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FROM TRADITIONAL TO COMPUTATIONAL ARCHAEOLOGY. AN INTERDISCIPLINARY METHOD AND NEW APPROACH TO VOLUME AND WEIGHT QUANTIFICATION

Summary. *The present study aims to show the effectiveness of a methodological procedure to estimate the volumetric capacity of archaeological ceramic vessels and the net and gross weights of their probable contents. This method can be easily applied, independently of cultural or chronological contexts, and, alongside other historical and economic conclusions, might serve to verify the possible existence of typological and metrological standardizations of domestic or commercial containers. This study gives a detailed description of a simple methodological protocol which uses profile drawings to calculate the approximate volume of any vessel, thus enabling assessment of its conformity to ancient weight systems. This article will illustrate the strength of the method using a sample of Ramon T-11213 amphorae made in the Bay of Cadiz during the fifth century BC, which, given its quantitative and qualitative strengths, proves to be an exemplary case study and a valid pilot.*

INTRODUCTION

The purpose of this paper is to present a methodological protocol, based on simple and easily reproducible archaeological and mathematical procedures, to estimate, with little margin of error, the capacities (volumes) and weights (gross and net) of any vessel or transport container made in antiquity, starting only with a profile drawing. This protocol is based on the contributions of a team of members trained in archaeology, mathematics and engineering, in an interdisciplinary collaboration that has resulted in the proposal of a proven, verified and scientifically well-founded procedure.

Through an analysis that goes beyond traditional, typological observations, this article outlines a methodological protocol that uses the data obtained by computational software (graphical representation, mathematical and statistical calculation) to calculate (indirectly) the values of possible standards of volume and weight with potentially important conclusions for the study of ancient trade. Using this empirical approach, it searches for models applicable to any historical period in any geographic-cultural context. In summary, this paper offers a methodology that allows for the study of the typological, morphometric and volumetric standardization of any vessel type in relation to the system of weights and measures commonly used in each particular area of study.

It should be noted that approximations of ancient weights and measures have thus far mainly been approached from the theoretical analysis of classical literary sources (Powell 1992; Pellicer i Bru 1997; Teodor 1998; 2000a; 2000b; Zamora 2003) and not from experimentation, direct or digital. The study of the metrological systems used in antiquity remains an underexplored area with great potential, due to the fact that the multitude of metrological references and their fractioning are not yet clear. In fact, as proven by recent studies (Moreno Pulido and Arévalo González 2017), research into ancient systems of weights and measures allows us to pursue, with interesting results, lines of parallel or related research which have hitherto been less accessible, but which will be discussed in this study.

This article describes in detail a valid methodological process, translatable to any cultural and chronological field, whose effectiveness will be demonstrated with data obtained from recent quantification studies of containers manufactured in Gadir (Cadiz, southern Spain) in the fifth century BC. The volumetric and weight investigation of ancient containers in the archaeology of Cadiz is still developing. Nevertheless, this kind of computational analysis is generating advances in fields such as the use and diffusion of systems of volume and weight, as well as changing relationships with the introduction and establishment of the monetary system (Moreno Pulido and Arévalo González 2017). This procedure also allows for the calculation of volume production and better estimations of the overall scale of these industrial-commercial practices (Sáez Romero and Moreno Pulido 2017).

The case study centres on the analysis of transport containers for trade in the Punic Bay of Cadiz, with the focus on a sample of 25 complete or nearly complete local T-11213 amphorae dating to around the fifth century BC. The choice of this group of archaeological objects and this particular geographical and historical scenario is due to two factors: first, the enormous importance of the ancient city of Gadir as the main political, cultural and economic focus of the southern Iberian peninsula; and second, the fact that Gadir was the leading centre in the region in the use of monetary economic systems, as the city had the first mint in the area at the beginning of the third century BC. Further justification for the choice is that currently, these archaeological remains are optimally systematized (Ramon Torres 1995). Also, this is a large set of complete and well-dated containers, which provides a synchronous representative sample that can yield statistically significant results (Sáez Romero and Moreno Pulido 2017).

FROM THE ARCHAEOLOGICAL RECORD TO MATHEMATICAL CALCULATIONS. PREVIOUS INTERDISCIPLINARY EXPERIENCES AND APPROACHES

Despite the intense development of digital technology and computer software in recent years, this progress has not yet had a commensurate impact on methodological applications in archaeological research. Even with their increasing diffusion and methodological implementation, the use of these new technologies in the documentary processing of archaeological objects still presents a major challenge. Moreover, paradoxically, we must admit that there is still a shortage of interdisciplinary teams made up of archaeologists, mathematicians and engineers. Due to this, work in this computational line still far from rivals other types of archaeological object analysis, such as purely typological approaches.

As an exception, the renowned work of the LAQU (Laboratori d'Arqueologia Quantitativa) project of the University of Barcelona focuses on the theoretical and methodological development of digital techniques for computer application in archaeology (Barceló 2007; 2009; Barceló and Vicente 2011), encouraging the development of new approaches combining mathematics and archaeology (Barceló and Bogdanovic 2014). In this innovative line of study, Professor Uzy Smilansky's Laboratory of Computer Archaeology at the Weizmann Institute of Science in Israel (Karasik *et al*. 2004a; 2004b; Mara and Sablatnig 2005; Sergi *et al*. 2012; Gilboa *et al*. 2013; Grosman *et al*. 2014) leads the way in the development of mathematical and computerized methods for archaeological research. The team has undertaken work on the volumetric principles, typology and classification of ancient ceramics, either from their profiles or by methods of precise orientation of the ceramic fragments for subsequent drawing and reconstruction.

Nevertheless, it should be noted that the calculation of the capacities of ancient ceramic vessels remains a relatively unexplored area, owing to difficulties resulting from the archaeological record itself, including the small number of complete vessels available. This has resulted in a general shortage of studies devoted to volume and weight, including the diachronic evolution of old metrological systems from data obtained through the computational analysis of archaeological containers.

