CMMHw4

Oliwia Lidwin

November 2024

1 Random Numbers

From part 1 of the random number assignment, it is visible that using a larger sample makes the results more uniform throughout all bins. This is due to the larger sample size. A random number generator was used to view the plots, after which the value was assigned to a bin. The number of elements in each bin was counted and returned on a histogram showing the distribution of numbers from 0 to 1. The resulting plots are shown in Figure 1 and Figure 2.

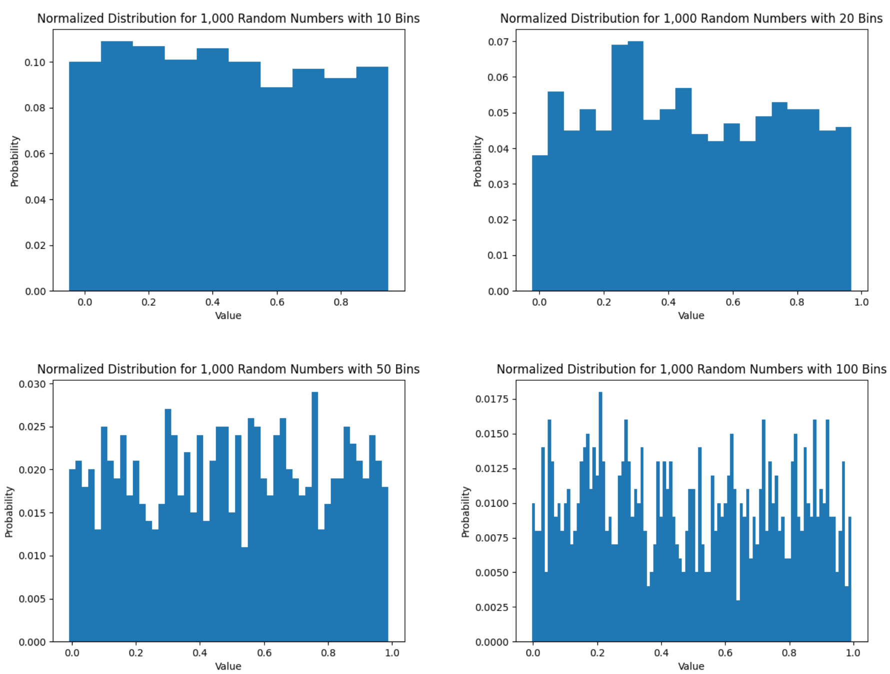

Figure 1: Plots of 1000 random numbers being generated and being split into bins of 10(top-left), 20(top-right), 50(bottom-left) and 100(bottom-right).

