Linking Cap-and-Trade Schemes Under Asymmetric Uncertainty

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Abstract

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1 Introduction

Globally, the number of cap-and-trade schemes to mitigate greenhouse gas emissions has been steadily growing. Reduced to its core, a cap-and-trade scheme caps CO_2 emissions by allocating allowances to emitters who are then allowed to trade their permits. Economists argue that such a policy combines the conservative certainty on emissions offered by more prescriptive command-and-control regimes with an efficient allocation of abatement efforts brought about by trading.

When multiple cap-and-trade schemes coexist, it is possible to establish linkages. Linking reciprocally enables the use of permits issued in one scheme to meet compliance obligations pursuant to another. Article 6 of the Paris Agreement – the last to be ratified – expressly provides for the possibility of linking regional permit markets, a provision soon to be exploited. On 1 January 2020, a link between the European Union's Emissions Trading System (EU ETS) and the Swiss Emisions Trading System went into force. Two years

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earlier, the cap-and-trade schemes of California and Quebec were linked.[1] Linkages between the Regional Greenhouse Gas Initiative (RGGI) and the Emissions Trading Systems of Virginia and Pennsylvania are currently on their way.

Linkages are thought to be efficient because they equalize regional marginal abatement costs without affecting marginal climate damages, which depend on global emissions only (Carbone et al., 2009; Flachsland et al., 2009; Tuerk et al., 2009; Doda and Taschini, 2017; Mehling et al., 2018; Doda et al., 2019; Holtsmark and Weitzman, 2020). An additional benefit may be that, through their increased cooperation, local planners take the effect of emissions in their own jurisdiction on global climate damages into account.

This paper proposes a simple theory of optimal linking. We build our analysis on the basic observation that a flow of allowances between two linked schemes signals information about the true abatement costs in both. Once this information is revealed, it will generally be optimal to adjust the global emissions cap. We formulate a policy that filters the maximum amount of information from the market and adjusts the global cap accordingly. Importantly, simple trading ratios (*c.f.* Holland and Yates, 2015) are never optimal. An efficient allocation of abatement efforts equalizes marginal abatement costs within and between schemes. If emissions cannot be traded one-for-one, firms have an incentive to trade beyond the perfect equalization of marginal abatement costs.

A key concept is asymmetric uncertainty. What matters for the optimal linking of cap-and-trade schemes is not how uncertain abatement costs in either scheme are per se. Rather, the degree to which uncertainty differs between systems turns out crucial. Our finding has a clear intuition. Trade flows signal a wedge in regional marginal abatement costs. At first, this may appear to be a relative observation only, pertaining to a differential in regional costs. We however show that planners can also learn something about absolute marginal costs by anchoring their updated beliefs about the most volatile scheme on those about the most predictable one. There is more scope for learning about more uncertain regions.[2]

Linking cap-and-trade systems across regions is related to linking cap-and-trade markets over time (Yates and Cronshaw, 2001). The latter type of dynamic integration was studied in Heutel (2020) and Pizer and Prest (2020) for flow externalities, and in Gerlagh and Heijmans (2020) for stock externalities. They show that smart dynamic instruments can (greatly) improve welfare. But it matters how the cap is endogenized. For EU ETS,

[1] Initially, the ETS of Ontatio bwas also part of this linked system. However, half a year after the link was formally established, the Ontario government revoked its cap-and-trade regulation, effectively withdrawing it from the linked carbon market.

[2] This is the same intuition that underlies statistical filtering, for instance through the Kalman filter.

Gerlagh and Heijmans (2019) and Gerlagh et al. (2020b) illustrate several undesirable side-effects of endogenous intertemporal emission caps. This literature therefore offers an important for linking regional cap-and-trade schemes: adjusting the aggregate cap to trade flows can be efficient, but the devil is in the details.

Mideksa and Weitzman (2019)

2 Model

Consider two regions, North and South, each operating its own cap-and-trade scheme. To each region $i = N, S$, the benefit of producing an amount $\tilde{e}_i$ of goods is $B_i(\tilde{e}_i; \theta_i)$, given by:

$$B_i(\widetilde{e}_i \mid \theta_i) = (p_i^* + \theta_i)(\widetilde{e}_i - e_i^*) - \frac{\beta_i}{2}(\widetilde{e}_i - e_i^*)^2. \tag{1}$$

The parameter θ_i is a fundamental of region i's economy that affects how much benefit is derived from any amount of production $\tilde{e}_i$ and is *private information*, though it is common knowledge that $\mathbb{E}[\theta_i] = 0$, $\mathbb{E}[\theta_i^2] = \sigma_i^2$, and $\mathbb{E}[\theta_N \theta_S] = \rho\sigma_N\sigma_S$. We interpret σ_i^2 as a measure for the uncertainty about region i's economy. To say that uncertainty is *asymmetric* is equivalent to saying that $\sigma_N \neq \sigma_S$.

