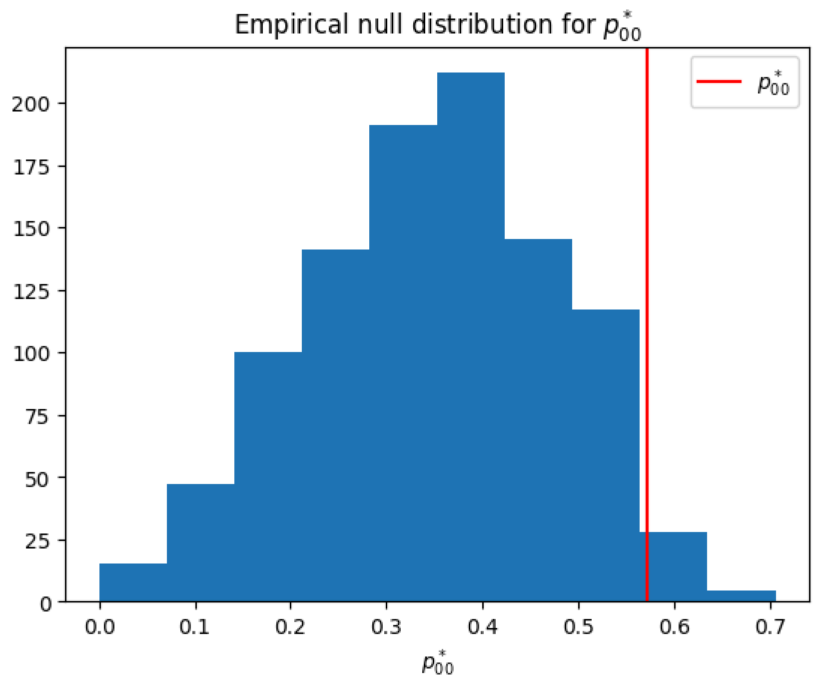
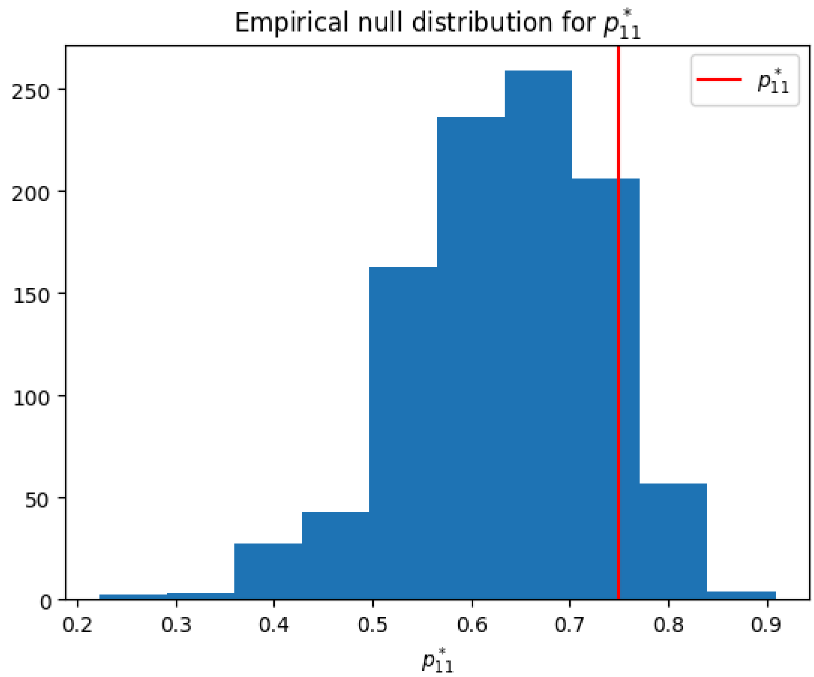
HW1

