Linking Cap-and-Trade Schemes Under Asymmetric Uncertainty

Roweno J.R.K. Heijmans*

Reyer Gerlagh†

Tilburg University

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Abstract

Recent years have seen a rapid increase in the number of cap-and-trade schemes to mitigate greenhouse gas emissions. With many independently operating systems, policy discussions have turned to the topic of linking. This paper offers a theory of optimal linking. We show that an efficient linkage adjusts the aggregate cap in response to inter-scheme trades of allowances. Compared to standard linking, our proposal has two major advantages. First, it increases global welfare by efficiently adjusting the cap in response to private information implicitly contained in inter-scheme trades. Second, post-linking price volatility is lower with an endogenous cap. The latter advantage may alleviate existing political barriers to linking such as imported price volatility. A key concept in our analysis is asymmetric uncertainty. Interestingly, while asymmetric information generally decreases welfare, asymmetric uncertainty compensates for part (or, in extreme cases, all) of that welfare loss.

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*Email: r.j.r.k.heijmans@uvt.nl

†Email: r.gerlagh@uvt.nl.

1 Introduction

The number of cap-and-trade schemes to mitigate greenhouse gas emissions has grown steadily. Reduced to its core, a cap-and-trade scheme caps CO_2 emissions by allocating allowances to emitters who are then allowed to trade their permits. Economists argue that such a policy combines the conservative certainty on emissions offered by more prescriptive command-and-control regimes with an efficient allocation of abatement efforts brought about by trading.

"The first major emissions trading system (ETS) for greenhouse gases – the European Emissions Trading System (EU ETS) – was established in 2005. To date, there are 20 ETSs in place across five continents and covering 27 jurisdictions which produce almost 40 % of global wealth (GDP). With over a dozen more governments considering or having already scheduled an ETS, emissions trading has emerged as a key instrument to cost effectively decarbonize our economies." (ICAP, 2020)

When multiple cap-and-trade schemes coexist, it is possible to establish linkages. Linking is seen as a promising development in cap-and-trade regulation (Mehling et al., 2018) and Article 6 of the Paris Agreement – the last to be ratified – expressly provides for it. Linking reciprocally enables the use of permits issued in one scheme to meet compliance obligations pursuant to another and has become increasinly prominent in recent years. California's cap-and-trade system linked with Quebec's on 1 January 2014 and the linked jurisdictions hold auctions together.[1] On 1 January 2020, a link between the European Union's Emissions Trading System (EU ETS) and the Swiss Emisions Trading System went into force. Linkages between the Regional Greenhouse Gas Initiative (RGGI) and the Emissions Trading Systems of Virginia and Pennsylvania are currently on their way, as are implicit linkages between California's ETS and Washintong State's Clean Air Rule.

Linkages are thought to be efficient because they allow for flows of allowances that lead to an ex post equalization of marginal abatement costs across jurisdictions without affecting marginal climate damages, which depend on global emissions only (Carbone et al., 2009; Flachsland et al., 2009; Doda and Taschini, 2017; Mehling et al., 2018; Doda et al., 2019; Holtsmark and Weitzman, 2020). An additional benefit may be that, through their increased cooperation, local planners are less likely to choose their policies noncooperatively (Mideksa and Weitzman, 2019), leading to a more efficient tradeoff between the costs of climate change and emission abatement.

[1] Starting from 2018, the ETS of Ontatio was also part of this linked system. However, half a year after the link was formally established, the Ontario government revoked its cap-and-trade regulation, effectively withdrawing it from the linked carbon market.

This paper proposes a simple theory of optimal linking. We build our analysis on the basic observation that a flow of allowances between two linked schemes signals information about the true abatement costs in both. Once this information is revealed, it may turn out to be optimal to adjust the global emissions cap. We formulate a policy that filters the maximum amount of information from the market and adjusts the global cap accordingly. Importantly, simple trading ratios (*c.f.* Holland and Yates, 2015) are never optimal. An efficient allocation of abatement efforts equalizes marginal abatement costs within and between schemes. If emissions cannot be traded one-for-one, firms have an incentive to trade beyond the perfect equalization of marginal abatement costs.

A key concept is asymmetric uncertainty. What matters for the optimal linking of cap-and-trade schemes is not how uncertain abatement costs in either scheme are per se. Rather, the degree to which uncertainty differs between systems turns out crucial. Our finding has a clear intuition. Trade flows signal a wedge in jurisdictional marginal abatement costs. At first, this is a relative observation only, pertaining to a differential in jurisdictional costs. We however show that planners can also learn something about absolute marginal costs by anchoring their updated beliefs about the most volatile scheme on those about the most predictable one. There is more scope for learning about more uncertain jurisdictions.[2]

Linking cap-and-trade systems across jurisdictions is related to linking cap-and-trade markets over time (Yates and Cronshaw, 2001). The latter type of dynamic integration was studied in Heutel (2020) and Pizer and Prest (2020) for flow externalities, and in Gerlagh and Heijmans (2020) for stock externalities. They show that smart dynamic instruments can (greatly) improve welfare. But it matters how the cap is endogenized. For EU ETS, Gerlagh and Heijmans (2019) and Gerlagh et al. (2020b) illustrate several undesirable side-effects of endogenous intertemporal emission caps. This literature therefore offers an important lesson for linking jurisdictional cap-and-trade schemes: adjusting the aggregate cap to trade flows can be efficient, but the devil is in the details.

