Efficient Epidemics: Contagion, Control, and Cooperation in a Global Game*

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Abstract

We study disease control in a game of imperfect information. While disease control games of perfect information have multiple equilibria, we show that even a vanishing amount of uncertainty forces selection of a unique equilibrium. This primal distinction leads to several new results. In well-identified cases, an epidemic will occur albeit it is inefficient and could be avoided. More harmful diseases are less likely to become an epidemic and may cause fewer deaths. We also study cooperation and let some players commit to control the disease whenever the expected benefit is sufficiently high. Cooperation facilitates selection of an efficient equilibrium.

Keywords: global games, epidemics, disease eradication, privately provided public goods

JEL Codes: I18, H41, C72

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1 Introduction

Epidemics are costly. Though intuition might attribute the rise of an epidemic to mere misfortune, that explanation leaves open to question how individual behavior affects the likelihood of an epidemic's onset. This paper highlights the strategic considerations leading up to an epidemic. Our game of imperfect information is distinct from existing studies in economics that either do not use game theory (Kremer, 1996; Geoffard and Philipson, 1997; Gersovitz and Hammer, 2003), or assume perfect information (Barrett, 2003). Our approach yields several new, and important, insights.

Game theory provides a tool to model and analyze strategic interactions between rational decision-makers. It captures the inherently strategic nature of disease control problems, where any private agent's optimal behavior critically depends on what other agents are doing. However, extant game theoretic analyses of disease control, see Barrett (2003) in particular, have multiple equilibria. Equilibrium multiplicity dwindles the predictive ability of these models. The outcome of a game with multiple equilibria is not, a priori, determined – there is no apparent connection between a disease's fundamental properties and its eventual fate. In such games, an epidemic is mere misfortune indeed.

The indeterminacy of outcomes in existing models comes about through a combination of perfect information and the dynamic of infections. A disease spreads when infected individuals infect others. Any individual's private efforts at controlling the spread of a disease thus boost the likelihood that other individuals' private efforts are successful (in the sense of them not contracting the disease). This type of mutual reinforcement, when combined with perfect information, oft leads to a manifold of equilibria and coordination failure (*c.f.* Van Huyck et al. (1990)).

While our game maintains the natural dynamic of infections, we study disease control under payoff uncertainty, which makes for a global game (Carlsson and van Damme, 1993; Morris and Shin, 1998). Global games are a class of imperfect information games where players are uncertain about some underlying fundamental of the game but receive private noisy signals of it. In our model, players are uncertain about the (net) expected benefit of controlling a disease.

There are at least two reasons to consider this type of uncertainty. First, epidemics have a highly multi-dimensional (negative) impact on society, making a comprehensive idea of their costs hard to grasp. The costs extend far beyond the mere expenditures on health care or illness-related loss of productivity (Shastry and Weil, 2003). They can lead to civil conflict (Cervellati et al., 2017, 2018) and cause impaired development in youth (Bleakley, 2003; Coffey et al., 2017). An epidemic may call the government's legitimacy into question (Flückiger and Ludwig, 2019) and can even affect a country's institutions (Acemoglu et al., 2001, 2003). Second, the epidemiological literature suggests that new diseases – by their nature subject to many uncertainties – will appear increasingly often in the near future (see Rappuoli, 2004, for a comprehensive review).

Even a vanishing amount of uncertainty leads to equilibrium uniqueness in our disease control game. While equilibrium uniqueness is well-established for global games generally, see especially Frankel et al. (2003), it is a new insight in the literature on epidemics. The advantage of a unique equilibrium over multiple equilibria is that it allows for sharper predictions. In our global game, a more harmful disease is less likely to become an epidemic and impairs fewer people. In terms of welfare, an epidemic can occur even though this is inefficient and could have been avoided. These results embed well-known predictions from the epidemiological studies within a strategic framework (*c.f.* Capasso and Serio (1978); Epstein et al. (2008); Fenichel et al. (2011)) and are supported empirically (Ahituv et al., 1996; Philipson, 1996).

After proving equilibrium uniqueness and deriving its implications, we consider a simple dynamic extension of the game to study cooperation. For reasons exogenous to the model, prior to the outbreak of a disease some subset of players forms a coalition. Membership to the coalition entails a pledge to take disease control measures whenever the expected benefit of control is above some exogenous threshold. This can be considered a type of ex ante cooperation among players that facilitates coordination when the need arises. To our knowledge, we are the first to study this type of commitment in a global game. We show that such a pledge helps selecting a more favorable equilibrium, decreasing the likelihood of an epidemic.

Our model yields additional insights on how societies can prepare for future disease outbreaks. Lowering the number of players in the game (for example, by planning eradication efforts at a more aggregated level) or lowering eradication costs also help to avoid future epidemics. Though intuitively plausible, extant game theoretic analysis of disease control do not yield these implications. Our model is the first to embed intuitive policy solutions within a strategic framework.

Our model is sufficiently general to allow for varying interpretations, ranging from community-level epidemics to full-fledged pandemics. If we think of players as citizens, the measures implemented to control the spread of a disease may consist of social distancing. If we think of players as countries, they can include a lockdown or border closures. Whichever of these interpretations is most suitable will depend on the specificities of the disease considered. Some diseases very easily spread globally (COVID-19 or the Spanish flu). Others will be more geographically restricted, for example because their transmission relies on specific vectors (malaria), or because the disease spreads through poor sanitary conditions only present in certain regions or continents (cholera).

