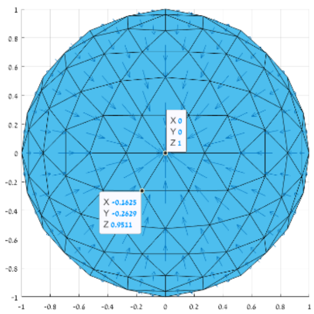
Digital Geometry Processing - 236329
Homework 3

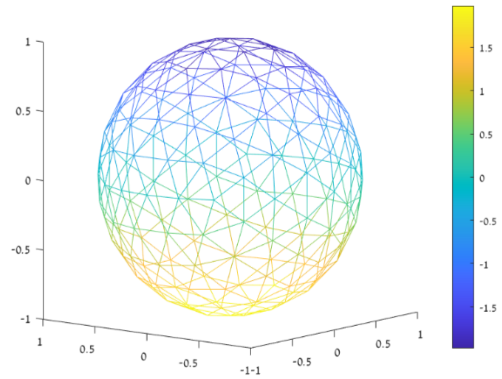
316948694 201631348

1. Implemented in method 'visualize_vec' of class 'MeshHandle'.
2.
   a) Implemented in methods 'calc_grad', 'calc_div' and 'calc_laplas' of class 'MeshHandle'.

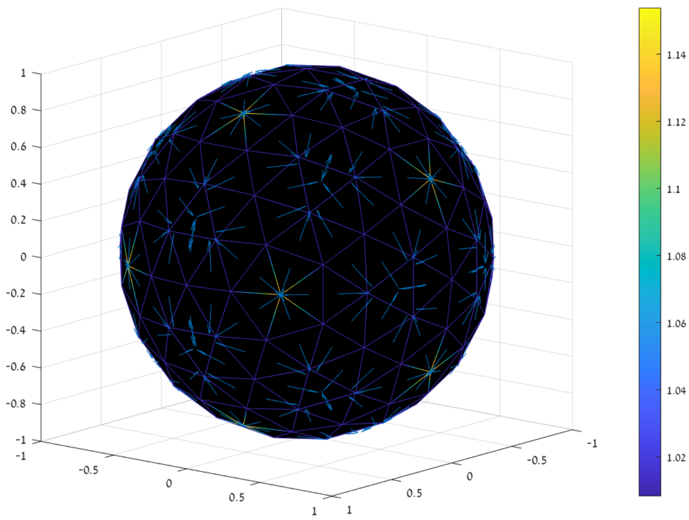
Gradient of sphere height:



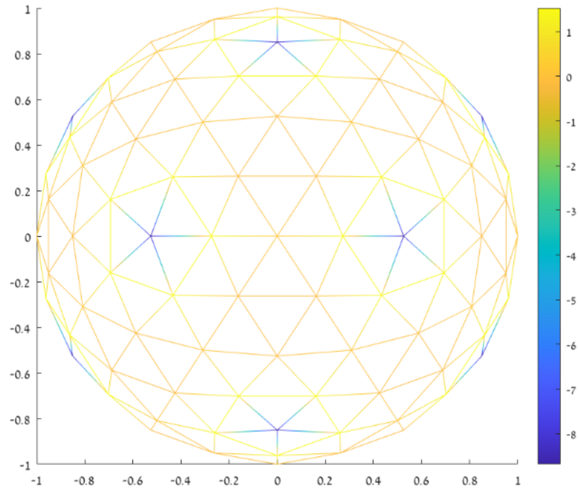
Divergence of gradient field:



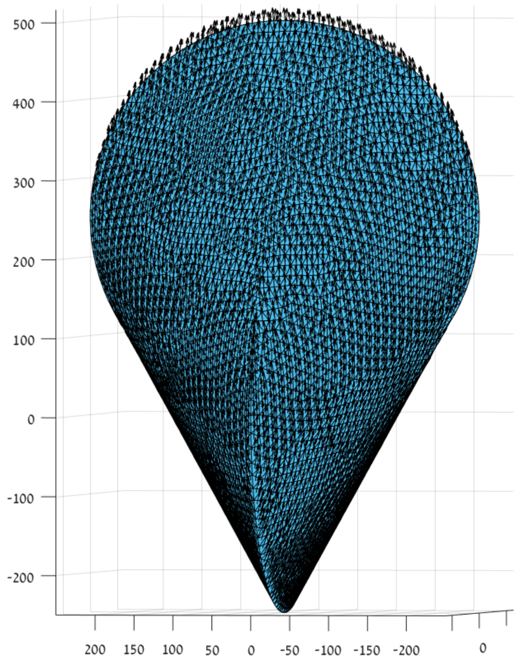
Gradient of gaussian curvature:



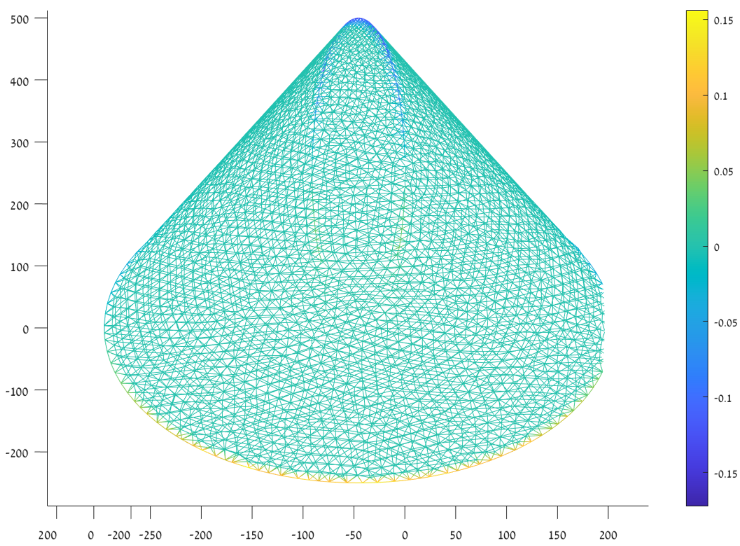
Divergence of gradient field:



Gradient of oloid height:

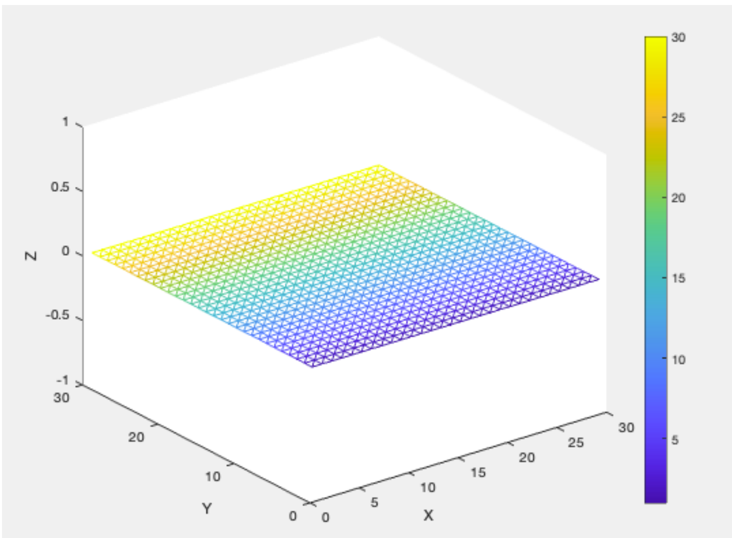


Divergence of gradient field:



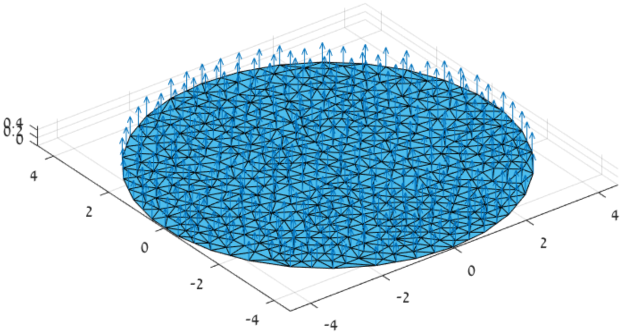
b) Implemented in method 'calc_laplas_cot' of class 'MeshHandle'. Comparison between the two Laplace calculations produces negligible differences (that occur due to numerical inaccuracies).

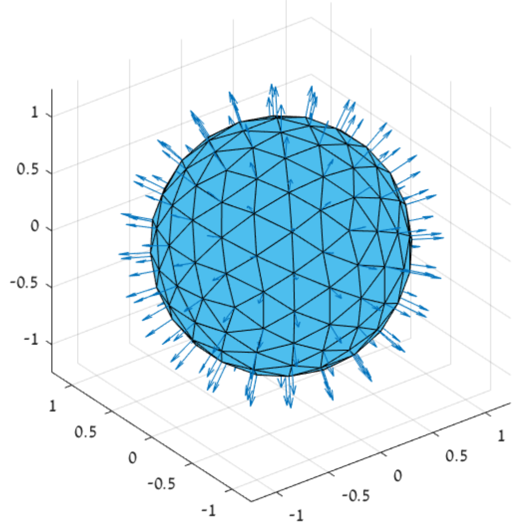
c) The quad mesh, with a linear function upon it can be seen in the following figure

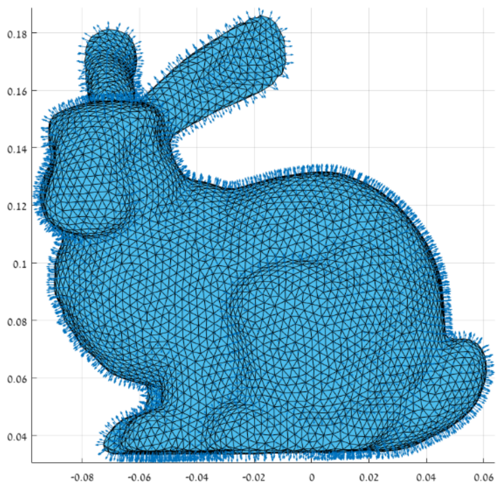


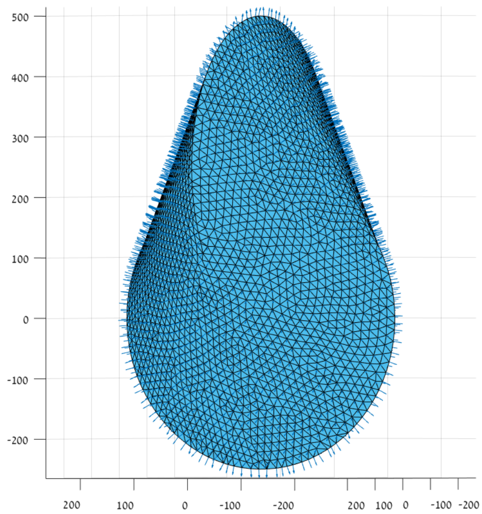
Computing its cot-Laplacian (the element-wise formula) is depend on a given function, thus cannot be deduced inductively. If the question would aim to the matrix form of the Laplacian operator, one would be able to construct the faces-area matrix (for example, with a constant function, all the areas would be 0.5), the vertex-area matrix (due to the recurrent structure) and the gradient matrix, and compute the operation matrix.