Despite the lack of investigation, the importance of these quantitative data for the study of economic history has not gone unnoticed in international research. In fact, it is a line that has been developing for several decades, especially in English-language literature (Lang and Crosby 1964; Wallace Matheson and Wallace 1982; Wallace 1984; Louise and Birnie 1994; Louise and Dunbar 1995; Senior and Birnie 1995; Anderson 1995a; 1995b; Thomas and Wheeler 2002; Engels *et al*. 2009; Cohen *et al*. 2013). Furthermore, as previously highlighted, there have been interesting contributions from Israeli research in recent years (Van Alfen 1996; Karasik and Smilansky 2006; Karasik *et al*. 2006; Zapassky *et al*. 2006; 2009; Kletter 2009; 2014), and it should be added that Russian archaeological research has also made remarkable progress in this area (Teodor 1998; 2000a; 2000b; Monachov 2005; Vodolazhskaya 2008).

The main objective of these studies has been the search for a general method for calculating the volume of ancient ceramic vessels, an issue which has for years been a priority, given extra impetus by the emergence of groups formed of researchers from the combined fields of humanities, mathematics and engineering (Rodríguez and Hastorf 2013). Particularly remarkable are the initiatives aimed at developing a free and universal computer program for the automatic calculation of container capacities, such as the *Kotyle* computer application, accessible only to registered users (see kotyle.readthedocs.org) (Costa 2013).

Finally, it should be emphasized that, despite the formulation of interdisciplinary initiatives to develop automated applications for quantifying the volumes of ancient containers, these techniques are currently a long way from being included in a standardized manner in conventional archaeological research methods, due to the difficulties they involve.

FROM LABORATORY TO AUTOCAD®: SELECTION, PROCESSING AND DIGITAL DOCUMENTATION OF ARCHAEOLOGICAL MATERIAL

The methodological protocol proposed by this study differs from those described above, because it has proven itself to be a fast and relatively easy system, requiring only a two-dimensional archaeological drawing of each item which is then digitized for further processing and analysis. This is advantageous as it allows the volume of the container to be quantified using only the existing bibliography, thus simplifying the laboratory work required to gain the primary information. However, it is imperative to keep in mind that the data collected from these secondary sources will lack accuracy in terms of quality, which is a fundamental factor in estimating the level of uncertainty in the results of the subsequent calculations.

The proposed method typically selects the vessels from the archaeological record most suitable for metrological study, giving priority to those containers which are complete or almost complete. After the typological and morphological data have been gathered and the selected vessels have been catalogued, they must then be photographed and a traditional two-dimensional drawing rendered of the profile.

As part of this data-gathering process, it is necessary to acquire accurate information on:

- Dimensions:

  Maximum and minimum body diameters
  Maximum diameter of the mouth
  Maximum preserved length
  Estimated non-preserved length

- Exterior appearance
- Density and thickness of the fabric?
- Technical or typological peculiarities
- Weight of the empty vessel
- Other observable data

All of these are factors for controlling the method of calculation and digitization. The gathering of this information is essential, especially in the case of complete or nearly complete items, particularly for reconstructing the capacity and weight of these specimens. If such specific data as the weight of the empty vessel or the density of the fabric are not recorded in the laboratory, the results obtained regarding the typified (net and gross) weight of the containers will be *a posteriori* highly uncertain and subject to an error whose significance will be difficult to assess.

The process must be repeated for each of the selected types, processing the largest possible number of examples from each typological group in order to gather a sufficient amount of data to carry out a statistically significant evaluation of the capacities and average weights per standardized type. This documentation is then archived in individualized files created for this purpose, including a high-resolution scan of the two-dimensional paper drawings of the selected ceramics.

The next step is to digitize the scanned drawings from the laboratory or from the published literature. For this purpose, the AutoCad vector drawing software has been proven to provide a powerful set of two-dimensional and three-dimensional design tools, making it the most useful digital design software both for drawing in engineering and architecture and as a tool for generating the type of precise two-dimensional drawing valid for use in archaeology. AutoCad allows for the importation of the scanned drawings into its work area; subsequently, the curve of the inner profile of each of the selected containers can be vectorized using the image as a base or template. For digitization, the axis of revolution of the item is drawn and guiding lines are placed at the beginning and end of the profile of each of the vessels, with the possibility of using the drawing method based on splines (soft curves) (Fig. 1).

At both the laboratory and AutoCad stages, special care must be taken with the reconstruction of any non-preserved curves when drawing the profile of a fragmentary vessel. A lack of accuracy in reconstructing a vessel according to its typological group could undermine or even invalidate the data thus obtained. In order to avoid this situation, when estimating the

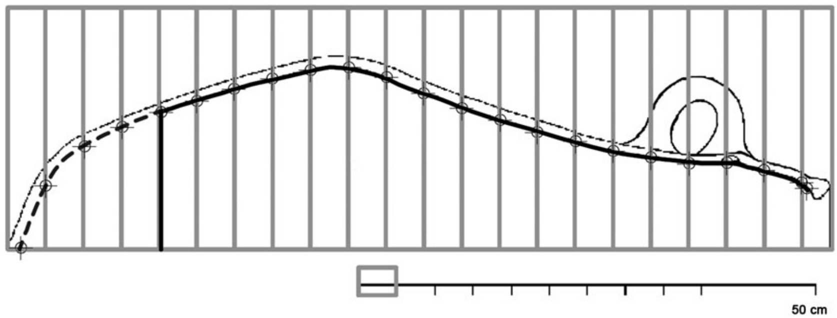


Figure 1
Example of digitizing the inner profile of one of the Gadir T-11213 amphorae (Amphora C) through *AutoCad*®.

reconstruction of each vessel, a typified curve, previously obtained by averaging the profiles of the largest possible number of complete vessels from the same typological group, must be accurately followed. In addition, as is usual and logical in the collection of any scientific data, it is imperative to carefully distinguish between the reconstructed and the preserved data in order to draw valid conclusions and discuss the results (Fig. 1).