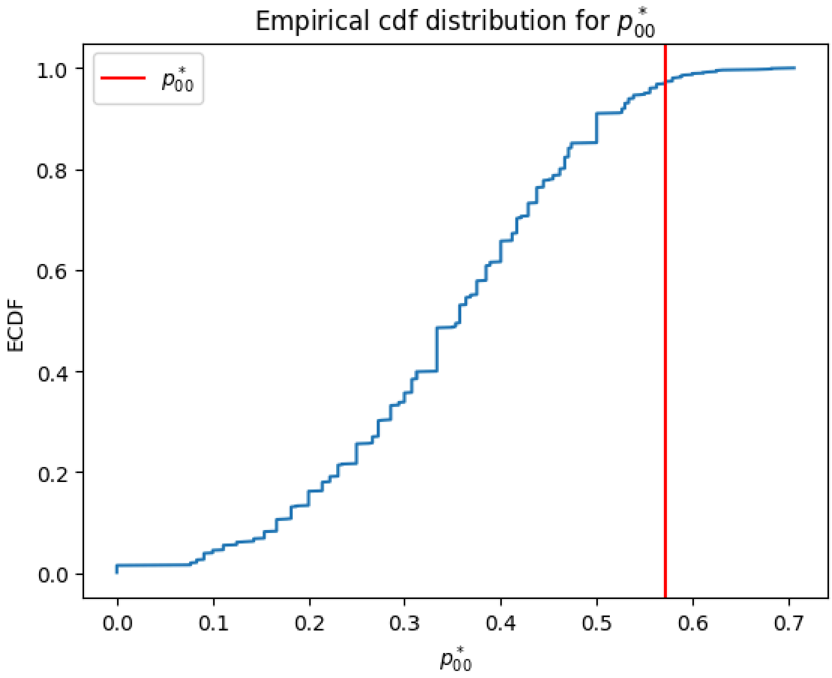
b)

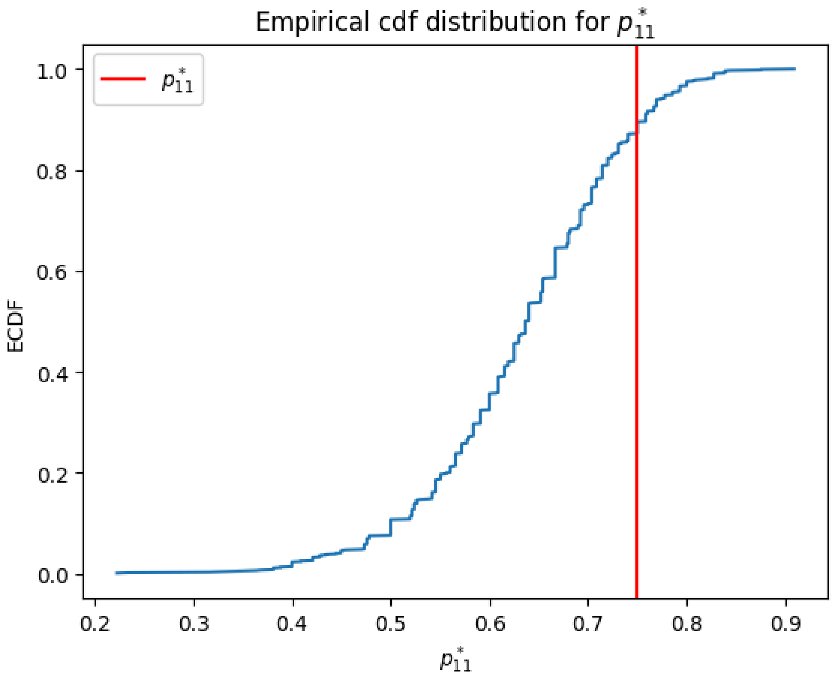


Empirical null distribution for p_{00}^*
p_{00}^*
0
25
50
75
100
125
150
175
200
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
p_{00}^*



Empirical null distribution for p_{11}^*
p_{11}^*
0
50
100
150
200
250
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
p_{11}^*