The remainder of this paper is organized as follows. Section 2 presents the building blocks of our model and the main results. Section 3 incorporates cooperation into the model. Section 4 concludes. The proofs of our main results are in the text and the proofs of additional results are in the Appendix.

2 The Model

Let there be N players, indexed i and acting simultaneously. Following Barrett (2003), player i can either exert effort to control the spread of the disease ($x_i = 1$), or not ($x_i = 0$). Throughout the paper, we will use the term "effort", though it is understood that this is an umbrella term describing any attempt toward containment or eradication. We write C for the cost of effort.

Conditional on n players exerting effort, let the probability that these efforts are successful be $p(n)$, where p is strictly increasing in n. We normalize $p(0) = 0$ and $p(N) = 1$. Note than $p(n+1) > p(n)$ and $p(n) < 1$ for all $n < N$, meaning that successful control becomes more likely as more players exert effort yet is not guaranteed unless all players do. This formalizes the idea that diseases spread when infected individuals infect others, or, on a larger scale, that a disease may spread to one country via another. As the reintroduction of smallpox in Botswana from South Africa illustrates, this possibility is real (Fenner et al., 1988). One can interpret $1 - p(n)$ as the (conditional) probability of an epidemic.

A player's benefit from successful control of the disease is B, drawn uniformly from $[\underline{B}, \overline{B}]$. We assume $\underline{B} < B_0 < B_1 < \overline{B}$, where $B_0 = C/p(N) = C$ and $B_1 = C/p(1)$ demarcate strict dominance regions. If $B < B_0$, the disease is harmless and it never pays to exert efforts towards controlling it, in view of the costs involved, e.g. childhood chickenpox (McKendrick, 1995). More dramatically, if $B > B_1$, the disease is so severe that a player will always want to control it, such as smallpox (Fenner et al., 1988). Finally, when $B \in (B_0, B_1)$, players play a coordination game – individual best-responses are mutually dependent and any player will want to exert effort if and only if sufficiently many others do so too. Figure 1 illustrates the a priori support of B.

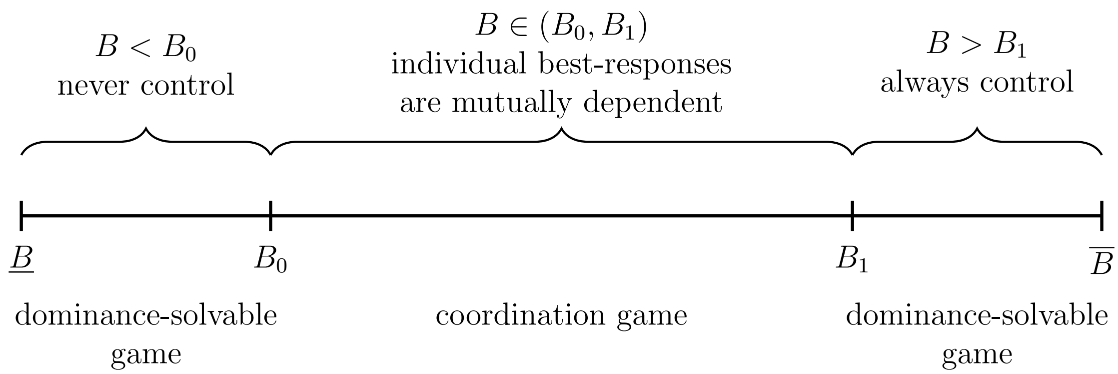

Figure 1: Support of benefit parameter B.

Given n players $j \neq i$ play $x_j = 1$, the payoff to player i is:

$$u_i(x_i; B, n) = [p(n + x_i) \cdot B - C] \cdot x_i, \tag{1}$$

where $p(n + x_i) \cdot B$ is the *expected benefit* from controlling the spread of the disease. We normalize payoffs relative to no effort ($x_i = 0$), so player i exerts effort if and only if the expected payoff thereof is positive: $u_i(x_i = 1; B, n) \geq 0$. Since p is increasing, the expected payoff to effort is increasing in the number of other players exerting effort.

Our normalization of (1) does not impose that player i's payoff to free-riding (playing $x_i = 0$) be constant in the number n of players exerting effort (playing $x_j = 1$). It is allowed that free-riding becomes more beneficial as n goes up. All we require is that the expected benefit of exerting effort increases *more*. For example, an individual always benefits from others wearing face masks but may benefit even more if he wears one himself. This type of strategic complementarity (Bulow et al., 1985) will not characterize any and all diseases or each and every type of effort (e.g. it does not appear to be an appropriate description of the decision to vaccinate). Our analysis applies to environments in which these incentives are at play. We believe this to be the case for example when players are more aggregated entities such as regions or countries. Recent work by Harrison and Jara-Moroni (2020) suggests that our main predictions continue to hold when individual effort choices are strategic *substitutes*.[1] We focus on strategic complements because this is the framework where our global games approach produces the sharper predictions than extant game-theoretic analyses.

Proposition 1 (Perfect information: equilibria)**.** *In the game of perfect information, for all $B \in (B_0, B_1)$, there are two pure strategy Nash equilibria, one in which $x_i = 1$ for all i, another in which $x_i = 0$ for all i.*

Proposition 1 establishes equilibrium multiplicity in the perfect information game and is equivalent to Barrett's (2003) Proposition 3. This means that no posterior on the probability of an epidemic is rationally favored over another. One cannot predict the likelihood of an epidemic from the benefit B.