Emissions are the byproduct of economic activity. We assume the severity of the externality to depend only on global emissions and given by $C(\tilde{e}_N + \tilde{e}_S)$. The costs of climate change are:

$$C(\widetilde{e}_N + \widetilde{e}_S) = p^*(\widetilde{e}_N + \widetilde{e}_S - e_N^* - e_S^*) + \frac{\gamma}{e}(\widetilde{e}_N + \widetilde{e}_S - e_N^* - e_S^*)^2. \tag{2}$$

What we are interested in is a cooperative kind of world where the cap in either scheme is set to internalize the *global* externality cost of *regional* emissions.[3] This assumption may appear somewhat out of the ordinary as it does not coincide with the noncooperative Nash assumption typically entertained when thinking about climate change, but in fact squares well with political reality. For example, signatories of international environmental agreements like the Paris Agreement or Kyoto protocol usually agree on limiting climate change, which has an effect on all countries anyway. This effectively means that regional caps are set with global damages in mind. Subtracting the costs of climate change from the sum of regional benefits, we obtain global welfare:

$$W = B_N(\widetilde{e}_N \mid \theta_N) + B_S(\widetilde{e}_S \mid \theta_S) - C(\widetilde{e}_N + \widetilde{e}_S). \tag{3}$$

[3] That is, we work with what Kotchen (2018) calls the 'Global Social Cost of Carbon'.

Note our simplifying assumption that emissions have local benefits but global costs. We might of course imagine more complicated settings where emissions also have local costs, such as air pollution, or global benefits. A model of this type, however, would have many moving parts which would distract us from the core question we are primarily concerned with: the optimal linking of cap-and-trade schemes to regulate a global externality.

For brevity of notation, where convenient we may write $\widetilde{E} = \widetilde{e}_N + \widetilde{e}_S$ and $E^* = e_N^* + e_S^*$. Our model is now characterized by eight parameters $(\beta_i, \gamma, p_i^*, e_i^*, p^*)$. For the system to be properly identified, we need two parameters per curve (slope and intercept) for three curves in total (2 benefit and 1 cost). This makes for a total of six parameters, while we have eight. Consequently, we may take the freedom to reduce the number of parameters through defining $p^* = p_N^* = p_S^*$, with the convenient implication that (p^*, e_N^*, e_S^*) is the vector of welfare-maximizing prices and emissions for region i, given $\theta_N = \theta_S = 0$. We label this the ex-ante optimum. This is clearly not an assumption, nor even a normalization; it is a definition.

Before proceeding to the analysis, we introduce some further notation. Superscripts will be scenario (instrument) labels for equilibrium outcomes. Moreover, let $\widetilde{x}^k$ denote the value of a variable x under policy k, then let $x^k := \widetilde{x}^k - x^*$ be the deviation of x under policy k from the ex-ante expected optimal value x^*, and let $\Delta^k x := \widetilde{x}^k - x^{SO}$ denote the difference between the value of x under scenario k and its ex post socially optimal value (to be derived shortly).

Our game has the following stages:

1. The barkeeper chooses an instrument to regulate the market for cigarettes.

2. Smokers observe their individual preference shock θ_N and θ_S.

3. Trade clears the market. Prices and quantities are chosen, jointly for both smokers, consistent with utility maximization by each smoker,

$$-\beta_i e_i^k + \theta_i = p_i^k, \tag{4}$$

   while the policy rules determine the relation between quantities and prices within and across smokers.

2.1 Global Social Optimum

By standard arguments, marginal benefits of emissions should equal marginal costs in efficient outcome. This implies $MB_N = MB_S$. Since marginal benefits equal prices, these are also the same, so $p_N^{SO} = p_S^{SO} = p^{SO}$. Labeling the symmetric information equilibrium as Social Optimum, we have the profit-maximization condition (4) and

$$\gamma(e_N^{SO} + e_S^{SO}) = p^{SO}$$

so the Social Optimum is fully characterized:

$$p^{SO} = \frac{\gamma(\beta_S\theta_N + \beta_N\theta_S)}{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S}, \tag{5}$$

$$e_i^{SO} = \frac{\beta_{-i}\theta_i + \gamma(\theta_i - \theta_{-i})}{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S}, \tag{6}$$

$$E^{SO} = \frac{\beta_S\theta_N + \beta_N\theta_S}{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S}, \tag{7}$$

where $i \in \{N, S\}$ and $-i$ simple means "the region that is not i".

Price-variation is:

$$\mathbb{E}\left[\left[p^{SO}\right]^2\right] = \left(\frac{\gamma}{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S}\right)^2 \left[\beta_S^2\sigma_N^2 + \beta_N^2\sigma_S^2 + 2\beta_N\beta_S\rho\sigma_N\sigma_S\right].$$

We note that increasing uncertainty translates in a more volatile price. We will return to this later.