3. Implemented in method 'vertex_normal' of class 'MeshHandle'.
   Examples:



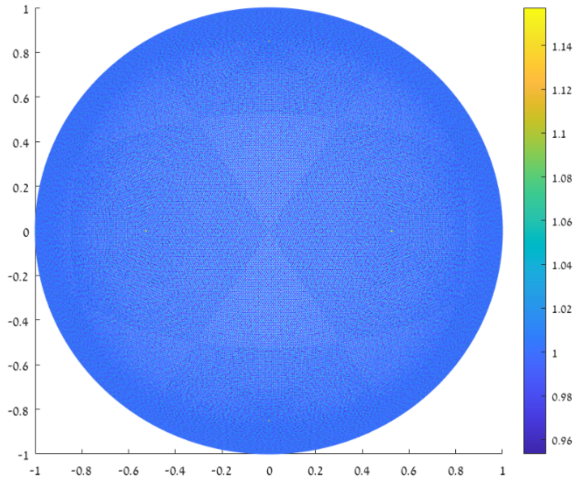




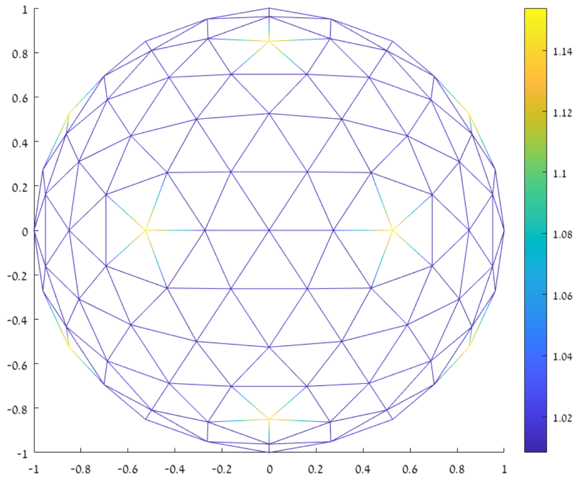


4. Implemented in method 'calc_mean_curv' and 'calc_gauss_curv' of class 'MeshHandle'.

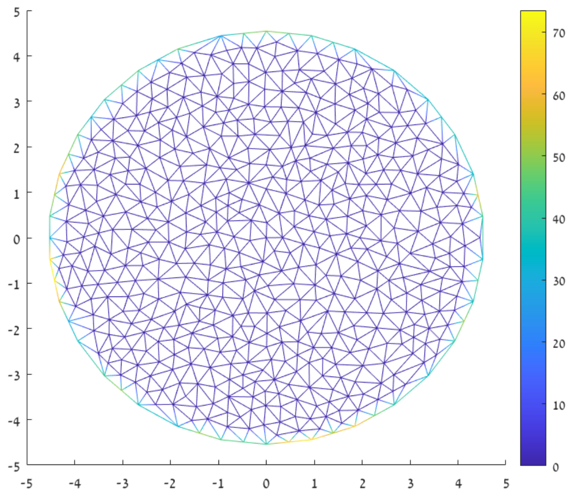
Gaussian curvature



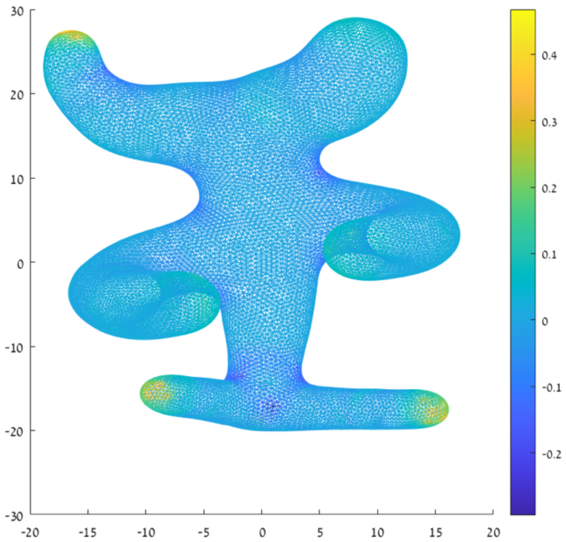
For a dense unit sphere, the gaussian curvature is approximately 1 at all vertices.



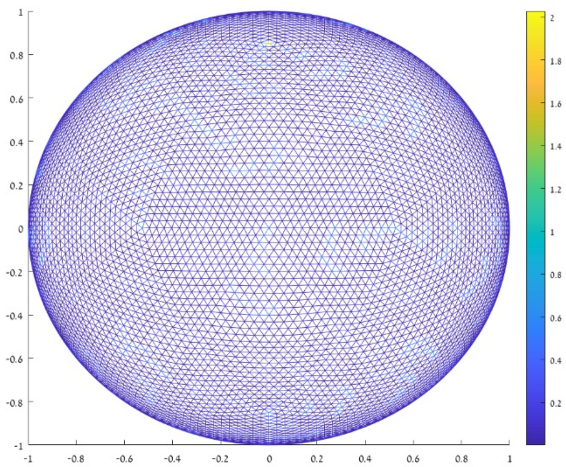
For a less dense unit sphere, whenever the valence is getting lower the curvature is getting higher.

