Risk-based Capital Allocation

Introduction

The present contribution is intended to serve as a survey of techniques of risk-based capital allocation. Before going into technical detail, however, some words have to be spent on the conceptions of risk-based capital and capital allocation.

As opposed to, for example, equity capital, regulatory capital, or capital invested, the conception of risk-based capital or risk-adjusted capital (RAC) is usually understood to be a purely internal capital conception. (In the literature a number of related notions are used, e.g., capital at risk or economic capital.) Culp [3, p. 16] defines risk-based capital as the smallest amount of capital a company must set aside to prevent the net asset value or earnings of a business unit from falling below some 'catastrophic loss' level. Because this capital is never actually invested, RAC is an *imputed* buffer against unexpected and intolerable losses. As well, the allocation of risk-based capital is usually understood as a *notional* or *pro forma* allocation of capital. (For the question of why risk-based capital is scarce and for the necessity of apportioning RAC cf. [3, p. 17] and [16].)

Both the determination and allocation of risk-based capital are elements of risk-adjusted performance management (RAPM), which is typically based on a performance measure of the RORAC (return on risk-adjusted capital)-type (cf. e.g. [1, p. 65] or [3, p. 10]).

$$RORAC = \frac{\text{net income}}{RAC}. \tag{1}$$

The RORAC performance measure can be determined for the entire company or the overall financial position respectively, on the one hand, and also for business segments or segments of financial positions respectively, on the other. A segment RORAC requires the determination of a segment RAC. This segment RAC can be the stand-alone RAC of the segment or an (pro forma) allocated portion of the overall RAC. Using the stand-alone RAC, ignores the consequences of stochastic dependencies between the segments of the overall position. These stochastic dependencies can only be taken into consideration on the basis of allocating the overall RAC to the respective segments. The remainder of this paper concentrates on techniques of capital allocation of this kind (for a critical assessment of capital allocation, cf. [27]). For the applications of capital allocation to risk-adjusted performance management, we refer to the literature. (For various applications, cf. e.g. [3, 6, 15, 18, 19, 21].)

Determination of Risk-based Capital and the Capital Allocation Process

Risk Exposure and Loss Variables

In the present contribution we use a unified approach, quantifying the risk exposure of a position by means of a (random) loss variable L. To illustrate this unified approach, we first consider a number of standard examples.

Example 1 (Insurance Liabilities: General Case) For the liabilities of a certain collective of insureds, we consider the accumulated claim $S \geq 0$ of the collective over a specified period of time (e.g. one year). The corresponding loss variable in this situation is defined as

$$L := S - \mathrm{E}(S). \tag{2}$$

(The subtraction of the expected value $\mathrm{E}(S)$ pays attention to the fact that the insurance company receives a (risk) premium, which is at a disposal to cover claims in addition to the (risk-based) capital. For details of this argument cf. [1, pp. 63–64].)

In the case of several segments (subcollectives) $i = 1, \ldots, n$ with corresponding **aggregated claims** $S_i \geq 0$ the segment loss variables are $L_i := S_i - \mathrm{E}(S_i)$ and the overall loss variable is given by (2) with $S := S_1 + \cdots + S_n$.

Example 2 (Homogeneous Collectives of Insurance Liabilities) Continuing Example 1, we now assume that the accumulated claim of the segment i consisting of k_i insureds is of the form

$$S_i = \sum_{j=1}^{k_i} X_{ij}, \tag{3}$$

where the X_{ij} are independent and identically distributed random variables, which are related to the accumulated claim of the jth insured in segment i.

The corresponding segment loss variable is as before defined by $L_i := S_i - \mathrm{E}(S_i)$.

Example 3 (Investment Portfolios) We first consider a single financial position (stock or bond investment, short or long position of an option) and the corresponding change of the market value over a (typically short) time interval. The corresponding loss variable is given by

$$L := v_t - V_{t+h}, \tag{4}$$

where v_t is the (known) market value of the position at time t and V_{t+h} is the (random) market value at time $t+h$.

Considering now a portfolio of financial positions, we have

$$L = \sum_{i=1}^{n} L_i = \sum_{i=1}^{n} x_i LF_i, \tag{5}$$

where LF_i corresponds to the periodic loss according to (4) for a unit of the ith financial position (e.g. one share, one bond) and x_i denotes the absolute number of (short or long) units of the ith position in the portfolio. $L_i := x_i LF_i$ is the periodic overall loss related to the ith financial position.