2.2 Welfare Costs

Suppose a policy k induces emission (or price) levels $\tilde{e}_i^k$ (pr $\tilde{p}_i^k$) in region i. Then from firms' equilibrium behavior (4), wee see that deviations in emissions from the social optimum scale with prices:

$$\Delta^k p_i = -\beta_i \Delta^k e_i. \tag{8}$$

Welfare losses relative to the social optimum are then given by:

$$\begin{aligned}\Delta^k W &= \mathbb{E}\left[\Delta^k B_N + \Delta^k B_S - \Delta^k C\right] \\ &= \frac{\gamma}{2}\mathbb{E}\left[\left(\Delta^k E\right)^2\right] + \sum_i \frac{\beta_i}{2}\mathbb{E}\left[\left(\Delta^k e_i\right)^2\right].\end{aligned} \tag{9}$$

Policies featuring equal prices across schemes admit the property that individual and aggregate emissions scale with the common price, so welfare losses can be written as a function of the price gap:

$$\Delta^k W = \frac{1}{2}\frac{(\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S)(\beta_N + \beta_S)}{\beta_N^2\beta_S^2}\mathbb{E}\left[\left(\Delta^k p\right)^2\right]. \tag{10}$$

3 Policies

3.1 Regional cap-and-trade

The simplest policy operates the two cap-and-trade schemes individually. In this case, the planner of scheme i sets a cap e_i to maximize expected global welfare:

$$\max_{e_i} \mathbb{E}W. \tag{11}$$

The resulting allocation is seen to be $e_N = e_S = 0$. Plugging this into (9), we obtain expected welfare losses when both regions operate their own cap-and-trade schemes individually:

$$\Delta^R W = \frac{1}{2}\frac{(\gamma + \beta_S)\sigma_N^2 + (\gamma + \beta_N)\sigma_S^2 - 2\gamma\rho\sigma_N\sigma_S}{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S}. \tag{12}$$

For future reference, it is important to note that regional cap-and-trade is the execution of this welfare program:

$$\max_{e_N, e_S} \mathbb{E} \quad W(e_N, e_S \mid \theta_N, \theta_S). \tag{13}$$

That is, regional cap-and-trade is the optimal choice of instrument under the constraint that caps must be set before any information is revealed and without using any information extracted from markets. It admits the desirable property that *expected* marginal benefits in each scheme equal marginal climate costs:

$$\mathbb{E}[MB_i|e_i] = MC. \tag{14}$$

We note that the RHS of (14) is perfectly known when caps are fixed, whereas marginal benefits are stochastic variables due to unknown fundamentals θ_i.

Regional cap-and-trade suffers from two inefficiencies. First, it does not guarantee an equalization of marginal abatement costs across jurisdictions (i.e. it satisfies this property only in expectations). Second, it ignores information revealed through the interactions between jurisdictions. The first of these, abatement cost equalization, is remedied by linking regional schemes.

3.2 Linking

When North and South link their cap-and-trade schemes, each planner sets the expected optimal cap in its jurisdiction but allowances issued in one scheme may be used to fulfill abatement obligations in another. Trading of allowances is subject to the constraint that global emissions are not affected:

$$e_N + e_S = Q = 0. \tag{15}$$

When schemes are linked, climate damages will be the same as when schemes operate in isolation. However, free trading between the schemes ensures that *realized* marginal benefits are equal in both:

$$p_N = p_S. \tag{16}$$

When two schemes are linked, firms can efficiently redistribute abatement efforts in response to (unforeseen) regional differences in abatement costs, subject to the constraint that total emissions are fixed. It follows:

$$\Delta^L W = \frac{1}{2}\frac{1}{\beta_N + \beta_S}\frac{\beta_S^2\sigma_N^2 + \beta_N^2\sigma_S^2 + 2\beta_N\beta_S\rho\sigma_N\sigma_S}{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S}. \tag{17}$$

The following proposition is now immediate:

Proposition 1. *Global welfare is always higher when cap-and-trade schemes are linked, compared to when they are not.*

Proof. Proof. We only need to compare welfare losses under a linked cap-and-trade regime, equation (17), to those under regional cap-and-trade, equation (12). Then linking outperforms regional cap-and-trade iff:

$$\frac{\beta_S^2\sigma_N^2 + \beta_N^2\sigma_S^2 + 2\beta_N\beta_S\rho\sigma_N\sigma_S}{\beta_N + \beta_S} < (\gamma + \beta_S)\sigma_N^2 + (\gamma + \beta_N)\sigma_S^2 - 2\gamma\rho\sigma_N\sigma_S$$

$$\Longleftrightarrow$$

$$\frac{2\rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2} < \frac{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S}{\gamma\beta_N + \gamma\beta_S},$$

which clearly is always true. Q.E.D.

Q.E.D.

