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# Climate-conscious consumers and the buy, bank, burn program

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# The Buy, Bank, Burn Program

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## Supplementary Notes

We first consider a two-period linear model that provides simple analytical results, used in the main text. Thereafter we show that the same qualitative results come out in a multi-period non-linear model.

### Two-period linear model.

First the base case: a two-period linear model. Demand is given by:

$$p_t = \bar{p}_t + \theta_t - 2cq_t \quad (1)$$

where  $p_t$  is the marginal productivity of allowances,  $\bar{p}_t$  is the expected choke price, and  $\theta_t$  is the demand shock. Market conditions follow an AR(1) process:

$$\theta_2 = \alpha\theta_1 + \mu \quad (2)$$

Firms are allowed to bank:

$$q_1 + b + k_1 = a_1 \quad (3)$$

$$q_2 + k_2 = a_2 + b \quad (4)$$

Free banking by individual firms ensures:

$$\mathbb{E}[p_2|\theta_1] = (1 + r)p_1 \quad (5)$$

The endogenous cap is implemented as:

$$a_2 = \bar{a}_2 - \zeta b \Rightarrow \quad (6)$$

$$q_2 = \bar{a}_2 + (1 - \zeta)b \quad (7)$$

Note, importantly, that in a two-period model, second-period allocations are adapted by definition only to previous-period banking (as there is only one pervious period) – for the multi-period setting, which we will discuss later, this need not be true. Note also that the decrease of second-period allowances only applies at the aggregate level. Individual savings are *not* cut. To solve for the market equilibrium, we substitute the response rate in banking. First we consider buy and burn,  $k_1 > 0, k_2 = 0$ .

$$(1 - \zeta)q_1 + q_2 = (1 - \zeta)(a_1 - k_1) + \bar{a}_2 \Rightarrow \quad (8)$$

$$(1 - \zeta)\frac{\bar{p}_1 - p_1 + \theta_1}{2c} + \mathbb{E}\frac{\bar{p}_2 - p_2 + \theta_2}{2c} = (1 - \zeta)(a_1 - k_1) + \bar{a}_2 \Rightarrow \quad (9)$$

$$(1 - \zeta)(\bar{p}_1 - p_1 + \theta_1) + \bar{p}_2 - (1 + r)p_1 + \alpha\theta_1 = 2c[(1 - \zeta)(a_1 - k_1) + \bar{a}_2] \Rightarrow \quad (10)$$

$$(1 - \zeta)\bar{p}_1 + \bar{p}_2 + (1 + \alpha - \zeta)\theta_1 - (2 + r - \zeta)p_1 = 2c[(1 - \zeta)(a_1 - k_1) + \bar{a}_2] \quad (11)$$

This gives:

$$\frac{dp_1}{dk_1} = \frac{2c(1 - \zeta)}{2 + r - \zeta} \Rightarrow \quad (12)$$

$$\frac{dq_1}{dk_1} = \frac{1 - \zeta}{2 + r - \zeta} \text{ and } \frac{dq_2}{dk_1} = \frac{(1 - \zeta)(1 + r)}{2 + r - \zeta} \Rightarrow \quad (13)$$

$$\frac{d(q_1 + q_2)}{dk_1} = \frac{(1 - \zeta)(2 + r)}{2 + r - \zeta} < 1, \quad (14)$$

We refer to the ratio on the RHS as  $\lambda_1$  in the main text. Note that in the main text, we set  $r = 0$  and take the value  $\zeta = 0.8$  from [1].

If, however, we have allowances burning in the second period, anticipated in the first period,  $k_1 = 0, k_2 > 0$ :

$$q_1 + b = a_1 \quad (15)$$

$$q_2 + k_2 = a_2 + b \quad (16)$$

Note that in our simplified two-period model, burning occurs in the second and last period, which need of course not be true for longer or even infinite time-horizons – all that matters

for the strategy to work is that at some future point in time the cap will become exogenous. Our main message is that parties need not wait burning their allowances until the ETS has ended, but can burn them as soon as the MSR does not take in new allowances, e.g. when private banking falls short of 833 MtCO<sub>2</sub> in EU ETS.