Example 4 (Credit Risk) For a portfolio of n **credit risks**, we consider the corresponding aggregated loss CL (credit loss) with $CL = \sum_{i=1}^{n} CL_i$ over a specified period of time as the relevant loss variable.

Risk-based Capital and Risk Measures

Given the overall loss variable L or the segment loss variables $L_1, \ldots, L_n$, respectively, representing the risk exposure, the next step is to quantify the corresponding risk potential. Formally, this is accomplished by the specification of a **risk measure**. Albrecht [2, Section 3] distinguishes two conceptions of risk measures. Risk measures R_{I} of the first kind are related to the magnitude of (one- or two-sided) deviations from a target variable. Risk measures R_{II} of the second kind conceive risk as the (minimal) necessary capital to be added to a financial position (in order to establish a riskless position or satisfy regulatory requirements). Obviously risk measures of the second kind can be used directly to define the risk-based capital RAC. With $R = R_{\mathrm{II}}$ we therefore define:

$$RAC(L) := R(L). \tag{6}$$

(However, risk measures of the first kind, which satisfy a one-to-one correspondence with risk measures of the second kind as explained in [2, Section 5.4] can be used, too, and RAC then is defined by $RAC = \mathrm{E}(L) + R_{\mathrm{I}}(L)$.)

For illustrative purposes, we consider three standard measures of risk (of the second kind) throughout the present contribution. First, the standard deviation–based risk measure ($a > 0$)

$$R(L) = \mathrm{E}(L) + a\sigma(L), \tag{7}$$

where $\mathrm{E}(L)$ denotes the expected value and $\sigma(L)$ the standard deviation of L. Second, the risk measure **Value-at-Risk (VaR)** at confidence level α, that is,

$$\mathrm{VaR}_\alpha(L) = Q_{1-\alpha}(L), \tag{8}$$

where $Q_{1-\alpha}(L)$ denotes the $(1-\alpha)$-quantile of the loss distribution. Finally, the Conditional Value-at-Risk (CVaR) at confidence level α, given by

$$\mathrm{CVaR}_\alpha(L) = \mathrm{E}[L | L > Q_{1-\alpha}(L)]. \tag{9}$$

In case L is normally distributed, Value-at-Risk and CVaR are only special cases of (7), with $a = N_{1-\alpha}$ (denoting the $(1-\alpha)$-quantile of the standard normal distribution) in case of the VaR and with $a = \varphi(N_{1-\alpha})/\alpha$ (where φ denotes the density function of the standard normal distribution) in case of the CVaR.

The Capital Allocation Process

In general, the process of capital allocation consists of the following steps:

1. Specification of a **multivariate distribution** for the vector of segment loss variables $(L_1, \ldots, L_n)$. (For illustrative purposes we typically consider the multivariate normal distribution.)
2. Selection of a risk measure (of the second kind) R.
3. Calculation of the overall risk-based capital $RAC(L) = R(L)$ as well as the stand-alone risk-based capital $RAC_i = R(L_i)$ of the segments.
4. In case of a positive diversification effect, that is, $R(L) < \sum R(L_i)$, application of an allocation rule to determine the risk-based capital RAC_i^* assigned to segment i.

Capital Allocation Procedures

Absolute Capital Allocation

Given the overall RAC, $RAC := R(L)$ and the stand-alone RAC, $RAC_i := R(L_i)$, the following relation is valid for many important cases:

$$D_R(L_1, \ldots, L_n) := \sum_{i=1}^{n} R(L_i) - R\left(\sum_{i=1}^{n} L_i\right)$$
$$= \sum_{i=1}^{n} RAC_i - RAC \geq 0, \quad (10)$$

where $D_R(L_1, \ldots, L_n)$ can be considered to be a measure of diversification. Relation (10) is valid for all subadditive risk measures, for instance. The standard deviation–based risk measure (7), for instance, is globally subadditive, the Value-at-Risk according to (8) is subadditive as long $(L_1, \ldots, L_n)$ follows a multivariate elliptical distribution (and $\alpha < 0.5$) and the CVaR according to (9) is subadditive, for instance, when $(L_1, \ldots, L_n)$ possesses a (multivariate) density function.