2 Model

Consider two jurisdictions, North and South, each operating its own cap-and-trade scheme. The assumption of two jurisdictions is not restrictive for our purposes. One may simply consider North a representative jurisdiction for two linked jurisdictions, East and West, where trade between North and South is essentially a reduced-form way of writing trade

[2]This is the same intuition that underlies statistical filters like the Kalman filter.

between East, West, and South.

Each jurisdiction i is populated by firms whose production of goods causes emissions as a byproduct. The benefit of producing an amount $\tilde{e}_i$ of goods is $B_i(\tilde{e}_i;\theta_i)$, given by:

$$B_i(\tilde{e}_i \mid \theta_i) = (p_i^* + \theta_i)(\tilde{e}_i - e_i^*) - \frac{b_i}{2}(\tilde{e}_i - e_i^*)^2. \tag{1}$$

Emissions yield benefits because they allow firms to produce goods and save on the cost of abatement. As an umbrella term, we refer to $B_i(\tilde{e}_|\theta_i)$ as abatement costs in jurisdiction i. The parameter θ_i is a fundamental of jurisdiction i's economy and is *private information* of its constituent firms, though it is common knowledge that $\mathbb{E}[\theta_i] = 0$, $\mathbb{E}[\theta_i^2] = \sigma_i^2$, and $\mathbb{E}[\theta_N\theta_S] = \rho\sigma_N\sigma_S$. We interpret σ_i^2 as a measure for the uncertainty about jurisdiction i's economy. We say that uncertainty is *asymmetric* if $\sigma_N \neq \sigma_S$.

Emissions cause climate change as an externality. Let the severity of the externality be denoted $C(\tilde{e}_N + \tilde{e}_S)$, given by:

$$C(\tilde{e}_N + \tilde{e}_S) = p^*(\tilde{e}_N + \tilde{e}_S - e_N^* - e_S^*) + \frac{c}{2}(\tilde{e}_N + \tilde{e}_S - e_N^* - e_S^*)^2. \tag{2}$$

Note our simplifying assumption that emissions have local benefits but global costs. We might of course imagine more complicated settings where emissions also have strictly regional costs like local air pollution. A model of this type, however, would have many moving parts which would distract us from the core question we are primarily concerned with: the optimal linking of cap-and-trade schemes to regulate a global externality.

Subtracting the costs of climate change from the sum of jurisdictional benefits, we obtain global welfare:

$$W = B_N(\tilde{e}_N \mid \theta_N) + B_S(\tilde{e}_S \mid \theta_S) - C(\tilde{e}_N + \tilde{e}_S). \tag{3}$$

Throughout the analysis, we assume that policies in both jurisdictions are chosen to maximize (3), implying that jurisdictions choose local caps to maximize global welfare even when the cap-and-trade schemes are not linked.[3] The (potential) welfare gains we describe should therefore be thought of as lower bounds compared to those obtained starting from a completely self-centered status quo in which regional, pre-linking caps are chosen to maximize regional welfare only (see Doda et al., 2019, for a decomposition of the welfare gains due to linking in this case). Our assumption appears to describe the political reality of linkages. For example, the independent but linked cap-and-trade schemes of

[3]That is, we work with what Kotchen (2018) calls the 'Global Social Cost of Carbon'.

California and Quebec organize joint auctions of allowances. For both schemed to agree, this effectively means that local but linked caps are set with joint damages in mind.

For brevity of notation, where convenient we may write $\tilde{E} = \tilde{e}_N + \tilde{e}_S$ and $E^* = e^*_N + e^*_S$. Our model is now characterized by three curves and eight parameters $(b_i, c, p^*_i, e^*_i, p^*)$. For the system to be identified, we need two parameters per curve (slope and intercept) for three curves in total (2 benefit and 1 cost). This makes for a total of six parameters. Consequently, we may take the freedom to reduce the number of parameters through defining $p^* = p^*_N = p^*_S$, with the convenient implication that (p^*, e^*_N, e^*_S) is the vector of welfare-maximizing prices and emissions for jurisdiction i, given $\theta_N = \theta_S = 0$. We label this the ex-ante optimum. This is clearly not an assumption, nor even a normalization – it is a definition.