To solve for the market equilibrium, we substitute the stabilization rule in banking, and (8)-(11) becomes:

$$(1 - \zeta)q_1 + q_2 = (1 - \zeta)a_1 - k_2 + \bar{a}_2 \Rightarrow \quad (17)$$

$$(1 - \zeta)\frac{\bar{p}_1 - p_1 + \theta_1}{2c} + \mathbb{E}\frac{\bar{p}_2 - p_2 + \theta_2}{2c} = (1 - \zeta)a_1 - k_2 + \bar{a}_2 \Rightarrow \quad (18)$$

$$(1 - \zeta)(\bar{p}_1 - p_1 + \theta_1) + \bar{p}_2 - (1 + r)p_1 + \alpha\theta_1 = 2c[(1 - \zeta)a_1 - k_2 + \bar{a}_2] \Rightarrow \quad (19)$$

$$(1 - \zeta)\bar{p}_1 + \bar{p}_2 + (1 + \alpha - \zeta)\theta_1 - (2 + r - \zeta)p_1 = 2c[(1 - \zeta)a_1 - k_2 + \bar{a}_2] \quad (20)$$

This gives

$$\frac{dp_1}{dk_2} = \frac{2c}{2 + r - \zeta} \Rightarrow \quad (21)$$

$$\frac{dq_1}{dk_2} = \frac{1}{2 + r - \zeta} \text{ and } \frac{dq_2}{dk_2} = \frac{1 + r}{2 + r - \zeta} \Rightarrow \quad (22)$$

$$\frac{d(q_1 + q_2)}{dk_2} = \frac{2 + r}{2 + r - \zeta} > 1, \quad (23)$$

which is abbreviated as  $\lambda_2$  in the main text.

### Multi-period non-linear model.

We next look at the effects of allowance burning indirectly through its effect on prices in a multi-period non-linear model. A larger price effect implies that cumulative emissions are reduced more. A smaller price effect implies that cumulative emissions are reduced less. We consider a sequence of periods starting at  $t = 1$  and the emission allowance market closing at  $t = T$ . For each period, we have that banking end-of-period,  $b_{t+1}$  equals total supply minus total demand:

$$b_{t+1} = b_t + a_t(b_t) - q_t(p_t, \theta_t) - k_t \quad (24)$$

where  $a_t(b_t)$  are the auctioned allowances, which decrease with past banking,  $a'_t < 0$  for  $b_t > 0$ , and  $a'_t = 0$  for  $b_t \leq 0$ , and  $q_t$  is demand dependent on prices  $p_t$  and on market conditions  $\theta_t$ , and  $k_t$  is the allowance burning bought by NGOs, Governments, private consumers, or any other party whose emissions are not covered by the ETS. Note from

the notation that the period  $t$  allocation depends only on banking during the previous period, i.e. on the bank at the very beginning of period  $t$ :  $a_t(b_t)$ . This period-specific relationship simplifies the mathematics. However, allocations can easily be made dependent on banking in earlier periods as well, though at the cost of increased analytical complication. Importantly, though, all our results will remain (qualitatively) true. Another point worth mentioning is the apparent implicit assumption banking will always occur, which of course need not happen in the real EU ETS, and this, it might seem, could change the model. However, if banking ends before the end of the ETS (because firms would like to borrow rather than bank, and this is not permitted), then effectively trading over time is not possible anymore and prices rise less than the interest rate (Hotelling). The analysis then remains the same, qualitatively, for it should be as if the ETS with banking stopped at that year. Equilibrium on the allowance market ensures that prices develop according to the Hotelling rule:

$$\mathbb{E}_t[p_i] = (1 + r)^{i-t} p_t \quad (25)$$

and that all allowances are exhausted at the last period,  $b_{T+1} = 0$ .