As already put forward in the 'Introduction', in case of a positive diversification effect, only a properly allocated risk capital

$$RAC_i^* := R(L_i; L), \quad (11)$$

where $R(L_i; L)$ denotes the (yet to be determined) contribution of the segment i to the overall risk $R(L)$, can form the basis of reasonable risk-adjusted performance management, properly reflecting the stochastic dependency between the segments. The basic requirements for the (absolute) capital allocation to be determined are

$$\sum_{i=1}^{n} R(L_i; L) = R(L), \quad (12)$$

that is, *full allocation*, and

$$R(L_i; L) \leq R(L_i), \quad (13)$$

that is, the allocated capital must not exceed the stand-alone RAC.

Denault [5] puts forward a general system of postulates for a reasonable absolute capital allocation. Denault requires full allocation according to (12) and the following sharpened version of (13)

$$\sum_{i \in M} R(L_i; L) \leq R\left(\sum_{i \in M} L_i\right) \quad \forall \subset M \text{ of } \{1, \ldots, n\}. \quad (14)$$

This condition, which is called '*no-undercut*', basically requires (13) for all unions of segments. A third condition is *symmetry*, which basically requires that within any decomposition, substitution of one risk L_i with an otherwise identical risk L_j does not change the allocation. Finally, a fourth condition is imposed (*riskless allocation*), requiring $R(c; L) = c$. This means that for deterministic losses $L_i = c$, the allocated capital corresponds to the (deterministic) amount of loss. An allocation principle satisfying all four postulates of full allocation, no-undercut, symmetry, and riskless allocation is called a *coherent allocation principle* by Denault.

In case of segments having a 'volume' – as, for example, in Examples 2 and 3 – we are not only interested in the determination of a risk capital $R(L_i; L)$ per segment but, in addition, in a risk capital per unit (*per unit allocation*), for example, $R(X_i; L)$ per insured in segment i (Example 2) or $R(LF_i; L)$ per investment unit (Example 3) respectively, satisfying

$$R(L_i; L) = k_i R(X_i; L) \quad (15)$$

or $$R(L_i; L) = x_i R(LF_i; L) \quad (16)$$

respectively. In the latter case this can be stated in the following alternative manner. (Cf. [5, 8, 24, 25] for this approach.) (A similar definition is possible for the insurance case (Example 2). But this will – because of the diversification effect within the collective – not result in a positive homogeneous risk measure, which would be essential for the validity of the results in the section 'Euler Principle' regarding (17).) Fixing the loss variables $LF_1, \ldots, LF_n$, the function

$$R(x_1, \ldots, x_n) := R\left(\sum_{i=1}^{n} x_i LF_i\right) \quad (17)$$

induces a risk measure on $\Re^n$. We are now interested in a capital allocation (*per unit allocation*) $R_i(x_1, \ldots, x_n)$ per unit of the ith basic financial position, especially satisfying the full allocation postulate, that is,

$$\sum_{i=1}^{n} x_i R_i(x_1, \ldots, x_n) = R(x_1, \ldots, x_n) \ \forall \ (x_1, \ldots, x_n). \quad (18)$$

Incremental Capital Allocation

Incremental capital allocation considers the quantities

$$R(L_i; L) := R(L) - R(L - L_i), \quad (19)$$

that is, the risk contribution of segment i corresponds to the total risk minus the risk of the overall position without segment i. However, an incremental capital allocation violates the condition (12) of full allocation and therefore is not a reasonable capital allocation procedure. The same holds true for the variant, where one defines $R(L_1;L) = R(L_1)$ and $R(L_i;L) := R(L_1 + \cdots + L_i) - R(L_1 + \cdots + L_{i-1})$, that is, the difference in risk capital caused by the inclusion of segment i, for $i = 2, \ldots, n$. This variant is satisfying the full allocation requirement, but now the risk capital required depends on the order of including the segments.