A perfectly informed social planner, who knows B and maximizes social welfare $W = \sum_i u_i(x_i; B, n)$, would dictate $x_i(B) = 0$ for all $B < B_0$ and $x_i(B) = 1$ for all $B \geq B_0$. When $B \in (B_0, B_1)$, an epidemic is inefficient. This is Barrett's Proposition 5.

2.1 Global Game

In the global game, let it be common knowledge that B is drawn from the uniform distribution on $[\underline{B}, \overline{B}]$. Rather than observe B directly, each player i receives a private noisy signal b_i of B, given by:

$$b_i = B + \varepsilon_i.$$

Here, ε_i is the noise in player i's signal, a random variable drawn i.i.d. from the uniform distribution on $[-\varepsilon, \varepsilon]$, with $\varepsilon > 0$ a measure of the uncertainties surrounding the disease. By construction, players' private signals are correlated as all have the same mean; however, conditional on this mean, signals are independent.

[1] See the discussion following Theorem 2 for more details.

In the global game, individual policies are chosen to maximize:

$$u_i^e(x_i; b_i, n) = \frac{1}{2\varepsilon} \int_{b_i-\varepsilon}^{b_i+\varepsilon} [p(n + x_i) \cdot B - C] \cdot x_i \, dB \tag{2}$$

which, ceteris paribus, is increasing in b_i. Observe that equation (2) is almost exactly equation (1) if ε becomes small.

We solve the global game by iterated dominance. For iterated dominance to work, there should be signals which support any of the two actions (effort vs. no effort) as a strictly dominant strategy. This means that:

$$0 < 2\varepsilon < \min\{B_0 - \underline{B}, \overline{B} - B_1\}. \tag{3}$$

Henceforth, we assume that (3) holds.

Theorem 1 says that uncertainty about the benefit B forces selection of a unique equilibrium. The equilibrium strategy is increasing: a player will try to control a disease only if the benefit is sufficiently large. It does not pay to run the risk of eliminating a fairly harmless disease. The contrast with the perfect information game is stark.

Theorem 1 (Unique equilibrium)**.** *Given C, for any $B \in [\underline{B}, \overline{B}]$, the game has a unique Bayesian Nash equilibrium. For all $i \in \{1, 2, ..., N\}$, let x_i^* denote the associated equilibrium strategy. Then there exists a unique $b^* \in (B_0, B_1)$ such that, for all $i \in \{1, 2, ..., N\}$:*

$$x_i^*(b_i) = \begin{cases} 1 & \textit{if} \quad b_i \geq b^* \\ 0 & \textit{if} \quad b_i < b^* \end{cases}. \tag{4}$$

Theoretically, Theorem 1 is a special case of theorem 1 in Frankel et al. (2003). Due to the generality of their result, however, we think intuition in our specific model may be served if we provide a simple constructive for the 2-player case. The case where

$n \geq 2$ is proven in the Appendix.

Proof. Start from B_0. If player $i = 1, 2$ observes a signal $b_i < B_0$, the action $x_i = 0$ is conditionally dominant. This means that, given a signal b_1, player 1 attaches a minimum posterior probability to the event that $x_2 = 0$, given by $\Pr[x_2 = 0; b_1] \geq \Pr[b_2 < B_0; b_1]$.

Since player 1's expected payoff to playing $x_1 = 1$ is highest when player 2 plays $x_2 = 1$, the *maximum* expected utility to playing $x_1 = 1$, given the signal b_1, is simply:

$$\Pr[b_2 < B_0; b_1] \cdot p(1) \cdot b_1 + [1 - \Pr[b_2 < B_0; b_1]] \cdot b_1 - C,$$

which is increasing in b_1. Let B_0^1 be the signal that solves

$$\Pr[b_2 < B_0; B_0^1] \cdot p(1) \cdot B_0^1 + [1 - \Pr[b_2 < B_0; B_0^1]] \cdot B_0^1 - C = 0.$$

Since $p(1) < 1$, it is immediate that $B_0^1 > B_0$. For all $b_1 < B_0^1$, the highest possible expected payoff to playing $x_1 = 1$ is negative, and so $x_1 = 0$ is dominant. The exact same argument can be made swapping player 1 for player 2. We conclude that *if* it is common knowledge that no player i will play $x_i = 1$ when $b_i < B_0$, *then* no player plays $x_i = 1$ when $b_i < B_0^1$.

Given player i plays $x_i = 0$ for all $b_i < B_0^1$, we can find a B_0^2 such that for all $b_i < B_0^2$ player i plays $x_i = 0$. Repeating this argument over and over, we construct a sequence $(B_0^k)_{k=0}^{\infty}$, consecutive elements of which are the solution to

$$\Pr[b_2 < B_0^k; B_0^{k+1}] \cdot p(1) \cdot B_0^{k+1} + [1 - \Pr[b_2 < B_0^k; B_0^{k+1}]] \cdot B_0^{k+1} - C = 0,$$

for all $k \geq 0$, where $B_0^0 = B_0$. This implies that $B_0^{k+1} > B_0^k$ for all $k \geq 0$. Note that $B_0^k \in [B_0, B_1]$ for all k. Hence, $(B_0^k)_{k=0}^{\infty}$ converges to a point $B_0^* \in [B_0, B_1]$. By iterated dominance, each player i plays $x_i = 0$ for all $b_i < B_0^*$. The top panel of Figure 2 illustrates.

