# ACT 7102

## Exemple 13

$$S = X_1 + \cdots + X_{10} \qquad X_i \sim \text{ComPois}(\lambda_i, F_{B_i})$$

| $i$ | $\lambda_i$ | $F_{B_i}$ |
|---|---|---|
| 1, 2, 3, 4, 5 | 0.1 | $F_{B_i} = \frac{7}{10} H(x, 1, 0.1) + \frac{2}{10} H(x, 2, 0.1) + \frac{1}{10} H(x, 3, 0.1)$ <br> $K_{1i}$: $q_{1i} = (0.7,\ 0.2,\ 0.1)$ |
| 6, 7, 8, 9, 10 | 0.2 | $F_{B_i} = \frac{1}{10} H(x, 1, 0.1) + \frac{4}{10} H(x, 2, 0.1) + \frac{5}{10} H(x, 3, 0.1)$ <br> $K_{2i}$ $q_{2i} = (0.1,\ 0.4,\ 0.5)$ |

$$X = \begin{cases} \sum_{k=1}^{M} B_k & M > 0 \\ 0, & M = 0 \end{cases} \qquad M \sim \text{Pois}(\lambda)$$

$$X = \begin{cases} \sum_{k=1}^{M^*} C_k, & M^* > 0 \\ 0, & M^* = 0 \end{cases}$$

$$M^* = \begin{cases} \sum_{j=1}^{M} K_j, & M > 0 \\ 0, & M = 0 \end{cases}$$

$$P_{M^*}(z) = P_M(P_K) \qquad P_M(z) = e^{\lambda(z-1)}$$

$$E[X^m] = \sum_{k=1}^{\infty} P_k \left(\frac{1}{\beta}\right) \prod_{j=0}^{m-1} (k+j)$$

1. $X = \sum_{j=1}^{M^*} C_j \qquad C_j \sim C \sim Exp(\beta)$

$$Var(X) = E(M^*)\,Var(C) + Var(M^*)\left(E(C)\right)^2$$

$$= \frac{1}{\beta^2} E(M^*) + \frac{1}{\beta^2} Var(M^*)$$

$$= \frac{1}{\beta^2}\left(E(M^*) + Var(M^*)\right)$$

$Cov(X_i, X_j) = \alpha_0\, E[B_i]\, E[B_j] \qquad X_i \sim ComPois(\lambda_i, F_{B_i})$

$X_i = \sum_{k=1}^{M_i} B_{i,k} \qquad N_i \sim Pois(\lambda_i)$

$$Cov(X_i, X_j) = Cov\left(\sum_{k=1}^{M_i} B_{i,k}, \sum_{k=1}^{M_j} B_{j,k}\right)$$

$J_0 \sim Pois(\alpha_0) \qquad J_i \sim Pois(\alpha_i = \lambda_i - \alpha_0) \qquad \begin{cases} M_1 = J_1 + J_0 \\ \vdots \\ M_n = J_n + J_0 \end{cases}$

$M_i \sim Pois(\lambda_i)$

$$X_i = \sum_{j=1}^{M_i} B_{i,j}$$

$$g_{m_1,m_2} = F_{M_1,M_2}(m_1, m_2) - F_{M_1,M_2}(m_1 - 1, m_2) - F_{M_1,M_2}(m_1, m_2 - 1) + F_{M_1,M_2}(m_1 - 1, m_2 - 1)$$

$Cov(X_i, X_j) = \sum_{k_1=1}^{M_i} \sum_{k_2=1}^{M_j} Cov(B_{i,k_1}, B_{j,k_2}) \qquad \begin{matrix} M_i = J_i + J_0 \\ M_j = J_j + J_0 \end{matrix}$

$M_i B_i$

$$= Cov\left(E(X_i|M_i), E(X_j|M_j)\right) + E\left(Cov(X_i|M_i, X_j|M_j)\right)$$

$$= Cov\left(M_i E(B_i), M_j E(B_j)\right) +$$

$$= E(B_i)\,E(B_j)\,Cov(M_i, M_j)$$

$Cov(X_i, X_j) = \alpha_0 E(B_i) E(B_j)$

$E(B_1) = 0.7\left(\frac{1}{0.1}\right) + 0.2\left(\frac{2}{0.1}\right) + 0.1\left(\frac{3}{0.1}\right)$