At a more fundamental level, linking can be seen as a policy that solves:

$$\max_{G} \quad \mathbb{E}\left[\max_{e_N, e_S} W(e_N, e_S \mid \theta_N, \theta_S) \quad \text{s.t.} \quad e_N + e_S = G\right]. \tag{18}$$

Practically, the planners of North and South cap local emissions at levels that, in expectations, maximize global welfare. When issued, permits can be traded on a one-to-one basis between schemes, as long as emissions overall remain fixed at the sum of the two regional caps. By linking their schemes, the planners of North and South effectively delay the determination of local emission caps until after regional abatement costs are known, guaranteeing an ex post efficiency gain through the equalization of marginal benefits, which in expectations equal marginal damages:

$$MB = MB_N = MB_S \tag{19}$$

$$\mathbb{E}[MB \mid e_N + e_S] = MC. \tag{20}$$

Though linking benefits global welfare (Proposition 1), regional effects are ambiguous. A simple thought experiment illustrates. We note that prices in equilibrium equate marginal benefits, so that the volatility of prices is equal to the volatility of marginal benefits. Hence if abatement costs in North are much less predictable than in South, $\sigma_N > \sigma_S$, North may import part of the price volatility to which South is subject. If this effect is strong enough, aggregate price volatility may increase and North may be harmed by linking with South (there are papers on this, cite here).

Linking suffers from another type on inefficiency. It disregards valuable information. Remember that allowances will be traded if, and only if, there is a wedge in regional marginal benefits under the local caps. A flow of allowances from one scheme to another therefore reveals information about θ_N and θ_S. The global cap on emissions ideally responds to these ex post observations, but does not under standard linking.

We will next develop a novel policy to regulate emissions. It combines the efficient allocation of emission reductions within a scheme with the equalization of marginal abatement costs brought about by linking. It mitigates the (relative) inefficiencies associated with standard linking. We call our policy endogenous cap-and-trade.

3.3 Endogenous Cap-and-trade

When cap-and-trade schemes link and a flow of allowances from one to the other is observed, valuable information is revealed. Our proposal is that the aggregate emissions cap be adjusted in response. In particular, we propose that if a total of e_N allowances issued in North are sold to firms in South, then the global cap on emissions is changed by an amount $f(e_N)$, where f is a cap-adjustment function chosen to maximize global welfare.[4] In our linear framework, the function f is simply linear in e_N:

$$(1-\delta)e_N = E \implies \delta e_N + e_S = 0, \tag{21}$$

where the cap-adjustment parameter δ is chosen to maximize welfare and given by:

$$\delta = \frac{\beta_N[\sigma_S^2 - \rho\sigma_N\sigma_S] + \gamma[\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S]}{\beta_S[\sigma_N^2 - \rho\sigma_N\sigma_S] + \gamma[\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S]}. \tag{22}$$

We return to the derivation of this policy shortly. Already here, we stress that our proposal really has two key properties. First, and most obviously, the global cap responds to inter-regional trade of allowances. This is done to make sure the information revealed in inter-scheme trades is incorporated into the global cap. Second, and more subtle though just as important, *regional schemes exchange allowances one-to-one*. This property is crucial and we cannot stress it enough. Linking creates efficiency gains because it incentives firms to equate marginal abatement costs globally. This desirable property would be lost if permits are not traded one-to-one between schemes, like with trading ratios (Holland and Yates, 2015). Indeed, only if emissions are traded ton-for-ton does individual profit maximization ensure equal marginal benefits in both regions:

$$p_S^{EC} = p_N^{EC}. \tag{23}$$

While free trade between regions stimulates the equalization of marginal abatement costs across all emitters, the global cap is adjusted to bring marginal benefits closer to marginal climate costs. If we plug (22) into (21), noting (23), one can show that the policy we propose implements the solution to the following program:

$$\mathbb{E}[MB \mid e_N, e_S] = MC. \tag{24}$$

To see this, note that from the observed flow of permits between regions we can construct

[4] It should be clear that anchoring of f on e_N is inconsequential; one could anchor on e_S instead.

the difference in marginal abatement cost innovations:

$$\mu \equiv \theta_S - \theta_N = \beta_N e_N - \beta_S e_S.$$

Using the demand equation (4) and plugging in μ, we find:

$$\begin{aligned}
\mathbb{E}[MB \mid e_N, e_S] &= \mathbb{E}[\theta_N|\mu] - \beta_N e_N \\
&= \mu \frac{\mathbb{E}[\mu\theta_N]}{\mathbb{E}[\mu^2]} - \beta_N e_N \\
&= \mu \frac{\rho\sigma_N\sigma_S - \sigma_N^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S} - \beta_N e_N \\
&= -\frac{\sigma_S^2\rho - \sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}\beta_N e_N - \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}\beta_S e_S
\end{aligned}$$

This can be equated to marginal damages:

$$\gamma(e_N + e_S) = -\frac{\sigma_S^2\rho - \sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}\beta_N e_N - \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}\beta_S e_S,$$

which for convenience we rewrite as

$$\delta e_N + e_S = 0.$$

Solving for δ, we obtain (22). This shows that our endogenous cap-and-trade policy indeed implements the solution to (24).