Starting at B_1, we can perform a symmetric procedure, constructing a sequence $(B_1^k)_{k=0}^{\infty}$ which is decreasing and has a limit $B_1^* \in [B_0, B_1]$, as shown in the bottom panel of Figure 2. Since both $(B_0^k)_{k=0}^{\infty}$ and $(B_1^k)_{k=0}^{\infty}$ are converging, we note that $\lim_{k\to\infty}|B_l^{k+1} - B_l^k|= 0$ for $l = 0, 1$, and so $\lim_{k\to\infty} \Pr[b_j < B_l^k; B_l^{k+1}] = 1/2$. The limits are therefore characterized by:

$$\frac{1 + p(1)}{2} \cdot B_0^* - C = \frac{1 + p(1)}{2} \cdot B_1^* - C = 0,$$

from which it is immediate that $B_0^* = B_1^* (= b^*)$. ■

For the intuition, recall that any $B \in (B_0, B_1)$ supports two Nash equilibria in the game *without* uncertainty, making it impossible for players to guess each other's actions. When B is not known and players receive a private noisy signal of it, a player knows neither the precise value of B nor the signal received by any other player. However, player i *does* know that the signal $b_j (j \neq i)$ must be drawn from $(b_i - 2\varepsilon, b_i + 2\varepsilon)$. For a high (low) b_i, player i therefore knows that the signal received by j includes a region that supports $x_j = 1$ ($x_j = 0$) as a strictly dominant strategy. Players in the global game are hence forced to construct upper or lower bounds on the probability that any other player will (not) exert effort, which is not the case in a complete information game. This starts the process of iterated dominance that leads to a unique equilibrium.

If player i observes signal $b_i = b^*$, then i's posterior distribution on the signal b_j received by any player $j \neq i$ is symmetric around b^*. In this case, and only in this case, player i believes j's signal to be either above or below b^* with equal probability. How is b^* characterized?

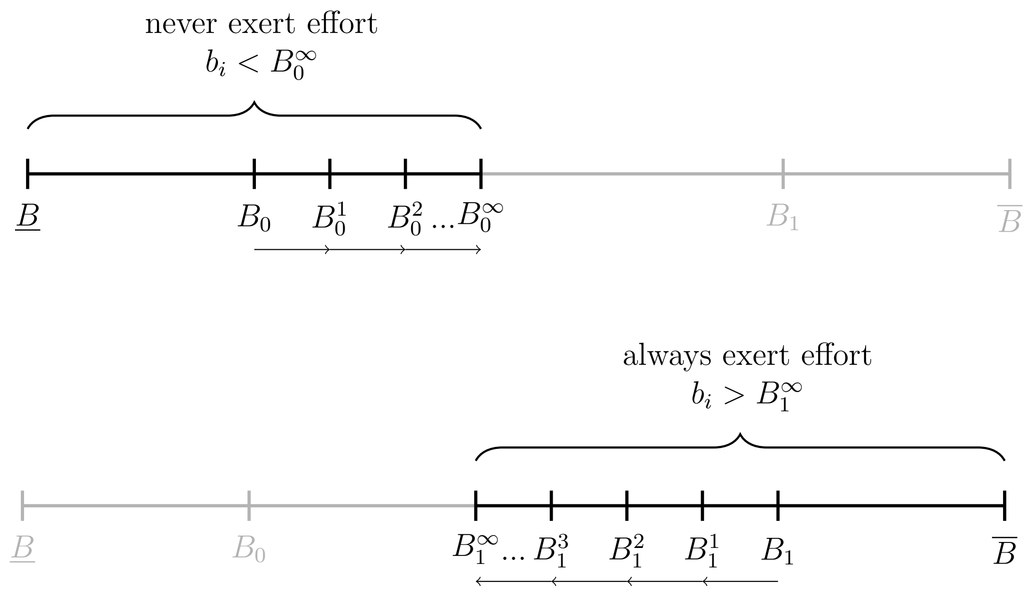

Figure 2: Iterated dominance illustrated.

Proposition 2 (Characterization)**.** *The threshold b^* is given by:*

$$b^* = \frac{2^{N-1}}{\sum_{k=0}^{N-1} \binom{N-1}{k} p(k+1)} \cdot C. \tag{5}$$

For a given number $N \geq 2$ of players, b^* is bounded by:

$$C < \frac{2^{N-1} \cdot C}{2^{N-1} - 1} < b^* < 2^{N-1} \cdot C. \tag{6}$$

In the special case that p is linear (i.e. $p(n) = n/N$), this simplifies to:

$$b^* = \frac{2N}{N+1} \cdot C, \tag{7}$$

which is increasing in N, the number of players. For $N = 1$, we have $b^* = C$. For $N \to \infty$, we have $b^* = 2C$.

Proof.

1. If i receives $b_i = b^*$, its posterior is that $b_j > b^*$ (and $b_j < b^*$) with probability $1/2$ (and $1/2$), for all $j \neq i$. Hence, i thinks that $x_j = 0$ or $x_j = 1$ each with probability $1/2$.

2. There are a total of N players, so there are $N-1$ players who are not player i. By the previous point, the probability of any vector $(x_j)_{j \neq i}$ is therefore $(1/2)^{N-1}$.

3. If there are $N-1$ other players, the number of different vectors $(x_j)_{j \neq i}$ that contain exactly k ones and $N-1-k$ zeroes is $\binom{N-1}{k}$.