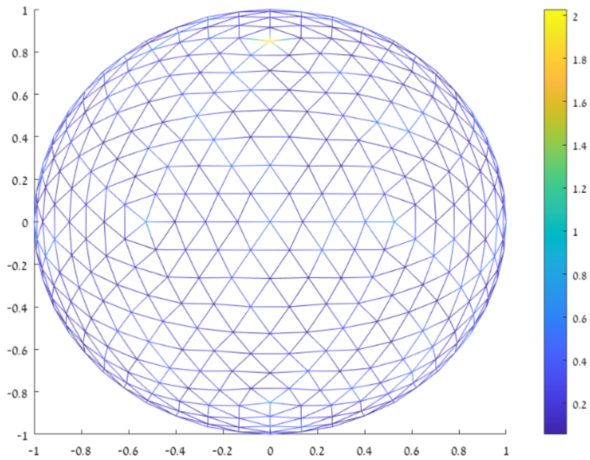


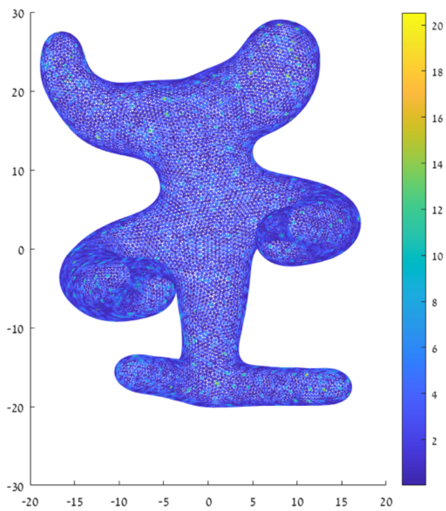
Disk curvature is approximately constant 0 except of the boundary.

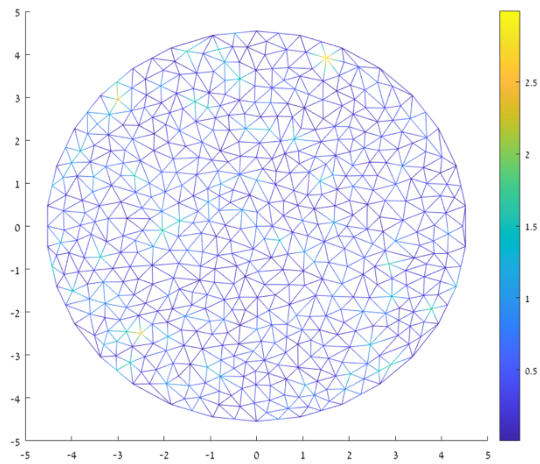


Mean curvature









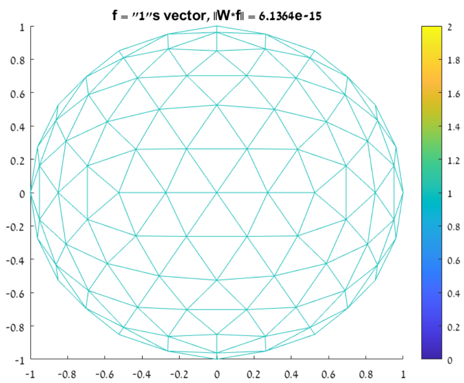
Analysis

1. \+ 2. ***** Conclusions are at the end *****

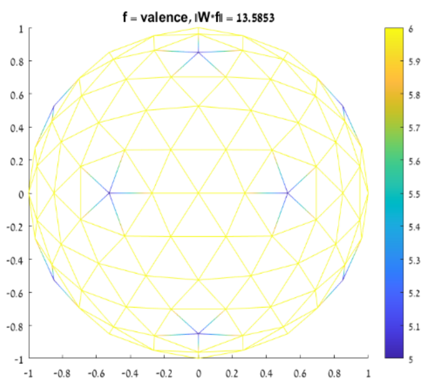
Sphere:

<u>Null –</u>

constant function



non-constant function



<u>Symmetry –</u>

$$\left\| W - W^T \right\|_F = 1.5622\text{e-}15$$

<u>Localization –</u>

\# non-zero elements = 1122 (out of 26244)

\# edges = 480

\# vertices = 162

➔ # non-zero elements = 2*# edges + # vertices

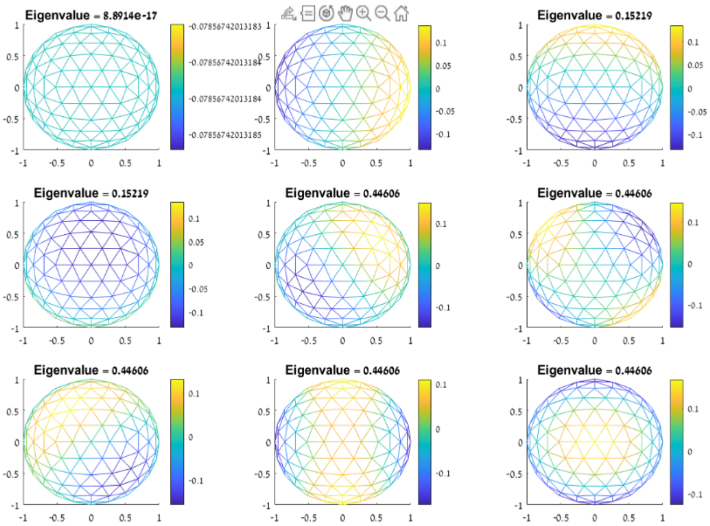
<u>Positivity –</u>

W contains non positive elements

<u>Positive semi-definite –</u>

W is positive semi-definite as all it's eigenvalues are positive. Smallest eigenvalue = 8.8914e-17, Largest eigenvalue = 5.3873