Firms are profit maximizers. Once a policy k caps emissions at a level $\tilde{e}^k_i$, individual firms trade allowances until marginal abatement costs for all are equal to:

$$p^k_i = -b_i e^k_i + \theta_i, \tag{4}$$

which we refer to as firms' demand equation.

Before proceeding to the analysis, we introduce some further notation. Superscripts will be scenario (instrument) labels for equilibrium outcomes. Moreover, let $\tilde{x}^k$ denote the value of a variable x under policy k, then let $x^k := \tilde{x}^k - x^*$ be the deviation of x under policy k from the ex-ante expected optimal value x^*, and let $\Delta^k x := \tilde{x}^k - x^{SO}$ denote the difference between the value of x under scenario k and its ex post socially optimal value (see subsection 2.1).

2.1 Global Social Optimum

By standard arguments, marginal benefits of emissions should equal marginal costs in an efficient outcome. This implies $MB_N = MB_S$. Since marginal benefits equal prices, these are also the same, so $p^{SO}_N = p^{SO}_S = p^{SO}$. Labeling the symmetric information equilibrium as Social Optimum, we have the profit-maximization condition (4) and

$$c \cdot (e^{SO}_N + e^{SO}_S) = p^{SO},$$

so the Social Optimum is fully characterized:

$$p^{SO} = \frac{c(b_S\theta_N + b_N\theta_S)}{cb_N + cb_S + b_N b_S}, \tag{5}$$

$$e_i^{SO} = \frac{b_{-i}\theta_i + c(\theta_i - \theta_{-i})}{cb_N + cb_S + b_N b_S}, \tag{6}$$

$$E^{SO} = \frac{b_S\theta_N + b_N\theta_S}{cb_N + cb_S + b_N b_S}, \tag{7}$$

where $i \in \{N, S\}$ and $-i$ simple means "the jurisdiction that is not i".

The variance of prices is given by:

$$\mathbb{E}\left[\left[p^{SO}\right]^2\right] = \left(\frac{c}{c \cdot b_N + c \cdot b_S + b_N b_S}\right)^2 \left[b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S\right]. \tag{8}$$

We note that increasing uncertainty translates in a more volatile price. We will return to this later.

2.2 Welfare Costs

Suppose a policy k induces emission (or price) levels $\tilde{e}_i^k$ (or $\tilde{p}_i^k$) in jurisdiction i. Then from firms' equilibrium behavior (4), wee see that deviations in emissions from the social optimum scale with prices:

$$\Delta^k p_i = -b_i \Delta^k e_i. \tag{9}$$

Expected welfare losses relative to the social optimum are then given by:

$$\begin{aligned}\Delta^k W &= \mathbb{E}\left[\Delta^k B_N + \Delta^k B_S - \Delta^k C\right] \\ &= \frac{c}{2}\mathbb{E}\left[\left(\Delta^k E\right)^2\right] + \sum_i \frac{b_i}{2}\mathbb{E}\left[\left(\Delta^k e_i\right)^2\right]. \end{aligned} \tag{10}$$

If a policy admits equal prices across regions, welfare losses can also be written as a function of the common price gap:

$$\Delta^k W = \frac{1}{2}\frac{(cb_N + cb_S + b_N b_S)(b_N + b_S)}{b_N^2 b_S^2}\mathbb{E}\left[\left(\Delta^k p\right)^2\right]. \tag{11}$$

3 Policies

3.1 Regional cap-and-trade

The simplest policy operates the two cap-and-trade schemes individually. In this case, the planner of scheme i sets a cap e_i to maximize expected global welfare:

$$\max_{e_i} \mathbb{E} \quad W. \tag{12}$$

The resulting allocation is $e_N = e_S = 0$. Plugging this into (10), we obtain expected welfare losses when both jurisdictions operate their own cap-and-trade schemes individually:

$$\Delta^R W = \frac{1}{2}\frac{(c+b_S)\sigma_N^2 + (c+b_N)\sigma_S^2 - 2c\rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S}. \tag{13}$$

For future reference, it is important to note that jurisdictional cap-and-trade is the execution of this welfare program:

$$\max_{e_N, e_S} \mathbb{E} \quad W(e_N, e_S \mid \theta_N, \theta_S). \tag{14}$$

That is, jurisdictional cap-and-trade is the optimal choice of instrument under the constraint that caps must be set before any information is revealed and without using any information extracted from markets. It admits the desirable property that *expected* marginal benefits in each scheme equal marginal climate costs:

$$\mathbb{E}[MB_i \mid e_i] = MC. \tag{15}$$

We note that the RHS of (15) is perfectly known when caps are fixed, whereas marginal benefits are stochastic variables due to unknown fundamentals θ_i.

