Space Mechanics (086287) – HW1

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HW from semester: Winter 2017-18

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Matlab ver.: 2024b

Initialization

```
clear
close all
clc
```

Constants

```
mu = 3.98603e5; %km^3/sec^2, from Appendix A
Re = 6378.16; %km
```

Question 1

Problem

Given:

```
T = 120*60; %sec
```

At descending node:

```
rx_d = 5300; %km
ry_d = 6100; %km

Vx_d = -1.5; %km/s
Vy_d = 2.2; %km/s
```

Find:

a. rz and Vz

b. a, e and inclination

Solution

Descending Node =>

```
rz_d = 0;
```

From Time Period:

$$T = 2\pi\sqrt{a^3/\mu} \Rightarrow a = \left(\frac{T^2\mu}{4\pi^2}\right)^{1/3}$$

```
a = (T^2*mu/4/pi^2)^(1/3);
```

From Energy Equation:

$$\frac{V^2}{2} - \frac{\mu}{r} = En \Rightarrow V^2 = 2\left(\frac{\mu}{r} + En\right)$$

For Elliptical orbit:

$$\mathrm{E}_n = {}^{-\mu}\!/_{2a}$$

```
En = -mu/2/a;

r_abs = norm([rx_d, ry_d, rz_d]);
V2 = 2 * (mu/r_abs + En);
```

At the descending node the z velocity component is negative

```
Vz_d = -sqrt(V2-Vx_d*Vx_d-Vy_d*Vy_d);
```

The Equation for Eccentricity:

$$e = \sqrt{1 + 2E_n h^2/\mu^2}$$

Where h is the angular momentum:

$$\vec{h} = \vec{r} \times \vec{V}$$

```
h_vec = cross([rx_d,ry_d,rz_d],[Vx_d,Vy_d,Vz_d]);
h_abs = norm(h_vec);
e = sqrt(1+2*En*h_abs^2/mu^2);
```

Inclination is the angle between the angular momentum vector and the z axis:

$$cos(i) = \frac{h_z}{h}$$

```
inclination = acos(h_vec(3)/h_abs)*180/pi; %deg
```

Arrange the output

```
str_ans = sprintf(['Q1 answers\npart a:\n' ...
    'rz = %4.3f [km], Vz = %4.3f [km/s]\n'...
    'part b:\n'...
    'a = %4.0f[km], e = %5.4f, inclination = %5.3f[deg]'],...
    rz_d, Vz_d, a, e, inclination);
```

```
disp(str_ans)
```

```
Q1 answers
part a:
rz = 0.000 [km], Vz = -6.489 [km/s]
part b:
a = 8059[km], e = 0.0965, inclination = 68.353[deg]
```

Question 2

Problem

Given:

```
T = 2*3600; %sec
alt_perigee = 500; %km
```

Calculate total velocity change for:

a. two pulse transfer from perigee

b. two pulse transfer from apogee

c. ine pulse transfer at intersection

Solution - precalculations

```
r_perigee_i = alt_perigee + Re;
```

Semimajor axis of initial and radus of target orbit:

$$T = 2\pi \sqrt{a^3/\mu} \Rightarrow a = \left(\frac{T^2\mu}{4\pi^2}\right)^{1/3}$$

```
a = (T^2*mu/4/pi^2)^(1/3);
```

Velocity at a point on ellictic orbit:

(1) $\frac{V^2}{2} - \frac{\mu}{r} = En \Rightarrow V = \sqrt{2\left(\frac{\mu}{r} - \frac{\mu}{2a}\right)}$

Velocity of circular orbit:

(2) $V_c = \sqrt{\frac{\mu}{r}}$

Solution a - transfer from perigee

Semimajor axis of transfer orbit from perigee:

It's apogee is the rads of the circular orbit.

```
aTp = (a+r_perigee_i)/2;
```

Using equations (1) and (2)

```
deltaV1 = sqrt(2*mu*(1/r_perigee_i-0.5/aTp))-sqrt(2*mu*(1/r_perigee_i-0.5/a))
```

```
deltaV1 =
-0.2435
```

```
deltaV2 = sqrt(2*mu*(1/a-0.5/aTp))-sqrt(mu/a)
```

```
deltaV2 =
-0.2837
```

```
deltaVtotal = abs(deltaV1)+abs(deltaV2)
```