4. Hence, the total probability that $\sum_{j \neq i} x_j = k$ is simply the probability of any vector times the number of possible vectors that contain precisely k zeroes: $(1/2)^{N-1} \cdot \binom{N-1}{k}$.

5. Given k players $j \neq i$ play $x_i = 1$, the expected benefit to player i (from playing $x_i = 1$), who has observed signal b^*, is $p(k+1) \cdot b^*$.

6. The expected payoff to player i from playing $x_i = 1$, given $b_i = b^*$, is therefore: $b^* \sum_{k=0}^{N-1} \frac{1}{2}^{N-1} \binom{N-1}{k} p(k+1) - C$.

7. Solving for the b^* that makes this expected payoff equal to zero, we obtain the proposition.

The boundaries in (6) follow directly from evaluating (5) at $p(n) = 0 \; \forall \; n < N$ and $p(n) = 1 \; \forall \; n > 1$, respectively. In the linear-p case, equation (5) reduces to:

$$b^* = \frac{2^{N-1} \cdot N}{\sum\limits_{k=0}^{N-1} \binom{N-1}{k} (k+1)} \cdot C.$$

The denominator of this expression can be rewritten as $\sum_{k=0}^{N-1}\binom{N-1}{k}(k+1) = \sum_{k=0}^{N-1}\binom{N-1}{k} + \sum_{k=0}^{N-1}k\binom{N-1}{k} = 2^{N-1} + (N-1)2^{N-2} = 2^{N-1}(2+(N-1)/2)$. Plugging this rewritten denominator into (2.1) yields the specification for a linear p. ■

Our characterization of b^* is qualitatively similar to that in Morris and Shin (1998). A direct analytical comparison is not possible due to difference between the models considered.

For two probability functions p_1 and p_2, let $p_1 \leq p_2$ mean that $p_1(n) \leq p_2(n)$ for all $n = 0, 1, ..., N$ with a strict inequality for at least one n. We say that p_2 is greater than p_1.

Proposition 3 (Comparative statics).

(i) The threshold b^* is monotone increasing in C;

(ii) The distance between B_0 and b^* is strictly increasing in C;

(iii) The threshold b^* is decreasing in the probability function p.

It is intuitive that b^* is unambiguously increasing in C, the cost of effort. When the costs are higher, a player is exposed to greater (more costly) risk when exerting effort. To compensate, the expected benefit should go up as well. In terms of policy implications, this implies that the probability of an epidemic can be decreased by lowering the cost of effort. Similarly, looking at (7), the threshold for taking effort is *increasing* in N, the number of players. Hence, if the game is played by fewer actors, the probability that an epidemic is avoided increases. For policy, this suggests that more aggregated decision making (e.g. at the level of the European Union rather than individual countries) may help prevent epidemics. Finally, observe that a higher p may be interpreted as stricter social distancing or border closures as both tend to decrease the probability that one individual infects another. Intuitively, our model suggests that strong social distancing avoids epidemics.

The decentralized equilibrium and the social planner solution coincide for $B \geq b^*$ and for $B < B_0$. They differ for all $B \in (B_0, b^*)$, when the social planner solution is to control a disease whereas the decentralized solution is not to. In these cases, an epidemic is inefficient.

Proposition 4 (Inefficiency)**.** *For all $B \in (B_0, b^*)$, an epidemic is inefficient. Moreover:*

(i) For all $B < b^* - \varepsilon$, there will *be an epidemic. For all $B > (b^* + \varepsilon)$, there* will not *be an epidemic;*

(ii) For all $B \in (b^* - \varepsilon, b^* + \varepsilon)$, the probability of an epidemic is monotone decreasing in B.

(iii) The ex ante (before B is drawn) probability of an inefficient epidemic is strictly increasing in C and strictly decreasing in the probability function p.

Proposition 5 (Speed bump effect)**.** *A more lethal disease ($B > b^* + \varepsilon$) causes fewer deaths than a less lethal one ($B < b^* - \varepsilon$).*

Proposition 4 says that a disease is more likely to be controlled if the benefit of successful control is higher. While this may sound obvious, note that such a stochastic dominance result is not true in a game of perfect information with multiple equilibria (Proposition 1). Relatedly, the proposition tells us that an inefficient epidemic will occur only for relatively low B.[2]

Proposition 5 states that more harmful diseases cause fewer deaths. This is true since a sufficiently harmful disease, for which $B > b^* + \varepsilon$, will certainly be controlled

[2]By the way, note that the only source of inefficiency, in equilibrium, is that an epidemic occurs even though the eradication benefits outweigh the costs. It is never possible, in equilibrium, that an epidemic is avoided even though society would be better off having one.

in equilibrium whereas a milder disease, for which $B < b^* - \varepsilon$, will not. While the benefit B is a generic term, encompassing many aspects of a disease's severity, it is likely correlated with its fatality rate. This would mean that a higher B tends to indicate a more deadly disease, motivating our interpretation of the Proposition.

Finally, our unique equilibrium result – and its implications such as the speedbump effect or the possibility of an inefficient epidemic-equilibrium – also holds true for the case of heterogeneous players. Let the cost of effort for player i be C_i. Furthermore, let the benefit from disease control to player i be B_i, with $(B_i)_{i=1}^N$ drawn uniformly from $[\underline{B}, \overline{B}]^N$. For each player i, the signal b_i is drawn uniformly from $[B_i - \varepsilon, B_i + \varepsilon]$. It is understood that $\underline{B} < B_0 < B_1 < \overline{B}$, where B_0 and B_1 demarcate strict dominance regions for *all* C_i, and this is common knowledge.

