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1. Assignment
Foundations of Mathematics and Statistics
WiSe 2025/26

Deadline: Oct 29, 23:59 (the midnight **before** the lecture)

The homework should be worked out individually, or in groups of 3-4 students. Pen & paper exercises need not be handed in. The solutions will be discussed in the tutorial sessions. Programming exercises must be submitted via Whiteboard. The file containing the submission must include the last names of all group members in alphabetic order, e.g. "AlbertRamakrishnan-Romano", for group members Mandy Albert, Mike Ramakrishnan, and Marcus Romano.

Pen & Paper Exericise 1 (The Birthday Problem)
Assuming that there are 60 students in our class, what is the probability that at least two share the same birthday?

Pen & Paper Exericise 2 (Conditional probability)
There are three cards. The first is green on both sides, the second is red on both sides and the third is green on one side and red on the other. We choose a card at random and we see one side (also chosen at random). If the side we see is green, what is the probability that the other side is also green?

Pen & Paper Exericise 3 (CDF and PDF)
Let X be a random variable with the CDF

$$F_X(x) = \begin{cases} 1 - e^{-x} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

a) Find its PDF $f_X(x)$.

b) Show that it is a valid PDF.

Pen & Paper Exericise 4 (Joint and marginal distributions)
Let (X, Y) have joint density

$$f_{X,Y}(x, y) = \begin{cases} c, & \text{for } 0 < x \leq y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a) Determine the value of the contant c. (5 pts)

b) What is $f_X(x)$ and $f_Y(y)$? Are X and Y independent? (5 pts)

Pen & Paper Exericise 5 (Minimum of Uniforms)
Let X and Y be independent and suppose that each has a Uniform(0,1) distribution. Let $Z = min\{X, Y\}$. Find the probability density function $f_Z(z)$ for Z.
Hint: It might be easier to first find $\mathbb{P}(Z > z)$.

Programming Exericise 1 (Simulation of minimum of uniforms)
(to be uploaded via Whiteboard)

i.) Write a program that samples values of X and Y and uses these to simulate the density of Z from the question above. There are three files provided on Whiteboard:

a) "Input.txt": the input file for the program. The first number in the input file is the 'seed' of the random number generator, the second is the number of samples N to be generated.

b) "Ex1RVs.py": the code skeleton handling the input. You should directly continue from the provided code.

c) 'ZSamples.txt": an example output file of the Z samples based on the provided input file, for you to check your solutions. It should be in the comma-separated text format using two digits after the comma (format '%1.2f'), e.g.

$$0.19, 0.30, 0.44, 0.01, ... \tag{1}$$

Submit the python code via the Whiteboard system.

ii.) (to be printed using plt.show() in the same python file as in i.)) Plot the samples of Z in a histogram, superimposed with the analytical solution. Make sure the two plots agree.

Good luck!