Jun.-Prof. Dr. Thomas Schultz
Michael Ankele (ankele@cs.uni-bonn.de)
Shekoufeh Gorgi Zadeh (gorgi@cs.uni-bonn.de)

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Bioinformatics II

Assignment Sheet 7

If you have questions concerning the exercises, please write to our mailing list:
vl-bioinf@lists.iai.uni-bonn.de.

We strongly encourage you to continuously work on the assignments and contact us with questions. However, you will only have to hand in your results (for all sheets of the second project) on January 31.

Exercise 1 (EM Algorithm for Image Segmentation, *17 Points*)

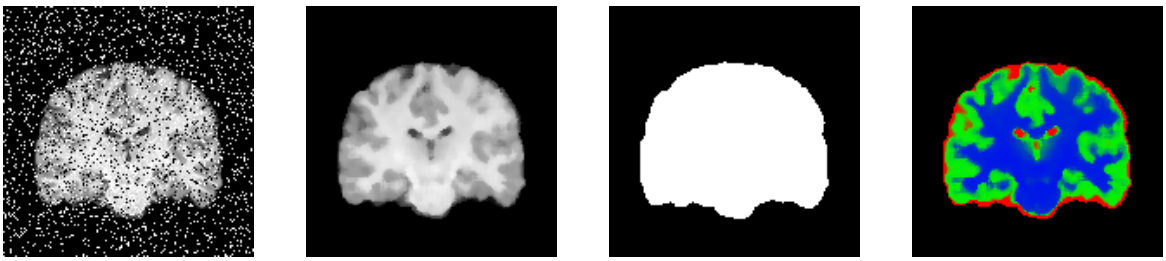

Figure 1: From left to right: The input image, after denoising, the binary mask, segmented brain image.

Last week, you implemented a simple thresholding-based image segmentation. This week, you will implement a Gaussian Mixture Model to produce a probabilistic segmentation, and to find suitable parameters automatically, by using the EM algorithm. Proceed in the following steps:

a) Load the same image `brain.png` and reduce the noise as in sheet 6. Produce a binary mask that marks all pixels with an intensity greater than zero. In all further steps, only treat pixels within that mask. (2P)
b) Initialize the parameters of a three-compartment Gaussian mixture model to some reasonable values and use them to compute the responsibilities ρ_{ik} of cluster k for pixel i. (4P)
c) Visualize the responsibilities by mapping the probabilities of belonging to the CSF, gray matter, and white matter clusters to the red, blue, and green color channels, respectively. Please submit the resulting image. (1P)
d) Use the update rules provided in the lecture to re-compute the parameters μ_k, σ_k, and π_k. (4P)
e) Iterate the E and M steps of the algorithm until convergence. Please submit the final parameter values, a visualization of the final responsibilities, and your code. (3P)
f) Create and submit a plot that illustrates the convergence of your algorithm. (3P)

Exercise 2 (Updating σ_k in the EM Algorithm, *4 Points*)

Derive the update rule for σ_k that is performed in the M-step of the EM algorithm:

$$\sigma_k^2 = \frac{\sum_{i=1}^{n} \rho_{ik}(x_i - \mu_k)^2}{N_k}$$

Hint: The derivation is very similar to the one for μ_k, which was shown in the lecture.

Exercise 3 (Convergence of K-means, *4 Points*)

The EM algorithm can be considered as an extension of K-means. Is the K-means algorithm guaranteed to converge after a finite number of steps? If so, why? Does your answer depend on the exact way in which collapsing clusters are detected and treated?

Good Luck!