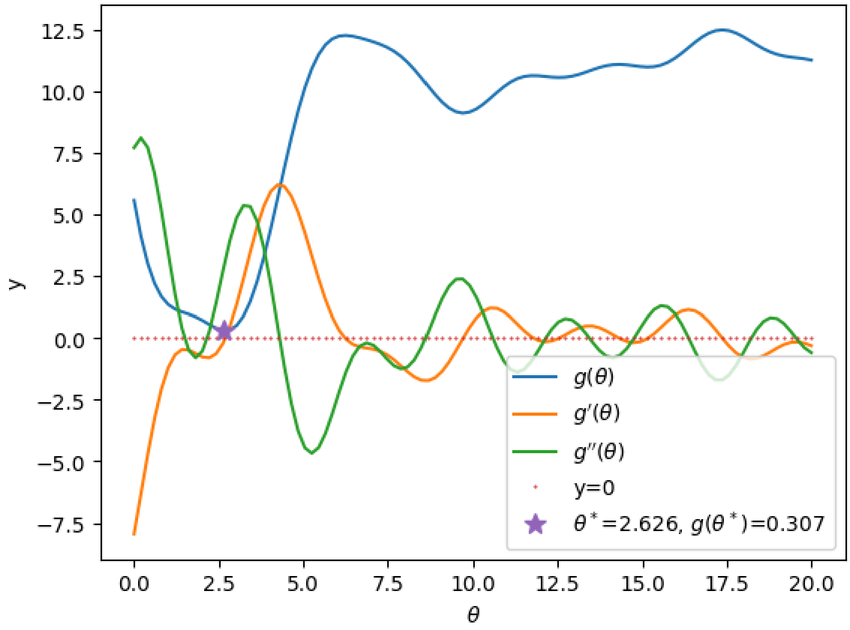

```
For p_00, the empirical p-value of 0.027 indicates that the observed transition probability is unlikely to arise from an i.i.d.
sequence, providing evidence for a first-order Markov model. Assuming an alpha of 0.05 we reject our null of a simple coin flip
model.

For p_11, the empirical p-value of 0.108 indicates that we do not have statistical significance, assuming an alpha of 0.05, to r
eject the null hypotesis that the data was generated via an simple coin flip model.
```

HW2

b)

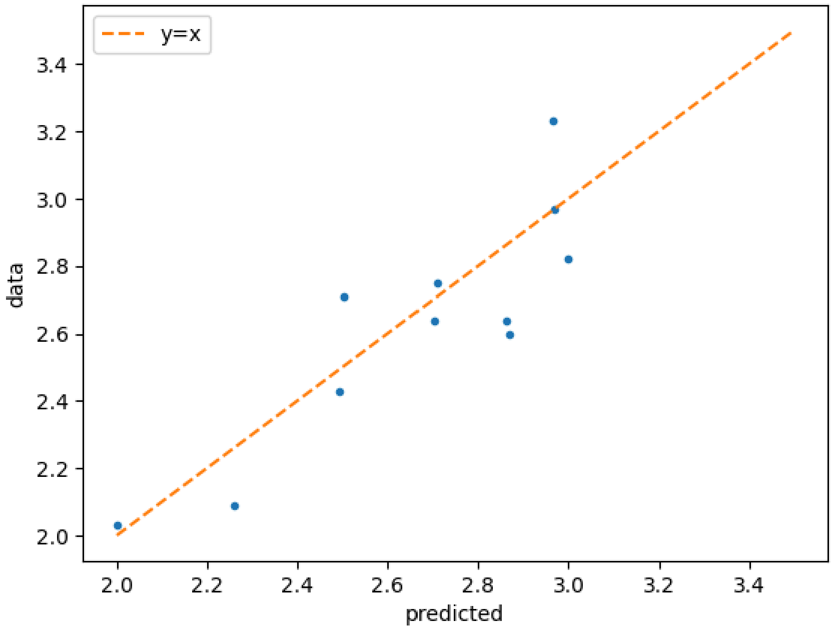
```
All optima are: [ 0.307,  9.104, 10.547, 10.972]
Respectively, the theta values corresponding to all optima are: [ 2.626,  9.697, 12.525, 15.152]
```

```
<matplotlib.legend.Legend at 0x11714fd60>
```