$= 14$

$S = X_1 + \quad + X_{10}$

$E(B_2) = 0.1\left(\frac{1}{0.1}\right) + 0.4\left(\frac{2}{0.1}\right) + 0.5\left(\frac{3}{0.1}\right)$

$= 24$

$E(S) = 5E(X_1) + 5E(X_6)$

$= 5\lambda_1 E(B_1) + 5\lambda_6 E(B_6)$

$E(S) = 5(0.1)(14) + 5(0.2)(24)$ $\quad \boxed{E(S) = 31}$

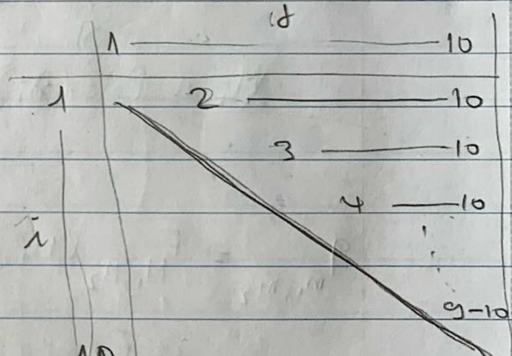
$Var(S) = Var\left(\sum_{i=1}^{10} X_i\right)$

$= \sum_{i=1}^{10} Var(X_i) + 2 \sum_{\substack{i>j \\ 1\le i,j \le 10}} Cov(X_i, X_j)$

$\sum \cancel{P}_k \frac{k}{\beta}$

$\sum_{i=1}^{10} Var(X_i) = 5(38) + 5(172) = 1050$

$\sum_{\substack{i>j \\ 1\le i,j\le 10}} Cov(X_i, X_j) = \alpha_0 \sum_{\substack{i\not\le j \\ 1\le i,j\le 10}} E(B_i)E(B_j)$




$i \in \{1, \ldots, 5\}$ $E(B_i) = 14$

$i \in \{6, \ldots, 10\}$ $E(B_i) = 24$

$= \alpha_0 \sum_{i=1}^{9} \sum_{j=i+1}^{10} E(B_i) E(B_j)$ $\quad 8\left(\frac{2+9}{2}\right)$

$= \alpha_0 \sum_{i=1}^{5} \sum_{j=i+1}^{10} 14 E(B_j) + \alpha_0 \sum_{i=6}^{9} \sum_{j=i+1}^{10} 24^2$

$= 14\alpha_0 \sum_{i=1}^{5} \left( \sum_{j=i+1}^{5} 14 + \sum_{j=6}^{10} 24 \right) + \alpha_0 24^2 \sum_{i=6}^{9} (10-i)$

$$= 14\alpha_0 \sum_{i=1}^{5} \left(14(5-i) + 24(5)\right) + 24^2 \alpha_0 \left(10(4) - 30\right)$$

$$= 14\alpha_0 \sum_{i=1}^{5} (190 - 14i) + \alpha_0 (5760)$$

$$= 14\alpha_0 (950 - 14(15)) + \alpha_0 (5760)$$

$$= 16\,120\, \alpha_0$$

$$\sum_{i=1}^{10} \sum_{j=1}^{10} E(B_i) E(B_j) =$$

$$\begin{cases} 14 & \text{si } i \leq 5 \\ 24 & \text{si } i \geq 6 \end{cases}$$

$$S = \sum_{i=1}^{10} X_i \qquad S \sim \text{ComPois}(\lambda_S, F_D)$$

$$\lambda_S = \lambda_1 + \dots + \lambda_n - (n-1)\alpha_0$$

$$X_i \sim \text{ComPois}(\lambda_i, F_{B_i})$$

$$B \sim \text{MixErl} \longrightarrow D \sim \text{MixErl}(\underline{\tau}; \beta)$$

