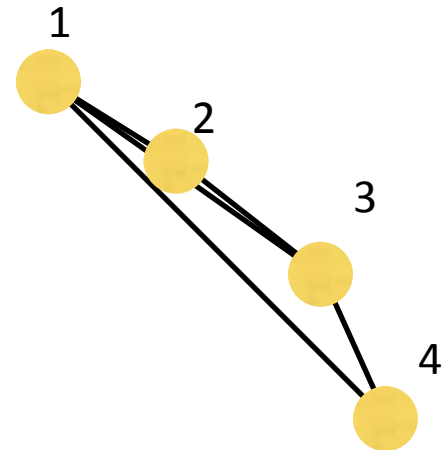
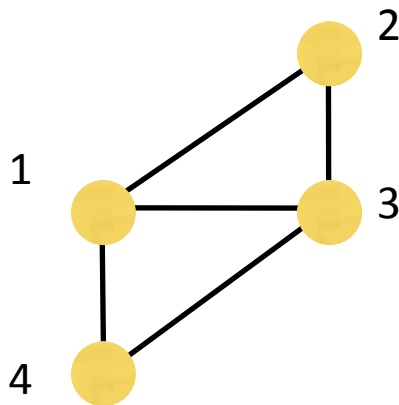
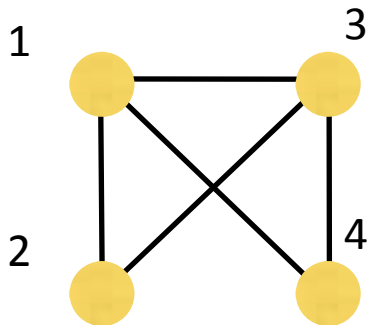
**facebook**

# **Section 5.1:**

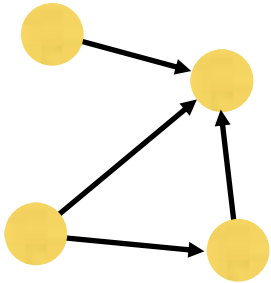
## **Basic Definitions**

# Graphs & Trees: Definitions

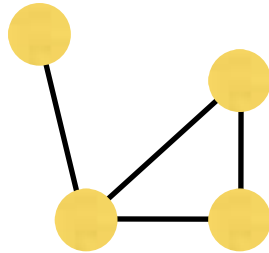
- A graph  $G$  consists of
  - a collection of nodes (or vertices)  $V$
  - a set of edges  $E$ , consisting of vertex pairs.
- An edge  $e_{xy} = (x,y)$  connects two nodes  $x$  and  $y$ .
  - Example:  $V=\{1,2,3,4\}$ ,  $E=\{(1,2),(1,3),(2,3),(3,4),(4,1)\}$



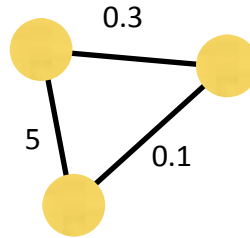
# Graphs & Trees: Definitions



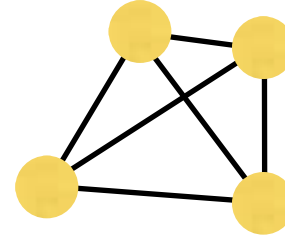
A directed graph



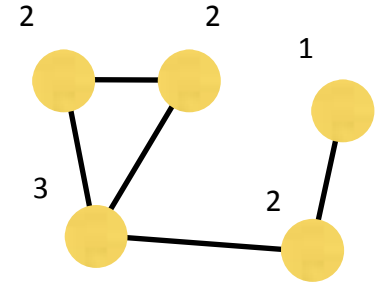
An undirected graph



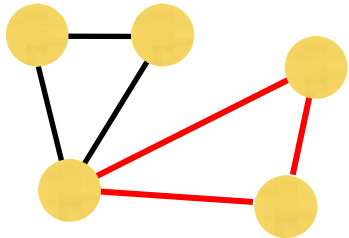
Weighted



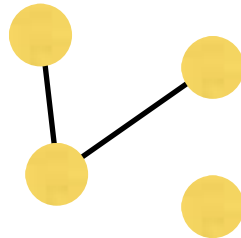
Complete



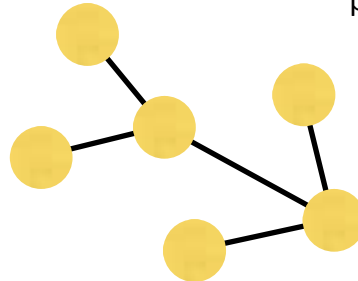
Node degrees



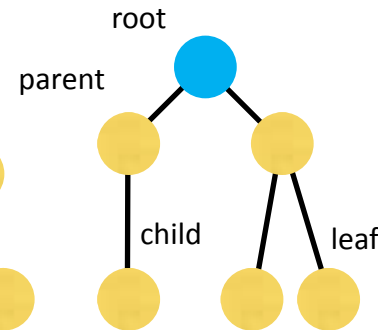
A cycle



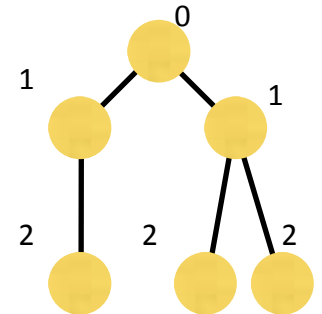
An acyclic graph



A connected acyclic graph,  
a.k.a. a tree



A rooted tree  
or hierarchy



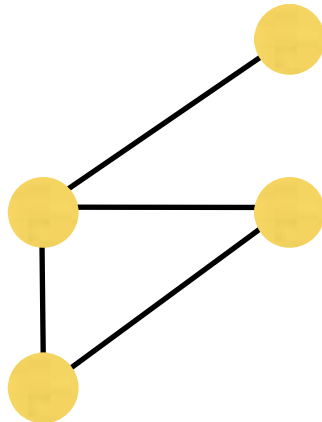
Node depths



# Graphs & Trees: Definitions

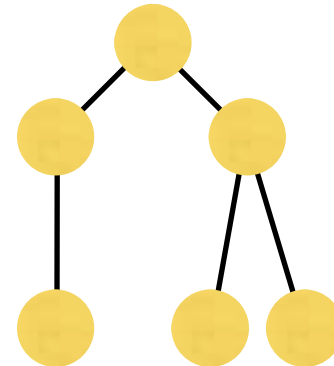
## Graphs

- models relations among data
- *nodes* and *edges*



## Trees

- graphs with a hierarchical structure
  - connected graph with  $n-1$  edges
- nodes as *parents* and *children*



# Graphs & Trees: Definitions

- Primary concern of graph drawing is the spatial layout of nodes and edges
- Often, the goal is to effectively depict the graph structure
  - connectivity, path-following
  - network distance
  - clustering
  - ordering (e.g., hierarchy level)

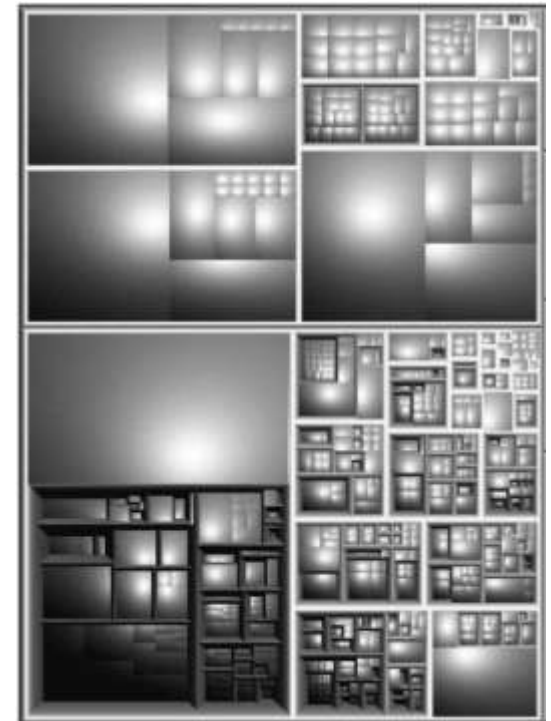
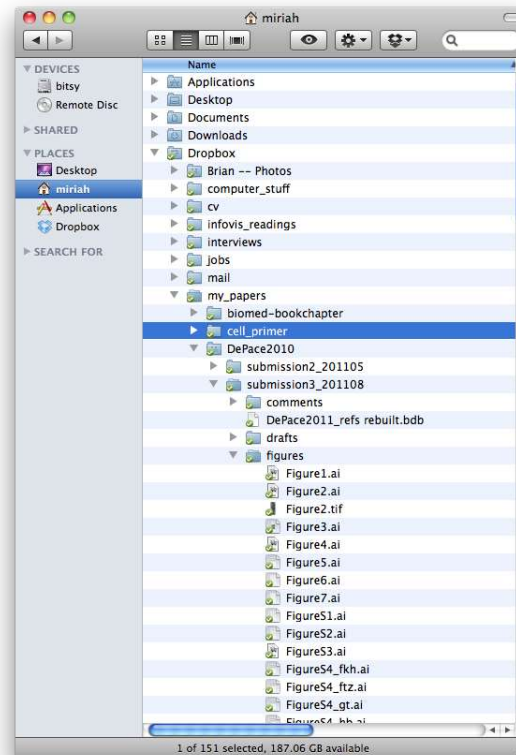
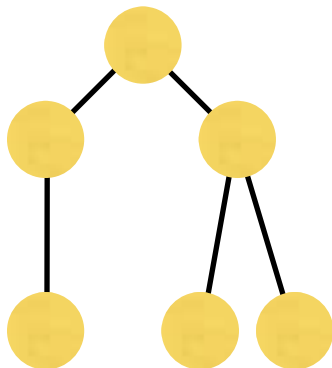
## **Section 5.2:**

# **Visualizing Trees / Hierarchies**

# Visualizing Trees

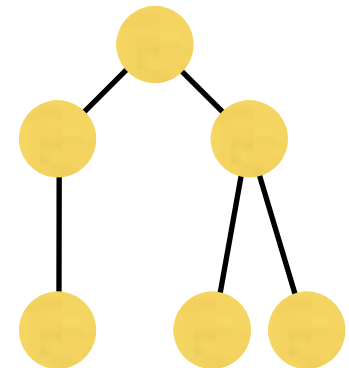
## Rooted trees

- Recursion makes it elegant and fast to draw trees
- Approaches:
  - node link
  - indentation
  - enclosure (treemaps)



# Visualizing Trees: Node-Link Diagrams

- Nodes are distributed in space, connected by straight or curved lines
- Typical approach is to use 2D space to break apart breadth and depth
- Frequent **design goals**:
  - Nodes at same depth share the same vertical position
  - Horizontal whitespace communicates hierarchy
  - Minimize required area
  - Minimize total length of edges
  - Achieve good aspect ratio