Once the vessel has been digitized, x/y coordinates are taken from a series of highlighted control points at standard intervals along the inner profile of the vessel under study. To do this, at regular distances (from *c*.5 cm), lines are plotted perpendicular to the axis of revolution defined by the base and mouth of the object, marking and annotating the x/y coordinates of each control point (Fig. 2). These coordinates will be obtained at *Cad* points, which must subsequently be scaled according to the measurements of each pot in cm or mm (Fig. 3; Table 1).

Regarding the calculation of the volume held by the containers, it is assumed that the contents would not completely fill the vessels, but that an empty space would be left for sealing the mouth with various resins or opercula. Therefore, it is generally estimated that the maximum filling-point of each container would be the end of the shoulder and the start of the mouth. The last control point, which measures the maximum estimated length for the calculation of the capacity of each container, would, accordingly, be placed at this point (Figs. 1 and 2). As this measure is not certain and is obviously relative, it should be as standardized and invariable as possible throughout

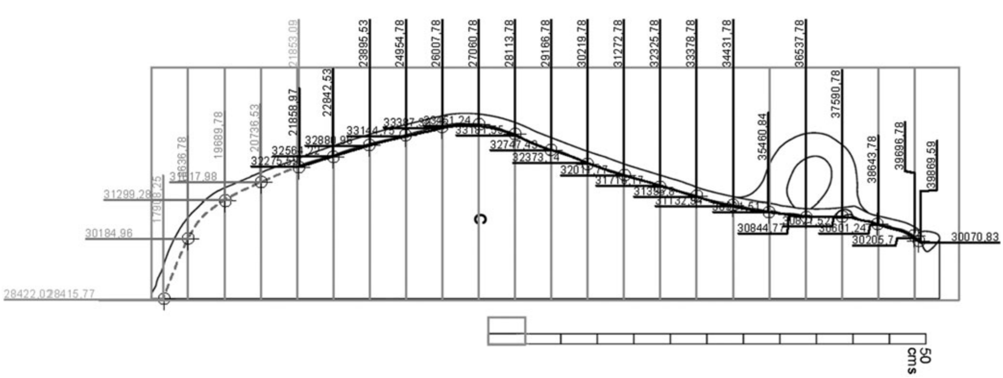


Figure 2
Example of taking coordinates of regular control points on the inner profile of amphora T-11213 C in *AutoCad*®.

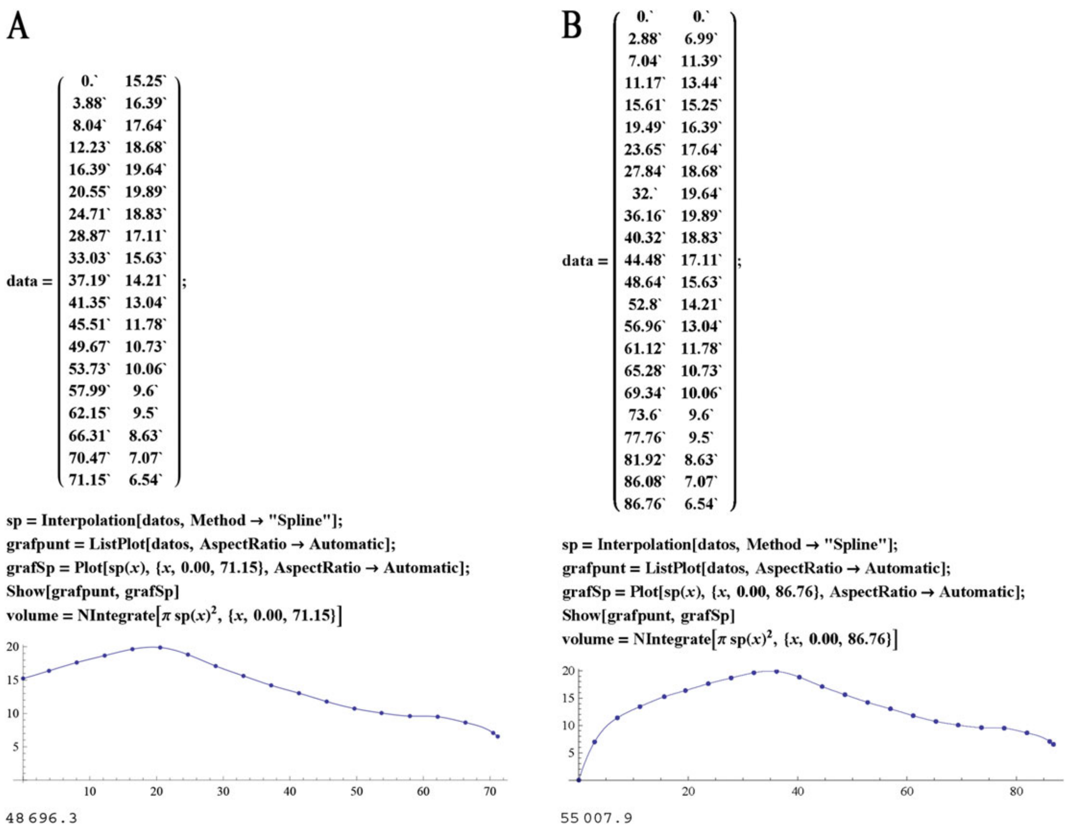


Figure 3
Example of the calculation of the volume of amphora T-11213 C in *Mathematica*®. A: results for preserved vessel; B: results for reconstructed vessel. [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1
Coordinate checkpoints taken from the inner profile of Amphora C