Regional cap-and-trade suffers from two inefficiencies. First, it does not guarantee an equalization of marginal abatement costs across jurisdictions (i.e. it satisfies this property only in expectations). Second, it ignores information revealed through the interactions between jurisdictions. The first of these, abatement cost equalization, can be remedied by linking schemes across jurisdictions.

3.2 Linking

When North and South link their cap-and-trade schemes, each planner sets the expected optimal cap in its jurisdiction but allowances issued in one scheme may be used to fulfill abatement obligations in another. Trading of allowances is subject to the constraint that global emissions are not affected:

$$e_N + e_S = E = 0. \tag{16}$$

When schemes are linked, climate damages will be the same as when schemes operate in isolation. However, free trading between the schemes ensures that marginal benefits are equal in both:

$$p_N = p_S. \tag{17}$$

When two schemes are linked, firms can efficiently redistribute abatement efforts in response to (unforeseen) jurisdictional differences in abatement costs, subject to the constraint that total emissions are fixed. It follows:

$$\Delta^L W = \frac{1}{2}\frac{1}{b_N + b_S}\frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S}{cb_N + cb_S + b_N b_S}. \tag{18}$$

The following proposition is now immediate:

Proposition 1. *Global welfare is always higher when cap-and-trade schemes are linked, compared to when they are not.*

Proof. We only need to compare welfare losses under a linked cap-and-trade regime, equation (18), to those under jurisdictional cap-and-trade, equation (13). Linking outperforms regional cap-and-trade iff:

$$\frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S\rho\sigma_N\sigma_S}{b_N + b_S} < (c + b_S)\sigma_N^2 + (c + b_N)\sigma_S^2 - 2c\rho\sigma_N\sigma_S$$

$$\iff$$

$$\frac{2\rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2} < \frac{cb_N + cb_S + b_N b_S}{cb_N + cb_S},$$

which is always true. □

[Insert here papers claiming this is not true, and explain what drives the difference.]

At a more fundamental level, linking can be seen as a policy that solves:

$$\max_{G} \quad \mathbb{E}\left[\max_{e_N, e_S} W(e_N, e_S \mid \theta_N, \theta_S) \quad \text{s.t.} \quad e_N + e_S = G\right]. \tag{19}$$

Practically, the planners of North and South cap local emissions at levels that, in expectations, maximize global welfare. When issued, permits can be traded on a one-to-one basis between schemes, as long as emissions overall remain fixed at the sum of the two jurisdictional caps. By linking their schemes, the planners of North and South effectively delay the determination of local emission caps until after jurisdictional abatement costs are known, guaranteeing an ex post efficiency gain through the equalization of marginal benefits, which in expectations equal marginal damages:

$$MB = MB_N = MB_S \tag{20}$$

$$\mathbb{E}[MB \mid e_N + e_S] = MC. \tag{21}$$

Though linking benefits global welfare (Proposition 1), jurisdictional effects are ambiguous. A simple thought experiment illustrates. We note that prices in equilibrium equate marginal benefits, so that the volatility of prices is equal to the volatility of marginal benefits. Hence if abatement costs in North are much less predictable than in South, $\sigma_N > \sigma_S$, North may import part of the price volatility to which South is subject. If this effect is strong enough, aggregate price volatility may increase and North may be harmed by linking with South (there are papers on this, cite here).

Linking suffers from another type on inefficiency. It disregards valuable information. Remember that allowances will be traded if, and only if, there is a wedge in jurisdictional marginal benefits under the local caps. A flow of allowances from one scheme to another therefore reveals information about θ_N and θ_S. The global cap on emissions ideally responds to these ex post observations, but does not under standard linking.

We will next develop a novel policy to regulate emissions. It combines the efficient allocation of emission reductions within a scheme with the equalization of marginal abatement costs brought about by linking. It mitigates the (relative) inefficiencies associated with standard linking. We call our policy optimal linking.

3.3 Optimal Linking

When cap-and-trade schemes link and a flow of allowances from one to the other is observed, valuable information is revealed. Our proposal is that the aggregate emissions cap be adjusted in response. In particular, we propose that if a total of e_N allowances issued in North are sold to firms in South, then the global cap on emissions is changed by an amount $f(e_N)$, where f is a cap-adjustment function chosen to maximize global welfare.[4] In our linear framework, the function f is simply linear in e_N:

$$(1-\delta)e_N = E \implies \delta e_N + e_S = 0, \tag{22}$$

where the cap-adjustment parameter δ is chosen to maximize welfare and given by:

$$\delta = \frac{b_N[\sigma_S^2 - \rho\sigma_N\sigma_S] + c[\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S]}{b_S[\sigma_N^2 - \rho\sigma_N\sigma_S] + c[\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S]}. \tag{23}$$

