Freie Universität Berlin
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3. Assignment
Foundations of Mathematics and Statistics
WiSe 2025/26

Deadline: Nov 19th, 23:59 (the midnight **before** the lecture)

The homework should be worked out individually, or in groups of 3-4 students. Pen & paper exercises need not be handed in. The solutions will be discussed in the tutorial sessions. Programming exercises must be submitted via Whiteboard. The file containing the submission must include the last names of all group members in alphabetic order, e.g. "AlbertRamakrishnan-Romano", for group members Mandy Albert, Mike Ramakrishnan, and Marcus Romano.

Pen & Paper Exercise 1 (Uniform MOM)
Derive the method-of-moment estimator $\hat{\theta}_{\text{MOM}}$ for the $Uniform(0, \theta)$ distribution. Is it an unbiased/consistent estimator?

Pen & Paper Exercise 2 (Gaussian MLE)
Derive the M.L.E estimator $\hat{\mu}_{MLE}$ and $\hat{\sigma}^2_{MLE}$ of the $Gaussian(\mu, \sigma^2)$ distribution. Show whether it is a biased estimator.

Pen & Paper Exercise 3 (Exponential MLE)
The PDF of the Exponential distribution is given by

$$f_X(x) = \lambda e^{-\lambda x}, \text{ for } x > 0,$$

Based on n i.i.d. samples $X_1, ..., X_n \sim Exponential(\lambda)$, derive the maximum likelihood estimator for the parameter λ.

Pen & Paper Exercise 4 (Binomial)
Derive the M.O.M estimator $\hat{p}_{MOM}$ and the M.L.E estimator $\hat{p}_{MLE}$ of the of the $Binomial(n, p)$.

Pen & Paper Exercise 5 (Uniform MLE)
Derive the MLE $\hat{\theta}_{MLE}$ for $Uniform(\theta, 0)$. Is it a biased estimator?

Programming Exercise 1 (Maximum Likelihood Estimator, 3+2 points)
(to be uploaded via Whiteboard)

a.) The input file "Input.txt" on Whiteboard contains data sampled from a uniform distribution $\mathcal{U}(a, 5)$. Derive the likelihood function $\mathcal{L}_n(\theta) = \prod_{i=1}^{n} f(X_i, \theta)$ for the parameter a based on the given data. Write a program that plots the likelihood $\mathcal{L}$.

b.) Compute/ state the maximum likehood estimator for a, $\hat{a}$, and extend your program so that it creates a plot of the given data overlaid with the PDF of $\mathcal{U}(\hat{a}, b)$.

c.) Create a plot that shows $\mathcal{L}_{10}(a)$ with a subsample of the given data vs. a.

Your code should print the requested plots (e.g. using plt.show()) as three subplots in one image/ plot. Call this program "Ex3MLE.py" and submit it via the Whiteboard system.

Good luck!