Note that if $\delta = 1$, endogenous cap-and-trade coincides to standard linking of schemes. Since the planners are free to set $\delta = 1$ but not required to do so, it is clear that endogenous cap-and-trade outperforms standard linking.

Proposition 2. *Expected welfare under endogenous cap-and-trade is always at least as high as under linking. The two policies coincide if and only if $\delta = 1$, which is a probability-zero event:*

$$\delta^* \lesseqqgtr 1 \iff \frac{\beta_N}{\beta_S} \lesseqqgtr \frac{\sigma_N^2 - \rho\sigma_N\sigma_S}{\sigma_S^2 - \rho\sigma_N\sigma_S}. \tag{25}$$

Another way to see that endogenous cap-and-trade is better than standard linking is to make a comparison between the programs they solve. Endogenous cap-and-trade implements the solution to $\mathbb{E}[MB \mid e_N, e_S] = MC$, which is different $\mathbb{E}[MB \mid e_N + e_S] = MC$, the program for linking. Because (e_N, e_S) provides more precise information than the mere aggregate $e_N + e_S$, endogenous cap-and-trade should give at least as good an outcome as standard linking.

In fact, endogenous cap-and-trade not 'just another' emissions policy. It is *the* most efficient implementable quantity-based regulation. It equalizes marginal costs and expected marginal benefits given the finest information available in observed trades.

Figure 1 plots the (normalized) cap-adjustment parameter δ as a function of regional uncertainties about fundamentals and their correlation. We observe that the cap-adjustment rate may be negative. This means that higher-than-expected emissions in one region may translate into higher-than-expected emissions in the other region too. This occurs for strongly positively correlated innovations and very asymmetric uncertainty: if benefits in one region are very unpredictable but strongly correlated to those in the other, predictable region, an increase in the value of emissions in the latter is likely to be matched by an equally strong increase in the former. A negative cap-adjustment rate bears some resemblance with putting negative weights on observations in making (econometric) predictions.

We also observe that the share of global emission reductions absorbed by a region is decreasing in its responsiveness of marginal benefits to emissions, that is, in β. For any adaption of global emissions to shocks, marginal costs change accordingly. Since trade leads to the ex-post equality of regional marginal benefits, and since an optimal mechanism equates regional marginal benefits to global marginal costs, for any realized pair (θ_N, θ_S) emissions change relatively less in the region with steeper marginal benefits.

It must be emphasized that the cap-adjustment parameter is not a trading ratio for pollution permits (*c.f.* Holland and Yates, 2015).

3.4 Asymmetric Uncertainty

Our potentially most interesting observation is that the cap-adjustment rate tends to increase, all else equal, if the uncertainty about benefits in South (σ_S) increases. Intuitively, this means that the endogenous emissions cap should be anchored on the demand for emissions in the most predictable region. Flexibility is warranted for the region we know least about.

Consider an extreme example. Suppose we know marginal benefits in North perfectly, so $\sigma_N = 0$, but we are uncertain about benefits in South, $\sigma_S > 0$. The planners clearly face an environment with asymmetric uncertainty.

Suppose now that we observe interregional trade of allowances. Since we know the exact marginal benefit curve in North, we also know marginal benefits in North for any e_N. And since permits will be traded until the point where marginal benefits in both regions are equal, we in fact know them for South as well. But this implies we in fact observe θ_S.

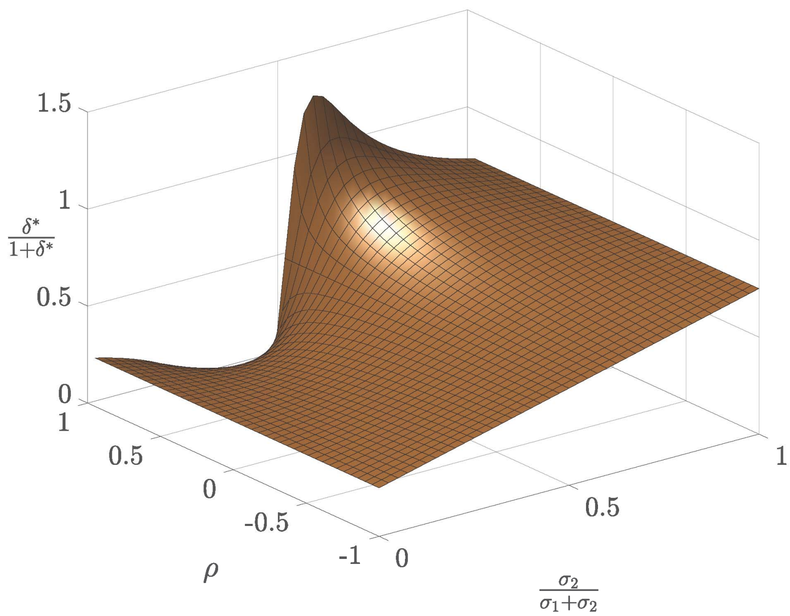

Figure 1: The (normalized) cap-adjustment parameter δ, as a function of fundamentals correlation ρ and relative uncertainty $\sigma_S/(\sigma_N+\sigma_S)$.