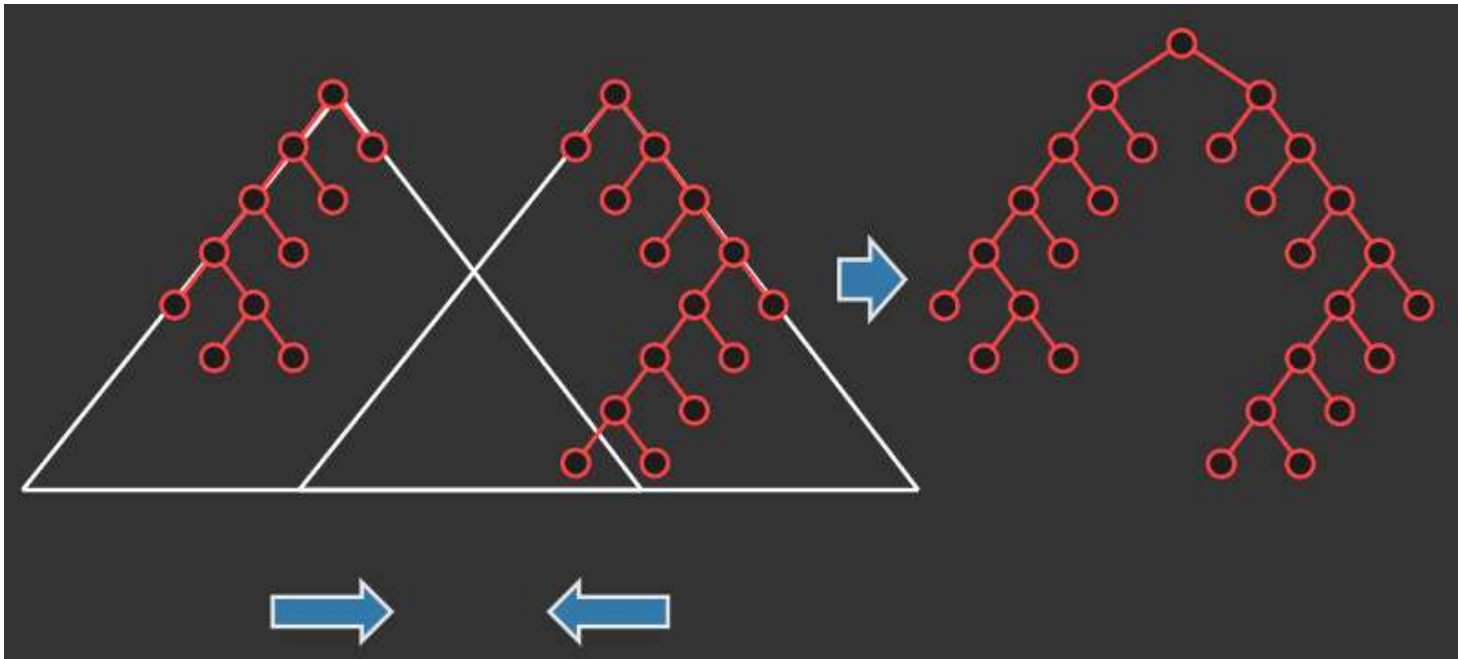
# Aesthetics of Reingold-Tilford

- *Tidier Drawings of Trees* [Reingold/Tilford 1981]
  - Formulation for binary trees, can be generalized
- **Aesthetic Goals:**
  - Nodes at the same level should be aligned
  - Maintain the relative ordering of left and right subtrees
  - Parent should be centered over the children
  - A tree and its mirror image should be drawn as reflections of each other
  - A subtree should be drawn the same way regardless of where it occurs in the tree



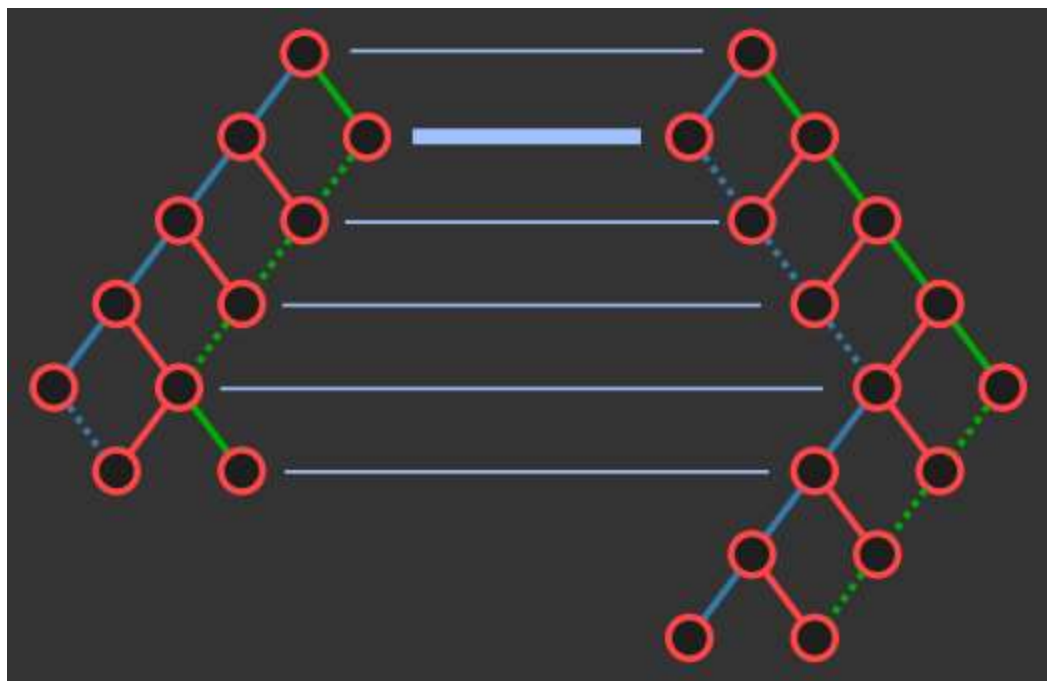
# Reingold-Tilford: Recursive Construction

- Assume left and right subtrees have already been drawn
- Shift them to a fixed horizontal distance
- Center parent between them



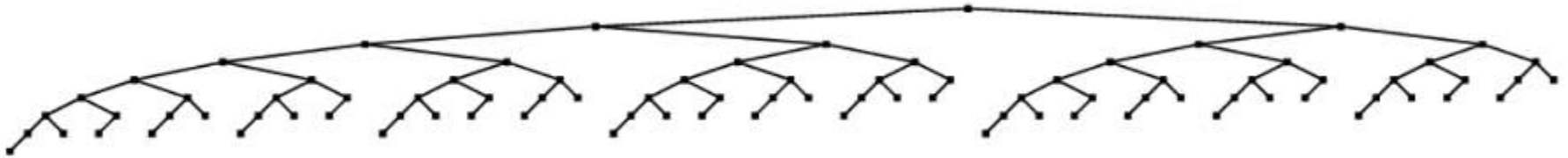
# Reingold-Tilford: Threading

- Finding the correct distance requires traversal of the contours of each subtree
- If a contour node at depth  $k$  is a leaf, store *thread* to contour node at depth  $k+1$

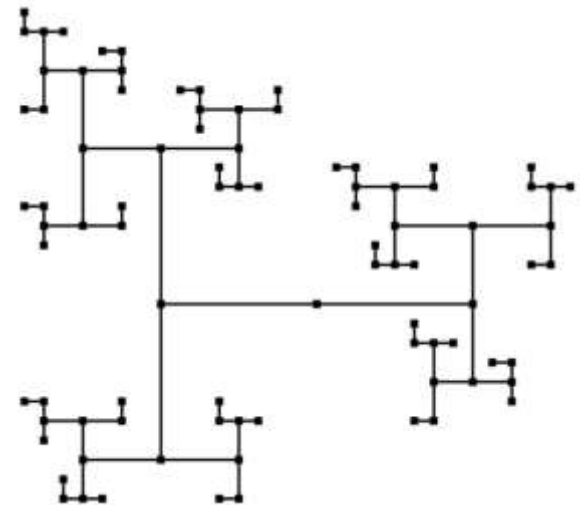


# Reingold-Tilford: Pros and Cons

- The Reingold-Tilford algorithm is
  - easy to understand and implement, but
  - can lead to poor aspect ratios:

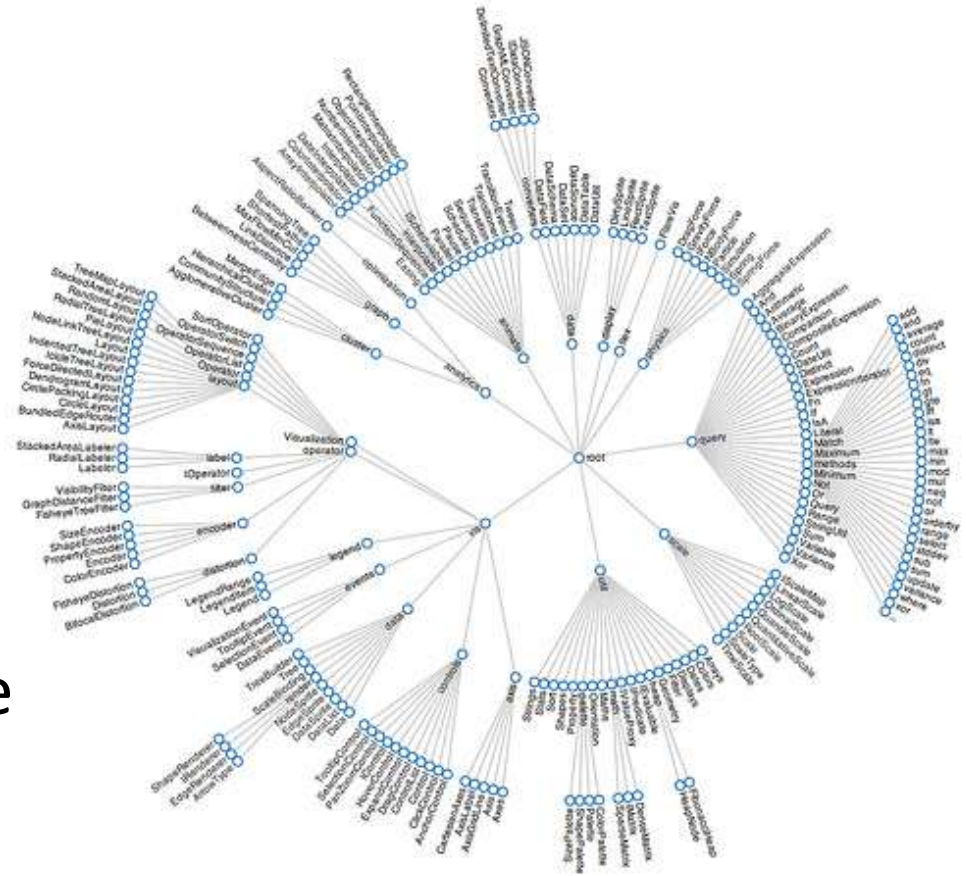


- alternative, non-level based layout of the same tree:



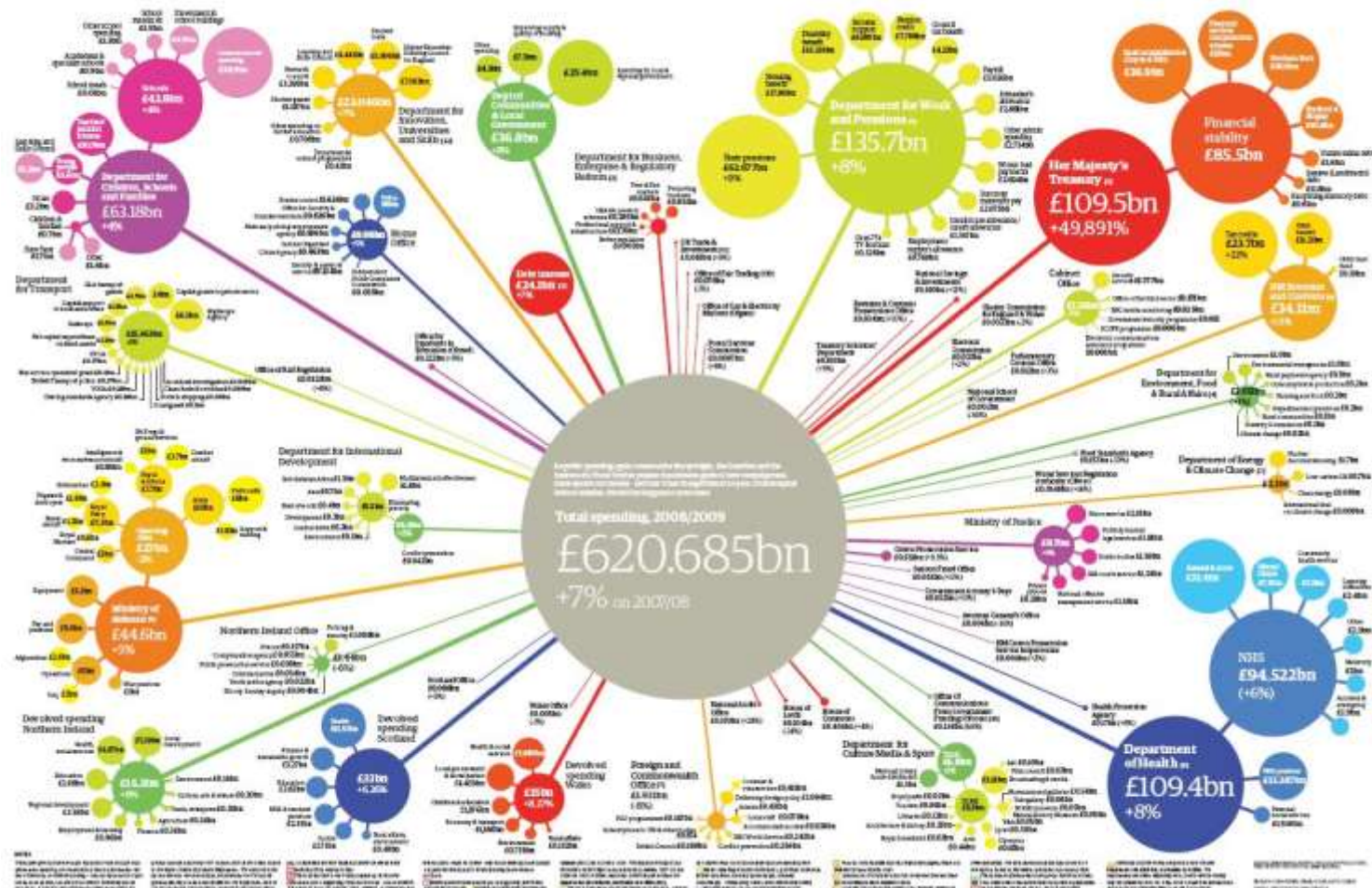
# Node-Link Diagrams: Radial Layout

- node-link diagram in polar coordinates
- radius encodes depth with root in center
- angular sectors assigned to subtrees
- Reingold-Tilford can be applied



# Node-Link Diagrams: Bubble Tree Layout

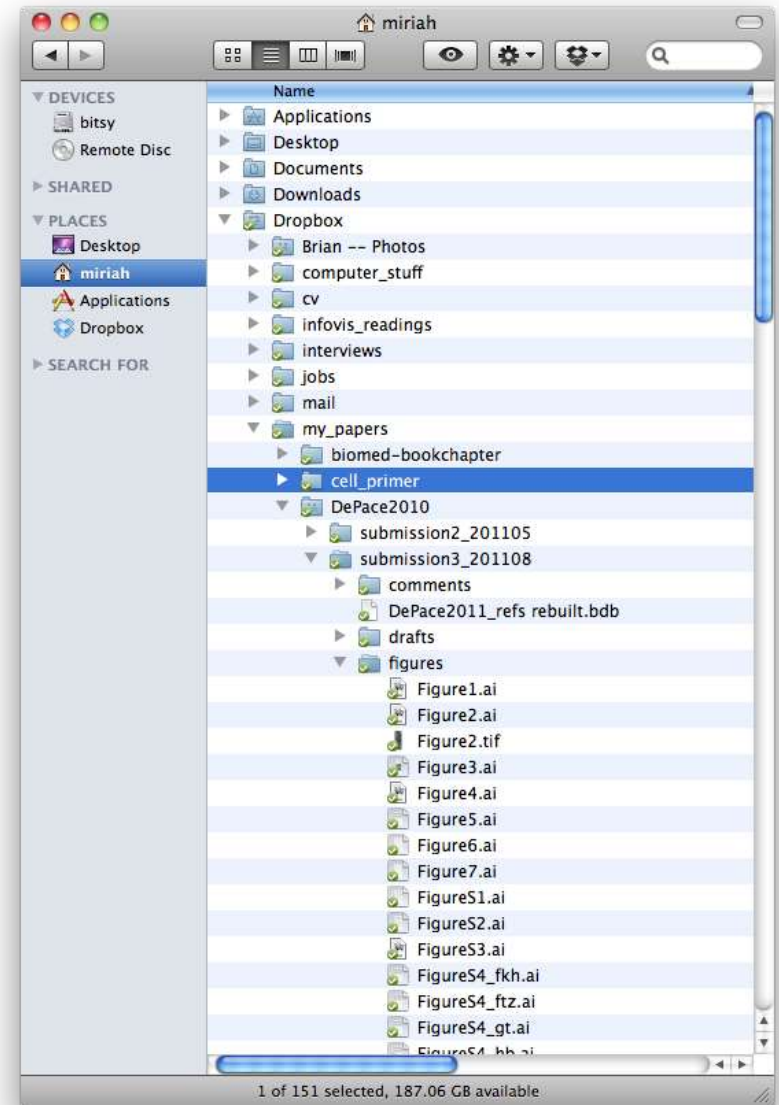
- Variant: Each inner node becomes the center of all its children, makes it easy to distinguish subtrees





# Visualizing Trees: Indentation

- place all items along vertically spaced rows
- indentation used to show parent/child relationships
- commonly used as a component in user interfaces
- breadth and depth contend for space
- often requires a great deal of scrolling





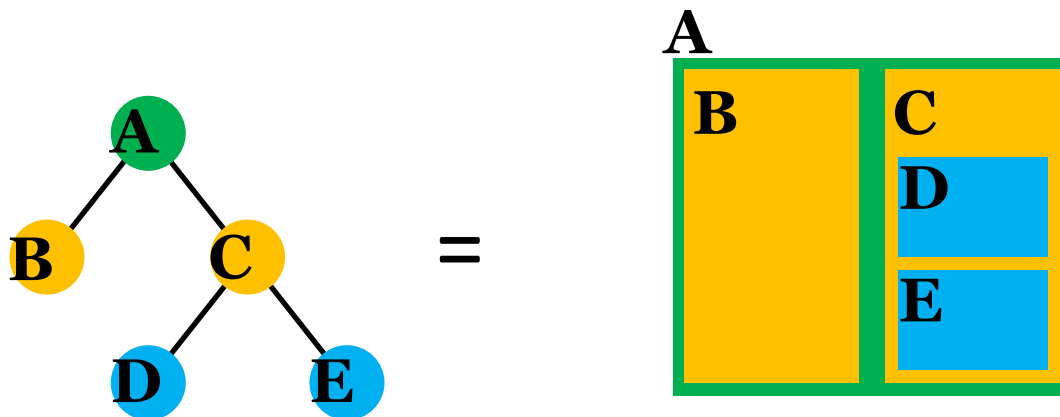
# Visualizing Trees: Indentation

- Trees are usually large and unbalanced
- How to use the rectangular screen space optimally?



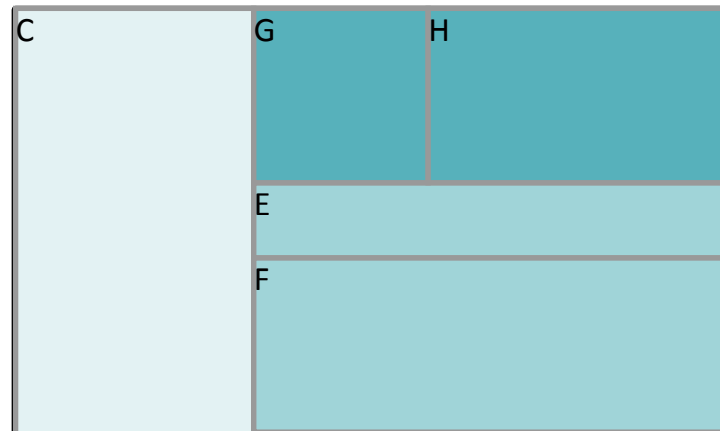
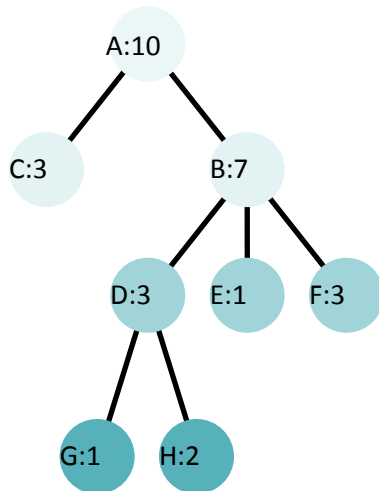
# Visualizing Trees: Treemaps

- encode structure using spatial enclosure
  - often referred to as **treemaps**
- benefits
  - provides single view of entire tree
  - easy to tell “size” of node
  - easy to encode additional attributes (color)
- problems
  - difficult to accurately read depth



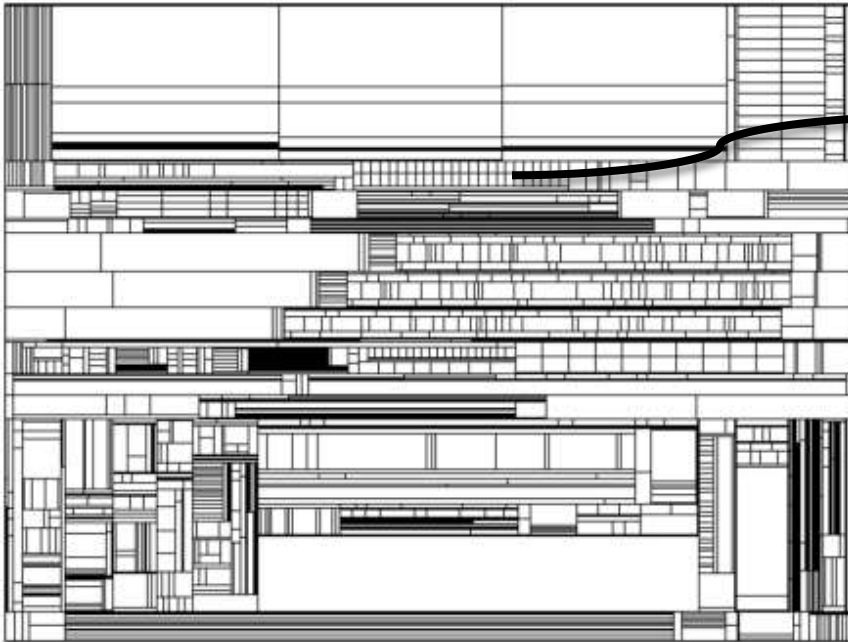
# Visualizing Trees: Treemaps

- recursively fill space based on a size metric for nodes
- enclosure indicates hierarchy
- additional measures can control aspect ratio of cells