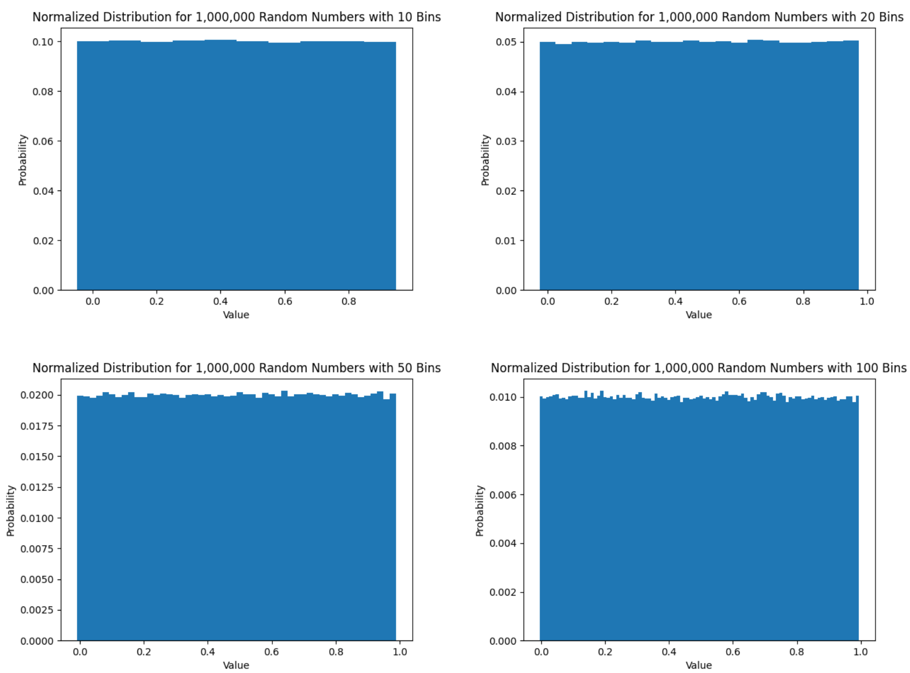

Figure 2: Plots of 1000000 random numbers being generated and being split into bins of 10(top-left), 20(top-right), 50(bottom-left) and 100(bottom-right).

For part 2, the generated random number value was assigned either to fit within the Gaussian distribution or not in which case it was rejected. This distribution was based on the equation:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

where $\sigma = 1.0$. The resulting plots are shown in Figure 3.

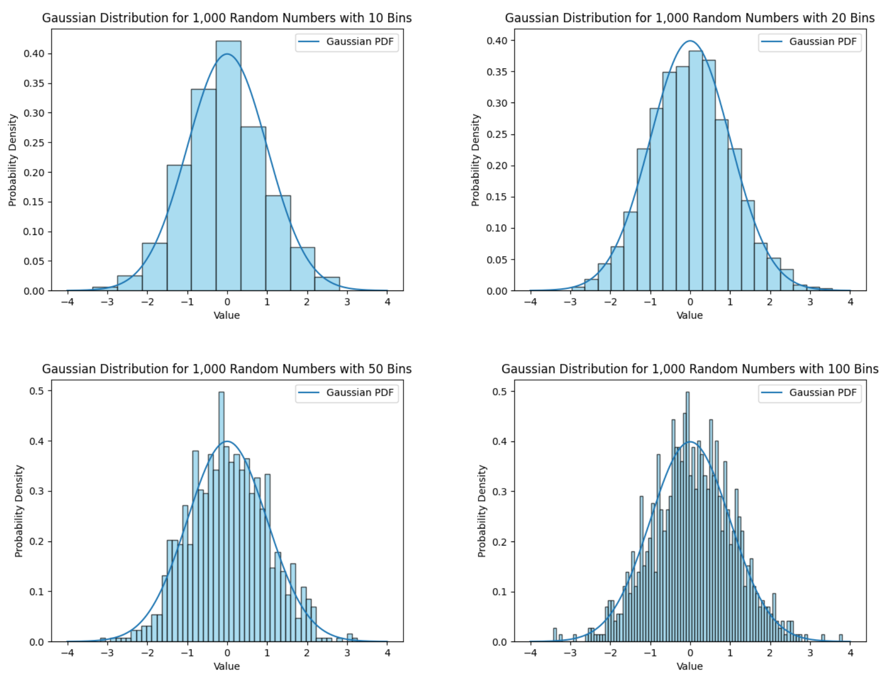

Figure 3: Plots of 1000 random numbers being generated, accepted, or rejected based on Gaussian distribution and split into bins of 10(top-left), 20(top-right), 50(bottom-left), and 100(bottom-right).

2 2 Dimensional Random Walk

As can be seen in Figure 5, as the number of steps or time increases, so does the mean square distance from the starting point such that there is a correlation between the radius to which the particle travels and the number of steps the particle takes. This positive correlation is also visible on the plot of the steps taken in the x-direction vs. the number of steps (Figure 5), however, this plot is irrelevant as only movement in the y-direction may occur. Specifically, it can be seen that the radius increases on an almost 1:1 to almost 1:1.4 ratio between the number of steps and the mean square distance.