Asymmetric uncertainty allows us to extract valuable information from the market.

We learn something important. Without paying attention to asymmetric uncertainty, a flow of allowances from one scheme to another merely indicates that there is a wedge in marginal abatement costs under the initial allocation of allowances. Efficiency is gained by linking since marginal benefits will be equated. The allocation is only constrained Pareto optimal, though: given a potentially suboptimal cap, emissions are allocated in an efficient way. Our endogenous cap-and-trade policy additionally adjusts the cap in response to the information revealed in allowance trading. We have shown that a key ingredient for smart cap-adjustments is asymmetric uncertainty. The lesson drawn from our extreme example is of course more generally true and applicable, even beyond our application to cap-and-trade schemes: there is *more* scope for learning about agents whose preferences are *more* uncertain.

Given the cap-adjustment parameter δ as in (22), we can solve for the associated level of expected global welfare.

Theorem 1. *Endogenous cap-and-trade is strictly welfare-superior to both regional cap-*

and-trade and linking, with welfare given by:

$$\Delta^{EC}W = \frac{1}{2}\frac{\beta_N + \beta_S}{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2 + \sigma_S^2 - 2\rho\sigma_N\sigma_S}. \tag{26}$$

Proof. In Appendix. Q.E.D.

Our extreme example to illustrate the power of asymmetric uncertainty is reflected in (26). If the planners have perfect knowledge about North (or South), $\sigma_N = 0$ (or $\sigma_S = 0$), then all preferences are revealed through trade and no welfare losses occur, $\Delta^{ST}W = 0$.

An implication of our theorem is that endogenous cap-and-trade reduces price volatility compared to standard linking:[5]

Proposition 3. *Endogenous cap-and-trade admits lower price volatility than linked cap-and-trade schemes:*

$$\forall i : \mathbb{E}\left[\left(p^{EC}\right)^2\right] \leq \mathbb{E}\left[\left(p^{L}\right)^2\right]. \tag{27}$$

Proof. In appendix. Q.E.D.

Proposition 3 shows how endogenous cap-and-trade mitigates the risk of regionally increased price volatility induced by classic linking.

3.5 Symmetric Uncertainty: No Information

The discussion above for the time being focused on the information value of trade conditional on asymmetries in uncertainty. We can further shape our intuition by reversing the argument: There is no information in trade when uncertainties are perfectly symmetric. If we are uncertain about all smokers, but equally so about each, observed aggregate behavior does not allow to learn *anything* about aggregate preferences. For inferences about aggregate preferences to possibly be made, differential uncertainty about smokers is a prerequisite.

Suppose we have a group of N identical individuals, which we split in two groups of size n and $N - n$ respectively. Trade between these groups cannot contain information on aggregate preferences; after all, we do not have any reason to assume that prior knowledge of one (sub-)group was more precise than the prior knowledge of the other (sub-)group (apart from the obvious scaling of uncertainty). Uninformative trade between the groups means that no information on 'aggregate preferences' of the population or market as a

[5] With inter*temporal* trading of permits, an endogenous cap can also lead to lower price volatility, see Gerlagh et al. (2020a).

whole can be filtered, and consequently that the allocation of cigarettes to the market should be independent of any such observed trades. This independence is achieved precisely when $\delta = 1$.

Formally, let a market be inhabited by n identical independent smokers for whom $\mathbb{E}[\theta_i^2] = \sigma_N^2$, with $i \in \{1, ..., n\}$. We want to describe the market as one representative agent that satisfies

$$-\beta_n e_n + \theta_n = p, \tag{28}$$

and to assess how β_n and σ_n scale with n. Note that e_n is the number of aggregate cigarettes consumed by the market, $e_n = \sum_{i=1}^n e_i$.

For all individuals i within the group (representative agent), we have:

$$-\beta_N e_i + \theta_i = p, \tag{29}$$

which is simply the demand equilibrium condition (4). Summing over all individuals i and dividing by group size n, we get:

$$-\frac{\beta_N}{n} e_n + \frac{1}{n} \sum_{i=1}^{n} \theta_i = p, \tag{30}$$

which immediately gives

$$\beta_n = \beta_N / n, \tag{31}$$

$$\mathbb{E}[\theta_n^2] = \sigma_N^2 / n \tag{32}$$

The same thought experiment also shows that for a group of N split into two groups of size n and $N - n$, we have $\beta_n = \beta_N/n, \sigma_n = \sigma_N/n$ and $\beta_{N-n} = \beta_N/(N-n), \sigma_{N-n} = \sigma_N/N - n$.