# Visualizing Trees: Treemaps

- **Problem:** Naïve splitting can lead to rectangles with poor aspect ratios

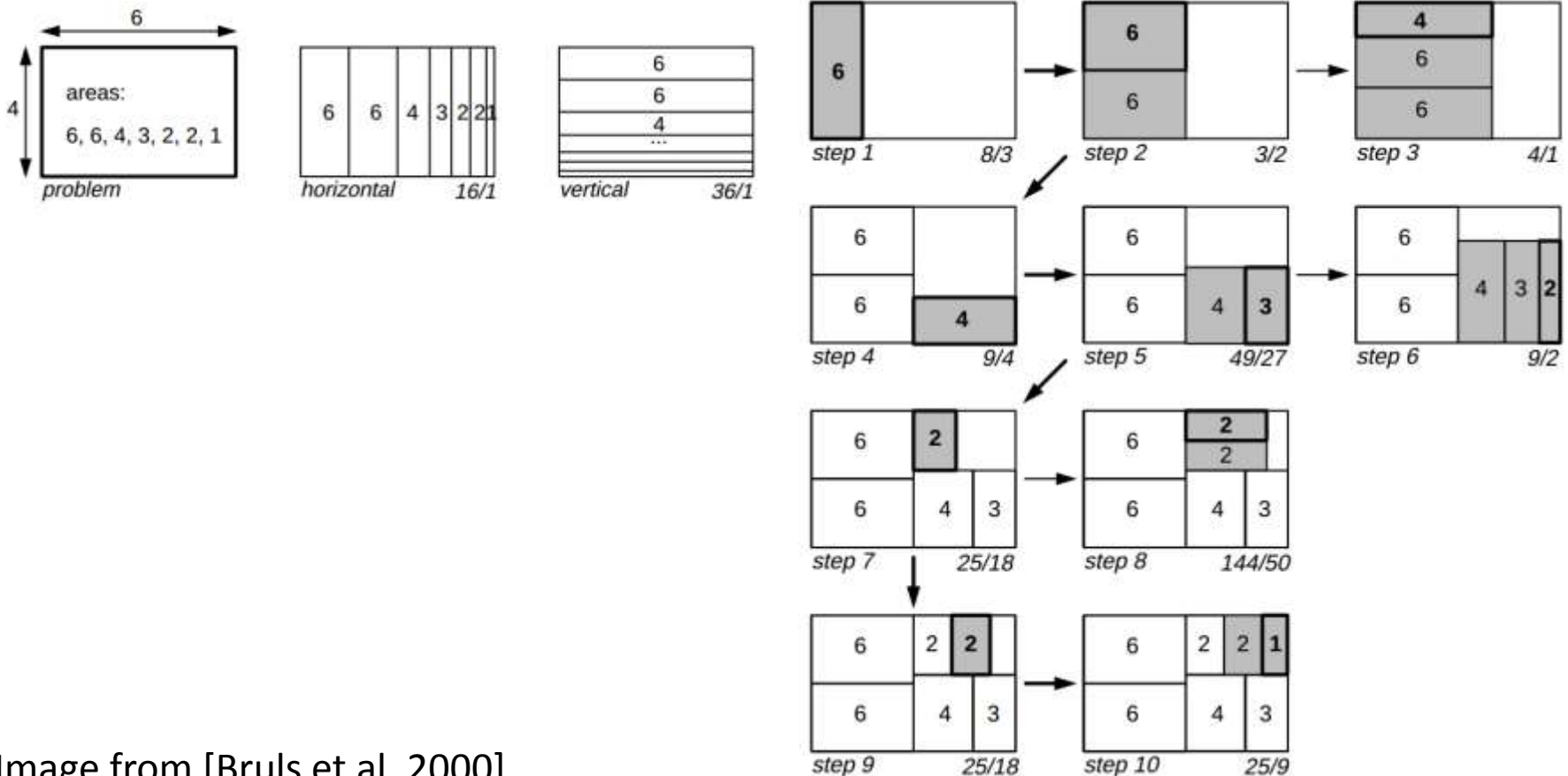


File system

Bruls, Mark, Kees Huizing, and Jarke J. Van Wijk.  
"Squarified treemaps." Data Visualization. 2000.

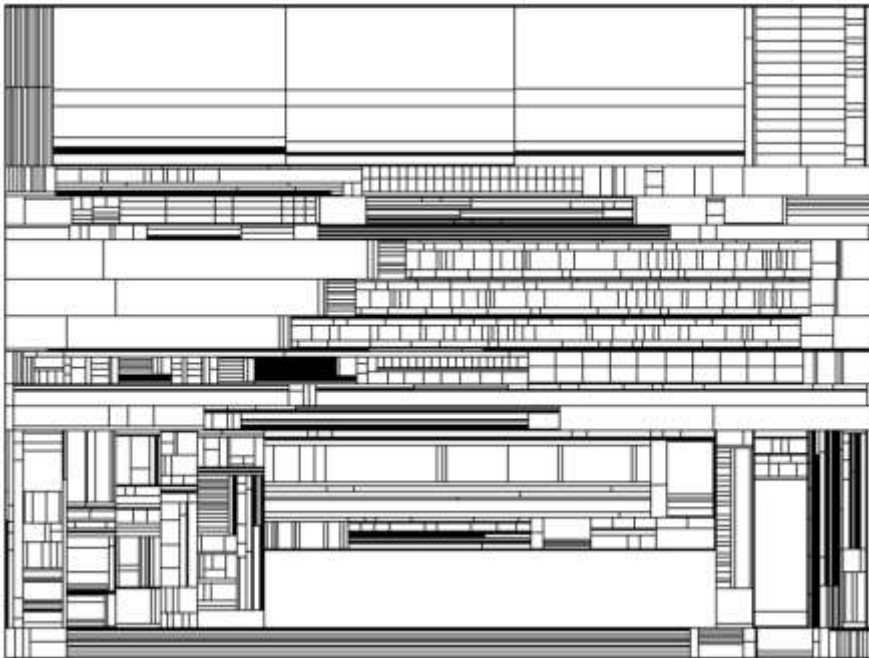
# Visualizing Trees: Squarified Treemaps

- **Squarified treemaps:** greedy heuristic to favor squares over elongated rectangles
  - finding an optimal solution is NP-hard

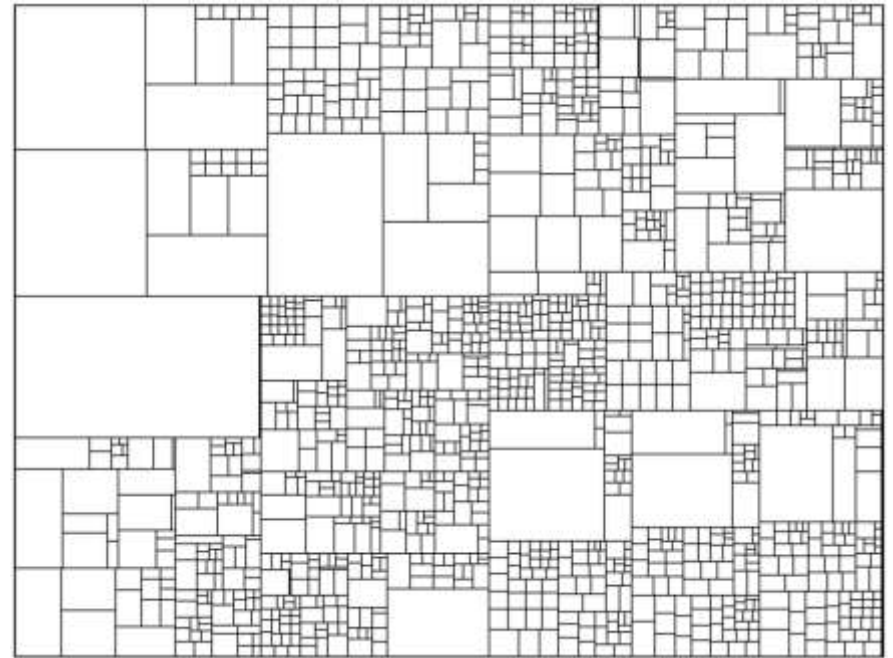


# Example Result: Squarified Treemap

- File system example from [Bruls et al. 2000]
  - **Drawback:** Hierarchical structure more difficult to perceive



Traditional Treemap

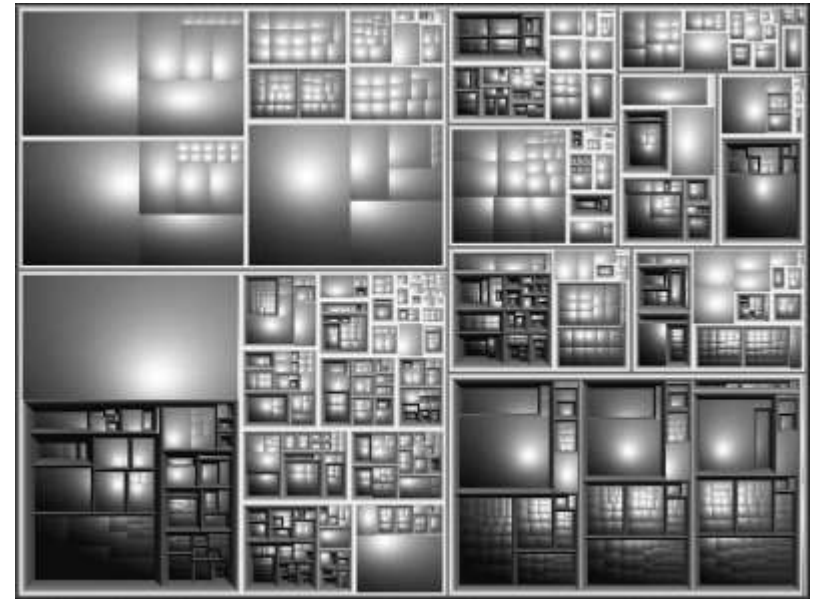
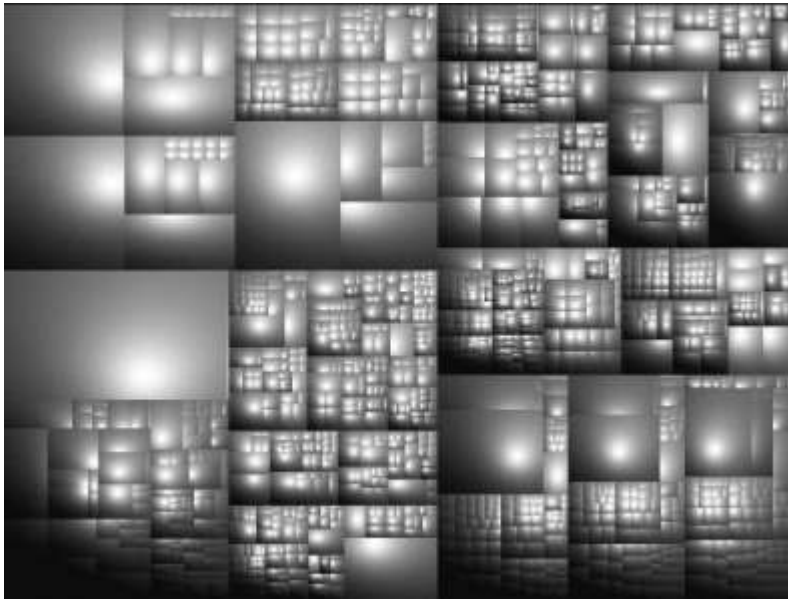
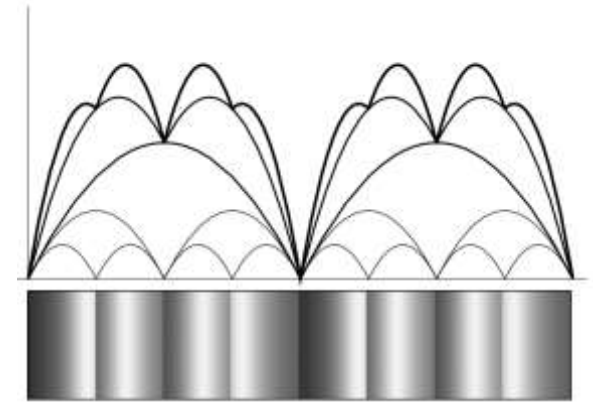