$$\tau_k = \left( \frac{\alpha_0}{\lambda_S} \varepsilon_k + \sum_{l=1}^{n} \frac{\lambda_1 - \alpha_0}{\lambda_S} \zeta_k^{(l)} \right)$$

$$F_D(x) = \frac{\alpha_0}{\lambda_S} \nu_0 + \sum_{l=1}^{n} \frac{\lambda_1 - \alpha_0}{\lambda_S} \zeta_0^{(l)}$$

$$+ \sum_{k=1}^{\infty} \left( \frac{\alpha_0}{\lambda_S} \nu_k + \sum_{l=1}^{n} \frac{\lambda_1 - \alpha_0}{\lambda_S} \zeta_k^{(l)} \right) H(x, k, \beta)$$

$$P(K_1 + \dots + K_n = k) = \nu_k$$

$$\varphi_{K_1}(t) = E(e^{itK_1}) \qquad \varphi_{K_1 + \dots + K_n}(t) = E(e^{it(K_1 + \dots + K_n)})$$

$$= \left(\varphi_{K_1}(t)\right)^n$$

$$B_i \sim \text{MixErl}\left(\underline{\zeta}^{(i)}, \beta\right)$$

$$i = 1, \dots, 5 \qquad \underline{\zeta}^{(i)} = (0.7,\ 0.2,\ 0.1) \leftarrow P(K_i = k)$$

$$i = 6, \dots, 10 \qquad \underline{\zeta}^{(i)} = (0.1,\ 0.4,\ 0.5)$$

$$M_{B_i}(t) = P_{K_i}\left(M_C(t)\right)$$

$$F_D(x) = \underbrace{\frac{\alpha_0}{\lambda_S} v_0}_{\tau_0} + \sum_{k=1}^{\infty} \underbrace{\left(\frac{\alpha_0}{\lambda_S} v_k + \sum_{l=1}^{n} \frac{\lambda_1 - \alpha_0}{\lambda_S} \zeta_k^{(l)}\right)}_{\tau_k} H(x, k, \beta)$$

$$P_{K_1 + \dots + K_5 + K_6 + \dots + K_{10}}(t) = \left(P_{K_1}(t)\right)^5 \left(P_{K_6}(t)\right)^5$$

| $l$ \ $k$ | 1 | 2 | 3 |
|---|---|---|---|
| 1 | | | |
| ⋮ | | | |
| 10 | | | |

$l = 1, \dots, 5 \rightarrow K_1 = (\cdot\ \cdot\ \cdot)$

$K_2 = (\cdot\ \cdot\ \cdot) \leftarrow l = 6, \dots, 10$

Par définition :

$$F_D(x) = \frac{\lambda_1 - \alpha_0}{\lambda_S} F_{B_1}(x) + \dots + \frac{\lambda_n - \alpha_0}{\lambda_S} F_{B_n}(x) + \frac{\alpha_0}{\lambda_S} F_{B_1 + \dots + B_n}(x)$$

$$\begin{cases} F_{B_1} \equiv \dots \equiv F_{B_5} \\ F_{B_6} \equiv \dots \equiv F_{B_{10}} \end{cases}$$

$$F_D(x) = \left(\sum_{l=1}^{5}\frac{\lambda_l-\alpha_0}{\lambda_s}\right)F_{B_1}(x) + \left(\sum_{l=6}^{10}\frac{\lambda_l-\alpha_0}{\lambda_s}\right)F_{B_6}(x) + \frac{\alpha_0}{\lambda_s}F_{5B_1+5B_6}(x)$$

