Comparing Predictions of a Disk Radius to Simulations

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Contents

1 Introduction

The question of the formation of planets has seen a resurgence in interest with the recent discoveries of exoplanets, which are only snapshots that show an end state of the initial system we look to study. Though it is understood that planet formation occurs in protoplanetary disks, the concept is still novel enough that there are uncertainties for where in the disk ideal conditions are present, as well as the timescales on which planet formation can occur. Further, protoplanetary disks in their many stages have not all been looked into thoroughly for planet formation, and much more work needs to be done to consider Class 0/I disks, which are fruitful places for the dust coagulation processes necessary.

Class 0/I disks consist of the infalling and and disk-forming phases - where the formation of a star requires an inside-out collapse from a cloud, and as the young star loses angular momentum, it is transferred to a disk forming around it. The total time it takes for the young star to start forming and the disk to take shape is on the order of a few hundreds of thousands of years, and has been discounted in the past as too short of a timescale for planet formation to begin in. When exploring Class 0/I disks and their importance for planet formation, it is further important to consider the correct initial conditions that allow for accurate exploration of our many questions.

Angular momentum has played a large role in realizing the initial conditions of a protoplanetary disk. A disk becomes crucial to relieving the forming star of its excess angular momentum, which in turn is given turbulence that aids in the coagulation of dust within the disk.

In this paper, we will discuss the importance of angular momentum to protoplanetary disk formation, as well the references that have built up modern ideas on how disks form (§2). Then, we will explore both the equations that can be used to predict a forming disk's radius, as well as the simulation that will be compared to our prediction (§3). Finally, we will summarize the work (§4) and discuss the outcomes of our labor.

2 The Importance of Angular Momentum in Disk Formation

In trying to understand the initial conditions from which planets can form, we must start at the beginning of even a disk's formation. The generally assumed model for a disk's formation involves a dense cloud of molecular gas in the interstellar medium, essentially an isothermal medium. Assuming the Bonnar-Ebert sphere as a model for prestellar cores, we imply that there is a finite amount of material from which our disk can be made and that beyond a certain mass, the cloud becomes unstable and collapses under gravity (Ebert (1955), Bonnar (1956)). In the core collapse process, the rotation of the core cannot be ignored, especially as angular momentum conservation is relevant as the core decreases in radius and the rotation rate of the forming disk increases (Shu *et al.* (1987)). Further, angular momentum aids in maintaining the shape of the disk, as the higher angular momentum material settles along the midplane of the protostar.

2.1 Disk Formation

Angular momentum is used in simulations to further ensure that the disk being studied is rotationally supported against gravitationally collapsing into its protostar. A method we will be employing in the analysis of the simulations is by taking the ratio of the gravitational and rotational forces:

$$\frac{F_{grav}}{F_{rot}} = \frac{GM_{\odot}}{R^2\Omega^2} \tag{1}$$

where G is the gravitational constant, M is the mass of the disk, and R is the radius of the disk. Solving for the Keplerian velocity Ω allows us to constrain the radius of the disk using a dynamical property. Using a similar measure, but with the consideration of angular momentum in the system, we can use:

$$\beta = \frac{E_{rot}}{|E_{grav}|} = \frac{\frac{1}{2}I\omega^2}{\frac{3}{5}\frac{GM^2}{R}} = \frac{1}{3}\frac{\Omega_{Kep}^2 R_c^3}{GM_c} \tag{2}$$

from Burkert & Bodenheimer (2000), where β is the rotational-to-gravitational energy ratio, Ω_{Kep} is the Keplerian angular velocity, R_c and M_c are the radius and mass of the molecular cloud core respectively, and G is the gravitational constant. This ratio is superior to the previous one as it accounts for a disk that is indeed rotationally supported, and will be important to consider for the simulations.

It is useful to think of the centrifugal radius with respect to the angular momentum, which shows us how far out a disk will be able to spread when opposing gravity. Though there is a vertical component of the disk that can be spread, to consider it here we will examine the radial opposing force by using:

$$R_{cent} = \frac{j^2}{GM_c} \tag{3}$$

We will further discuss how these last two equations are used in the prediction subsection of section 3.

2.2 Planet Formation

Angular momentum is also able to find relevance when it comes to the full lifetime of a disk, and when planets may begin to form. Exploring theories on how disks may allow dust to coagulate, especially in turbulent regions, requires validation of the disk environment with angular momentum. The issue of dust being spread widely and steadily enough to stick together has still not thoroughly been solved. Understanding whether the dust and gas within a disk are undergoing a steady state drift, and how angular momentum is transported between the dust and gas within a disk, can be an avenue to solve this (Youdin & Goodman (2005)).

3 Comparing Predictions to Simulations

In trying to better understand how the Burkert and Bodenheimer calculation for angular momentum is able to predict the radius of a forming disk around a young and forming star, we can perform quick calculations to measure our simulation values against.