X axis (Cad Points) Reconstructed	Y axis (Cad Points) Reconstructed	X axis (scaled in cm) Reconstructed	Y axis (scaled in cm) Reconstructed	X axis (scaled in cm) Preserved	Y axis (scaled in cm) Preserved
0	0	0.00	0.00	-	-
728.53	1769.19	2.88	6.99	-	-
1781.53	2883.51	7.04	11.39	-	-
2828.28	3402.21	11.17	13.44	-	-
3950.72	3859.82	15.61	15.25	0.00	15.25
4934.28	4148.45	19.49	16.39	3.88	16.39
5987.28	4465.19	23.65	17.64	8.04	17.64
7046.53	4728.98	27.84	18.68	12.23	18.68
8099.53	4971.62	32.00	19.64	16.39	19.64

(Continues)

TABLE 1
(Continued)

X axis (Cad Points) Reconstructed	Y axis (Cad Points) Reconstructed	X axis (scaled in cm) Reconstructed	Y axis (scaled in cm) Reconstructed	X axis (scaled in cm) Preserved	Y axis (scaled in cm) Preserved
9152.53	5035.47	36.16	19.89	20.55	19.89
10205.53	4765.78	40.32	18.83	24.71	18.83
11258.53	4331.66	44.48	17.11	28.87	17.11
11258.53	4331.66	44.48	17.11	28.87	17.11
12311.53	3957.37	48.64	15.63	33.03	15.63
13364.53	3596	52.80	14.21	37.19	14.21
14417.53	3300.4	56.96	13.04	41.35	13.04
15470.53	2981.03	61.12	11.78	45.51	11.78
16523.53	2717.17	65.28	10.73	49.67	10.73
17552.59	2545.74	69.34	10.06	53.73	10.06
18629.53	2429	73.60	9.60	57.99	9.60
19682.53	2405.75	77.76	9.50	62.15	9.50
20735.48	2185.47	81.92	8.63	66.31	8.63

the sample. Moreover, it should be taken into account when considering the mean of the maximum lengths of the sample and the possible error in the volume and weight calculation of each vessel.

The digital drawing and the gathering of the control points for the x/y coordinates of the internal profile are systematically repeated throughout the sample of selected vessels (Table 1), in order to create a sufficiently diverse dataset for subsequent statistical analysis. Likewise, for the sake of greater transparency in the treatment of the figures, both actual (preserved) and estimated (reconstructed) data will be included for fragmentary vessels.

The maximum diameter of each specimen should also be noted, so that individual measurements can be compared against averages within groups as part of a typological evaluation. The range of values for the maximum preserved and reconstructed lengths must be recorded, since the relationship between the length and diameter of a vessel obviously determines its volume. This

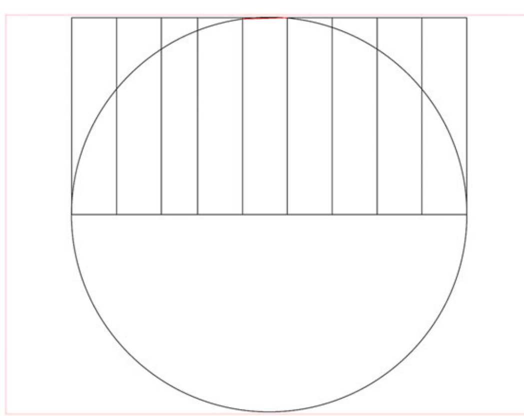


Figure 4
Calibration of the method from the drawing of a semicircle in *AutoCad*®. [Colour figure can be viewed at wileyonlinelibrary.com]

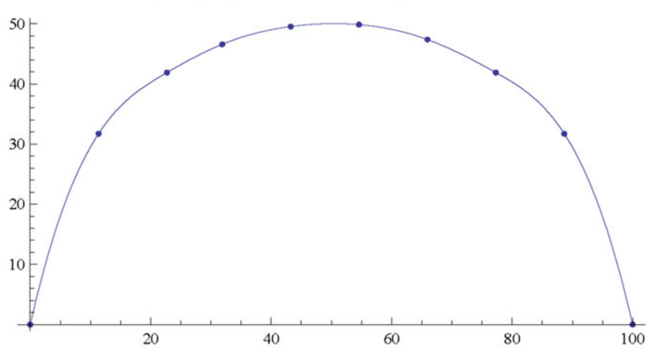
relationship was well known in the pottery workshops, as evinced by ancient sources such as the Egyptian formulae of the Rhind mathematical papyrus, pBM 10057-8, problems 41-7, and the Greek formula of Heron (Vodolazhskaya 2008). Although the application of formulae relating lengths, diameters and volumes is still controversial (Kletter 2009), they must be taken into account in order to try to advance the understanding of the processes followed by artisans in manufacturing standard vessels in the pottery workshops, as they must have undergone some kind of control and regulation, the nature of which, to this day, remains unclear.

FROM AUTOCAD® TO WOLFRAM MATHEMATICA®: INTEGRAL CALCULATION OF VOLUME OF REVOLUTION

The coordinates obtained by the method described in AutoCad, or in an alternative vector-drawing program, are transferred to the computational software Wolfram Mathematica, or to another calculation program such as the well-known SAGE. In order to determine the capacities of the selected ceramic vessels, a relatively simple mathematical method is employed, using the conveniently scaled control points of each two-dimensional digitized profile to obtain a vector

```
data = ( 0.`      0.`
         11.35`   31.73`
         22.71`   41.9`
         31.87`   46.6`
         43.23`   49.54`
         54.58`   49.86`
         65.94`   47.39`
         77.29`   41.9`
         88.65`   31.73`
         100.`    0.` );

sp = Interpolation[datos, Method → "Spline"];
grafpunt = ListPlot[datos, AspectRatio → Automatic];
grafSp = Plot[sp(x), {x, 0., 100}, AspectRatio → Automatic];
Show[grafpunt, grafSp]
volume = NIntegrate[π sp(x)^2, {x, 0., 100.}]
```



518 666.