Theorem 2 (Heterogeneous players)**.** *Given $(C_i)_{i=1}^N$, for any $(B_i)_{i=1}^N \in [\underline{B}, \overline{B}]^N$, the game has a unique Bayesian Nash equilibrium. For all $i \in \{1, 2, ..., N\}$, let x_i^* denote the associated equilibrium strategy. Then there exists a unique $(b_i^*)_{i=1}^N \in (B_0, B_1)^N$ such that, for all $i \in \{1, 2, ..., N\}$:*

$$x_i^*(b_i) = \begin{cases} 1 & \text{if} \quad b_i \geq b_i^* \\ 0 & \text{if} \quad b_i < b_i^* \end{cases} . \tag{8}$$

Players might also differ is their individual contribution to the probability of successful control. We do not analyze this case explicitly. However, it is fairly straightforward to see that Theorem 2 will continue to hold qualitatively. One need only realize that a player's posteriors on B and b_j, all $j \neq i$, would still be monotone continuous increasing in b_i; the game clearly preserves strategic complementarity. Hence, the same argument that underlies Theorems 1 and 2 still applies.

Harrison and Jara-Moroni (2020) recently showed that global games with strategic substitutes and sufficiently precise signals will have a unique equilibrium if players are heterogeneous. Similar to Theorem 2, this equilibrium is in increasing strategies. While we do not study strategic substitutes explicitly, their result shows that our predictions, including the comparative statics and the speed bump effect, are not specific to our base model of strategic complements. They appear to be fundamental properties of modeling disease control as a global game.

3 Committed Coalitions

In the previous section, we showed that the decentralized equilibrium outcome may be inefficient. An important question is then how one can increase the probability that the decentralized equilibrium coincides with the social planner solution. One way of trying to achieve this could be the provision of public information about B, as discussed in Angeletos and Pavan (2007). We propose another option: ex ante cooperation.

For reasons exogenous to the model, prior to the outbreak of a disease $\bar{n} \leq N$ players form a coalition. (Note that $\bar{n}$ effectively returns our static game from the previous section). Each player i in the coalition credibly commits to exerting efforts whenever a disease arises for which control is perceived to be of some minimum benefit $b^c \in [B_0, b^*]$. That is, each i in the coalition commits to playing strategy $x_i^*(b_i) = 1$ for all $b_i \geq b^c$. Precisely what constitutes a credible commitment lies beyond the scope of our analysis. Rather, the question of interest is how, *given* such commitments are made, the equilibrium and its properties will be affected. Without loss of generality, reshuffle the set of players so that the coalition consists of all players $i \in \{1, 2, ..., \bar{n}\}$. Players still act simultaneous. To our knowledge, we are the first to study this type of commitment in a global game.

Theorem 3 (Equilibrium with a coalition)**.** *Given $\bar{n}$, the game has a unique Bayesian*

Nash equilibrium. For all $i \in \{\bar{n}+1, ..., N\} \supseteq \{N\}$*, let* $x_i^*(\cdot; \bar{n})$ *denote the associated equilibrium strategy. Then, conditional on* $\bar{n}$*, there exists a unique* $b^*(\bar{n})$ *such that, for all* $i \in \{\bar{n}+1, ..., N\}$*:*

$$x_i^*(b_i) = \begin{cases} 1 & \textit{if} \quad b_i \geq b^*(\bar{n}) \\ 0 & \textit{if} \quad b_i < b^*(\bar{n}) \end{cases}. \tag{9}$$

Moreover, $b^*(\bar{n})$ *is monotone decreasing in* $\bar{n}$*, with* $b^*(0) = b^*$ *and* $b^*(N) = b^c$*.*

While the unique equilibrium is passed on from the static to the dynamic game, its properties are importantly different.

Proposition 6 (Inefficiency with a coalition)**.** *For all* $B \in (B_0, b^*(\bar{n}))$*, an epidemic is inefficient. Moreover,*

(i) The probability of an inefficient epidemic is decreasing in $\bar{n}$*, the number of players in the coalition;*

(ii) The probability of an epidemic is monotone decreasing in the (true) benefit B*.*

Theorem 3 and Proposition 6 qualify a call for strong ex ante cooperation. If players can commit to try and control diseases for which the benefit thereof is sufficiently high, the results show that inefficient epidemics are more likely to be avoided.

4 Conclusion

This paper studies disease control in a global game. Our approach stands in contrast to the existing game theoretic literature. While "eradication games" of perfect information have multiple equilibria, the global game has a unique equilibrium. This primal distinction leads to several derivative, yet important, results. First, our model can predict when an (in)efficient epidemic occurs. Second, diseases for which the benefit of successful control is higher are more likely to be controlled (or eradicated). Third, and paradoxically, diseases that are more costly a priori end up being less costly to society, precisely because these get eradicated. Our analysis is the first to embed these well-known predictions from epidemiology models within a strategic framework.

Our results clearly demonstrate that an epidemic may occur even when this is inefficient. One way to avoid this dismal outcome is to lower eradication costs. Another possibility is to reduce the set of players, for example by having countries coordinate policies through more aggregated bodies such as the European or African Unions. Third, strong social distancing or hard border closures can contribute to contain a disease. While intuitive, extant game theoretic models do not support these conclusions.