Squarified Treemap



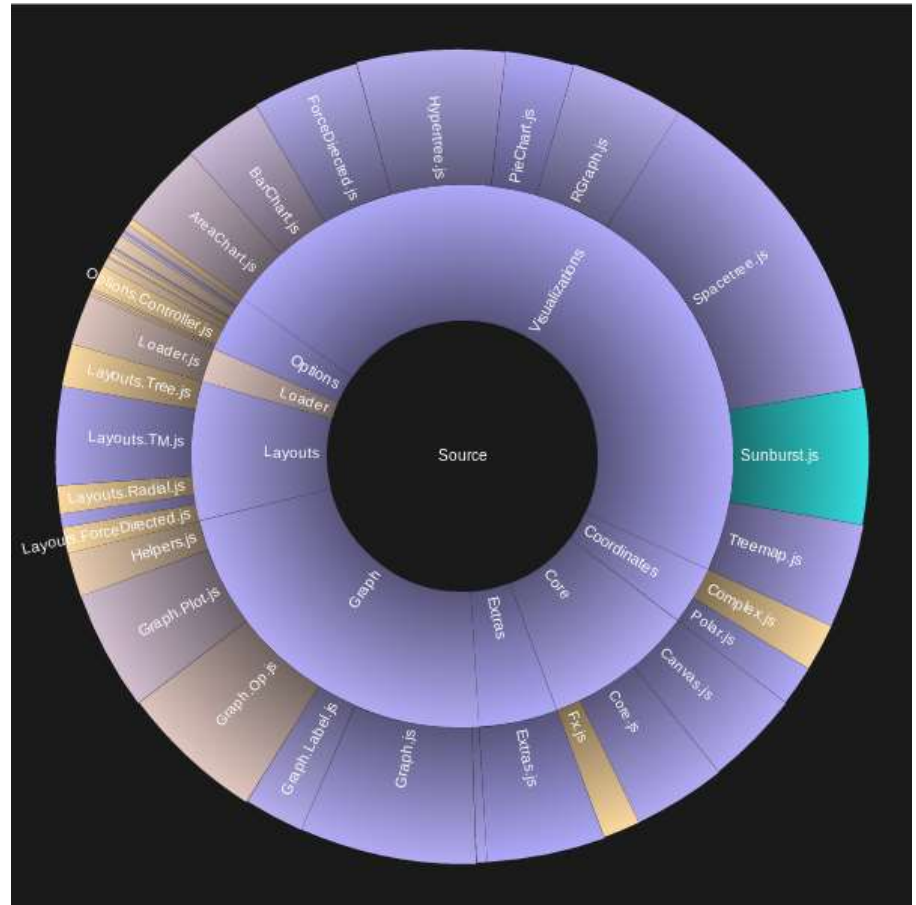
# Visualizing Trees: Cushion Shading

- **Cushion shading** supports perception of hierarchies
- $I = I_a + I_d \max(0, \mathbf{n} \cdot \mathbf{l})$
- Can add **frames and interaction**



# Sunburst Displays

- **Sunburst displays**
  - Root at center, nested rings around it
  - Space-filling like treemaps
  - Layout similar to radial node-link diagrams



# Summary: Visualizing Trees / Hierarchies

- Main strategies for visualizing trees:
  - Node-link diagrams
    - Reingold-Tilford
    - Radial layouts
  - Indentation
  - Containment (treemaps)
    - Squarification
    - Cushions and frames

## **Section 5.3: General Graphs**

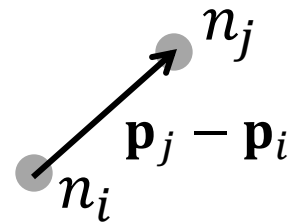
# Force-Directed Layouts

- **Goal:** Place groups of strongly connected nodes close to each other, preserve minimum distance between nodes
  - *Spring force* that node  $n_j$  exerts on  $n_i$  (natural spring length  $s_{ij}$ , tension  $k_{ij}$ ) if an edge connects them

$$\mathbf{f}_{ij}(x) = k_{ij}(\|\mathbf{p}_j - \mathbf{p}_i\| - s_{ij}) \frac{\mathbf{p}_j - \mathbf{p}_i}{\|\mathbf{p}_j - \mathbf{p}_i\|}$$

- *Electrical repulsion* that node  $n_j$  exerts on  $n_i$  (repulsion strength  $r_{ij}$ )

$$\mathbf{g}_{ij}(x) = -\frac{r_{ij}}{\|\mathbf{p}_j - \mathbf{p}_i\|^2} \frac{\mathbf{p}_j - \mathbf{p}_i}{\|\mathbf{p}_j - \mathbf{p}_i\|}$$

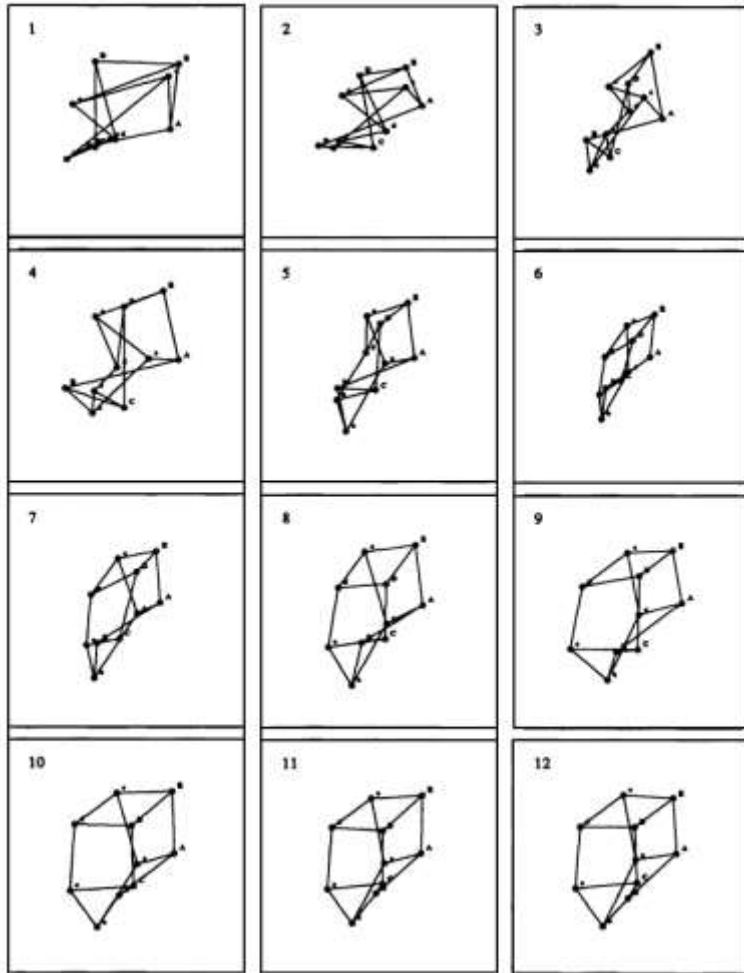


# Force-Directed Layouts: General Algorithm

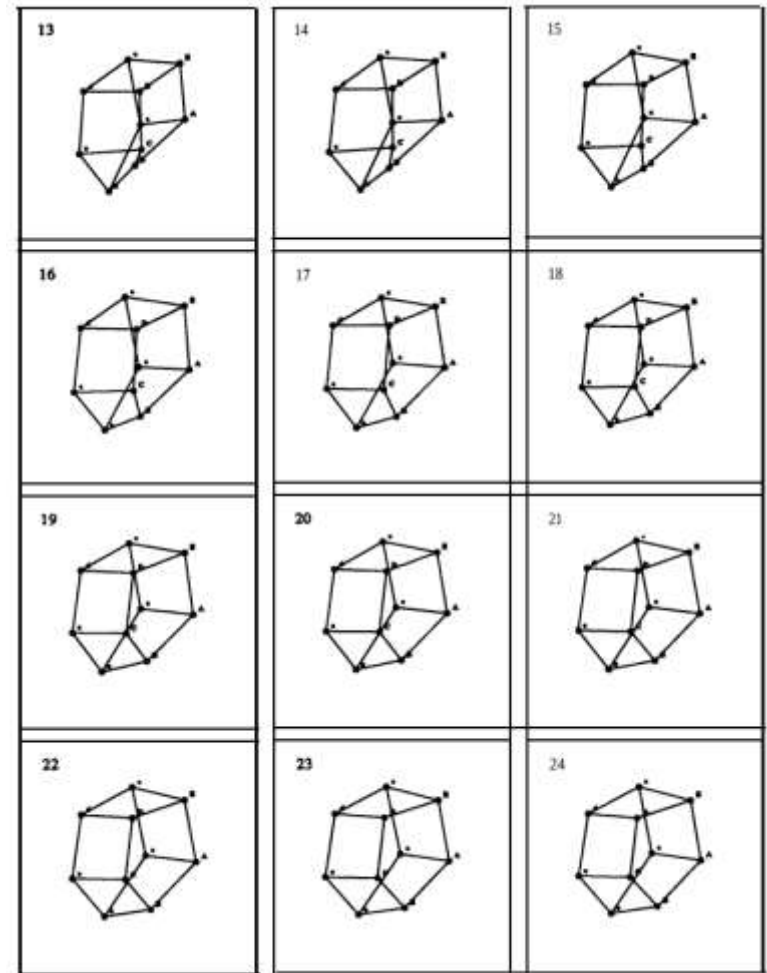
- Initialize (randomly or through a heuristic)
- Iterate:
  - For each node:
    - Sum all attractive and repulsive forces
    - Multiply overall force by stepsize (“temperature”)
    - Impose a maximum displacement (e.g., keep within image)
    - Move node
  - Adjust temperature



# Quenching and Simmering



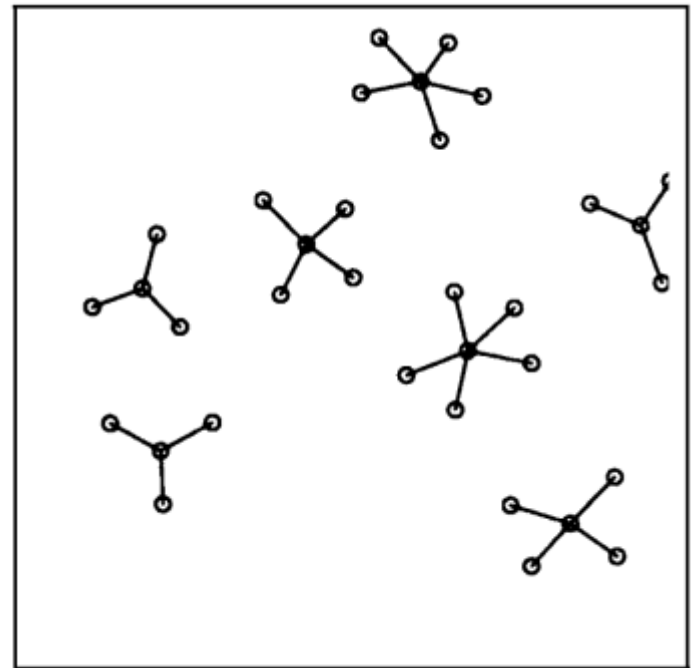
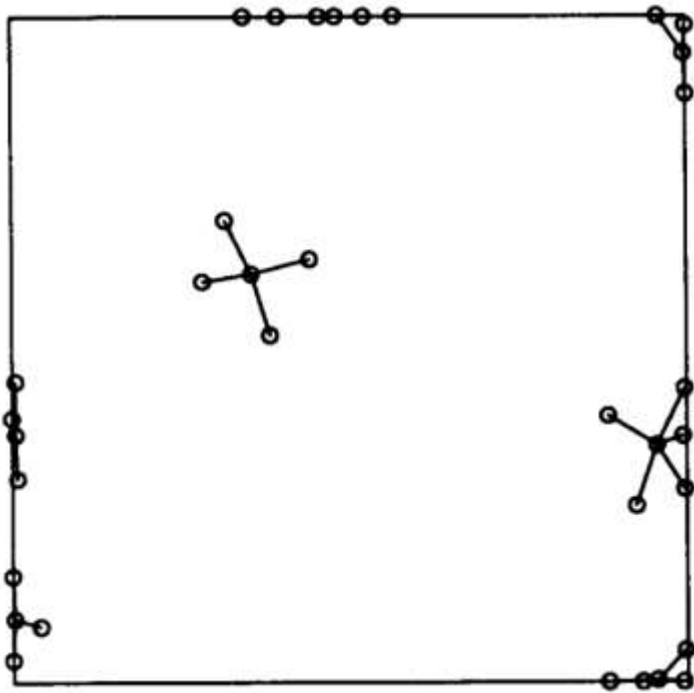
**Quenching:** Rapid cooling