Marginal Capital Allocation

Marginal capital allocation considers the impact of marginal changes of positions on the necessary risk capital. This approach is especially reasonable in a situation as in Example 3, where a risk measure $R(x_1, \ldots, x_n)$ according to (17) is defined on a subset of $\Re^n$. One then considers the marginal quantities (Deltas)

$$D_i(x_1, \ldots, x_n) : = \frac{\partial R(x_1, \ldots, x_n)}{\partial x_i}, i = 1, \ldots, n, \tag{20}$$

which require the existence of the respective partial derivatives. However, the marginal quantities D_i are, in general, primarily relevant for a sensitivity analysis and not for capital allocation purposes. However, there is a close link to absolute capital allocation in case of a risk measure R, which is positively homogeneous, that is, $R(cx) = cR(x)$ for all $c > 0$ and $x \in \Re^n$, and which, in addition, is totally differentiable. In this case, the following fundamental relation is valid due to a theorem of Euler:

$$R(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i D_i(x_1, \ldots, x_n). \tag{21}$$

Defining $R_i(x_1, \ldots, x_n) := D_i(x_1, \ldots, x_n)$, we therefore obtain a per unit capital allocation, which satisfies the full allocation condition (18). In this context, we can subsume marginal capital allocation under absolute capital allocation. We will pursue this approach in the section 'Euler-principle'.

Allocation Principles

Proportional Allocation

A first (naive) allocation rule is given by

$$R(L_i; L) := \frac{R(L_i)}{\sum_{j=1}^{n} R(L_j)} R(L), \tag{22}$$

where the diversification effect is distributed proportionally to the segments. This approach guarantees the full allocation condition (12). Because allocation is only oriented at the stand-alone quantities $RAC_i = R(L_i)$, it, however, ignores the stochastic dependencies between the segments, when allocating capital.

Covariance Principle

Here we consider the risk contributions

$$\begin{aligned} R(L_i; L) &:= \mathrm{E}(L_i) + \frac{\mathrm{Cov}(L_i, L)}{\mathrm{Var}(L)}[R(L) - \mathrm{E}(L)] \\ &= \mathrm{E}(L_i) + \beta_i[R(L) - \mathrm{E}(L)], \end{aligned} \tag{23}$$

where the *beta factor* $\beta_i = \beta(L_i; L)$ is defined as $\beta_i = \mathrm{Cov}(L_i, L)/\mathrm{Var}(L)$. Owing to $\sum \mathrm{E}(L_i) = \mathrm{E}(L)$ and $\sum \beta_i = 1$, the condition (12) of full allocation is satisfied. Obviously, the allocation rule is independent of the underlying risk measure used to determine the overall RAC. The allocation factors β_i intuitively result from the following decomposition of the variance

$$\begin{aligned} \mathrm{Var}(L) &= \mathrm{Var}\left(\sum_{i=1}^{n} L_i\right) = \sum_{i=1}^{n}\left[\sum_{j=1}^{n} \mathrm{Cov}(L_i, L_j)\right] \\ &= \sum_{i=1}^{n} \mathrm{Cov}(L_i, L) \end{aligned} \tag{24}$$

and a subsequent normalization of the risk contributions $\mathrm{Cov}(L_i, L)$ by dividing by the overall risk $\mathrm{Var}(L)$. The covariance principle therefore allocates (independent of the risk measure) the diversification effect with respect to $R(L) - \mathrm{E}(L)$ on the basis of the covariance structure of the $L_1, \ldots, L_n$.

The covariance principle possesses the advantage of being generally applicable as long as $R(L) >$

$\mathrm{E}(L)$. In principle, however, the allocation is performed as if $R(L) - \mathrm{E}(L)$ and $\mathrm{Var}(L)$ were identical. The allocation of the diversification effect with respect to $R(L) - \mathrm{E}(L)$ on the basis of the covariance structure can be considered to be reasonable primarily in the multivariate elliptical case.

In the case – as in Examples 2 and 3 – where the segments have a 'volume', the covariance principle can be applied as well. In the insurance case (Example 2), we have $L_i = \sum_{j=1}^{k_i} X_{ij}$, where the X_{ij} are independent and identically distributed according to X_i. Defining $\beta(X_i; L) := \mathrm{Cov}(X_i, L)/\mathrm{Var}(L)$, we have

$$\begin{aligned}\beta(L_i; L) &= \mathrm{Cov}\left(\sum_{j=1}^{k_i} X_{ij}, L\right) \Big/ \mathrm{Var}(L) \\ &= \sum_{j=1}^{k_i} \mathrm{Cov}(X_{ij}, L)/\mathrm{Var}(L) \\ &= k_i \mathrm{Cov}(X_i, L)/\mathrm{Var}(L) \qquad (25)\end{aligned}$$

