TODO NOTES

1. Linear systems

Goal: solve $Ax = b$ with $A \in \mathbb{R}^{n\times n}$ (possibly psd A), $b \in \mathbb{R}^n$.
A fixed point equation that corresponds to the linear system is

$$x = x - M^{-1}(Ax - b) \tag{1.1}$$

for any regular matrix $M \in \mathbb{R}^{n\times n}$. This equation suggests a successive iteration method

$$x_{k+1} = x_k - M^{-1}(Ax_k - b) \tag{1.2}$$

It converges whenever $\rho(I - M^{-1}A) < 1$.

Y: *Explain why and explain connection of* (1.2) *to gradient descent.*

If we represent $A = A_L + A_D + A_U$ as a sum of lower triangular, diagonal and upper triangular matrices and take $M = A_D + A_L$, we obtain the Gauss-Seidel method:

$$x_{k+1} = (A_D + A_L)^{-1}(b - A_U x_k) \tag{1.3}$$

Let in (1.1) $M = A_D$. Then we will arrive at the Jacobi method:

$$x_{k+1} = A_D^{-1}(b - (A_L + A_U)x_k) \tag{1.4}$$

You can read more on such and similar methods, for instance in this book, Chapter 4 or any other online material you will find

2. Adaptive stepsizes

(1) Read the paper (the first 6 pages) and understand its analysis.
(2) Improve its analysis for a convex quadratic function $f(x) = \frac{1}{2}\langle Ax, x\rangle - \langle b, x\rangle$.
(3) Use these new stepsize in the setting of Section 1.