3.1 Predictions

For predicting the centrifugal radius of the resulting disk from a collapsing molecular cloud, we are going to use equation 2 in conjunction with the specific angular momentum:

$$j = R^2\omega \tag{4}$$

and rearranging this to solve for the angular velocity of the core, we are able to put this back into the rotational energy component of β:

$$E_{rot} = \frac{1}{2}I(\frac{j}{R_c^2})^2 \tag{5}$$

Given that we are exploring the ability of equation 2 to predict the radius of the forming disk based on the initial molecular cloud core, for the moment of inertia of the cloud core, we will assume a uniform sphere (which is indeed a gross simplification). We can set $I = \frac{2}{5}M_cR_c^2$, and when we set β up again we get:

$$\beta = \frac{E_{rot}}{|E_{grav}|} = \frac{\frac{1}{2}\frac{2}{5}\frac{M_cj^2}{R_c^2}}{\frac{3}{5}\frac{GM_c^2}{R_c}} = \frac{1}{3}\frac{j^2}{GR_cM_c} \tag{6}$$

Again rearranging, but to solve for the angular momentum, and then plugging it into equation 3, we end up with:

$$R_{cent} = \frac{j^2}{GM_c} = 3\beta R_c \tag{7}$$

So to predict our ultimate disk radius, we can use our chosen simulation initial conditions which are (spoiler alert!) $\beta = 4\%$ and a $R_c = 4125$ au, which gives us an expected disk radius of roughly 495 au.

3.2 Simulations

For our simulations, we employ the RAMSES code (Teyssier (2002)) in its non-ideal magnetohydrodynamic iteration, with ambipolar diffusion considered by Marchand *et al.* (2016). The central protostar is modeled using sink particles as has been done in Lebreuilly *et al.* (2021), and the dust dynamics are included by the dustycollapse dust solver (Lebreuilly *et al.* (2021)), using an initial grain size of 0.1, representative of interstellar medium dust.

The central protostar is modeled as a $1M_\odot$ dense core following the Boss & Bodenheimer (1979). We fix the radius of the initial uniform gas-dust cloud by imposing a thermal-to-gravitational energy ratio α:

$$\alpha \equiv \frac{5}{2}\frac{R_0k_BT_0}{GM_\odot\mu_g m_H} \tag{8}$$

with G as the standard gravitational constant, k_B as the Boltzmann constant, m_H and $\mu_g = 2.31$ the mass of an Hydrogen atom and the mean molecular weight respectively, and T_0 as the initial cloud temperature. In this work, we consider α to be 0.4, corresponding to a cloud radius of roughly 4125 au.

As mentioned in section 2, the initialization for the collapsing cloud can be set by using the rotational-to-gravitational energy ratio β, which was selected to be 4%. This is 1% higher than the value Burkert and

Bodenheimer use in their 1999 paper, but it might be useful as it can lead to a higher chance of fragmentation. Further, the rotational axis of the disk and the magnetic field are misaligned by 30°, and the initialized magnetic field is fixed using the ratio:

$$\mu = \frac{\frac{M_0}{\Phi}}{(\frac{M}{\Phi})_c} \tag{9}$$

where $\frac{1}{\mu} = 0.3$, roughly $3 * 10^{-4}$ G.

For analysis, we select the disk according to criteria given in Joos & Ciardi (2012) and Lebreuilly *et al.* (2021):

- $n > 10^9 cm^{-3}$ (gas number density)
- $|v_\phi| > 2|v_r|$ (v_ϕ is the azimuthal velocity, v_r is the radial velocity)
- $|v_\phi| > 2|v_z|$ (v_z is the vertical velocity)
- $\frac{1}{2}v_\phi^2 > 2P$ (P is the thermal pressure)

As the disk takes a few outputs to initialize, for the sake of analysis, the first three outputs were not considered. Since there were roughly 26 outputs left to search, we selected four of those to look at. The outputs, their corresponding simulation time, and the farthest radius they reach, are listed in the table below:

Output Number	Time (kyr)	Maximum Radius (au)
850	82.95	29
1200	88.1	28.7
1500	89.45	32
1900	91.47	41.5

Table 1: Selected disk information

These criteria give a disk radius of roughly 41 au at the disk's last output point in the simulation, which is only 0.0099 times the expected value of the disk radius. Much smaller than the "small disk" issue that theorists found when disks were first being observed.

4 Discussion, Summary, and Future Work

Though the predicted disk size was expected to be larger than the simulated disk size, we did not expect the disk to be far below half the predicted size. A few things may be occurring that caused the simulated disk to be much smaller than expected.

4.1 Incorrect Calculations

There is always the possibility that we (I) incorrectly calculated the predicted values. In fact, that is expected as the author of this paper is tired. If the math done in rearranging the equations is indeed correct, it is likely that the incorrect value for the initial molecular cloud radius, or R_c, was used and that it should have been related to the star at the center of the forming disk.

4.2 Differing β Values

As discussed in subsection 3.2, the core-collapse simulation was run using a $\beta = 4\%$ value - 1% higher than was used in the Burkert and Bodenheimer paper from 1999. While the higher value is useful for fragmentation, it may mean that on paper it leads to a much higher-than-actual disk radius value.

