GRADIENT DESCENT WITH A PRECONDITIONER

Our goal is to solve $\min_x f(x)$, where $f : \mathbb{R}^n \to \mathbb{R}$ is a convex differentiable function.

Let P be an $n \times n$ symmetric positive definite matrix. We can define the (weighted) inner product $\langle x, y \rangle_P = \langle Px, y \rangle$. This new inner product defines a new gradient $\nabla_P f(x)$ (why?), so we may as well consider a new gradient descent:

$$x_{k+1} = x_k - \alpha \nabla_p f(x_k) \tag{1}$$

TODO:

- Find an explicit form of $\nabla_P f(x)$.
- Think of other ingredients/assumptions you need (such as the Lipschitzness of $\nabla_P f$) and prove the convergence of (1).
- Hint: you should understand well main ingredients of the standard proof of GD in the convex case and adjust them to your setting.