To study the effect of a change in  $dk_1$  on banking and prices, taking expectations is an immaterial complication. First, we note that if supply is independent of banking,  $a'_t = 0$ , then first-period and announced last-period allowances burning are fully equivalent. By cumulating the supply-demand equation for that case we get:

$$\left( - \sum_{t=0}^T (1 + r)^{t-1} q'_t \right) dp_1 = \sum_{t=0}^T k_t \quad (26)$$

Now, we will show that with responsive auctioning,  $a'_t < 0$ , first-period allowances burning becomes less effective, unless the allowances are banked and only annihilated in the last period.

We take full derivatives for each period:

$$db_{t+1} - (1 + a'_t)db_t + (1 + r)^{t-1} q'_t dp_1 + dk_t = 0 \quad (27)$$

We first consider allowances burning in the first period,  $dk_1 > 0$ , and  $dk_t = 0$  for  $t > 1$ .

We define  $\xi_t \equiv -db_t/dp_1 > 0$ . By backwards induction, using  $db_{T+1} = 0$ , we find

$$-(1 + a'_T)db_T + (1 + r)^{T-1}q'_T dp_1 = 0 \Rightarrow \xi_T = \frac{-(1 + r)^{T-1}q'_T}{1 + a'_T} > 0 \quad (28)$$

$$db_T - (1 + a'_{T-1})db_{T-1} + (1 + r)^{T-2}q'_{T-1} dp_1 = 0 \Rightarrow \xi_{T-1} = \frac{\xi_T - (1 + r)^{T-1}q'_T}{1 + a'_{T-1}} > 0 \quad (29)$$

$$[\dots] \Rightarrow \xi_2 > 0 \quad (30)$$

From the above, we see clearly that a more responsive supply (more negative  $a'_t$ ) implies a larger  $\xi_t$ . Using  $db_1 = 0$ , for the first period we find:

$$db_2 + q'_1 dp_1 + dk_1 = 0 \Rightarrow \quad (31)$$

$$\frac{dp_1}{dk_1} = \frac{1}{\xi_2 - q'_1} > 0 \quad (32)$$

The responsive supplies reduce the effectiveness of allowance burning in the first period. If we compare a situation with fixed allowance auctioning  $a'_t = 0$ , with one with responsive auctioning  $a'_t < 0$ , larger  $\xi_t$ , we see that the responsive auctioning reduces the price effect. The effect of allowance burning in the first period is that prices increase, but banking decreases as well and thus more allowances are auctioned in next periods, offsetting part of allowances burning.

Now consider delayed allowance burning. The NGO takes allowances out of the market in period 1, but only annihilates these in period  $T$ . The only difference with the above is that the NGO carbon is counted as savings. This is equivalent to an announced allowance burning in the last period,  $k_T > 0$ . We now must work with forward induction and define  $\zeta_t \equiv db_t/dp_1 > 0$ :

$$db_2 + q'_1 dp_1 = 0 \Rightarrow \zeta_2 = -q'_1 > 0 \quad (33)$$

$$db_3 - (1 + a'_2)db_2 + (1 + r)q'_2 dp_1 = 0 \Rightarrow \zeta_3 = (1 + a'_2)\zeta_2 - (1 + r)q'_1 > 0 \quad (34)$$

$$[\dots] \Rightarrow \zeta_T > 0 \quad (35)$$

And then for the last period  $t = T$ , we get:

$$-(1 + a'_T)db_T + (1 + r)^{T-1}q'_T dp_1 + dk_T = 0 \quad (36)$$

$$\Rightarrow \frac{dp_1}{dk_T} = \frac{1}{(1 + a'_T)\zeta_T - (1 + r)^{T-1}q'_T} > 0 \quad (37)$$

A more responsive allocation  $a'_t < 0$  reduces the terms  $\zeta_t$ , thus increases the price response  $dp_1/dk_T$ .

## References

- [1] Grischa Perino, *New EU ETS Phase 4 rules temporarily puncture waterbed*, Nature Climate Change, 2018.