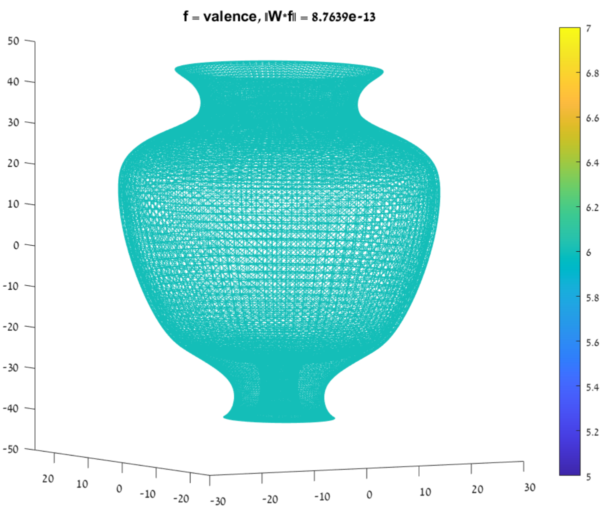
Eigenfunctions examples –



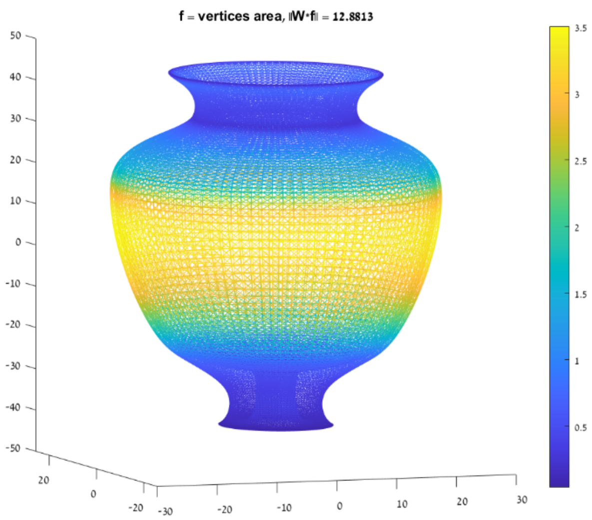
Vase:

Null –

constant function



non-constant function



Symmetry –

$\left\| W - W^T \right\|_F = 4.4793\text{e-}15$

Localization –

non-zero elements = 153216 (out of 479084544)

edges = 65664

vertices = 21888

➔ # non-zero elements = 2*# edges + # vertices

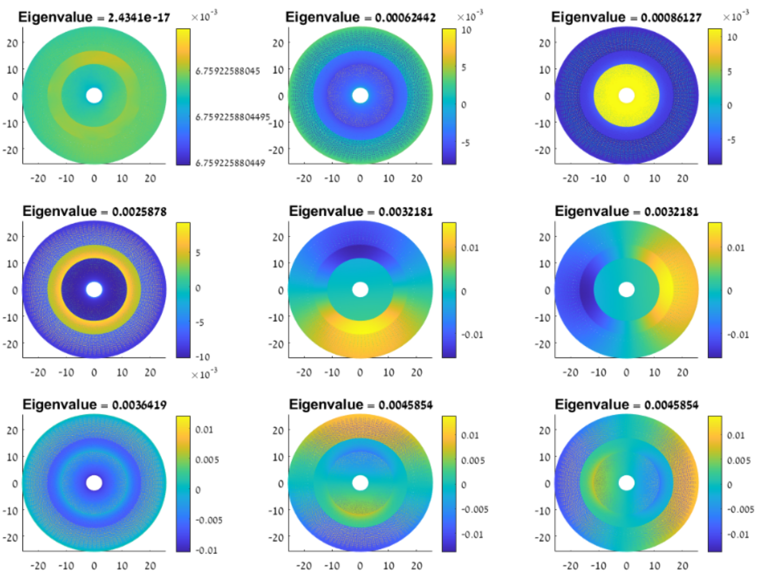
Positivity –

W contains non positive elements

Positive semi-definite –

W is positive semi-definite as all it's eigenvalues are positive. Smallest eigenvalue = 5.3428e-17, Largest eigenvalue = 14.3993