**Simmering:** Constant low temperature

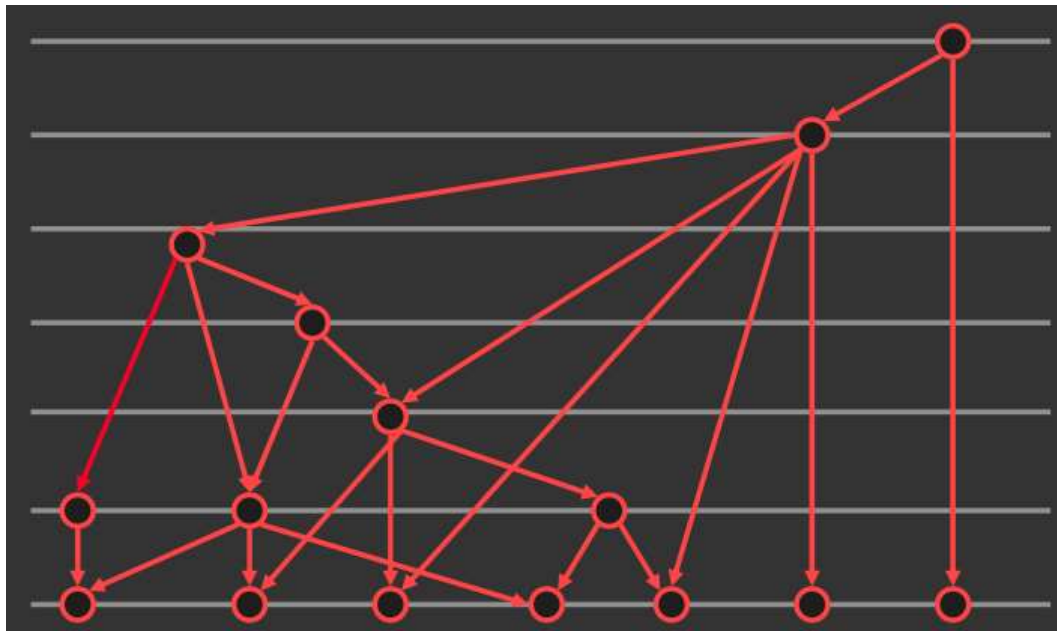
# Grid Variant

- **Speedup:** Divide plane into grid cells and only compute repulsive forces between nodes in the same cell
  - Produces better results for disconnected graphs



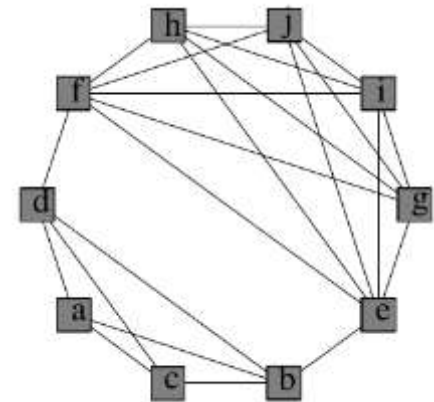
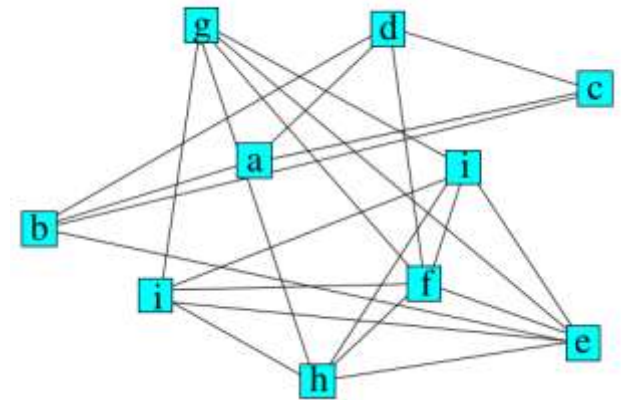
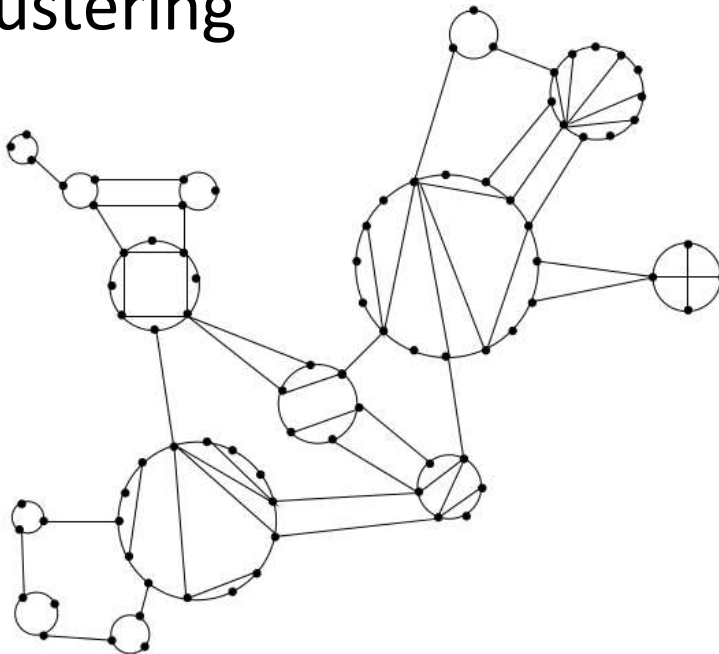
# Layered Layouts

- Nodes of **Directed Acyclic Graphs** can be organized in layers
  - Assign vertical position to each layer
  - Suggests a hierarchy, similar to tree visualization



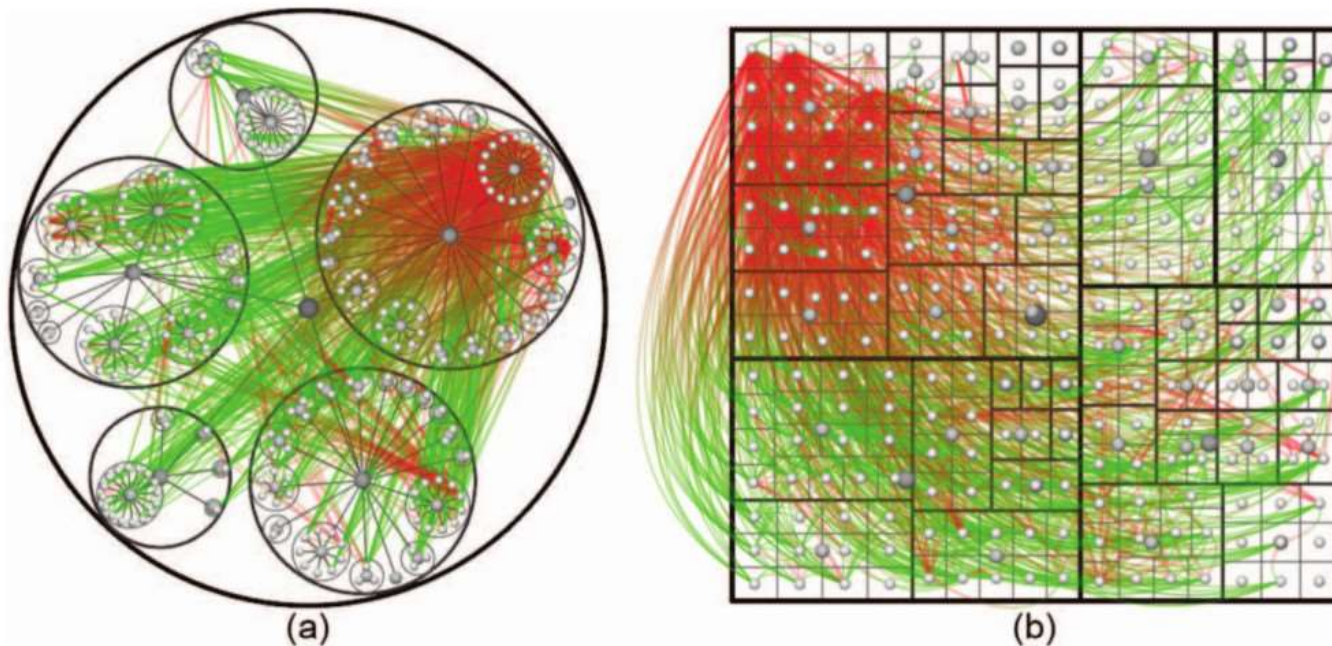
# Circular Layouts

- **Circular layouts** distribute nodes along the circumference of a circle
  - Try to minimize edge crossings
  - Can be combined with clustering