An alternative solution is strong ex ante cooperation (rather than aggregation) between players. We support this claim in a dynamic extension of our static game, in which a subset of players forms a coalition prior to the outbreak of a disease. Members of the coalition credibly commit to exert efforts towards disease control whenever the benefit is perceived to be sufficiently high. We show that credible commitment catalyzes coordination on eradication efforts and decreases the likelihood of inefficient epidemics.

There are at least two ways to think about a coalition. First, as the outcome of intensive ex ante cooperation among players. This is perhaps most intuitively, though not exclusively, thought of as international cooperation between countries through a supra-national entity such as the World Health Organization. Second, as a reduced-form description of a sequential global game, where a subset of players determines its actions first, and only then do the remaining choose theirs.

This paper offers a new perspective on the economics of disease control. Existing studies either do not rely on game theory (Kremer, 1996; Geoffard and Philipson, 1996, 1997; Gersovitz and Hammer, 2003; Epstein et al., 2008; Fenichel et al., 2011), or make the strong assumption of perfect information (Barrett, 2003). Our global game incorporates the strategic incentives underlying the spreading of a disease while taking explicit account of uncertainty. Our results provide useful insights on how societies can prepare for future disease outbreaks, highlighting the need for increased cooperation to avoid future epidemics.

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Appendix

Proof of Theorem 1

Denote $n = \sum_{j \neq i} x_j$. Let $F_i(B; b_i)$ denote i's posterior density of B, given its signal b_i. Let $G_i(n; B, \beta)$ denote the density of n, conditional on B *and* assuming all players $j \neq i$ play strategy $x_j(b_j) = 0$ for all $b_j < \beta$ and $x_j(b_j) = 1$ for all $b_j \geq \beta$. For given B, the density $F_i(B; b_i)$ is continuously (weakly) decreasing in b_i. Moreover, for given B, $G_i(n; B, \beta)$ is continuously (weakly) increasing in β, and given β, $G_i(n; B, \beta)$ is continuously (weakly) decreasing in B.

Given the "strategy" summarized by β, player i's rationally expected payoff is given by $\iint u_i(x_i; B, n)\, dG(n; B, \beta)\, dF(B; b_i)$, which is strictly and continuously increasing in b_i and n.

Having set the stage, we proceed with the proof in three steps and each step corresponds to a lemma in the proof. The first step starts at $b_i = B_0$ and uses iterated elimination of strictly dominated strategies to find the signal that makes player i indifferent between eradicating the disease or not; The second step does the analogous exercise starting from $b_i = B_1$; The third and last step shows that the signals found in the previous steps are the same.

Proof. Since the payoff to exerting effort toward controlling the disease is increasing in

n, the *maximum expected payoff* to exerting effort for player i is obtained by assuming all players $j \neq i$ exert effort unless doing so is a dominated strategy. Because $x_j(b_j) = 0$ is dominant for all $b_j < B_0$, the initial density of the maximum global disease control efforts is therefore $G(n; B, B_0)$.

Lemma 1 (Maximum payoff)**.** *There exists a unique $\bar{b}$ such that the maximum expected payoff to exerting effort is 0 iff $b_i = \bar{b}$, for all i, where $\bar{b}$ solves: $\iint u_i(x_i; B, n)\, dG(n; B, \bar{b})\, dF(B; \bar{b}) = 0$.*

Proof. Start at $b_i = B_0$. It is immediate that $\iint u_i(x_i; B, n)\, dG(n; B, B_0)\, dF(B; b_i = B_1) > \int u_i(x_i; B, n = 0)\, dF(B; b_i = B_1) = 0$, where the equality follows from the fact that B_1 demarcates the strict dominance of $x_1 = 1$ (and the fact that noise is distributed uniformly). Define B_1^1 as the point that solves: $\iint u_i(x_i; B, n)\, dG(n; B, B_0)\, dF(B; b_i = B_0^1) = 0$. That is, assuming no j plays a dominated strategy, then i's minimum payoff to playing $x_i(b_i)$ is positive for all $b_i < B_0^1$. Since this is true for all players, all $i \in \{1, 2, ..., N\}$ will play $x_i(b_i) = 0$ for all $b_i < B_0^1$. Hence, knowing this, the maximum expected payoff to exerting effort for i is obtained by assuming that all players j for whom $b_j \geq B_0^1$ play $x_j(b_j) = 1$.

Proceeding this way, for all $k \geq 1$ let us inductively define $\iint u_i(x_i; B, n)\, dG(n; B, B_0^k)\, dF(B; b_i = B_0^{k+1}) = 0$. We thus obtain a sequence $(B_0, B_0^1, B_0^2, ...)$, where $B_0 < B_0^k < B_0^{k+1}$ for all $k > 0$. A monotone sequence defined on a closed real interval converges to a point in the interval, which we call $\bar{b}$. Since $\bar{b}$ is the limit of our sequence $(B_0, B_0^1, B_0^2, ...)$, it by definition solves $\iint u_i(x_i; B, n)\, dG(n; B, \bar{b})\, dF(B; \bar{b}) = 0$. ■

The *minimum expected payoff* to exerting disease control effort is obtained by assuming no player $j \neq i$ will exert effort unless it is a dominant strategy. Since $x_j(b_j) = 1$ is dominant for all $b_j \geq B_1$, the initial distribution of n consistent with the minimum expected payoff to exerting effort is given by $G(n; B, B_1)$.