and therefore $\beta(L_i; L) = k_i \beta(X_i; L)$. Defining $R(X_i; L) = \mathrm{E}(X_i) + \beta(X_i; L)[R(L) - \mathrm{E}(L)]$, we then have a per unit capital allocation satisfying $R(L_i; L) = k_i R(X_i; L)$ according to (15). In the investment case (Example 3) we similarly define $\beta(LF_i; L) := \mathrm{Cov}(LF_i, L)/\mathrm{Var}(L)$ and $R(LF_i; L) = \mathrm{E}(LF_i) + \beta(LF_i; L)[R(L) - \mathrm{E}(L)]$. It has to be pointed out that the $\mathrm{Var}(L)$ terms of the two examples are different and so are the beta factors. In the insurance case, we have $\mathrm{Var}(L) = \sum_{i=1}^{n} k_i \mathrm{Var}(X_i) + \sum_{j \neq i} k_i k_j \mathrm{Cov}(X_i, X_j)$ and in the investment case we have $\mathrm{Var}(L) = \sum_{i=1}^{n} x_i^2 \mathrm{Var}(LF_i) + \sum_{j \neq i} x_i x_j \mathrm{Cov}(LF_i, LF_j)$. The difference (linear respective quadratic contributions to the first term) results from the fact that there is a diversification effect within the segment in the insurance case, while in the investment case there is none.

Conditional Expectation Principle

Considering conditional expectation, the relations $L = \mathrm{E}[L|L] = \sum_{i=1}^{n} \mathrm{E}[L_i|L]$ and $\mathrm{E}[L|L = R(L)] = R(L)$ are valid, which results in

$$R(L) = \mathrm{E}[L|L = R(L)] = \sum_{i=1}^{n} \mathrm{E}[L_i|L = R(L)]. \qquad (26)$$

This suggests the following definition of the segment risk capital

$$R(L_i; L) = \mathrm{E}[L_i|L = R(L)], \qquad (27)$$

which satisfies the condition (12) of full allocation.

In the case of a multivariate elliptical distribution we have (cf. [7] and [11, p. 12])

$$\mathrm{E}[L_i|L] = \mathrm{E}(L_i) + \frac{\mathrm{Cov}(L_i, L)}{\mathrm{Var}(L)}[L - \mathrm{E}(L)], \qquad (28)$$

which results in

$$R(L_i; L) = \mathrm{E}(L_i) + \beta_i[R(L) - \mathrm{E}(L)], \qquad (29)$$

where the beta factor β_i is defined as in the section 'Covariance Principle'. In the considered case, the conditional expectation principle therefore is identical to the covariance principle. Considering the standard deviation–based risk measure (7) we obtain (still for the elliptical case)

$$R(L_i; L) = \mathrm{E}(L_i) + a\beta_i\sigma(L). \qquad (30)$$

In addition, in the (multivariate) normal case (30) is valid for the Value-at-Risk and the CVaR, with $a = N_{1-\alpha}$ and $a = \varphi(N_{1-\alpha})/\alpha$ respectively.

In the case of segments with volume (Examples 2 and 3) the per unit risk capital can similarly be defined by $R(X_i; L) := \mathrm{E}[X_i|L = R(L)]$ and $R(LF_i; L) := \mathrm{E}[LF_i|L = R(L)]$ respectively, thus guaranteeing (15).

Conditional Value-at-Risk Principle

Because of $\mathrm{E}[L|L > Q_{1-\alpha}(L)] = \sum_{i=1}^{n} \mathrm{E}[L_i|L > Q_{1-\alpha}(L)]$ a direct linear composition of the CVaR exists, which suggests the segment allocation capital

$$R(L_i; L) = \mathrm{E}[L_i|L > Q_{1-\alpha}(L)]. \qquad (31)$$

In the (multivariate) elliptical case we can again use (28) to obtain

$$R(L_i; L) = \mathrm{E}(L_i) + \beta_i[\mathrm{CVaR}_\alpha(L) - \mathrm{E}(L)]. \qquad (32)$$

This again is a special case of the covariance principle (23) for $R(L) = \mathrm{CVaR}_\alpha(L)$.