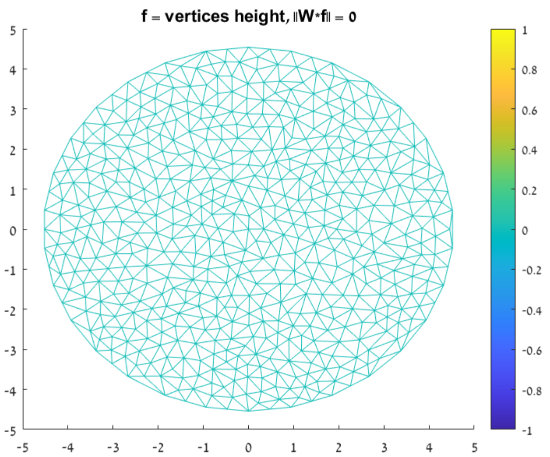
Eigenfunctions examples –



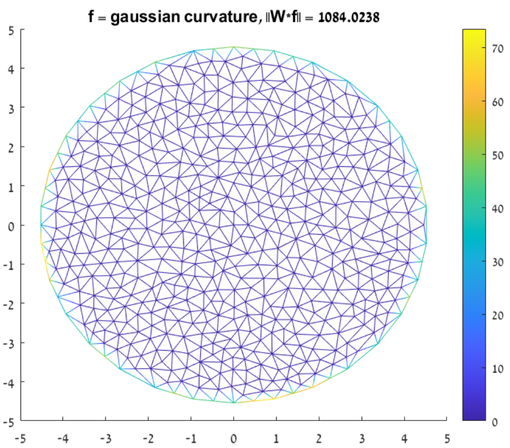
Disk:

Null –

constant function



non-constant function



Symmetry –

$\left\|W - W^T\right\|_F = 4.4793\text{e-}15$

Localization –

non-zero elements = 3672 (out of 291600)

edges = 1566

vertices = 540

➔ # non-zero elements = 2*# edges + # vertices

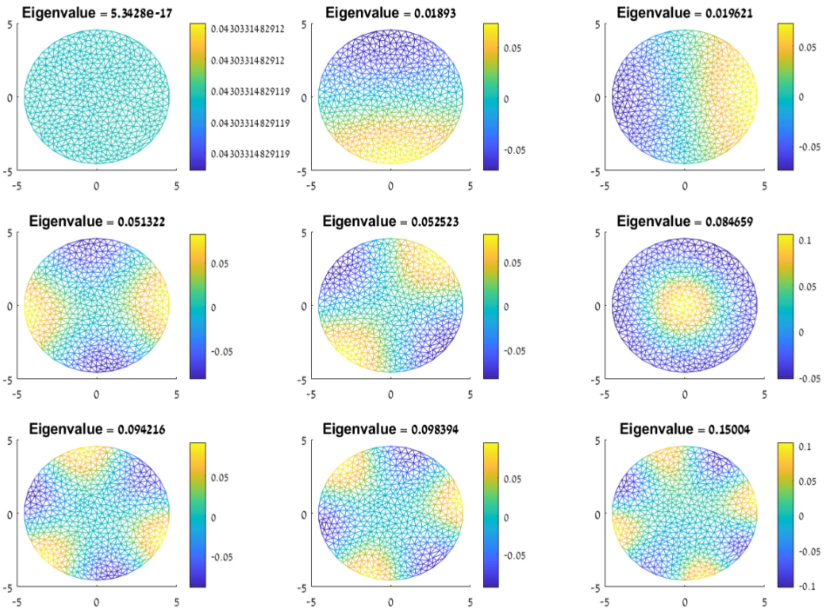
Positivity –

W contains non positive elements

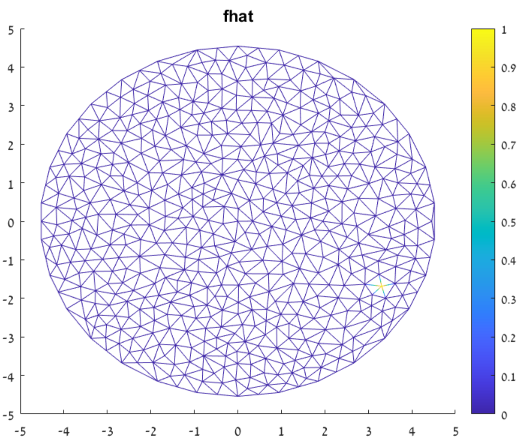
Positive semi-definite –

W is positive semi-definite as all it's eigenvalues are positive. Smallest eigenvalue = 5.3428e-17, Largest eigenvalue = 14.3993

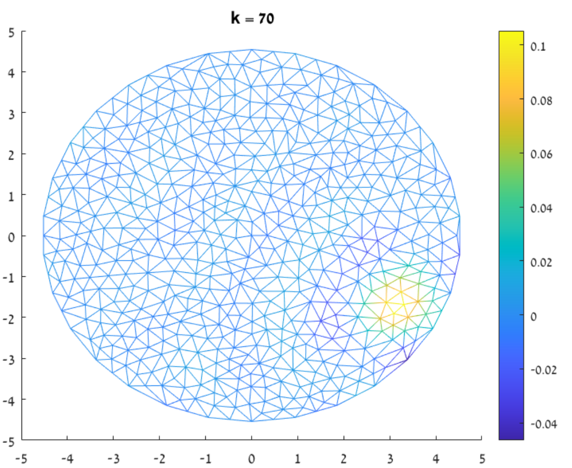
Eigenfunctions examples –



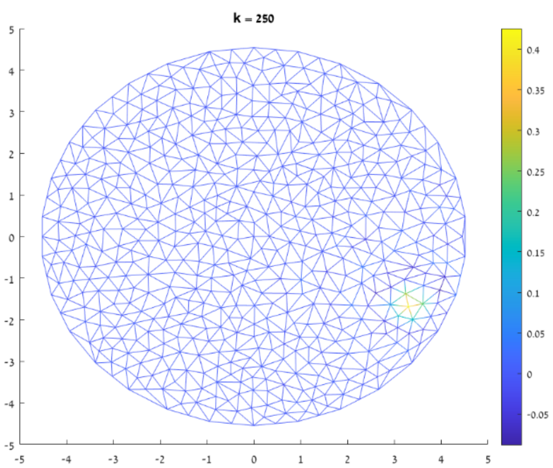
Hat function:



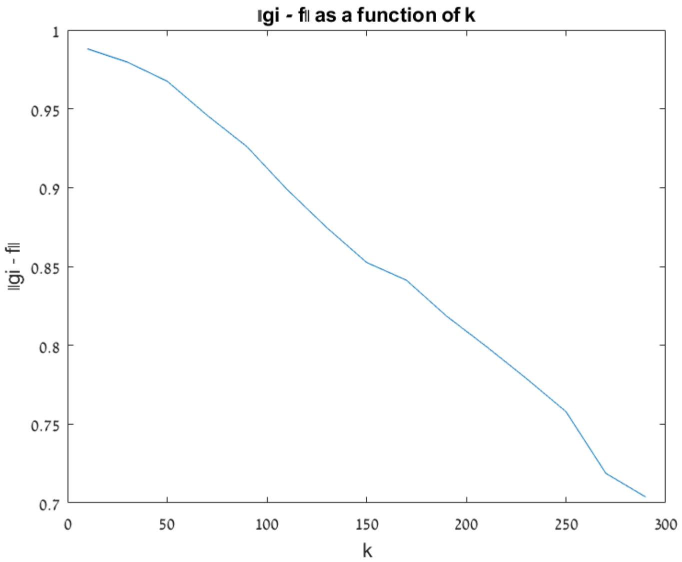
Approximation with 70 eigenvectors:



Approximation with 250 eigenvectors:



Approximation error as a function of k – number of eigenvectors that are used for approximation:



Conclusions:

Null – as expected, for constant function $Wf = 0$.

Symmetry – W is a symmetrical matrix. We showed that $\left\| W - W^T \right\|_F \simeq 0$, where the Frobenius norm defined as $\|A\|_F \equiv \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2}$.

Sparse metrix – For all examples, the amount of the non-zero elements is significantly smaller than the total number of matrix-elements. Also, the following relation is being held:

non-zero elements = 2*# edges + # vertices

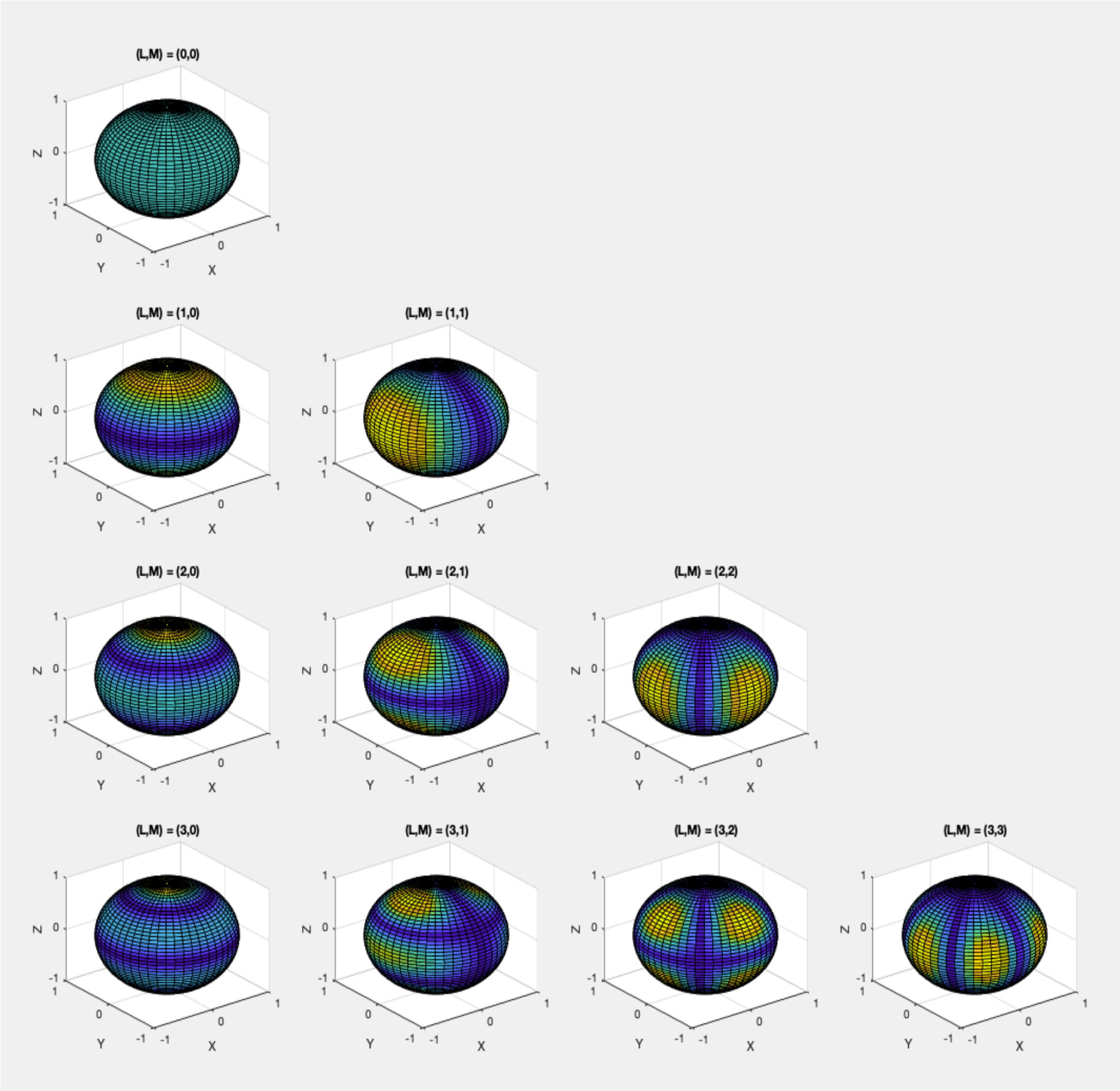
Positivity – W contains non-positive elements.