$(0.7H_1 + 0.2H_2 + 0.1H_3)$ $\quad k_1$

$(0.1H_1 + 0.4H_2 + 0.5H_3)$ $\quad k_2$

$\sum v_k H_k$

$$= \underbrace{\frac{\alpha_0}{\lambda_s}v_0}_{\tau_0} + \underbrace{\left(0.7\sum_{l=1}^{5}\frac{\lambda_l-\alpha_0}{\lambda_s} + 0.1\sum_{l=6}^{10}\frac{\lambda_l-\alpha_0}{\lambda_s} + \frac{\alpha_0}{\lambda_s}v_1\right)}_{\tau_1}H_1$$

$$+ \underbrace{\left(0.2\sum_{l=1}^{5}\frac{\lambda_l-\alpha_0}{\lambda_s} + 0.4\sum_{l=6}^{10}\frac{\lambda_l-\alpha_0}{\lambda_s} + \frac{\alpha_0}{\lambda_s}v_2\right)}_{\tau_2}H_2$$

$$+ \underbrace{\left(0.1\sum_{l=1}^{5}\frac{\lambda_l-\alpha_0}{\lambda_s} + 0.5\sum_{l=6}^{10}\frac{\lambda_l-\alpha_0}{\lambda_s} + \frac{\alpha_0}{\lambda_s}v_3\right)}_{\tau_3}H_3$$

$$+ \frac{\alpha_0}{\lambda_s}v_4 H_4$$

$$\vdots$$

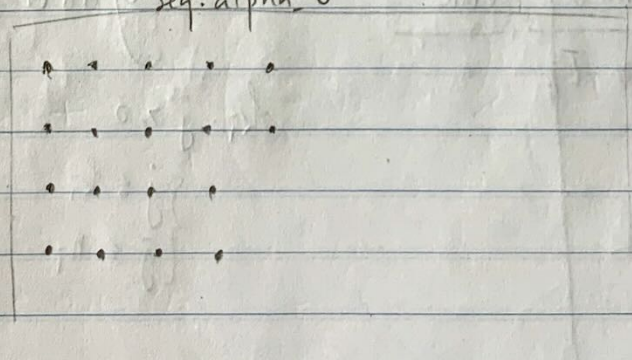
$S \sim \text{ComPois}(\lambda_s, F_D)$ $\qquad D \sim \text{MixErl}(\underline{\tau}, \beta)$

$\Rightarrow \quad S \sim \text{MixErl}(\underline{\xi}, \beta)$

$0 \leq \alpha_0 \leq \min(\lambda_1, \dots, \lambda_n) \quad \Rightarrow \quad 0 \leq \alpha_0 \leq 0.1$

Seq. alpha_0

Seq. kappa



$$S \sim \text{ComPois}(\lambda_S, F_D) \rightarrow S = \begin{cases} \sum_{j=1}^{N} D_j & N > 0 \\ 0 , & N = 0 \end{cases}$$

$N \sim \text{Pois}(\lambda_S) \qquad D_j \sim D.$

$$TVAR_\kappa(S) = \frac{1}{1-\kappa} \sum_{k=1}^{\infty} P(N=k)\, \boxed{E\left[ (D_1 + \cdots + D_k) \times 1_{\{D_1 + \cdots + D_k > VaR_\kappa(S)\}} \right]}$$

$X \times 1 \quad x > d$

$$J_{-i} = \sum_{\substack{l=1 \\ l \neq i}}^{n} J_l \sim \text{Pois}(\lambda_{-i}) \qquad J_i \sim \text{Pois}(\alpha_i = \lambda_i - \alpha_0)$$

$$J_0 \sim \text{Pois}(\alpha_0)$$

$$\lambda_{-i} = \sum_{\substack{l=1 \\ l \neq i}}^{n} \lambda_l - \alpha_0$$

$\alpha_1 + \cdots + \alpha_{10}$

$\lambda_1 - \alpha_0 + \cdots + \lambda_{10} - \alpha_0$

$$f_{M_i, N_{-i}}(k_i, n_{-i}) = \sum_{j=0}^{\min(k_i, n_i)} f_{J_i}(k_i - j)\, f_{J_{-i}}(n_i - j)\, f_{J_0}(j)$$

$$\begin{cases} M_i = J_i + J_0 \\ N_{-i} = J_{-i} + J_0 \end{cases} \qquad M_i \sim \text{Pois}(\lambda_i) \qquad N_{-i} \sim \text{Pois}\left( \sum_{\substack{l=1 \\ l \neq i}}^{n} \lambda_l \right)$$

R//

VaR

?

| kappa | value | alpha_0 | TVaR |
|---|---|---|---|
| | | | |

good!