4.3 Computational and Real Time Restriction

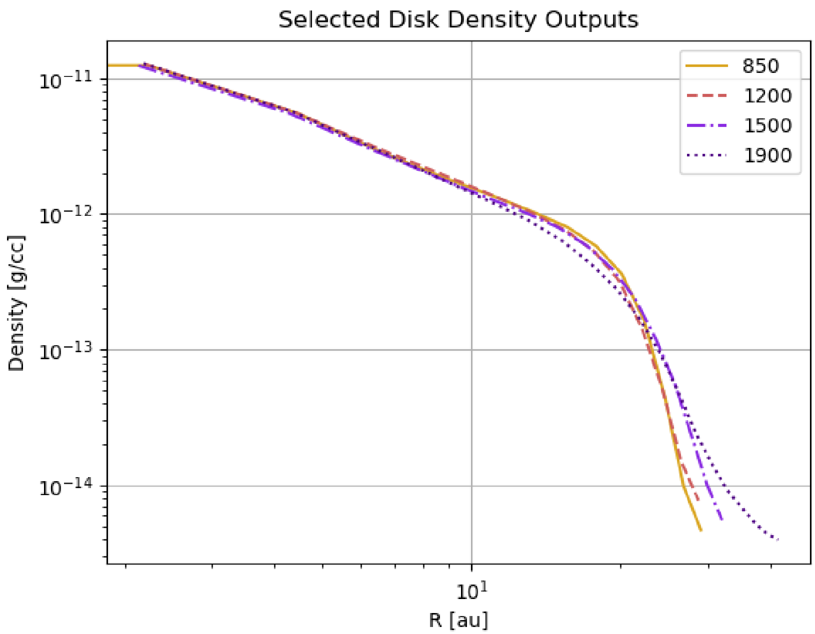
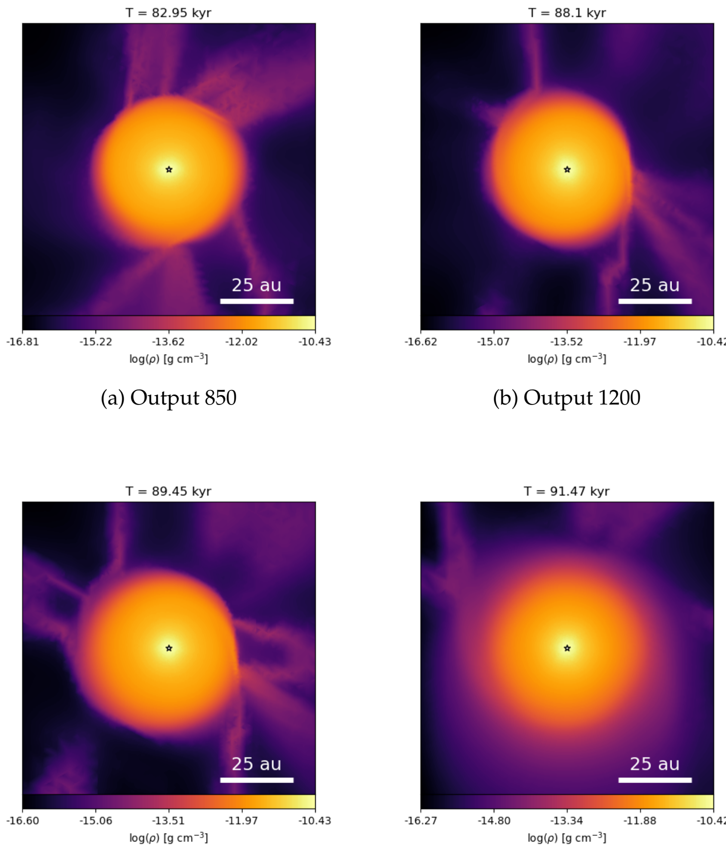


Figure 1: Selected outputs on a radial density plot.

Taking a look at the radial plots of the total density of the disk model in figure 1, it seems as though the disk sits around the same value in radius over time. Though this is interesting, it also is likely due to the fact that the outputs in total only span roughly 31 kyr. Though the total expected amount of time the Class 0/I disk phase takes is (based on the free-fall time) about 230,000 years, any changes and expansion within a disk may occur in later, more turbulent stages of the star's formation. The star has begun forming within 30,000 years of the initial inside-out collapse, which means that the disk in our simulation has not been around for longer than around 60,000 years by the end of the run. When considering plot slices of the time evolution of the disk, as in figure 2, it is useful also to see the expansion and "puffiness" of the dust and gas disk, shown explicitly in the last output.



(a) Output 850

(b) Output 1200

(c) Output 1500

(d) Output 1900

Figure 2: The time evolution of the disk model.

4.4 Summary and Discussion

The value for the radius that we calculated using the Burkert and Bodenheimer equation was far larger than the value for the radius seen in the disk simulations. Whether the magnitude of that difference was due to error or not, it is clear that approximating an initial molecular cloud as a uniform, solid sphere that injects angular momentum in an instance to a forming disk within it is not a total solution to understanding the initial conditions for a planet formation model, although it is a good start.

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