Plugging all this into equation (22), we see that δ^* continues to be 1 after aggregating individual identical smokers into any two sets of n and $N - n$ identical smokers. This confirms the intuition that no information can be obtained from trade between *symmetrically uncertain* smokers.

Corollary 1. *No aggregate information can be obtained from trade between smokers about whom we are symmetrically uncertain. That is, only trade between smokers with asymmetrically uncertain preferences allows for aggregate information filtering. Labeling* *smokers or groups as '1' and '2':*

$$\rho = 0 \quad \textit{and} \quad \sigma_N/\beta_N = \sigma_S/\beta_S \Rightarrow \delta = 1. \tag{33}$$

4 Summary

We propose fairly simply manipulations of standard linking between cap-and-trade schemes that increase global and regional welfare. The core of our argument is that the trade of allowances between regional permit markets signals (Harstad and Eskeland, 2010) valuable information which an efficient policy tries to incorporate. Practically, our proposal is that the aggregate cap of two linked schemes should be adjusted in response to trade flows between the schemes. We pin down an exact analytic formulation for this cap-adjustment.

Optimal linkages allow permits to be exchanged one-to-one. A "trading ratio" that differs from one, though de facto shifting the aggregate cap indeed, would only disturb individuals firms' incentives away from the exact equalization of marginal abatement costs across schemes, which is inefficient.

A key concept we exploit is asymmetric uncertainty.

Cap-and-trade schemes have become a prominent policy instrument in the fight against climate change. In Europe alone, roughly 45% of greenhouse gas emissions are regulated by EU ETS. Linkages between regional schemes already exist, and more are currently being contemplated. Given the fundamental importance of strong climate policies and the enormous sums of money involved, it is important that policies be

Our theory of optimal linking follows a purely quantity-based approach. We take (flows of) emissions as an input to get aggregate emissions as an output. One could instead focus on prices, or a combination of prices and emissions (Roberts and Spence, 1976). Flachsland et al. (2020), for example, argue that EU ETS can be much improved by including a price floor in its design. We leave analyses of linking with price or hybrid instruments for future research.

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A Derivations and Proofs

DERIVATION OF (17):
Combining the definition with the firms' FOCs, (4), we find the change in permit use by region:

$$\Delta^T e_N = \frac{\theta_N - \theta_S}{\beta_N + \beta_S} \tag{34}$$

$$\Delta^T e_S = \frac{\theta_S - \theta_N}{\beta_N + \beta_S}. \tag{35}$$

PROOF OF PROPOSITION 2; DERIVATION OF (22):
Regional and global deviations from Socially Optimal permit use are given by:

$$\Delta^{ST} e_N = \frac{\beta_S}{\beta_N + \delta\beta_S} \frac{[\delta\beta_S - \gamma(1-\delta)]\theta_N + [\beta_N + \gamma(1-\delta)]\theta_S}{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S} \tag{36}$$

$$\Delta^{ST} e_S = \frac{\beta_N}{\beta_N + \delta\beta_S} \frac{[\delta\beta_S - \gamma(1-\delta)]\theta_N + [\beta_N + \gamma(1-\delta)]\theta_S}{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S} \tag{37}$$

$$\Delta^{ST} Q = \frac{\beta_N + \beta_S}{\beta_N + \delta\beta_S} \frac{[\delta\beta_S - \gamma(1-\delta)]\theta_N + [\beta_N + \gamma(1-\delta)]\theta_S}{\gamma\beta_N + \gamma\beta_S + \beta_N\beta_S}. \tag{38}$$

Define

$$\xi := \frac{\beta_N + \gamma(1-\delta)}{\beta_N + \delta\beta_S} \implies 1 - \xi := \frac{\delta\beta_S - \gamma(1-\delta)}{\beta_N + \delta\beta_S}. \tag{39}$$

Welfare losses can now be written as:

$$\begin{aligned}\Delta^{ST}W &= \frac{1}{2}\frac{\gamma(\beta_N+\beta_S)^2+\beta_N^2\beta_S+\beta_N\beta_S^2}{(\gamma\beta_N+\gamma\beta_S+\beta_N\beta_S)^2}\mathbb{E}\left[(1-\xi)\theta_N+\xi\theta_S\right]^2 \\ &= \frac{\beta_N+\beta_S}{2}\frac{(1-\xi)^2\sigma_N^2+\xi^2\sigma_S^2+2\xi(1-\xi)\rho\sigma_N\sigma_S}{\gamma\beta_N+\gamma\beta_S+\beta_N\beta_S}. \end{aligned} \tag{40}$$