$k_i - j \geq 0$

$$\begin{cases} j \leq k_i \\ j \leq n_i \end{cases}$$

## ⊛ Calcul des contributions : $TVaR_{\kappa}(X_i, S)$

$$V_i^B = \sum_{l=1}^{k_i} B_{i,l} \sim MixErl(\underline{v}, \beta) \qquad V_i^B = B_{i,1} + \cdots + B_{i,k_i}$$

$$\mathcal{L}_{V_i^B} = \left(\mathcal{L}_{B_i}\right)^{k_i} = \left(P_{K_i}(\mathcal{L}_{C_i})\right)^{k_i} = P_{\underbrace{K_1 + \cdots + K_{k_i}}}(\mathcal{L}_{C_i})$$

$$S_{-i} = \begin{cases} \sum_{j=1}^{J_0} C_{-i,j}, & J_0 > 0 \\ 0, & J_0 = 0 \end{cases} + \begin{cases} \sum_{j_{-i}=1}^{J_{-i}} D_{-i,j_{-i}}, & J_{-i} > 0 \\ 0, & J_{-i} = 0 \end{cases}$$

$$F_{C_{-i}}(x) = F_{\sum_{l=1, l\neq i}^{n} B_l}(x) \qquad F_{D_{-i}}(x) = \sum_{l=1, l\neq i}^{n} \frac{\lambda_l - \alpha_0}{\sum_{l=1, l\neq i}^{n} (\lambda_l - \alpha_0)} F_{B_l}(x)$$

$$\underbrace{\sum_{l=1}^{k_i} B_{i,l}} + \sum_{m=1}^{n_i - j} D_{-i,m} + \sum_{r=1}^{j} C_{-i,r}$$

$$D_{-i} \sim MixErl\left(\underline{\tau}^{(-i)}, \beta\right) \qquad \tau_k^{(-i)} = \sum_{l=1, l\neq i}^{n} \frac{\lambda_l - \alpha_0}{\sum_{l=1, l\neq i}^{n} (\lambda_l - \alpha_0)} \, p_k^{(l)}$$

$P_k$

$$\sum_{l=1}^{k_i} B_{i,l} \equiv \sum_{l=1}^{Z_{k_i}} C_{i,l} \qquad Z_{k_i} = K_{i_1} + \quad + K_{k_i}$$

$$\sum_{\substack{l=1 \\ l \neq i}}^{n} \lambda_l - \alpha_0 =$$

$X + Y + Z$

MixErl

$$a * b_k = \sum_{r=0}^{k} a_r * b_{k-r}$$

$$P_{K_i}(s) = \sum_{k=0}^{\infty} \zeta_k^{(i)} s^k \qquad P(K_i = k) = \zeta_k^{(i)}$$

$$\zeta_l^{(i)*k_i} \equiv \left(P(K_i = l)\right)^{k_i}$$

$$\sum_{l=1}^{k_i} B_{i,l}$$

Coefs

$$B_i \sim \text{MixErl}\left(\underline{\zeta}^{(i)}, \beta\right)$$

$$\sum_{k=1}^{\infty} \sum_{l=1}^{k} \zeta_l^{(i)*k_i} \, v^{(-i)*l} * \tau_{k-l}^{(-i)*(n_i - l)} \, \frac{l}{\beta} \, \overline{H}\left(VaR_{kappa}(S); k+1, \beta\right)$$

$$v^{(-i)*l} * \tau_m^{(-i)*n_i - l} = \sum_{u=0}^{m} v_u^{(-i)*l} \, \tau_{m-u}^{(-i)*n_i - l}$$