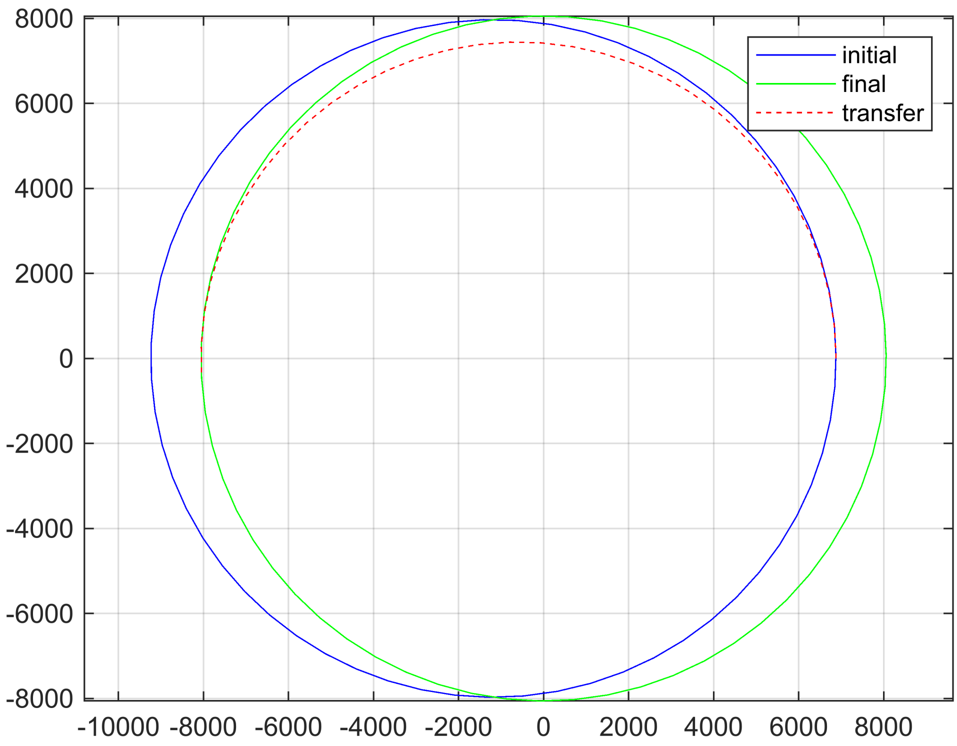
```
deltaVtotal =
0.5272
```

Calculate other geometric parameters:

```
e_initial = 1 - r_perigee_i/a;
b_initial = a*sqrt(1-e_initial^2);
c_initial = e_initial*a;

e_transfer = 1 - r_perigee_i/aTp;
b_transfer = aTp*sqrt(1-e_transfer^2);
c_transfer = e_transfer*aTp;
```

Draw initial and target orbits:

```
figure(1)

theta_vec_full = 0:.1:(2*pi+.1);
theta_vec_transfer = 0:.1:(pi+.1);

x_vec_initial = -c_initial+a*cos(theta_vec_full);
y_vec_initial = b_initial*sin(theta_vec_full);
plot(x_vec_initial,y_vec_initial,'b');
hold all

x_vec_circle = a*cos(theta_vec_full);
y_vec_circle = a*sin(theta_vec_full);
plot(x_vec_circle,y_vec_circle,'g');
hold all

x_vec_transfer = -c_transfer+aTp*cos(theta_vec_transfer);
y_vec_transfer = b_transfer*sin(theta_vec_transfer);
plot(x_vec_transfer,y_vec_transfer,'r--');
hold all

legend('initial','final','transfer')
grid
axis equal
```



Solution b - transfer from apogee

Semimajor axis of transfer orbit from perigee:

It's perigee is the rads of the circular orbit.

```
r_apogee_i = 2*a - r_perigee_i;

aTa = (a+r_apogee_i)/2;
```

Using equations (1) and (2)

```
deltaV1 = sqrt(2*mu*(1/r_apogee_i-0.5/aTa))-sqrt(2*mu*(1/r_apogee_i-0.5/a))
```

```
deltaV1 =
0.2721
```

```
deltaV2 = sqrt(2*mu*(1/a-0.5/aTa))-sqrt(mu/a)
```

```
deltaV2 =
0.2361
```

```
deltaVtotal = abs(deltaV1)+abs(deltaV2)
```