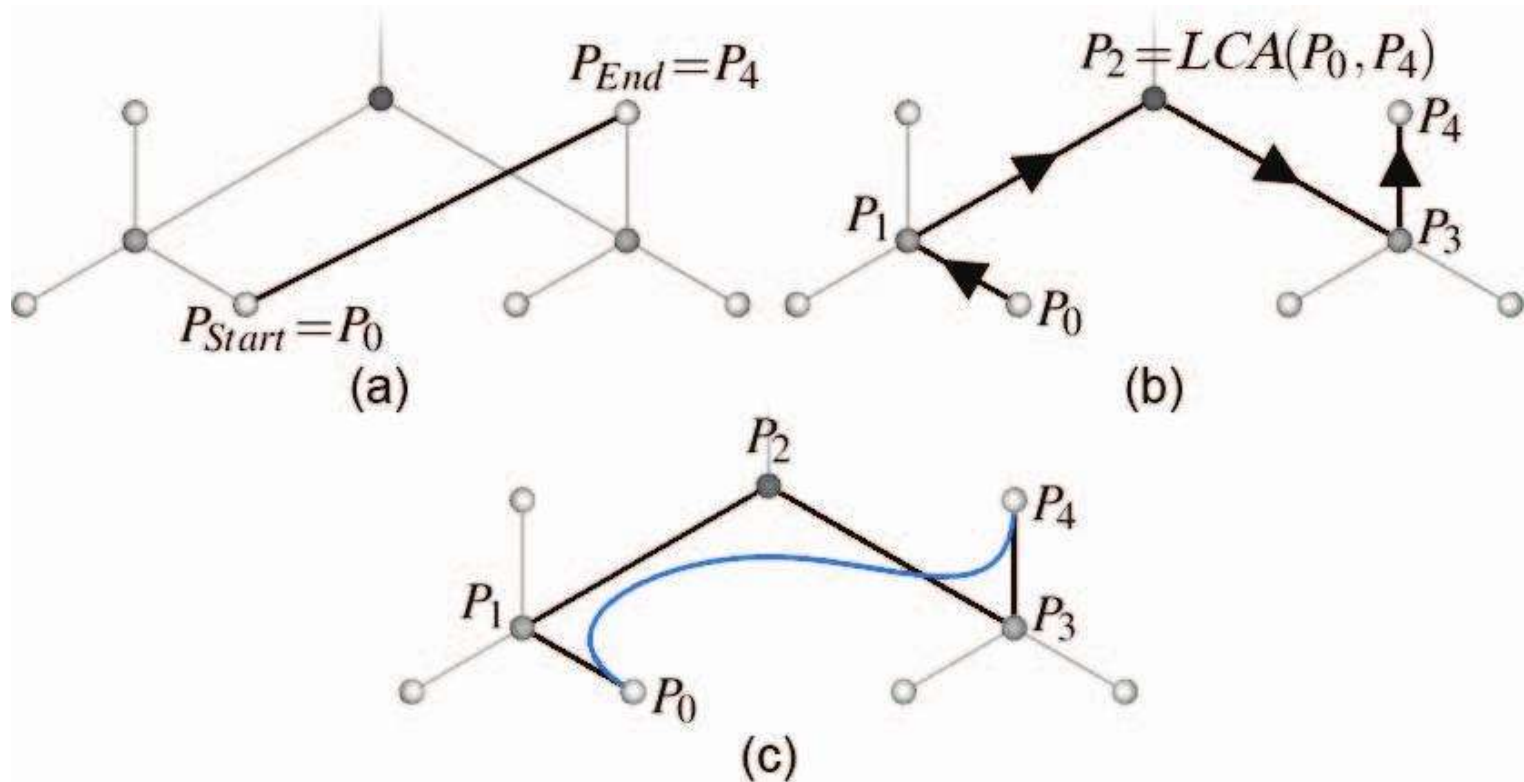
# Hierarchical Edge Bundling

- What if you have hierarchical (*inclusion*) and non-hierarchical (*adjacency*) relationships at the same time?
  - *Example*: software system; caller green, callee red



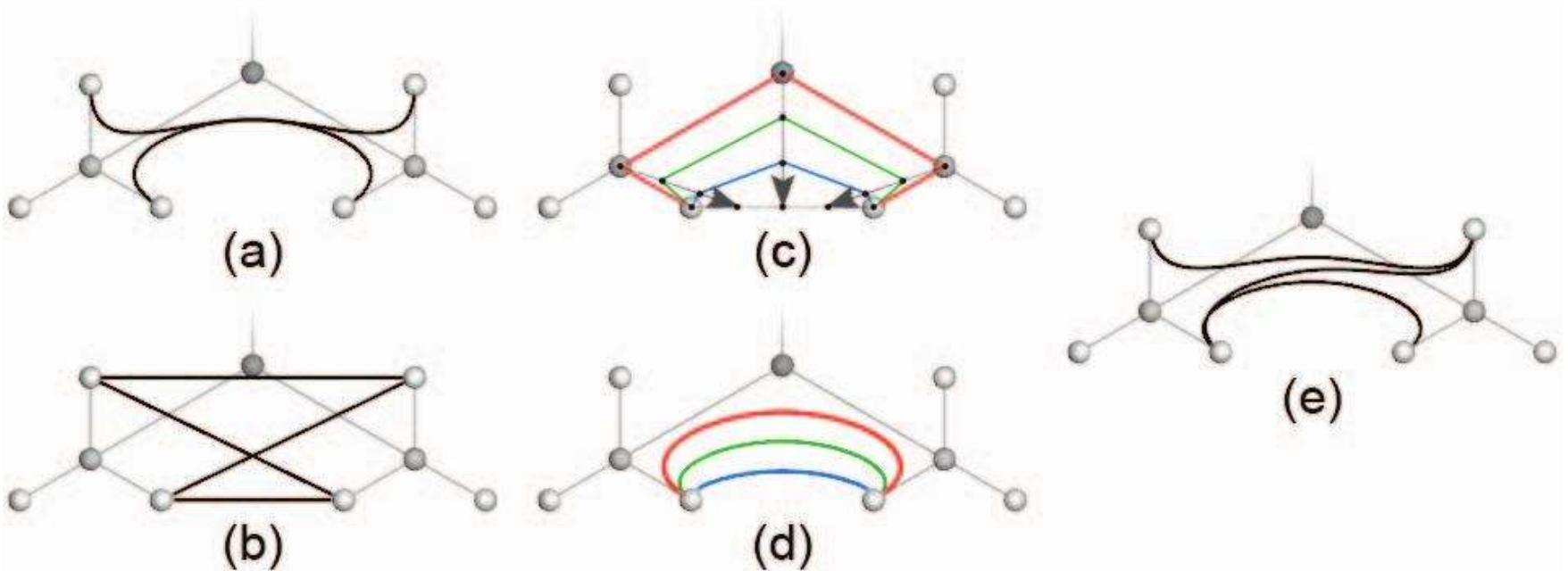
# Hierarchical Edge Bundling: Concept

- **Idea:**
  - Use standard tree layout for hierarchy
  - Path along tree as control polygon for graph edge



# Edge Bundling Strength

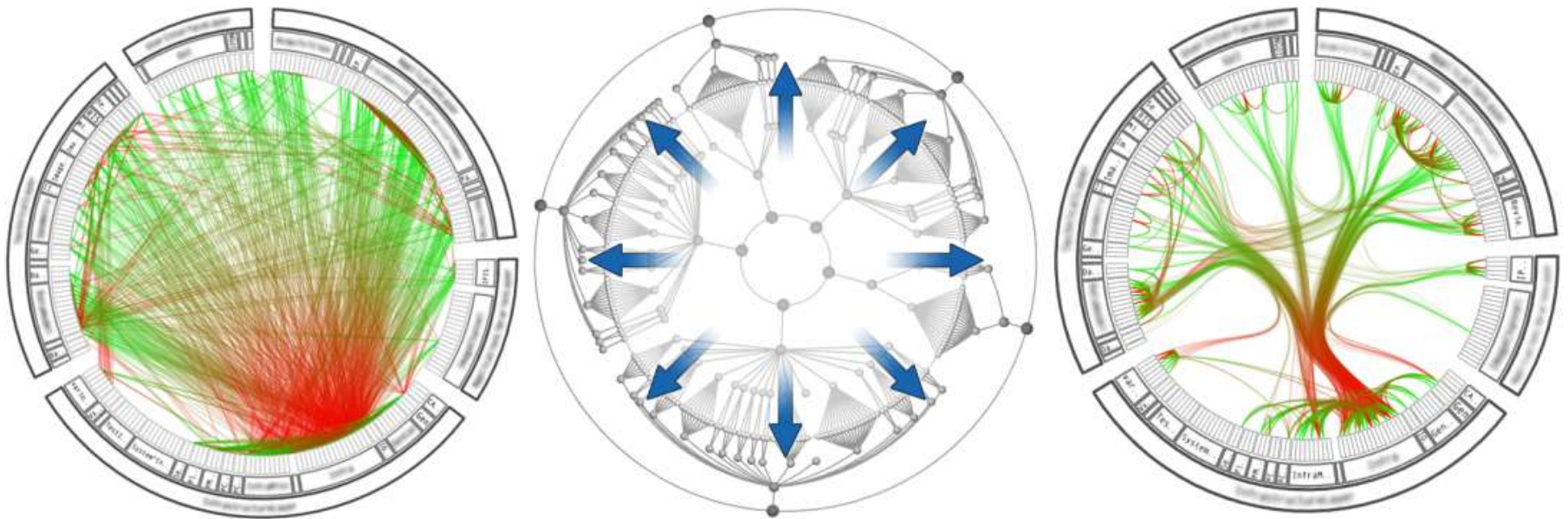
- Bundling can lead to ambiguities
- Continuously straightening edges using a **bundling strength**  $\beta \in [0,1]$  resolves this





# Edge Bundling: Radial Layout

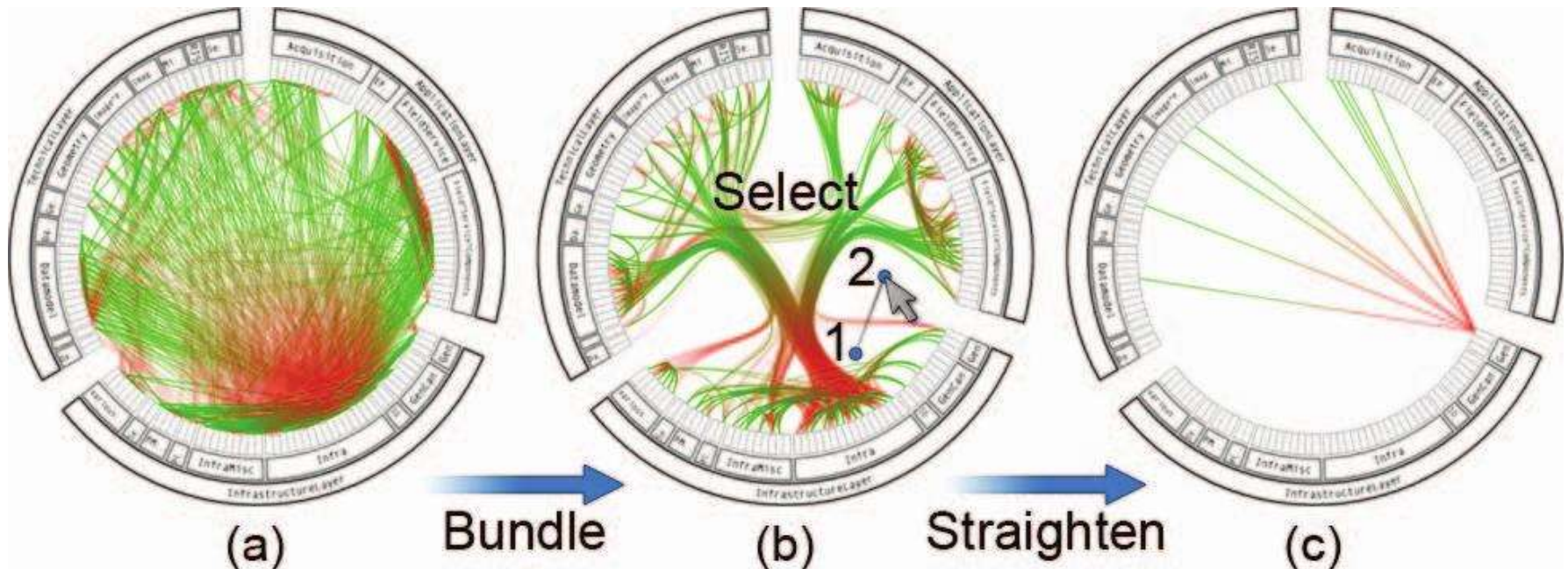
- **Radial Graph Layout** can be used for control polygons; nodes mirrored to the outside for labeling