$1 \leq l \leq k$

$1 \leq l \leq 100$

$$P(M_i = k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

## Exemple 12

$$S = X_1 + X_2 + \dots + X_{n_1+n_2}$$

$X_i \sim \text{ComPois}(\lambda_i, F_{B_i})$

| $i$ | $\lambda_i$ | $B_i \sim \text{Gamma}(\gamma_i, \frac{1}{1000})$ |
|---|---|---|
| $1, \dots, n_1$ | 0.003 | $\gamma_i = 2$ |
| $n_1+1, \dots, n_2$ | 0.004 | $\gamma_i = 1$ |

$$X_i = \sum_{j=1}^{M_i} B_{j,i}$$

$$Var(X_i) = E(M_i)\,Var(B_i) + Var(M_i)\,E(B_i)^2$$
$$= \lambda_i(\gamma_i \times 10^6) + \lambda_i(\gamma_i \times 10^3)^2$$
$$= \lambda_i \gamma_i 10^6 (1 + \gamma_i)$$

$$F_{X_i}(x) = P(M_i = 0) + \sum_{k=1}^{\infty} P(M_i = k)\, P(B_{i1} + \dots + B_{ik} \le x)$$

$$f_j^{(i)*m_i} = \Pr(K_{i,1} + \cdots + K_{i,m_i} = j)$$

$$X = \sum_{k=1}^{M} B_k \qquad B_k \sim B \sim \text{Gamma}(\alpha, \beta)$$

$q_k$

$$TVaR_\kappa(X) = \frac{1}{1-\kappa} \sum_{k=1}^{\infty} q_k \frac{k\alpha}{\beta} \overline{H}\left(VaR_\kappa(X); \alpha k+1, \beta\right)$$

$$F_X(x) = q_0 + \sum_{k=1}^{\infty} q_k H(x, \alpha k, \beta)$$

<u>Exple 12</u> $B \sim \text{Gamma}\left(8, \frac{1}{1000}\right)$

<u>Proposition 6</u>

$$B_i \sim \text{Gamma}(\alpha_i, \beta)$$

$$F_S(x) = q_{0,\ldots,0} + \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \cdots \sum_{m_n=0}^{\infty} q_{m_1,\ldots,m_n}$$

$$\times H\left(x, \sum_{i=1}^{n} m_i\alpha_i, \beta\right)$$

3 3 3

$$q_{0,\ldots 0} = e^{-\alpha_0} \prod_{i=1}^{n} e^{-(\lambda_i - \alpha_0)} = e^{-\alpha_0} e^{-\sum_{i=1}^{n}(\lambda_i - \alpha_0)}$$

$$= e^{-\alpha_0 - \sum_{i=1}^{n}\lambda_i + n\alpha_0}$$

$$= e^{(n-1)\alpha_0 - \sum_{i=1}^{n}\lambda_i}$$

TVaR_kap_ $X_i$, $S$

than vec

$$25\alpha_0 (14)(24) + 10\alpha_0 (24^2) + 10\alpha_0 (14^2) = \underline{32240}$$

$$\underline{Var(S) = 1050 + 32240\,\alpha_0}$$

**Exemple 12**

$$S = X_1 + \dots + X_{n_1+n_2}$$

$X_i \sim ComPois(\lambda_i ; F_{B_i})$ $\qquad B_i \sim Gamma(\gamma_i, 10^{-3})$

$$Var(S) = \sum_{i=1}^{n_1+n_2} Var(X_i) + 2\sum_{i=2}^{n_1+n_2}\sum_{j=1}^{i-1} Cov(X_i, X_j)$$

$$= 18000\,n_1 + 8000\,n_2 + \underbrace{2\alpha_0 \sum_{i=2}^{n_1+n_2}\sum_{j=1}^{i-1} E(B_i)E(B_j)}_{A}$$

$E(B_i) = 10^3\gamma_i$

$$A = (2\alpha_0)10^6 \underbrace{\sum_{i=2}^{n_1+n_2}\sum_{j=1}^{i-1} \gamma_i\gamma_j}_{T} \qquad \gamma_i = \begin{cases} 2 & i = 1 \dots, n_1 \\ 1 & i > n_1 \end{cases}$$

$$T = \sum_{i=2}^{n_1}\sum_{j=1}^{i-1} \underbrace{\gamma_i\gamma_j}_{4} + \sum_{i=n_1+1}^{n_1+n_2}\sum_{j=1}^{i-1} \gamma_i\gamma_j$$

$$= \sum_{i=2}^{n_1} (i-1)4 +$$

044 66 ←
02244

Exemple 12

$S = X_1 + \cdots + X_{n_1+n_2}$ $\quad X_i \sim \text{ComPois}(\lambda_i, F_{B_i})$

| $i$ | $\lambda_i$ | $F_{B_i}$ |
|---|---|---|
| 1 ⋮ $n_1$ | 0.003 | $\gamma_i = 2$ |
| $n_1+1$ ⋮ $n_1+n_2$ | 0.004 | $\gamma_i = 1$ |