Lemma 2 (Minimum payoff)**.** *There exists a unique $\underline{b}$ such that the minimum expected payoff to exerting effort is 0 iff $b_i = \underline{b}$, for all i, which solves: $\iint u_i(x_i; B, n)\, dG(n; B, \underline{b})\, dF(B; \underline{b}) = 0$.*

Proof. Start from $b_i = B_1$. The proof then follows the structure in Lemma 1, but assumes minimum (rather than maximum) rational disease control efforts. That is, we define B_0^1 as the solution to $\iint u_i(x_i; B, n)\, dG(n; B, B_1)\, dF(B; b_i = B_1^1) = 0$. Thereafter, for all $k \geq 1$, we inductively define B^{k+1} to solve $\iint u_i(x_i; B, n)\, dG(n; B, B_0^k)\, dF(B; b_i = B_0^{k+1}) = 0$. We thus obtain a sequence $(B_1, B_1^1, B_1^2, ...)$, where $B_1 > B_1^k > B_1^{k+1}$ for all $k > 0$. A monotone sequence on a closed real interval converges to a point in the interval. We call this point $\underline{b}$. Being the limit of the inductively defined sequence, it is the unique solution to $\iint u_i(x_i; B, n)\, dG(n; B, \underline{b})\, dF(B; \underline{b}) = 0$. ■

Lemma 3. $\bar{b} = \underline{b}$.

Proof. By definition:

$$\iint u_i(x_i; B, n)\, dG(n; B, \bar{b})\, dF(B; \bar{b}) = 0 = \iint u_i(x_i; B, n)\, dG(n; B, \underline{b})\, dF(B; \underline{b}),$$

■

so $\underline{b} = \bar{b}$. We label $b^* = \underline{b} = \bar{b}$. Since even the minimum payoff to exerting effort is positive for all $b_i \geq b^*$, any rational strategy x_i^* must satisfy $x_i^*(b_i) = 1 \iff b_i \geq b^*$. Moreover, since even the maximum gain to exerting effort is negative for all $b_i < b^*$, any rational strategy must satisfy $x_i^*(b_i) = 0 \iff b_i < b^*$. ■

Proof of Proposition 4

Proof. Inefficiency follows from that fact that $B > C$.

(i) The probability mass of players i with signal $b_i < \bar{b}$ is decreasing in B for all $\bar{b}$.

Hence, in particular it is for $\bar{b} = b^*$. Finally, since p is increasing in n, the number of players for whom $b_i < b^*$ decreases the probability of successful disease control.

(ii) For all $B < b^*$, there exists a $\varepsilon > 0$ small enough such that $B + \varepsilon < b^*$. But $b_i \leq B + \varepsilon$ for all i. Yet $x_i^*(b_i) = 0$ for all $b_i < b^*$. Given $C < B$, the result follows. ■

Proof of Proposition 5

Proof. Immediate from combining the fact that $p(0) = 0$ and $p(N) = 1$ with the equilibrium strategy given in Theorem 1. ■

Proof of Theorem 2

Proof. Follows readily from the proof of Theorem 1. ■

Proof of Theorem 3

Proof. Fix $\bar{n}$. $F(B; b_i)$ still corresponds to the posterior on B, conditional on b_i. Now, however, let $G(n; B, \beta, \bar{n})$ denote the density of n, conditional on B *and* assuming all players $j\{\bar{n}+1, \bar{n}+2, ..., N\}$ play strategy $x_j(b_j) = 0$ for all $b_j < \beta$ and $x_j(b_j) = 1$ for all $b_j \geq \beta$, while all players $i \in \{1, 2, ..., \bar{n}\}$ play the strategy as given by assumption for the coalition. The posterior G is still continuous in all of its arguments. It is immediate that $G(n; B, \beta, \bar{n}) \geq G(n; B, \beta, \bar{n}') \iff \bar{n}' \leq \bar{n}$, for all B, where the inequality is strict if $\bar{n}' < \bar{n}$ and $\beta \in [B - \varepsilon, B + \varepsilon]$. Moreover, note that the case of no coalition (Theorem 1) corresponds to $\bar{n} = 0$ in the extended model.

It therefore must be that for $\bar{n} > 0$, for all b_i, we have $\iint u_i(x_i; B, n)\, dG(n; B, \beta, \bar{n})\, dF(B; b_i) \geq \iint u_i(x_i; B, n)\, dG(n; B, \beta, 0)\, dF(B; b_i)$, which inequality is strict whenever $b_i \in [\beta - 2\varepsilon, \beta + 2\varepsilon]$. But this means that:

$$\iint u_i(x_i; B, n)\, dG(n; B, b^*, \bar{n})\, dF(B; b^*) > \iint u_i(x_i; B, n)\, dG(n; B, b^*, 0)\, dF(B; b^*) = 0, \tag{10}$$

implying that there exists a $b^*(\bar{n}) < b^*$ for which $\iint u_i(x_i; B, n)\, dG(n; B, b^*(\bar{n}), \bar{n})\, dF(B; b^*(\bar{n})) = 0$. But defining $b^*(\bar{n})$ this way, we can conclude (by the same argument as for Lemma 3 in the proof of Theorem 1) it is a unique equilibrium strategy for all players $i \in \{\bar{n}+1, \bar{n}+2, ..., N\}$ to play the strategy given in Theorem 3.

Finally, from the monotonicity of G it is clear that $b^*(\bar{n})$ is decreasing in $\bar{n}$. ■

Proof of Proposition 6

Proof. Immediate from the fact that $b^*(\bar{n}) \leq b^*$ ($B_0 < b^*$), combined with equilibrium strategies for players outside the coalition (in the coalition). ■