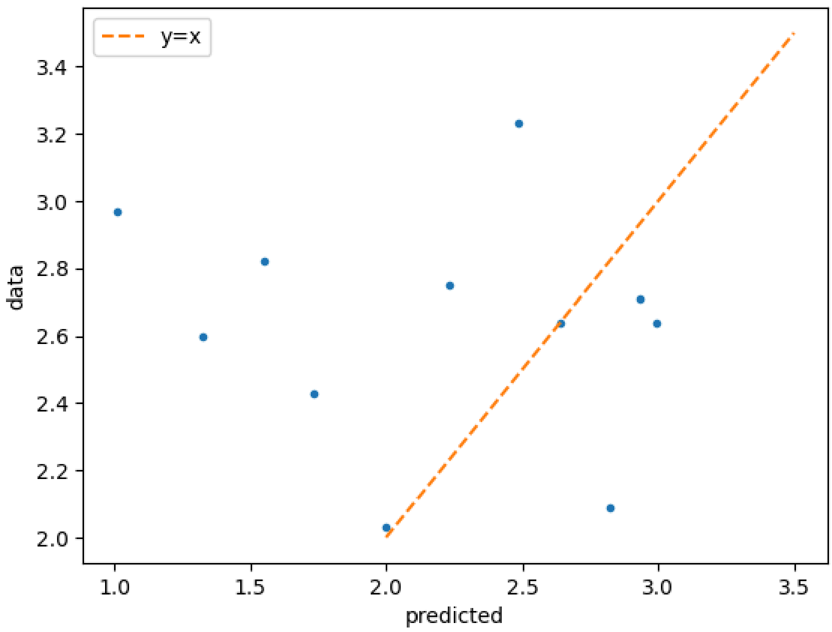


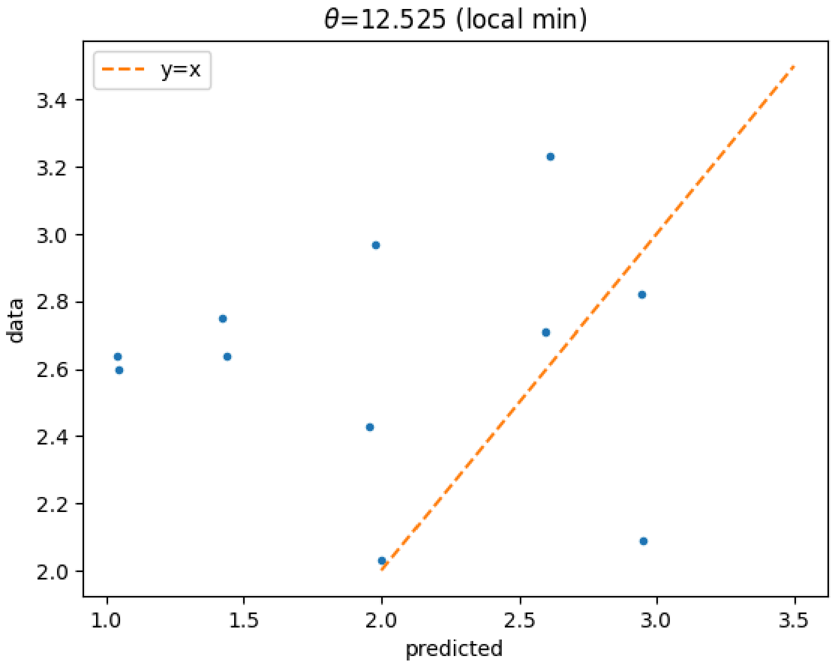
c)