We return to the derivation of this policy shortly. Already here, we stress that our proposal really has two key properties. First, and most obviously, the global cap responds to inter-jurisdictional trade of allowances. This is done to make sure the information revealed in inter-scheme trades is incorporated into the global cap. Second, and more subtle though just as important, *jurisdictional schemes exchange allowances one-to-one*. This property is crucial and we cannot stress it enough. Linking creates efficiency gains because it incentives firms to equate marginal abatement costs globally. This desirable property would be lost if permits are not traded one-to-one between schemes, like with trading ratios (Holland and Yates, 2015). Indeed, only if emissions are traded ton-for-ton does individual profit maximization ensure equal marginal benefits in both jurisdictions:

$$p_N^{OL} = p_S^{OL}. \tag{24}$$

While free trade between jurisdictions stimulates the equalization of marginal abatement costs across all emitters, the global cap is adjusted to bring marginal abatement costs more in line with marginal climate costs. If we plug (23) into (22), noting (24), one can show that the policy we propose implements the solution to the following program:

$$\mathbb{E}[MB \mid e_N, e_S] = MC. \tag{25}$$

To see this, note that from the observed flow of permits between jurisdictions we can construct the difference in marginal abatement cost innovations:

$$\mu \equiv \theta_S - \theta_N = b_N e_N - b_S e_S.$$

[4]It should be clear that anchoring of f on e_N is inconsequential; one could anchor on e_S instead.

Using the demand equation (4) and plugging in μ, we find:

$$\begin{aligned}
\mathbb{E}[MB \mid e_N, e_S] &= \mathbb{E}[\theta_N|\mu] - b_N e_N \\
&= \mu \frac{\mathbb{E}[\mu\theta_N]}{\mathbb{E}[\mu^2]} - b_N e_N \\
&= \mu \frac{\rho\sigma_N\sigma_S - \sigma_N^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} - b_N e_N \\
&= -\frac{\sigma_S^2\rho - \sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_N e_N - \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_S e_S
\end{aligned}$$

This can be equated to marginal damages:

$$c(e_N + e_S) = -\frac{\sigma_S^2\rho - \sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_N e_N - \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} b_S e_S,$$

which for convenience we rewrite as

$$\delta e_N + e_S = 0.$$

Solving for δ, we obtain (23), which shows that our optimal linking policy indeed implements the solution to (25).

Note that if $\delta = 1$, optimal linking coincides with standard linking of schemes. Since the planners are free to set $\delta = 1$ but not required to do so, it is clear that optimal linking outperforms standard linking. In fact, looking at (23), we see that δ will generally *not* be equal to 1:

$$\delta \lesseqgtr 1 \iff \frac{b_N}{b_S} \lesseqgtr \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_S^2 - \rho\sigma_N\sigma_S}. \tag{26}$$

Only in very exceptional parametric conditions do we find $\delta = 1$. Since δ is chosen to maximize welfare it immediately follows that linking with an endogenous cap always outperforms standard linking, in most cases strictly.

Theorem 1. *Optimal linking is strictly welfare-superior to both jurisdictional cap-and-trade and standard linking. Welfare is given by:*

$$\Delta^{OL} W = \frac{1}{2} \frac{b_N + b_S}{cb_N + cb_S + b_N b_S} \frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}. \tag{27}$$

Proof. In Appendix. □

Another way to see that optimal linking is better than standard linking is to think in terms of efficient use of information. This can be done by looking at the mathematical program either policy solves. Endogenous cap-and-trade implements the solution to $\mathbb{E}[MB \mid e_N, e_S] = MC$, which is different $\mathbb{E}[MB \mid e_N + e_S] = MC$, the program for linking. Because (e_N, e_S) provides more precise information than the mere aggregate $e_N + e_S$, optimal linking should give at least as good an outcome as standard linking. From this discussions follows an important lessen. Optimal linking not 'just another' cap-and-trade policy. It is *the* most efficient implementable quantity-based regulation across jurisdictions. It equalizes marginal costs and expected marginal benefits given the finest information available in observed trades.

The fact that optimal linking increases global welfare compared to standard linking or jurisdictional-cap-and-trade does not guarantee it to be a politically viable avenue for future carbon markets. Though the welfare of North and South jointly increases, individual jurisdictions may be worse off after linking.[5] The reason is that a relatively stable jurisdiction may, through linking, expose itself to the volatile abatement costs in the other jurisdiction, importing a variable allowance price (Holtsmark and Weitzman, 2020). However, our optimal linking policy mitigates this concern by lowering price volatility compared to standard linking:[6]

Proposition 2. *Endogenous cap-and-trade admits lower price volatility than linked cap-and-trade schemes:*

$$\mathbb{E}\left[\left(p^{OL}\right)^2\right] \leq \mathbb{E}\left[\left(p^{L}\right)^2\right]. \tag{28}$$

Proof. In appendix. □

3.4 Optimal Cap Adjustments And Asymmetric Uncertainty

First of all, it must be emphasized that the cap-adjustment parameter is not a trading ratio for pollution permits (*c.f.* Holland and Yates, 2015).