If for notational convenience, we define:

$$\psi := \frac{1}{2}\frac{\beta_N+\beta_S}{\gamma\beta_N+\gamma\beta_S+\beta_N\beta_S}, \tag{41}$$

it is straightforward to derive:

$$\frac{\partial}{\partial\xi}\frac{\Delta^{ST}W}{\psi} = 2\xi\sigma_S^2-2(1-\xi)\sigma_N^2+2(1-\xi)\rho\sigma_N\sigma_S-2\xi\rho\sigma_N\sigma_S. \tag{42}$$

The welfare-maximizing ξ^* therefore satisfies:

$$\xi^* = \frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}. \tag{43}$$

From the definition of ξ, the optimal stabilization rate δ^* follows:

$$\delta^* = \frac{(\beta_N+\gamma)[\sigma_S^2-\rho\sigma_N\sigma_S]+\gamma[\sigma_N^2-\rho\sigma_N\sigma_S]}{(\beta_S+\gamma)[\sigma_N^2-\rho\sigma_N\sigma_S]+\gamma[\sigma_S^2-\rho\sigma_N\sigma_S]}, \tag{44}$$

as stated.

PROOF OF THEOREM 1:

Proof. Plugging (43) in (40), we find:

$$\begin{aligned}\frac{\Delta^{ST}W}{\psi} &= \left[\frac{\sigma_S^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]^2\sigma_N^2+\left[\frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]^2\sigma_S^2 \\ &\quad+\left[\frac{\sigma_S^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]\left[\frac{\sigma_N^2-\rho\sigma_N\sigma_S}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S}\right]\rho\sigma_N\sigma_S \\ &= \frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S} \\ &\Longrightarrow \\ \Delta^{ST}W &= \frac{1}{2}\frac{\beta_N+\beta_S}{\gamma\beta_N+\gamma\beta_S+\beta_N\beta_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S},\end{aligned}$$

as stated. This is strictly lower than the welfare loss under traditional Trading if and only if:

$$2\Delta^T W - 2\Delta^{ST} W \geq 0$$
$$\Longrightarrow$$
$$\frac{1}{\beta_N+\beta_S}\frac{\beta_S^2\sigma_N^2+\beta_N^2\sigma_S^2+2\beta_N\beta_S\rho\sigma_N\sigma_S}{\gamma\beta_N+\gamma\beta_S+\beta_N\beta_S} - \frac{\beta_N+\beta_S}{\gamma\beta_N+\gamma\beta_S+\beta_N\beta_S}\frac{(1-\rho^2)\sigma_N^2\sigma_S^2}{\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S} \geq 0$$
$$\Longrightarrow$$
$$(\sigma_N^2+\sigma_S^2-2\rho\sigma_N\sigma_S)(\beta_S^2\sigma_N^2+\beta_N^2\sigma_S^2+2\beta_N\beta_S\rho\sigma_N\sigma_S) - (1-\rho^2)(\beta_N^2+\beta_S^2+2\beta_N\beta_S)\sigma_N^2\sigma_S^2 \geq 0$$
$$\Longrightarrow$$
$$[(\beta_S\sigma_N^2-\beta_N\sigma_S^2)+(\beta_N-\beta_S)\rho\sigma_N\sigma_S]^2 \geq 0,$$

which is always true. Q.E.D.

PROOF OF PROPOSITION 3:

Proof. We derived quantity derivations under both policies. Prices are equal in both regions, so without loss of generality we can solve for price deviations in region 1:

$$\Delta^T p_N = \frac{\beta_S\theta_N+\beta_N\theta_S}{\beta_N+\beta_S}$$
$$\Delta^{ST} p_N = \frac{\delta\beta_S\theta_N+\beta_N\theta_S}{\beta_N+\delta\beta_S}.$$

Thus:

$$\mathbb{E}\left[\left(\Delta^T p\right)^2\right] = \frac{\beta_S^2\sigma_N^2+\beta_N^2\sigma_S^2+2\beta_N\beta_S\rho\sigma_N\sigma_S}{\beta_N^2+\beta_S^2+2\beta_N\beta_S}$$
$$\mathbb{E}\left[\left(\Delta^{ST} p\right)^2\right] = \frac{\delta^2\beta_S^2\sigma_N^2+\beta_N^2\sigma_S^2+2\delta\beta_N\beta_S\rho\sigma_N\sigma_S}{\beta_N^2+\delta^2\beta_S^2+2\delta\beta_N\beta_S}.$$

Writing these out, we obtain:

$$\mathbb{E}\left[\left(\Delta^{ST} p\right)^2\right] < \mathbb{E}\left[\left(\Delta^T p\right)^2\right] \iff (\delta-1)\left[\beta_S\left(\sigma_N^2-\rho\sigma_N\sigma_S\right)-\beta_N\left(\sigma_S^2-\rho\sigma_N\sigma_S\right)\right] < 0.$$

We now invoke Proposition 2 and establish that this condition is always satisfied. Q.E.D.