$B_i \sim \text{Gamma}(\gamma_i, 10^{-3})$

Posons $N = n_1 + n_2$

$$\boxed{S \sim \text{ComPois}(\lambda_S, F_D)} \qquad S = \begin{cases} \sum_{j=1}^{N_S} D_j & N_S > 1 \\ 0 & N = 0 \end{cases}$$

$$\lambda_S = \lambda_1 + \cdots + \lambda_N - (N-1)\alpha_0$$

$$M_S(r) = e^{\lambda_S (M_D(r) - 1)}$$

$$M_D(r) = \frac{\alpha_0}{\lambda_S} \prod_{l=1}^{N} M_{B_l}(r) + \sum_{l=1}^{n} \frac{\lambda_l - \alpha_0}{\lambda_S} M_{B_l}(r)$$

$$F_S(x) = \underbrace{P(N_S = 0)} + \sum_{k=1}^{\infty} \underbrace{P(N_S = k)}\, P(\underline{D_1 + \cdots + D_k} \le x)$$

$$M_{B_l}(r) = \left(\frac{10^{-3}}{10^{-3} - r}\right)^{\gamma_l}$$

$$\prod_{l=1}^{N} M_{B_l}(r) = \prod_{l=1}^{N} \left(\frac{10^{-3}}{10^{-3} - r}\right)^{\gamma_l} = \left(\frac{10^{-3}}{10^{-3} - r}\right)^{\sum_{l=1}^{N} \gamma_l} \sim \underline{\text{Gamma}\left(\sum_{l=1}^{N} \gamma_l, 10^{-3}\right)}$$

$$B_i \sim \text{Gamma}(\gamma_i, 10^{-3}) \equiv \underline{\text{Erl}(\gamma_i, 10^{-3})}$$

$$F_{B_i}(x) = H(x, \gamma_i, 10^{-3})$$

$i - 1 - n_1 - 1 + 1$

$$F_{B_1} = H(x, 2, 10^{-3}) \equiv H_2 \qquad q = \begin{array}{ccc} 0 & 1 & 2 \\ \downarrow & \downarrow & \downarrow \\ (0, & 0, & 1) \end{array} \searrow S^{(i)}$$

$$F_{B_{n_1+1}} = H(x, 1, 10^{-3}) \equiv H_1 \qquad q = \begin{array}{ccc} 0 & 1 & 2 \\ \downarrow & \downarrow & \downarrow \\ (0, & 1, & 0) \end{array} \nearrow$$

$$F_D(x) = \frac{\lambda_1 - \alpha_0}{\lambda_s} F_{B_1}(x) + \cdots + \frac{\lambda_n - \alpha_0}{\lambda_s} F_{B_N}(x) + \frac{\alpha_0}{\lambda_s} F_{B_1 + \cdots + B_N}(x)$$

$$F_D(x) = \left( \sum_{l=1}^{n_1} \frac{\lambda_l - \alpha_0}{\lambda_s} \right) H_2 + \left( \sum_{l=n_1+1}^{N} \frac{\lambda_l - \alpha_0}{\lambda_s} \right) F_{B_{n_1+1}}(x)$$

$$+ \frac{\alpha_0}{\lambda_s}$$

$$\sum_{i=1}^{n_2-1} i \qquad \frac{n_2(n_2-1)}{2}$$

$$2 \times 10^2 \left( 2(5)(4) + 2(5)(5) + \frac{5(4)}{2} \right)$$

$$2 \times 10^2 \left( \underbrace{40 + 50 + 10} \right)$$