Looking at (23), we observe that the cap-adjustment rate may be negative. This means that higher-than-expected emissions in one jurisdiction may translate into higher-than-expected emissions in the other jurisdiction too. This occurs for strongly positively correlated innovations and very asymmetric uncertainty: if benefits in one jurisdiction are very unpredictable but strongly correlated to those in the other, predictable jurisdiction,

[5] Of course, since the sum of welfares has gone up, it is possible to define a system of transfers so that both jurisdictions are still strictly better off in the end. Such a system of transfers appears to be politically infeasible.

[6] With inter*temporal* trading of permits, an endogenous cap can also leads to lower price volatility, see Gerlagh et al. (2020a).

an increase in the value of emissions in the latter is likely to be matched by an equally strong increase in the former. A negative cap-adjustment rate bears some resemblance with putting negative weights on observations in making (econometric) predictions.

We also observe that the share of global emission reductions absorbed by a jurisdiction is decreasing in its responsiveness of marginal benefits to emissions, that is, in β. For any adaption of global emissions to shocks, marginal costs change accordingly. Since trade leads to the ex-post equality of jurisdictional marginal benefits, and since an optimal mechanism equates jurisdictional marginal benefits to global marginal costs, for any realized pair (θ_N, θ_S) emissions change relatively less in the jurisdiction with steeper marginal benefits.

Our potentially most interesting observation is that the cap-adjustment rate tends to increase, all else equal, if the uncertainty about benefits in South (σ_S) increases. Intuitively, this means that the endogenous emissions cap should be anchored on the demand for emissions in the most predictable jurisdiction. The opportunity for learning is greatest for jurisdiction we know least about. Asymmetric uncertainty should be used as an important input into policy making.

An extreme example can illustrate the power of asymmetric uncertainty. Suppose we know marginal benefits in North perfectly, so $\sigma_N = 0$, but we are uncertain about benefits in South, $\sigma_S > 0$. The planners clearly face an environment with asymmetric uncertainty.

Suppose now that we observe an interjurisdictional trade of allowances. Since we know the exact marginal benefit curve in North, we also know marginal benefits in North for any e_N. And since permits will be traded until the point where marginal benefits in both jurisdictions are equal, we in fact know them for South as well. But this implies we in fact observe θ_S. Asymmetric uncertainty allows us to extract valuable information from the market.

Our extreme example to illustrate the power of asymmetric uncertainty is reflected in Theorem 1. If the planners have perfect knowledge about North (or South), then all abatement costs are revealed through trade and no welfare losses occur, $\Delta^{OL}W = 0$.

Corollary 1. *In the extreme case with perfect information about one of the two linked schemes, optimal linking implements the ex post optimal level of emissions in both jurisdictions.*

Without paying attention to asymmetric uncertainty, a flow of allowances from one scheme to another merely indicates that there is a wedge in marginal abatement costs under the initial allocation of allowances. Efficiency is gained by linking since marginal benefits will be equated. The allocation is only constrained Pareto optimal, though: given a potentially suboptimal cap, emissions are allocated in an efficient way. Our optimal linking policy additionally adjusts the cap in response to the information revealed in allowance trading, moving it closer to unconstrained optimal levels. We have shown that a key ingredient for smart cap-adjustments is asymmetric uncertainty. Intuitively, there is *more* scope for learning about agents whose abatement costs is *more* uncertain. If the planners want to use as much information as possible, they must allows themselves to learn differentially about either scheme in response to one and the same trade flow.

4 Prices vs. Quantities

We saw how jurisdictions can optimally link their cap-and-trade schemes. But rather than implement a cap-and-trade scheme, each jurisdiction might also a levy a carbon tax. When is one instrument favored over the other (Weitzman, 1974)?

We assume that emissions are taxed at an expected optimal rate:

$$\max_{p_N, p_S} \quad \mathbb{E} \quad B_N(e_N \mid \theta_N) + B_S(e_S \mid \theta_S) - C(e_N + e_S) \tag{29}$$

$$\text{s.t.} \quad p_i = -b_i e_i + \theta_i, \tag{30}$$

where the constraint is firms' profit maximization rule (4). To simplify the analysis, we assume that both jurisdictions have the same benefit function B, so $b_N = b_S = b$ and $\sigma_N = \sigma_S = \sigma$. Plugging this into (27), expected welfare losses when jurisdictions optimally link their cap-and-trade schemes become:

$$\frac{1}{2c+b}\frac{1+\rho}{2}\sigma^2. \tag{31}$$

Similarly, if $B_N = B_S$ then the solution to (29) will have $p_N = p_S$, that is, each jurisdiction imposes the same tax. In this case, we can plug (8) into (11) which, after some rewriting yields the following expression for expected global welfare losses when both jurisdictions tax emissions:

$$\left(\frac{2c}{b}\right)^2 \frac{1}{2c+b}\frac{1+\rho}{2}\sigma^2. \tag{32}$$

If the choice between instruments is made on the basis of welfare maximization, then taxes are favored over optimally linked cap-and-trade if and only if the following condition is satisfied:

$$b \geq 2c. \tag{33}$$

5 Summary

We propose a simple theory of optimal linking. Fairly straightforward manipulations of standard linking between cap-and-trade schemes are shown to increase global and jurisdictional welfare. The core of our argument is that the trade of allowances between jurisdictional permit markets signals valuable information which an efficient policy incorporates. Practically, our proposal is that the aggregate cap of linked schemes should be adjusted in response to trade flows between the schemes. We pin down an exact analytic formulation for this endogenous cap-adjustment and discuss its intuitive properties.

Optimal linkages allow permits to be exchanged one-to-one. A "trading ratio" that differs from one, though de facto shifting the aggregate cap indeed, would only disturb individuals firms' incentives away from the exact equalization of marginal abatement costs across schemes, which is inefficient. To implement an optimal linkage, the aggregate cap must be adjusted in response to trade, but not through trade. A straightforward way to achieve this is to either inject new permits or buy back already issued ones as called for by the observed trade flows (Hintermayer, 2020, analyzes a buyback policy in a dynamic model of EU ETS). Upward adjustment of the cap may be easy to achieve as it only requires the issuing of extra permits. Adjusting the cap downward may be more problematic, however, calling for planners to spend money on buying allowances back. This may cause distributional problems: which scheme ought to spend the money? A possible way to mitigate this concern is by having linked schemes hold joint auctions, using the revenue of the joint auction to buy back permits. Joint auctions have a political track record: they are used by the linked schemes of California and Quebec.

A key concept we exploit is asymmetric uncertainty. When two schemes are linked, information on *relative* abatement costs is revealed through trade. But when the schemes are asymmetrically uncertain, this information on relative abatement costs be used to infer (sharp) predictions about *absolute* abatement costs in either jurisdiction. The same is not possible when trade between symmetrically uncertain schemes is observed. Our results suggest that the study of asymmetric uncertainty deserves a more prominent place in environmental economics.

Cap-and-trade schemes have become a major policy instrument in the fight against climate change. In Europe alone, roughly 45% of greenhouse gas emissions are regulated by EU ETS, the world's largest market for carbon. With ever more cap-and-trade being erected, linking of regional markets has become a prominent policy development. Linkages between jurisdictional schemes already exist, and more are currently being contemplated. The are linkages between EU ETS and the Swiss ETS, between RGGI and Quebec, between

Quebec and California. The up-and-coming carbon markets of China and the post-Brexit UK will create even more possibilities for linking. Given the large amount of money and CO_2 involved in cap-and-trade, and given the increasing prevalence of linkages between jurisdictional schemes, constructive ideas on optimal linking are needed. This paper offers some thoughts.

Our theory of optimal linking follows a purely quantity-based approach. We take (flows of) emissions as an input to get aggregate emissions as an output. One could instead focus on prices, or a combination of prices and emissions. We leave such exercises for future research.

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A Derivations and Proofs

DERIVATION OF (18):

Combining the definition with the firms' FOCs, (4), we find the change in permit use by jurisdiction:

$$\Delta^L e_N = \frac{\theta_N - \theta_S}{b_N + b_S} \tag{34}$$

$$\Delta^L e_S = \frac{\theta_S - \theta_N}{b_N + b_S}. \tag{35}$$

DERIVATION OF (23):

Regional and global deviations from Socially Optimal permit use are given by:

$$\Delta^{OL}e_N = \frac{b_S}{b_N+\delta b_S}\frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S} \tag{36}$$

$$\Delta^{OL}e_S = \frac{b_N}{b_N+\delta b_S}\frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S} \tag{37}$$

$$\Delta^{OL}Q = \frac{b_N + b_S}{b_N+\delta b_S}\frac{[\delta b_S - c(1-\delta)]\theta_N + [b_N + c(1-\delta)]\theta_S}{cb_N + cb_S + b_N b_S}. \tag{38}$$

Define

$$\xi := \frac{b_N + c(1-\delta)}{b_N + \delta b_S} \implies 1-\xi := \frac{\delta b_S - c(1-\delta)}{b_N+\delta b_S}. \tag{39}$$