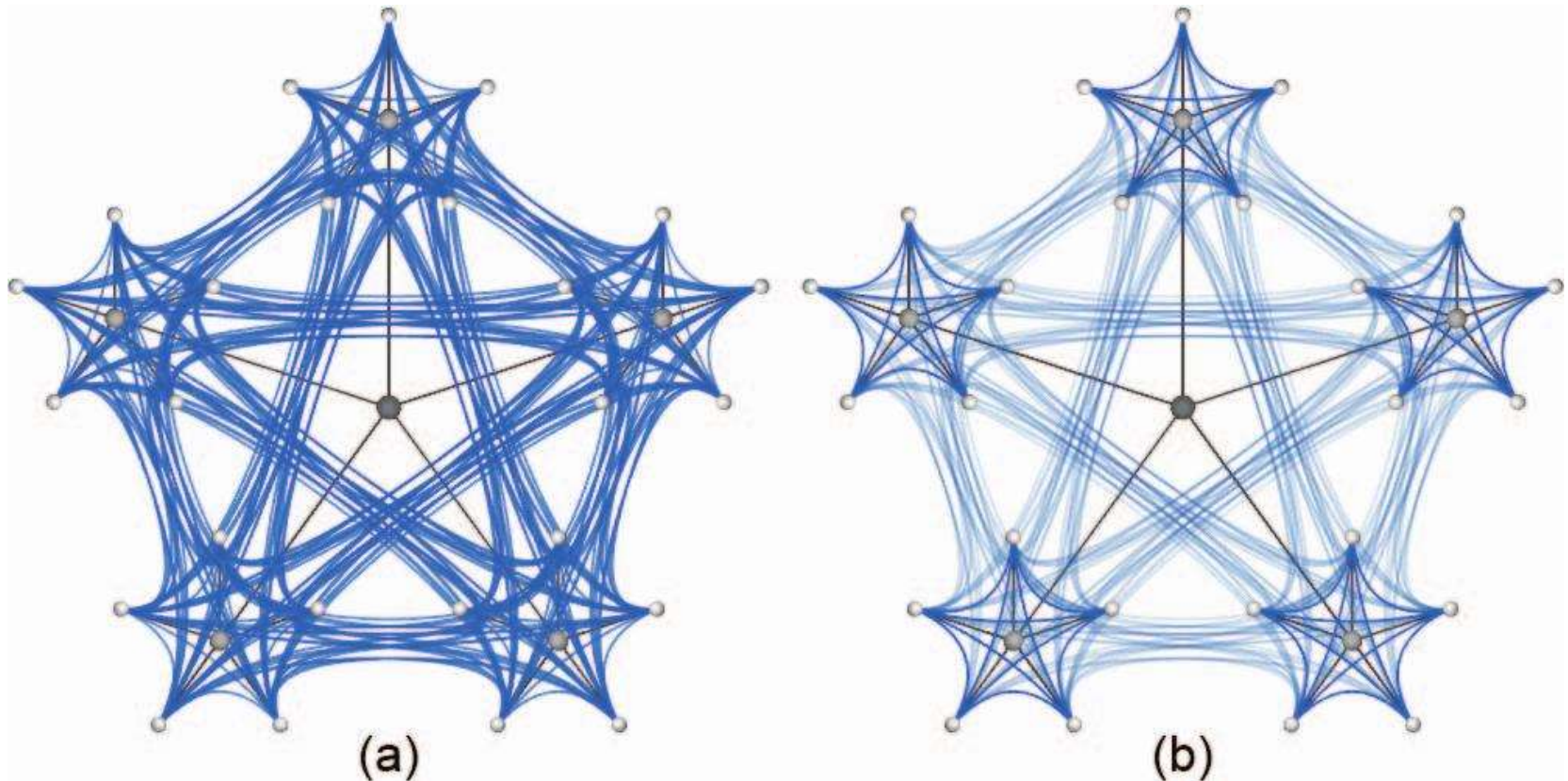
# Edge Bundling: Interaction

- **Interactive filtering** and straightening helps with disambiguation:



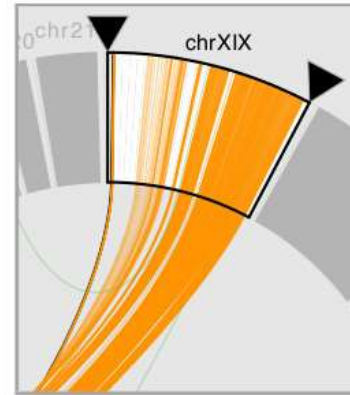
# Edge Bundling: Transparency

- Making longer bundles more **transparent** allows us to more easily see shorter edges

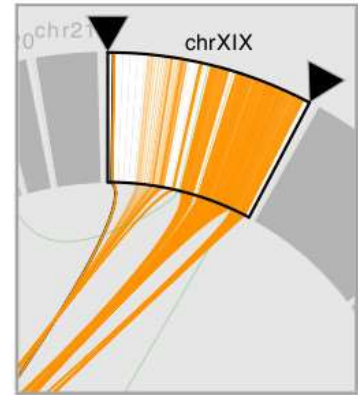


# Edge Bundling in Comparative Genomics

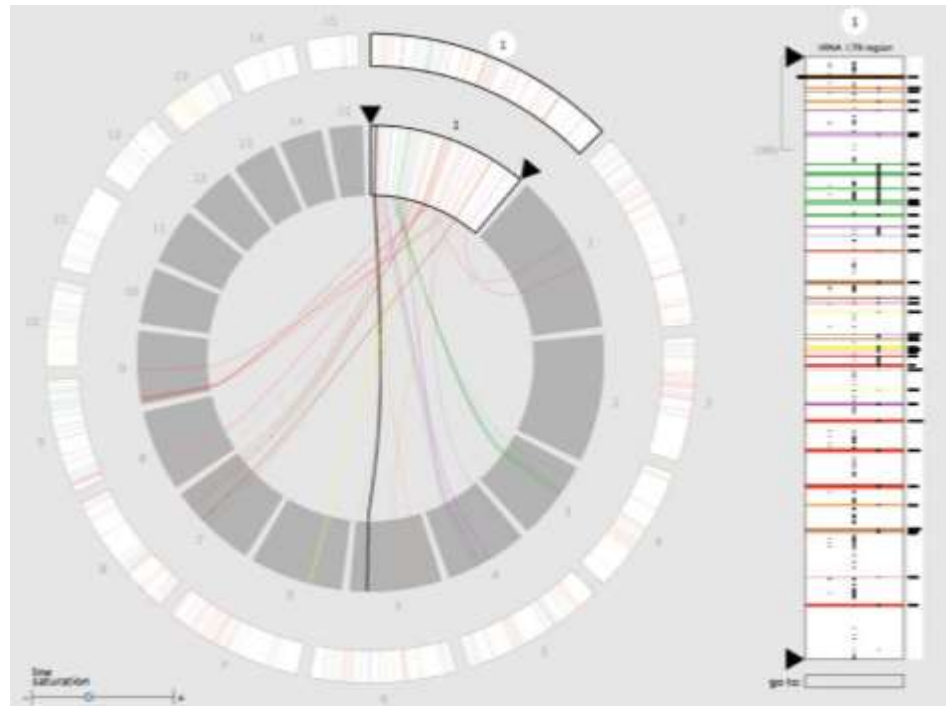
- MizBee [Meyer et al. 2009] bundles edges from the same source block that are preserved in the same target chromosome



(a)



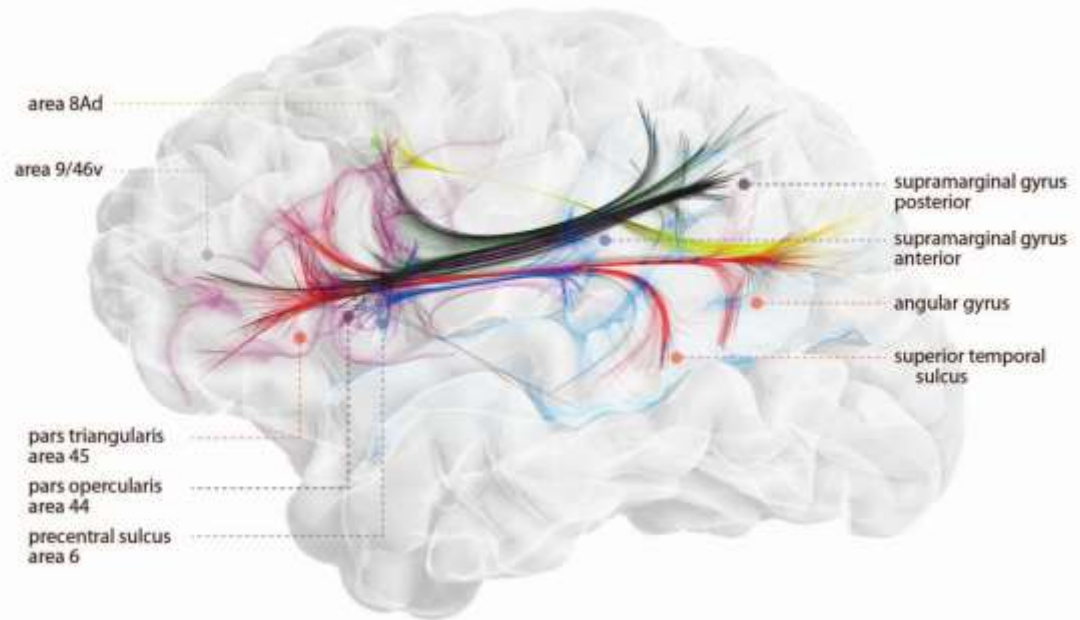
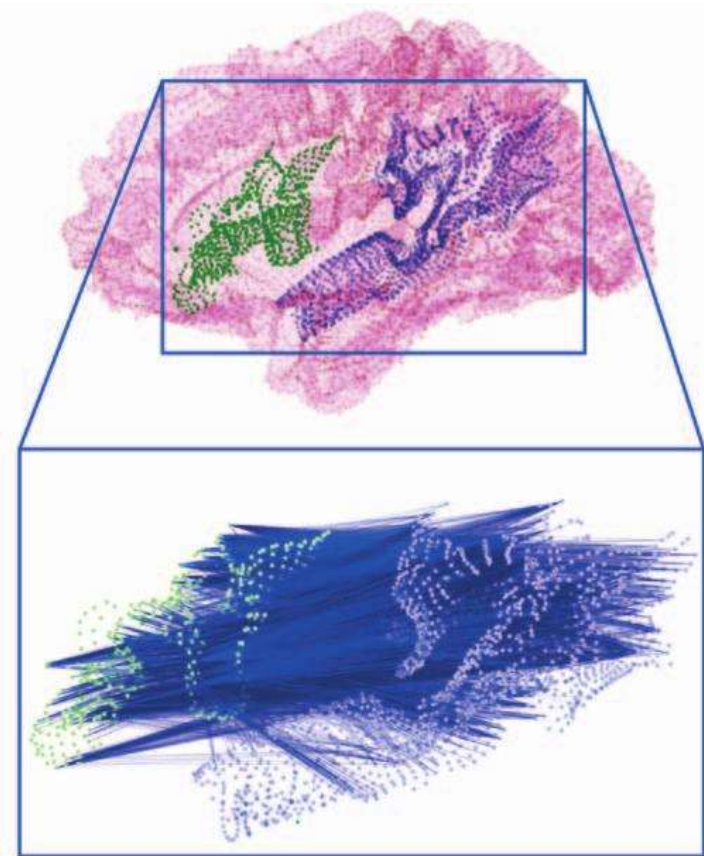
(b)





# Edge Bundling in Neuroimaging

- [Böttger et al. 2014] iteratively move similar edges in 3D fMRI connectivity graphs towards each other



# Summary: Visualizing Graphs

- **Node-and-link** visualizations of general graphs
  - Force-directed layouts
- Techniques for **specific classes of graphs**
  - Layered layouts for directed acyclic graphs
  - Circular layouts
- **Edge bundling** can clean up visualizations of graphs with many edges
  - First introduced for “inclusion+adjacency” graphs
  - Idea has been applied widely

# References

- Matthew O. Ward, Georges Grinstein, Daniel Keim. **Interactive Data Visualization: Foundations, Techniques, and Applications.** A. K. Peters/CRC Press, 2010
- Roberto Tamassia (Ed). **Handbook of Graph Drawing and Visualization.** CRC Press, 2013