Figure 5
Calibration of the method from the calculation of the volume of the proposed semicircle in *Mathematica*®. [Colour figure can be viewed at wileyonlinelibrary.com]

spline (interpolation of a smooth curve from the defined points) of the interior of the vessel, from which its hypothetical volume can then be estimated in cm^3 (Fig. 3).

For calibration purposes, in order both to verify the validity of the method and to estimate the relative error of the procedure, a semicircle with a 50 cm radius was drawn in AutoCad (Fig. 4) following the systematic procedure previously described. Only 10 control points were considered and used for the Mathematica program (Fig. 5). The volume calculated by the software for the sphere of revolution presented a relative error of the order of +/- 1%, validating the method.

This shows that, from the first step of taking data from the internal profile of the vessel to its subsequent use in the calculation of the volume, there is hardly any error. This validates the procedure and assures that it can be transferred to any vessel produced by revolution with great accuracy. This calculation is systematically repeated for all the specimens studied, in order to create a sufficiently contrasted dataset for later statistical consideration (Fig. 6).

The method of digitization and subsequent calculation used in the pilot study had already been utilized in previous studies (Moreno Pulido and Arevalo González 2017; Sáez Romero and Moreno Pulido 2017). The volumetric capacities were estimated for 25 complete or nearly complete amphorae of the series T-11213 manufactured in Gadir in the fifth century BC (Table 2). Measurement data were obtained for each of the containers, the average volume being estimated at 52.93 litres.

FROM THE VOLUME TO THE WEIGHT: HOW TO CALCULATE MASS?

Once the capacities of each of the studied vessels have been obtained, the method is faced with the obvious problem that it is not possible to estimate the weights, understood here as synonymous with mass (calculated in kilograms) rather than as force (mass by acceleration of gravity, expressible in Newtons), of full or empty pots based only on the calculated volumes. The weight of each vessel will vary according to the density of the contents. Moreover, in order to

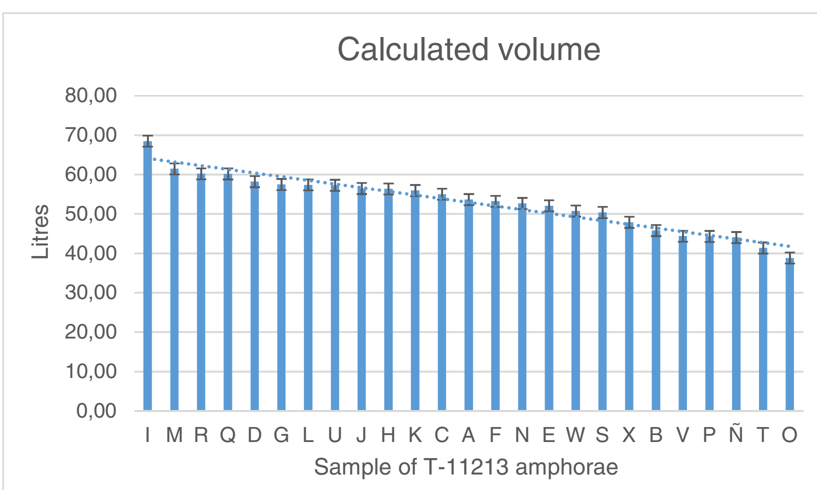


Figure 6
Calculated volumes of the sample of amphorae using the described method. [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 2
Measurement data for the sample of amphorae (data reworked from Sáez Romero and Moreno Pulido 2017)

Amphora	Complete	Max. preserved length (cm)	Estimated total length (cm)	Maximum body radius (cm)	Calculated volume (litres)
I	No	88.78	92.14	22.16	68.47
M	No	86.56	93.47	20.35	61.43
R	Yes	102.78	102.78	21.59	60.18
Q	Yes	93.31	93.31	20.66	60.15
D	No	59.16	89.05	21.11	58.19
G	Yes	92.11	92.11	19.94	57.47
L	No	66.75	95.39	20.35	57.37
U	No	67.09	97.65	20.65	57.26
J	No	67.60	93.32	20.22	56.46
H	No	77.67	91.71	20.04	56.34
K	No	80.10	94.09	20.62	55.94
C	No	71.15	86.76	19.89	55.00
A	Yes	84.77	84.77	20.56	53.65
F	Yes	91.45	91.45	19.91	53.20
N	Yes	92.65	92.65	19.72	52.66
E	No	71.64	90.63	20.23	52.05
W	No	65.14	92.49	19.95	50.73
S	No	70.63	90.43	18.85	50.37
X	No	66.23	90.37	19.02	47.87
B	No	55.82	85.64	19.73	45.78
V	No	68.67	92.18	18.30	44.34
P	Yes	94.82	94.82	17.78	44.29
Ñ	No	61.83	93.38	18.62	44.00
T	No	77.73	86.06	19.02	41.36
O	Yes	91.00	91.00	17.29	38.82
Average		**78.35**	**91.91**	**19.86**	**52.93**
Standard deviation		**12.55**	**3.83**	**1.12**	**7.05**

estimate the gross weight of these receptacles, it will be necessary to know the density of both the content and the container, since it is well known that mass (weight) is equivalent to volume × density (specific weight).