Welfare losses can now be written as:

$$\begin{aligned}\Delta^{OL}W &= \frac{1}{2}\frac{c(b_N+b_S)^2 + b_N^2 b_S + b_N b_S^2}{(cb_N + cb_S + b_N b_S)^2}\mathbb{E}\left[(1-\xi)\theta_N + \xi\theta_S\right]^2 \\ &= \frac{b_N+b_S}{2}\frac{(1-\xi)^2\sigma_N^2 + \xi^2\sigma_S^2 + 2\xi(1-\xi)\rho\sigma_N\sigma_S}{cb_N+cb_S+b_Nb_S}. \end{aligned} \tag{40}$$

If for notational convenience, we define:

$$\psi := \frac{1}{2}\frac{b_N + b_S}{cb_N + cb_S + b_N b_S}, \tag{41}$$

it is straightforward to derive:

$$\frac{\partial}{\partial\xi}\frac{\Delta^{OL}W}{\psi} = 2\xi\sigma_S^2 - 2(1-\xi)\sigma_N^2 + 2(1-\xi)\rho\sigma_N\sigma_S - 2\xi\rho\sigma_N\sigma_S. \tag{42}$$

The welfare-maximizing ξ^* therefore satisfies:

$$\xi^* = \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}. \tag{43}$$

From the definition of ξ, the optimal stabilization rate δ^* follows:

$$\delta^* = \frac{(b_N+c)[\sigma_S^2 - \rho\sigma_N\sigma_S] + c[\sigma_N^2 - \rho\sigma_N\sigma_S]}{(b_S+c)[\sigma_N^2 - \rho\sigma_N\sigma_S] + c[\sigma_S^2 - \rho\sigma_N\sigma_S]}, \tag{44}$$

as stated.

PROOF OF THEOREM 1:

Proof. Plugging (43) in (40), we find:

$$
\begin{aligned}
\frac{\Delta^{OL}W}{\psi} &= \left[\frac{\sigma_S^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]^2\sigma_N^2+\left[\frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]^2\sigma_S^2 \\
&+\left[\frac{\sigma_S^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]\left[\frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]\rho\sigma_N\sigma_S \\
&= \frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S} \\
&\Longrightarrow \\
\Delta^{OL}W &= \frac{1}{2}\frac{b_N+b_S}{cb_N+cb_S+b_Nb_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S},
\end{aligned}
$$

as stated. This is strictly lower than the welfare loss under traditional Trading if and only if:

$$
\begin{gathered}
2\Delta^L W - 2\Delta^{OL}W \geq 0 \\
\Longrightarrow \\
\frac{1}{b_N+b_S}\frac{b_S^2\sigma_N^2+b_N^2\sigma_S^2+2b_Nb_S\rho\sigma_N\sigma_S}{cb_N+cb_S+b_Nb_S}-\frac{b_N+b_S}{cb_N+cb_S+b_Nb_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\geq 0 \\
\Longrightarrow \\
(\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S)(b_S^2\sigma_N^2+b_N^2\sigma_S^2+2b_Nb_S\rho\sigma_N\sigma_S)-(1-\rho^2)(b_N^2+b_S^2+2b_Nb_S)\sigma_N^2\sigma_S^2\geq 0 \\
\Longrightarrow \\
[(b_S\sigma_N^2-b_N\sigma_S^2)+(b_N-b_S)\rho\sigma_N\sigma_S]^2\geq 0,
\end{gathered}
$$

which is always true. □

PROOF OF PROPOSITION 2:

Proof. We derived quantity derivations under both policies. Prices are equal in both jurisdictions, so without loss of generality we can solve for price deviations in jurisdiction 1:

$$
\begin{aligned}
\Delta^L p_N &= \frac{b_S\theta_N+b_N\theta_S}{b_N+b_S} \\
\Delta^{OL} p_N &= \frac{\delta b_S\theta_N+b_N\theta_S}{b_N+\delta b_S}.
\end{aligned}
$$

Thus:

$$\mathbb{E}\left[\left(\Delta^{L}p\right)^{2}\right] = \frac{b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2b_N b_S \rho\sigma_N\sigma_S}{b_N^2 + b_S^2 + 2b_N b_S}$$
$$\mathbb{E}\left[\left(\Delta^{OL}p\right)^{2}\right] = \frac{\delta^2 b_S^2\sigma_N^2 + b_N^2\sigma_S^2 + 2\delta b_N b_S \rho\sigma_N\sigma_S}{b_N^2 + \delta^2 b_S^2 + 2\delta b_N b_S}.$$

Writing these out, we obtain:

$$\mathbb{E}\left[\left(\Delta^{OL}p\right)^{2}\right] < \mathbb{E}\left[\left(\Delta^{L}p\right)^{2}\right] \iff (\delta - 1)\left[b_S\left(\sigma_N^2 - \rho\sigma_N\sigma_S\right) - b_N\left(\sigma_S^2 - \rho\sigma_N\sigma_S\right)\right] < 0.$$

We now invoke Proposition **??** and establish that this condition is always satisfied. $\square$