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2. Assignment
Foundations of Mathematics and Statistics
WiSe 2025/26

Deadline: Nov 5th, 23:59 (the midnight **before** the lecture)

The homework should be worked out individually, or in groups of 3-4 students. Pen & paper exercises need not be handed in. The solutions will be discussed in the tutorial sessions. Programming exercises must be submitted via Whiteboard. The file containing the submission must include the last names of all group members in alphabetic order, e.g. "AlbertRamakrishnan-Romano", for group members Mandy Albert, Mike Ramakrishnan, and Marcus Romano.

Pen & Paper Exericise 1 (Variance)
Prove the following two statements assuming that the variance $\mathbb{V}(X)$ is well defined:

a) $\mathbb{V}(X) = \mathbb{E}(X^2) - \mu^2$

b) $\mathbb{V}(aX + b) = a^2\mathbb{V}(X)$, where $a, b \in \mathbb{R}$.

Pen & Paper Exericise 2 (The Uniform distribution)
Let $X \sim Uniform(a, b)$. Derive $\mathbb{E}(X)$ and $\mathbb{V}(X)$.

Pen & Paper Exericise 3 (Maximum of Uniforms)
Let X and Y be independent and suppose that each has a Uniform(0,1) distribution. Let $Z = max\{X, Y\}$. Find the expected value $\mathbb{E}(Z)$ and variance $\mathbb{V}(Z)$.

Pen & Paper Exericise 4 (Covariance and Independence)

Let X and Y have the joint density

	$f_{X,Y}(x,y)$	y -1	0	1	
x	-1	0	0.5	0	
	1	0.25	0	0.25	

a) Find the covariance and the correlation of X and Y.

b) Are X and Y independent, and why? How does independence relate to the correlation?

Pen & Paper Exericise 5 (Variance)
Let

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{3}(x+y) & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Find $\mathbb{V}(2X - 3Y + 8)$.

Programming Exericise 1 (Central limit theorem, 3+2 points)
(to be uploaded via Whiteboard)

a) In this exercise we aim to verify the central limit theorem by simulation. Write a program that computes ten sample mean values $\overline{X}_N^i, i = 1, 2, ..., 10$ of N samples from a Poisson distribution with rate 1. Repeat this five times, each time with an increased number of samples.

Let your program read the input file ("Input.txt") provided on Whiteboard. The first number in the input file is the 'seed' of the random number generator, the following 5 numbers are the sample sizes N. Then, for each N, compute the variance of the sample mean values $\mathbb{V}(\overline{X}_N)$.

Save the values of $\overline{X}_N^i$ as well as their variance and write them into the file "MeanVar.txt". The output text-file should be in the comma-separated text format using two digits after the comma (format '%1.2f'), e.g.

$$2.45, 2.30, 2.34, 2.33, ... \tag{2}$$

$$3.01, 1.20, 1.09, 0.99, ... \tag{3}$$

The first 5 rows should contain the 10 sample means for each of the values of N, respectively. The last line should contain the variances for all N. Call this program "Ex2CLT.py" and submit it via the Whiteboard system.

b) (to be printed by your code)

a) Make a plot with the sample means $\overline{X}_N^i$ on the y-axis and the number of samples N on the x-axis. Comment on the trend you observe with increasing N.

b) Make another plot of the variance $\mathbb{V}(\overline{X}_N)$ versus a function of N that should give you a straight line with slope 1 (cf. central limit theorem). (Don't worry if it's not perfect - we are working with finite samples after all.)

Good luck!