For the content, therefore, the calculated estimation will vary according to the available data about what would have been contained in the vessels under study. It will depend on prior research and also on data collected from archaeometric analysis and historical texts on the subject. In previous papers on the estimation of the weights of T-11213 amphorae manufactured in Gadir (Moreno Pulido and Arévalo González 2017; Sáez Romero and Moreno Pulido 2017), it was assumed that these containers were usually reserved for the transportation of salted fish in a 50/50 mixture of salt and fish broth, this being a rather conservative estimation based on the information provided by literary sources. Therefore, the individual net weights of each of the containers could be estimated by multiplying half the volume of each vessel by the density of the salted fish (estimated at 1.058 kg/l) and the other half by that of the salt (estimated at 1.1 kg/l) (Table 3; Fig. 7).

Weighing complete or nearly complete empty ceramic vessels in the laboratory is of enormous importance in estimating the average tare weight and is fundamental to any gross weight quantification study. However, archaeometric studies do not currently focus on calculating the density of the ceramics, which allows the estimation of the average tare weight by calculating the difference between the volumes obtained by applying the same described method to the splines of the external and internal profiles of each vessel.

TABLE 3
Method of obtaining the net and gross weights of amphora T-11213 C

Calculated volume (litres)	Half volume (litres)	Weight of salted fish content (1/2 V × 1.058) (kg)	Weight of salt content (1/2 V × 1.1) (kg)	Total net weight (kg)	Total gross weight with 14 kg of estimated average tare (kg)
55.00	27.50	29.10	30.25	59.35	73.35

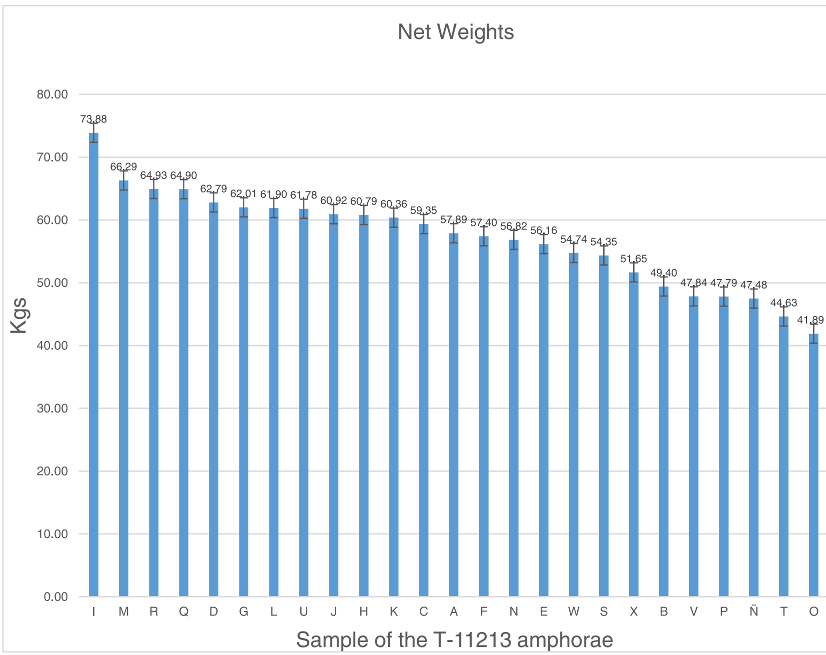


Figure 7
Calculated weights of the sample of amphorae using the described method. [Colour figure can be viewed at wileyonlinelibrary.com]

In the case study of Gadir T-11213 amphorae, the weight of a single complete empty amphora (G), 14 kg, was known. This weight was also given for vessels from Corinth published by Zimmerman Munn (2003), and in the absence of any other data, this was estimated to be the average weight of the tare of the selected amphorae. However, from the maximum diameters and lengths of the sample vessels, it was clear that the larger the size and capacity of the amphorae, the greater the weight would be. Therefore, in order to identify the tare more precisely, an estimated ratio of the tare was calculated from the weight and volume ratio of each of the samples, thus obtaining a series of weights approximate to the real ones, though clearly not testable. The mean results of the gross weights of the sample calculated on the basis of the hypothetical average tare of 14 kg (a mean of the gross weight of 71.05 kg) and the calculated tare weight (an average of 69.82 kg) were very similar; both hypotheses are proposed as approximate but equally valid (Table 4).

TABLE 4
Calculated net and gross weights of the sample of amphorae (data reworked from Moreno Pulido and Arévalo González 2017)

Amphora	Complete	Volume (litres)	Net weight of salted fish (kg)	Average tare (14 kg)	Gross weight of salted fish (kg)	Tare in a ratio of weight/ vol. (0.24 kg/l)	Gross weight of salted fish (kg)
I	No	68.47	73.88	14.00	87.88	16.43	90.31
M	No	61.43	66.28	14.00	80.28	14.74	81.03
R	Yes	60.18	64.93	14.00	78.93	14.44	79.38
Q	Yes	60.15	64.90	14.00	78.90	14.44	79.34
D	No	58.19	62.79	14.00	76.79	13.97	76.75
G	Yes	57.47	62.01	14.00	76.01	14.00	76.01
L	No	57.37	61.90	14.00	75.90	13.77	75.67
U	No	57.26	61.78	14.00	75.78	13.74	75.53
J	No	56.46	60.92	14.00	74.92	13.55	74.47
H	No	56.34	60.79	14.00	74.79	13.52	74.31
K	No	55.94	60.36	14.00	74.36	13.43	73.78
C	No	55.00	59.35	14.00	73.35	13.20	72.55
A	Yes	53.65	57.87	14.00	71.87	12.87	70.74
F	Yes	53.20	57.40	14.00	71.40	12.77	70.17
N	Yes	52.66	56.82	14.00	70.82	12.64	69.46
E	No	52.05	56.16	14.00	70.16	12.49	68.65
W	No	50.73	54.74	14.00	68.74	12.18	66.91
S	No	50.37	54.35	14.00	68.35	12.09	66.44
X	No	47.87	51.65	14.00	65.65	11.49	63.14
B	No	45.78	49.40	14.00	63.40	10.99	60.38
V	No	44.34	47.84	14.00	61.84	10.64	58.48
P	Yes	44.29	47.79	14.00	61.79	10.63	58.42
Ñ	No	44.00	47.48	14.00	61.48	10.56	58.04
T	No	41.36	44.63	14.00	58.63	9.93	54.55
O	Yes	38.82	41.89	14.00	55.89	9.32	51.20
Average		**52.93**	**57.12**	**14.00**	**71.05**	**12.70**	**69.82**
Standard deviation		**7.05**	**7.61**		**7.60**	**1.69**	**9.30**