Positive semi-definite – As all the eigenvalues of W are positive, W is a positive semi-definite matrix.

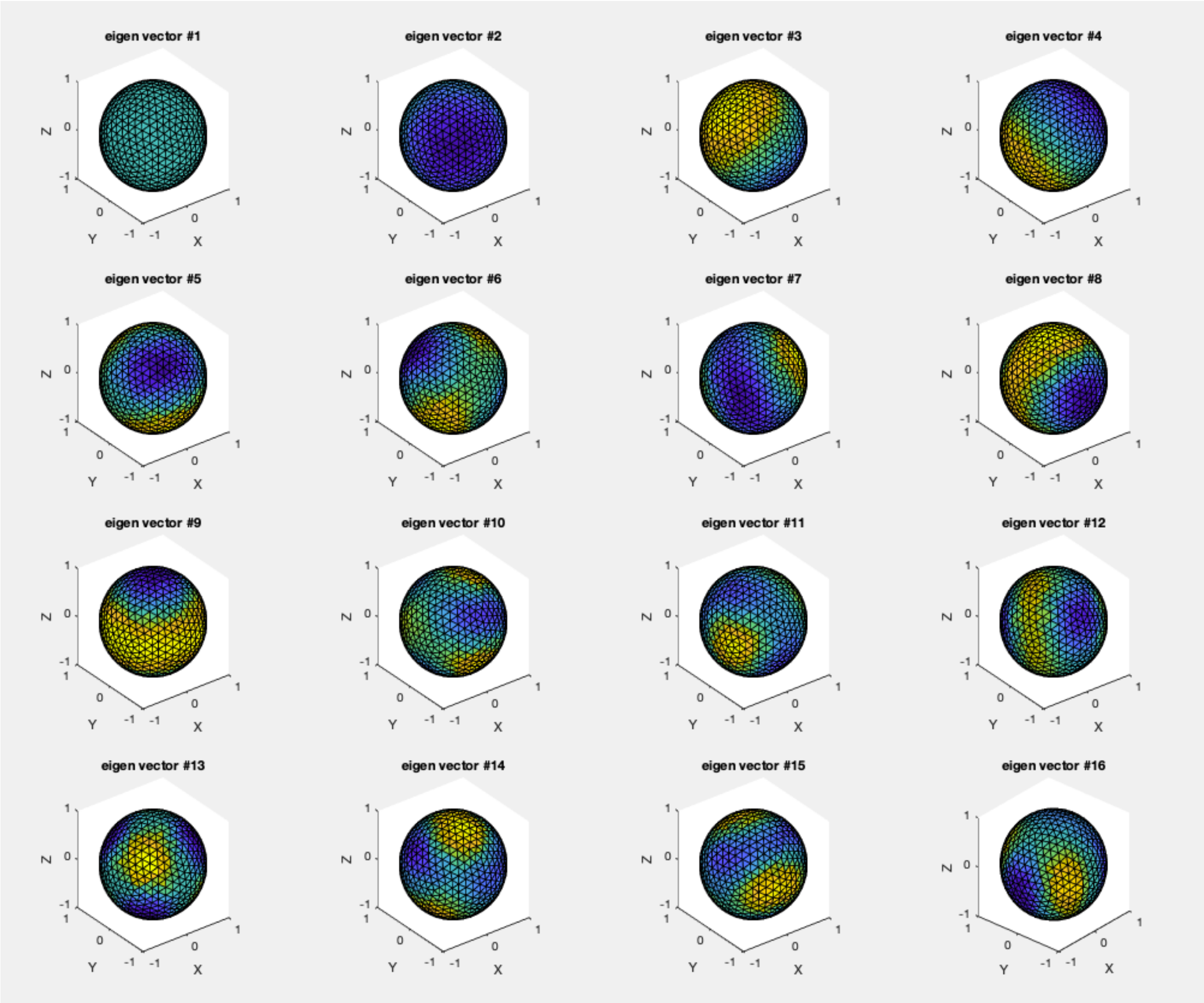
Eigenfunction examples – The eigenfunctions seems to look like harmonic components (the first one is constant, and later it contains some peeks and lows).

Function approximation – One can notice from the disk example that for a higher k number, the desired function is better approximated. It can be seen from the snapshots as well as from the graph that describes that approximation error as a function of k.

3. The first 10 spherical harmonics on a sphere mesh:



The first 10 eigen vectors on a sphere mesh:



As expected, up to order, there are similar spheres in the above figures.