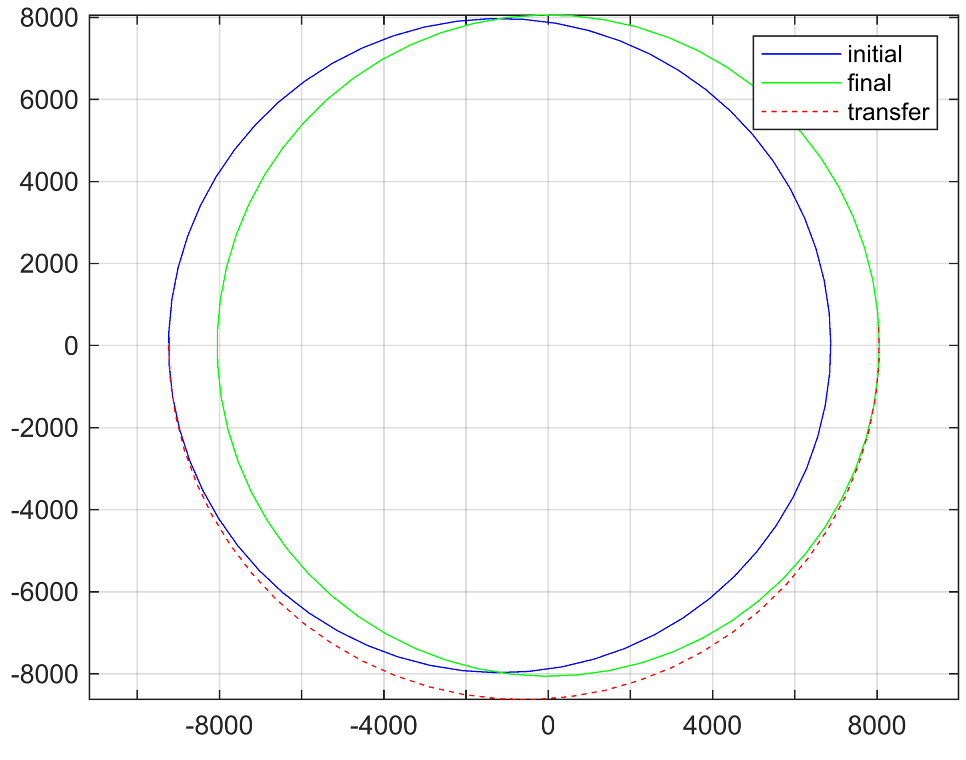
```
deltaVtotal =
0.5082
```

Draw initial and target orbits:

```
figure(2)
```

Calculate other geometric parameters:

```
e_transfer = 1 - a/aTa;
b_transfer = aTa*sqrt(1-e_transfer^2);
c_transfer = e_transfer*aTa;

theta_vec_full = 0:.1:(2*pi+.1);
theta_vec_transfer = pi:.1:(2*pi+.1);

plot(x_vec_initial,y_vec_initial,'b');
hold all

plot(x_vec_circle,y_vec_circle,'g');
hold all

x_vec_transfer = -c_transfer+aTa*cos(theta_vec_transfer);
y_vec_transfer = b_transfer*sin(theta_vec_transfer);
plot(x_vec_transfer,y_vec_transfer,'r--');
hold all

legend('initial','final','transfer')
grid
axis equal
```



Solution c - single pulse at intersection

At intersectionradius and semimajor axis of both orbits are equal. The velocity vector needs to be rotated by angle γ.

(2) $V_c = \sqrt{\frac{\mu}{r}}$

(3) $tan(\gamma) = \frac{e}{\sqrt{1-e^2}}$

(4) $\Delta V_{total} = 2Vsin(\gamma/2)$

```
V  = sqrt(mu/a)
```

```
V =
7.0328
```

```
gamma = atan(e_initial/sqrt(1-e_initial^2))
```

```
gamma =
0.1471
```

```
deltaVtotal = 2*V*sin(gamma/2)
```

```
deltaVtotal =
1.0333
```

Question 3

Get expressions for radial and normal components of the velocity vector vs. the true anomaly angle, parameter p and ecsentricity e. Find the extremum valies and their positions on the orbit.

Solution

In polar coordinates:

$$\bar{r} = \begin{Bmatrix} r \\ 0 \\ 0 \end{Bmatrix}; \dot{\bar{r}} = \begin{Bmatrix} \dot{r} \\ r\dot{\theta} \\ 0 \end{Bmatrix} \text{ -> } v_r = \dot{r}; v_\perp = r\dot{\theta}$$

Differentiating the expression for r:

$$r = \frac{p}{1+e\cos\theta} \Rightarrow \dot{r} = \frac{-pe\sin\theta}{(1+e\cos\theta)^2}\dot{\theta}$$

The differential of the angle:

$$\left.\begin{array}{l} r^2\dot{\theta} = h \\ p = \dfrac{h^2}{\mu} \end{array}\right\} \Rightarrow \dot{\theta} = \frac{\sqrt{p\mu}}{r^2}$$

Therefore:

$$v_r = \dot{r} = \frac{-pe\sin\theta}{(1+e\cos\theta)^2}\frac{\sqrt{p\mu}}{r^2} = -e\sin\theta\cdot\sqrt{\frac{\mu}{p}}$$

$$v_\perp = r\dot{\theta} = \frac{\sqrt{p\mu}}{r} = (1+e\cos\theta)\sqrt{\frac{\mu}{p}}$$

The extrema:

$$\theta = 0(perigee) \Rightarrow \sin\theta = 0, \cos\theta = 1 \Rightarrow v_r = 0, v_\perp = \max$$
$$\theta = \pi(apogee) \Rightarrow \sin\theta = 0, \cos\theta = -1 \Rightarrow v_r = 0, v_\perp = \min$$
$$\theta = \pi/2, 3\pi/2 \Rightarrow |\sin\theta| = 1, \cos\theta = 0 \Rightarrow |v_r| = \max$$