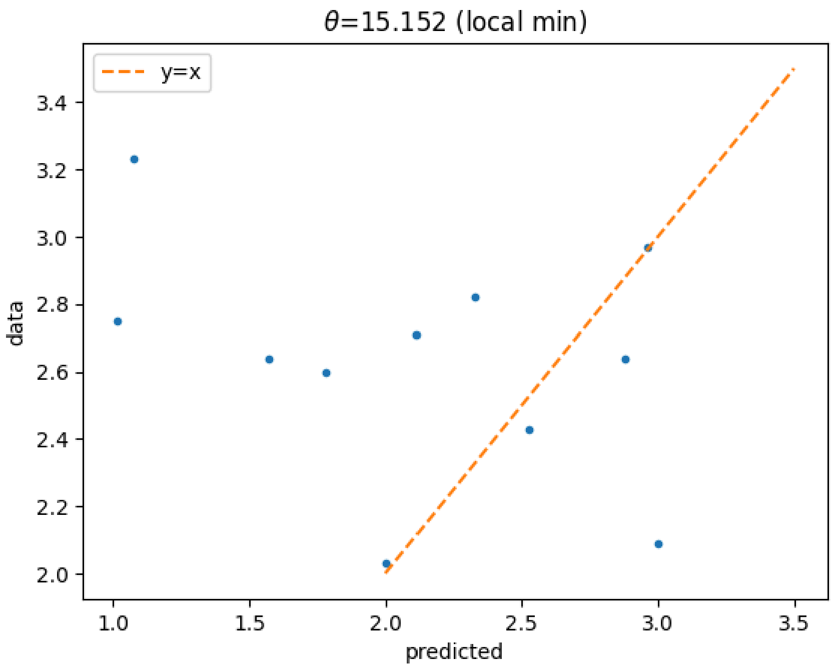
θ=2.626 (global min)



θ=9.697 (local min)







According to the comparison between the solution obtained with the Newton method and the parameter corresponding to the global minimum, it seems that the Newton method is not guaranteed to provide the global optimal parameter. θ has a bit different values:

```
Newton method: 2.702120
Global minimum value parameter for the objective function: 2.626263
```

Additionally, it is important to correctly initialize the Newton method function, otherwise it would give the parameter associated with the local, not global minimum. As we have seen on the data vs predicted plots, the parameters θ associated with the local minumum do not allow us to model the data correctly.

```
'The solution for the objective function theta* associated with the local minimum after running the Newton method with the start
ing point theta_0 equals to 10: 9.711984372034813'
```

HW3

$y_i = x_{i|\theta} + \eta_i$ and $\eta_i \sim Norm(0, \sigma_i^2)$

$L(y_i|\theta) = \prod_{i=1}^{n} p(y_i|\theta)$

where $p(y_i|\theta) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp(\frac{-(y_i - x_{i|\theta})^2}{2\sigma_i^2})$

$L(y_i|\theta) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp(\frac{-(y_i - x_{i|\theta})^2}{2\sigma_i^2}) = \frac{1}{(2\pi)^{\frac{n}{2}} \sigma_1 \cdot \ldots \cdot \sigma_n} \exp(-\sum_{i=1}^{n} \frac{(y_i - x_{i|\theta})^2}{2\sigma_i^2})$

$l(y_i|\theta) = \log L(y_i|\theta) = \underbrace{\frac{n}{2}\log(\frac{1}{2\pi})}_{constant} - \underbrace{\sum_{i=1}^{n} \log(\sigma_i)}_{no\ \theta} - \underbrace{\sum_{i=1}^{n} \frac{(y_i - x_{i|\theta})^2}{2\sigma_i^2}}_{2\ is\ a\ constant}$

As $\theta^* = \arg\min_{\theta}(-l(y_i|\theta))$

Therefore

As $\theta^* = \arg\min_{\theta}(\sum_{i=1}^{n} \frac{(y_i - x_{i|\theta})^2}{\sigma_i^2})$

The objective function is $g(\theta) = \sum_{i=1}^{n} \frac{(y_i - x_{i|\theta})^2}{\sigma_i^2}$

HW4

$N = 2$

$t = (1, 3)$

$y = (6, 7)$

$(\theta_1, \theta_2)^\top = (1, 1)$

$x_i = \theta_1 e^{\theta_2 t_i}$

$g(\theta) = \sum_{i=1}^{N} (x_{i|\theta} - y_i)^2 = \sum_{i=1}^{N} (\theta_1 e^{\theta_2 t_i} - y_i)^2$