FROM STATISTICS TO THE PATTERN: THE RECONSTRUCTION OF ANCIENT METROLOGICAL SYSTEMS

New hypotheses in the study of ancient metrological systems can be discussed through comparison of the available literary and archaeological sources with the results produced by the method presented here. The diachronic evolution of the systems, as well as its relation to the monetary and weight systems in each cultural area under study, is remarkable. This is still a developing area of study, but its results should allow for a better understanding of exchange before the invention of coinage and the process of monetization of the ancient economies.

The application of the protocol described here in particular cases where archaeological contexts, written testimonies and other historical evidence are known, allows the advancement of new hypotheses regarding the reconstruction of ancient metrological (weight, volumetric, longitudinal and monetary) systems, which varied culturally and chronologically. The procedure followed in our case study serves as a valid example and as a pilot which can be transferred to other environments and historical periods.

According to recently obtained data (Moreno Pulido and Arévalo González 2017), the averages of the sample volumes of amphorae analysed in our case study seem to fit very well with the Hebrew volumetric system, which had connections with a hypothetical earlier system (Ugaritic) which was used on the Syrian-Palestinian coast at least from the Final Bronze Age and would later have been exported to Gadir by the Phoenicians.

The volumetric system used in Ugarit is not yet well known. However, recent contributions (based on quantification methods and mathematical calculation of the volumes contained in *lmlk* amphorae from Canaan) seem to present sufficient evidence to propose that the Ugaritic system was based on the weight of the volume of water measured according to the talent, thought to be 28.2 kg (Pellicer i Bru 1997; Parise 2006). This is based on several archaeological testimonies, particularly the weight of the ingots carried in wrecks such as Uluburun or Cape Gelidonya (Petruso 1992).

The manufacture of the Canaanite amphorae could also have followed a standardized procedure based on the capacity of the *kd*, the measurement of which is estimated at between 12 and 14 litres and appears to theoretically approximate the intention to contain 14.1 litres of liquid. The measure of the *kd* would, in practice, support the relationship between weight and volume found in the ancient systems of measurement (Pellicer i Bru 1997), since it would weigh half a talent. However, this Ugaritic pattern has not been sufficiently compared with epigraphic and archaeological sources (Zamora 2003), and therefore remains hypothetical. It would be interesting to see more studies of the volumes and weights of the amphorae found in the winery of Minet-el-Beida, the port of Ugarit, based on the application of the methodological protocol outlined here.

Unlike the Ugaritic system, the Hebrew metrological system was reconstructed by Hultsch in 1882 and later revised by other researchers (Docter (1988–90) and Pellicer i Bru (1997), among others), based on the testimonies provided later by Flavius Josephus (Josephus *Jewish* Antiquities 8.2.9 (57)). In spite of the greater reliability of these data, potential changes in the system, which is known only indirectly, through later testimony, must be taken into account. In addition, the possible accumulation of errors in the reconstruction of these measures and their transfer to other systems must also be considered; the late Hellenistic equivalencies given by Josephus clearly accommodate Roman metrology. All these issues would eventually explain some of the differences between the theoretical values and those actually calculated.

Taking into account all of these difficulties, it can be said that the ancient Hebrew system of capacity measures was based on the dimension of the *koros* (393.81 litres), which was subdivided into 30 *sata* (1 *saton* = 13.127 litres), 180 *kabs* (1 *kab* = 2.188 litres) or 720 *logs* (1 log = 0.547 litres). The measurement of the *saton*, it can clearly be seen, is very close to that proposed hypothetically for the *kd*, a fact which is unsurprising given the connections and equivalences between the systems, which were close both culturally and geographically.

As previously noted, the average volume of the Gadir T-11213 amphorae approaches 53 litres. When this measure is converted to *sata*, it is found that these amphorae could have been manufactured with the intention of containing an average capacity of almost 4 *sata*. This result clearly seems to indicate that these vessels were manufactured in Gadir in the fifth century BC in a standardized form. This standard was related to a pattern of official measures with very close equivalents to the Hebrew system used in Judaea in the first century AD.

However, the sample of amphorae in our case study had a standard deviation of 7.05 (Table 4). As these results were not completely satisfactory, it was decided that more detailed statistical analysis was needed. For this, the sample was divided into ranges organized from minimal to maximal volume, based on the approximation of the capacity of full amphorae. This resulted in the finding that there were at least four different groups of capacity readily expressible as 4.5, 4, 3.5

and 3 *sata*. Therefore, these containers would have been manufactured in a standardized way, with different sizes to meet the different market needs.

Also, thanks to this methodology and to its statistical evaluation, observations can be made regarding the net and gross average weights of the studied amphorae. In this case it has been assumed from the weight system used for the production of its first coins, that Gadir knew the metrological system of Ugarit, which was based on the talent of 28.2 kg. This question has previously been addressed by García-Bellido (2003; 2013), based on the weight of the silver *hemishekels* issued in Gadir at the beginning of the third century BC. These *hemishekels* had an average weight of 4.5/4.7 g and were adjusted to the *shekel nsf*, also referred to as Syrian, micro-Asian or Ugaritic (Elayi and Elayi 1997; Parise 2006), weighing 9.4 g. Moreover, this *shekel* testifies to the use of a complex metrological system, equivalent to 3000 parts of a talent of 28.2 kg, or to 50 parts of a *minē* of 470 g.