Euler Principle

The Euler principle (cf. [19, p. 65] for this terminology) unfolds its importance in the context of segments with a portfolio structure as in Example 3. The allocation itself is then based on relation (21), the theorem of Euler. The interesting fact about this principle of capital allocation now is that there are certain optimality results to be found in the literature.

So, for instance, Denault [5] shows on the basis of the theory of **cooperative (convex) games** with frictional players that for a positive homogeneous, convex, and totally differentiable risk measure, the gradient $(D_1, \ldots, D_n)$ according to (10) corresponds to the Aumann–Shapley value, which, in addition, is a unique solution. Therefore, in this context, the gradient $(D_1, \ldots, D_n)$ can be considered to be the unique fair capital allocation per unit. In case of a coherent and differentiable risk measure, Denault in addition shows that this allocation principle satisfies the postulates to be satisfied by a coherent allocation principle as outlined in the section 'Absolute Capital Allocation'.

If the risk measure is only positive homogeneous and differentiable, Tasche [24] and Fischer [8] show that only the Euler principle satisfies certain conditions for a 'reasonable' performance management based on the RORAC quantity (1).

For the application of the Euler principle, differentiability of the risk measure is a key property. This property is globally valid for the standard deviation–based risk measure (7), but not for the VaR and the CVaR. For the latter two, one, for instance, has to assume the existence of a multivariate probability density (for generalizations cf. [24, 25]).

We now consider a standard application to the investment case, concentrating on the multivariate normal case and the risk measure $R(L) = \sigma(L)$. The induced risk measure is $\sigma(x_1, \ldots, x_n) = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \mathrm{Cov}(LF_i, LF_j)\right]^{1/2}$. By differentiation we obtain

$$\frac{\partial \sigma}{\partial x_i} = \frac{\sum_{j=1}^{n} x_j \mathrm{Cov}(LF_i, LF_j)}{\sigma(L)} = \frac{\mathrm{Cov}(LF_i, L)}{\sigma(L)} = \frac{\mathrm{Cov}(LF_i, L)}{\sigma^2(L)}\sigma(L) = \beta_i \sigma(L), \tag{33}$$

where $\beta_i = \beta(LF_i; L) := \mathrm{Cov}(LF_i, L)/\mathrm{Var}(L)$. Therefore $R_i(x_1, \ldots, x_n) = \beta_i \sigma(L)$ is the capital allocation per investment unit demanded. Obviously, this is the variant of the covariance principle for the investment case considered at the end of the section 'Covariance Principle'.

Considering the risk measure (7) we obtain

$$R_i(x_1, \ldots, x_n) = \mathrm{E}(L_i) + a\beta_i \sigma(L), \tag{34}$$

subsuming the risk measures VaR and CVaR in the multivariate normal case. In the Value-at-Risk literature the quantities $\beta_i \sigma(L)$ are called component VaR or marginal VaR.

We close with two results for the VaR and the CVaR assuming the existence of a (multivariate) probability density. In case of the VaR we obtain (cf. e.g. [10, p. 229])

$$\frac{\partial R}{\partial x_i} = \mathrm{E}[LF_i | L = \mathrm{VaR}_\alpha(L)], \tag{35}$$

which is a variant of the conditional expectation principle for the portfolio case. In the case of the CVaR, we obtain (cf. [22])

$$\frac{\partial R}{\partial x_i} = \mathrm{E}[LF_i | L > Q_{1-\alpha}(L)], \tag{36}$$

which is a special case of the CVaR principle. The conditional expectations involved can be determined on the basis of Monte Carlo methods or by statistical estimation, for example, using kernel estimators.

Additional Approaches

Firm Value-based Approaches

The approaches considered so far are based on a purely internal modeling of the relevant loss variables. In the literature a number of approaches are discussed, which rely on an explicit model of the firm value, typically in a capital market context. Respective results on capital allocation exist in the context of the capital asset pricing model (cf. e.g. [1, section 4.3]) (CAPM), option pricing theory (cf. e.g. [4, 16, 17]) and special models of the firm value (cf. e.g. [9, 20, 23]).

Game Theoretic Approaches

Beyond the results of Denault [5] reported in this contribution the results from the game theoretic approach to cost allocation (cf. for a survey [26] and for insurance applications [12, 13]) can easily be applied to the situation of the allocation of risk costs (cf. e.g. [14]).

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(*See also* **Capital Allocation for P&C Insurers: A Survey of Methods**)

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