$\frac{dg(\theta)}{d\theta 1} = 2\sum_{i=1}^{N} (\theta_1 e^{\theta_2 t_i} - y_i) e^{\theta_2 t_i} = 2\sum_{i=1}^{N} (1e^{1t_i} - y_i) e^{t_i} = 2(e - 6)e + 2(e^3 - 7)e^3$

$\frac{dg(\theta)}{d\theta 2} = 2\sum_{i=1}^{N} (\theta_1 e^{\theta_2 t_i} - y_i) e^{\theta_2 t_i} \theta_1 t_i = 2\sum_{i=1}^{N} (e^{t_i} - y_i) e^{t_i} t_i = 2e(e - 6) + 6e^3(e^3 - 7)$

$\frac{d^2 g(\theta)}{d\theta_1^2} = 2\sum_{i=1}^{N} e^{\theta_2 t_i} = 2e^2 + 2e^{2\cdot 3} = 2(e^2 + e^6)$

$\frac{d^2 g(\theta)}{d\theta_1 d\theta_2} = 2\sum_{i=1}^{N} (\theta_1 e^{2\theta_2 t_i} 2t_i - y_i e^{\theta_2 t_i} t_i) = 2\sum_{i=1}^{N} (1e^{2t_i} 2t_i - y_i e^{1t_i} t_i) = 2(2e^2 - 6e) + 2(6e^6 - 7 \cdot 3e^3) = 2(2e^2 - 6e) + 2(6e^6 - 21e^3)$

$\frac{d^2 g(\theta)}{d\theta_2^2} = 2\sum_{i=1}^{N} (\theta_1^2 e^{2\theta_2 t_i} 2t_i^2 - y_i \theta_1 e^{\theta_2 t_i} t_i^2) = 2(1e^2 2 - 6e) + 2(e^{2\cdot 3} 2 \cdot 9 - 7e^3 9) = 2(2e^2 - 6e + 18e^6 - 63e^3)$

```
d g(theta)/ d theta1: 507.8188003171955
d g(theta)/ d theta2: 1559.1389404388808

d^2 g(theta)/ d theta1^2: 821.6356991833313
d^2 g(theta)/ d theta1 dtheta2: 3994.4898135931526
d^2 g(theta)/ d theta1^2: 11989.595755871029
```

$$H = \begin{bmatrix} \frac{d^2 g(\theta)}{d\theta_1^2} & \frac{d^2 g(\theta)}{d\theta_1 \theta_2} \\ \frac{d^2 g(\theta)}{d\theta_1 \theta_2} & \frac{d^2 g(\theta)}{d\theta_2^2} \end{bmatrix} = \begin{bmatrix} 821.6 & 3994.5 \\ 3994.5 & 11989.6 \end{bmatrix}$$

If we consider H as: $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$H^{-1} = \frac{1}{a \cdot c - b^2} \begin{bmatrix} c & -b \\ -b & a \end{bmatrix} = \frac{1}{821.6 \cdot 11989.6 - 3994.5^2} \begin{bmatrix} 11989.6 & -3994.5 \\ -3994.5 & 821.6 \end{bmatrix} = \frac{1}{-6105374.8} \begin{bmatrix} 11989.6 & -3994.5 \\ -3994.5 & 821.6 \end{bmatrix} = \begin{bmatrix} -0.001964 & 0.00065426 \\ 0.00065426 & -0.0001346 \end{bmatrix}$$

$$\theta^{s+1} = \theta^s + \Delta\theta = \theta^s - H^{-1} \cdot \nabla g(\theta^s) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.001964 & 0.00065426 \\ 0.00065426 & -0.0001346 \end{bmatrix} \begin{bmatrix} 507.82 \\ 1559.14 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.0228 \\ 0.1224 \end{bmatrix} = \begin{bmatrix} 0.9772 \\ 0.8776 \end{bmatrix}$$