Thanks to the weight of the Gadir coins, it was demonstrated that this system was well known in the city. However, these data have not been compared with the hypothetical average weights of (or measured for) these amphorae (Table 4), since this information remained unknown until very recently. Thanks to this methodology it has been possible to clearly observe that the average net weights of the T-11213 amphorae were very close to the weight of two talents (Table 5), which confirms the usefulness of the described method and clearly expands its possibilities for historical research. Moreover, it was found that the average weight of an empty vessel was close to half a talent, a discovery which should lead to a rethink about the manufacturing processes of these containers in the workshop.

Likewise, given the high standard deviation of the average weights of the studied sample (Table 2), the same ranges expressed in *sata* were used to ascertain whether their measurements corresponded logically to some type of standardization or metrological normalization. Subsequently, it was observed that the sample reflected the existence of groups of amphorae weighing around 3 talents 15 minas, 2 talents 45 minas, 2 talents 30 minas and 2 talents respectively.

Therefore, we were able to hypothesize that the monetary value of the salted fish contained in the T-11213 amphorae justified the preparation of vessels according to different sizes, following a weight standard marked by the measurement of the Ugaritic talent. If the Ugaritic pattern was used in Gadir for the manufacture of these amphorae and for the minting of its coins, it is still necessary to go a step further and explore the monetary value of the goods, and try to formulate approximations of the prices that these products may have reached in antiquity. Following this same methodology, it will also be interesting to ascertain whether these same weight systems were used for the manufacture of other objects and to measure the value of other tradable products. Thus, this method and data could lead to the development of many interesting lines of research for future consideration.

TABLE 5

Average volume and net and gross weights of the sample expressed in litres, kilogrammes, *sata* and talents

Mean values of the sample of T-11213 amphorae			
	Calculated averages (litres or kilos)	**Value of the theoretical units**	**Number of *Sata* or talents**
Volume	52.93 l	1 *saton* = 13.27 l	3.9 ≈ 4 *sata*
Net weight of the salt fish	56.21 kg	1 talent = 28.20 kg	1.9 ≈ 2 talents
Gross weight with average tare of 14 kg (*circa* half a talent)	70.21 kg	1 talent = 28.20 kg	2.4 ≈ 2.5 talents

REFLECTIONS ON AND IMPROVEMENT OF THE METHOD

Thanks to the digital tools which are universally available today, the method of quantification proposed in this paper is easily applicable to all types of ceramic containers of any chronological and cultural context. Its utility is based both on its relative ease and speed and on the enormous possibilities for its use in diverse lines of investigation. Errors in volumetric quantification using this methodology are based primarily on the collection of data in the laboratory. This is an extremely important factor that must be taken into account when carrying out a protocol for studying ceramic vessels beyond purely typological factors involving the analysis of quantitative data essential for the study of ancient economic history.

It could be argued that a limitation of the method lies in the quality of the data provided by the archaeological record, which mostly offers fragmented and partial objects. It must be admitted that by including vessels with reconstructed profiles, the method utilizes estimative data. The data would only be totally accurate when the specimens are complete, a circumstance which is usually quite anomalous in archaeology. Therefore it is important to emphasize again the significance of meticulous treatment of the complete specimens that have been recovered and the detailed collection of data on their weight, dimensions, etc. at the laboratory. Obviously, the sample with the greatest number of complete specimens will be the most reliable and the means obtained in the volumetric calculation and the reconstruction of the spline of the internal profile of each item should serve as a template and a control method for the digital reconstruction of the fragmented pots.

It is possible to go a step further and argue that in the case of the T-11213 amphorae, the typological classification proposed so far is insufficient, since all the specimens included in the analysed sample have been considered *a priori* as equals, belonging to the same typological group. This classification was carried out according to profile, length and external and morphological characteristics, without taking capacity into account. However, the application of this method makes it possible to distinguish at least four different groups within this sample of T-11213 according to the results of the calculation of the volume in terms of Ugaritic and Gaditanian metrology. Therefore, the quantification of the volume contained by any archaeological vessel should be considered key to the compendium of any typological organization, and capacity must be considered a factor of importance equal to other details such as the external typological characteristics or the quality of the ceramic.

Furthermore, volume should be considered as the essential differentiating feature of transport vessels at the time of their manufacture and filling, as they were, after all, intended to carry a valuable product for trading. For this reason, when establishing ceramic typological categories, it must be taken into account that vessels would have been manufactured with higher or lower capacities and with the intention of carrying a specific volume according to metrological measures based on standardized values within a concrete system.

It is important to approximate the measures of reconstructed containers according to the pattern followed by each typological series and to reconstruct the curve of their profile accordingly. The results must then be checked to see if, as a result, they are better approximated to the means of these normalized measures. It could also be noted that, logically, it would have been strange to use different systems for the manufacture of the same type of container at any one time since this would have resulted in a delay in the profit of the sale of the traded product, and this loss would have been avoided by all possible means.

In conclusion, it is clear that the reconstruction of incomplete ceramic pot curves must be made according to the longitudinal means of the full vessel, its spline and the volumetric groups obtained after its comparison with the known metrological systems. This paper has proposed creating a standard profile curve according to the metrology and typology of each container, so laboratories can have a more accurate reference against which to compare archaeological fragments.

Equally, the great possibilities that the application of this simple mathematical method can offer for our knowledge of any chronological period and cultural region are clear. Once these data are obtained, conclusions may be reached regarding economic, commercial and consumption patterns, as well as the labour forces necessary for the manufacture of a given product, the use and dissemination of weight patterns, etc., among many other lines of research hitherto not quantifiable in strictly numerical terms.

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