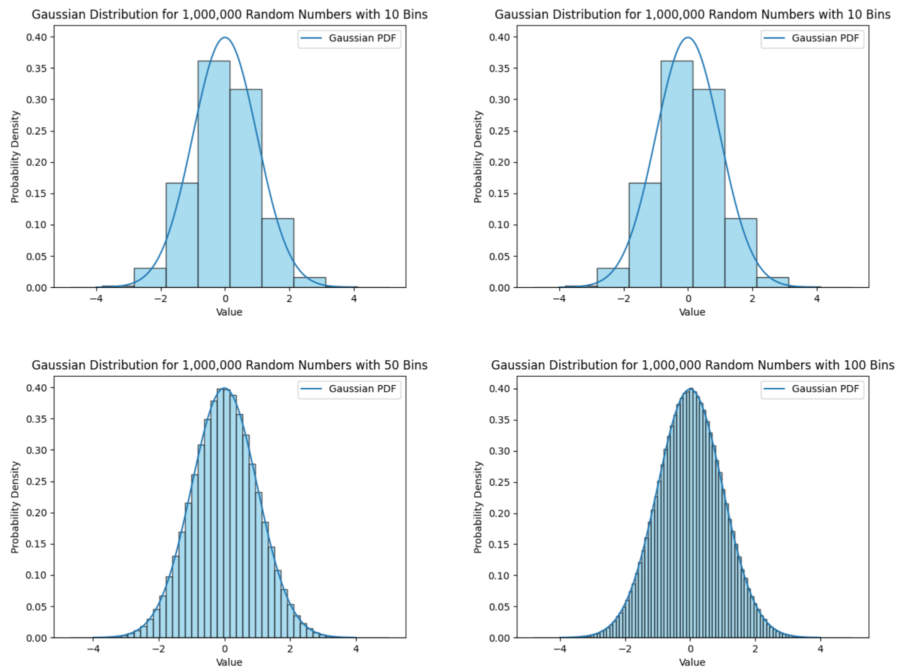

Figure 4: Plots of 1000000 random numbers being generated, accepted, or rejected based on Gaussian distribution and split into bins of 10(top-left), 20(top-right), 50(bottom-left), and 100(bottom-right).

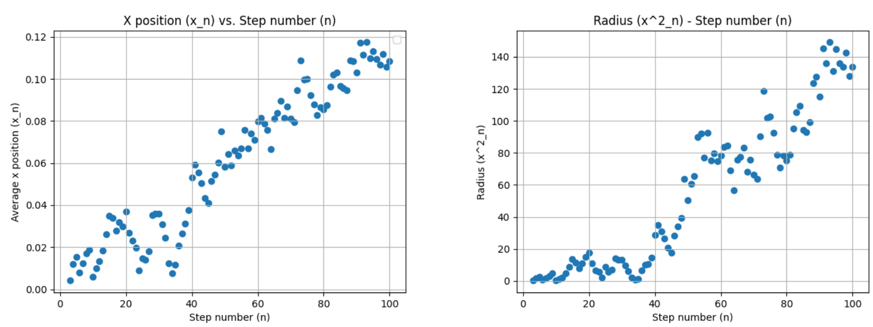

Figure 5: Average position and radius based on the number of steps attempted.

3 Diffusion Equation

The expected value using $< x(t)^2 >$ of the 1 Dimensional Normal Distribution is equal to $\sigma(t)^2$. This is derived from the Normal Distribution:

$$\rho(x,t) = \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right)$$

$$< x^2(t) >= \int_{-\infty}^{\infty} x^2 \rho(x,t)\, dx$$

$$< x^2(t) >= \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi\sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right) dx$$

$$< x^2(t) >= \frac{1}{\sqrt{2\pi\sigma(t)^2}} \sqrt{2\pi\sigma(t)^2} * \sigma(t)^2$$

$$< x^2(t) >= \sigma(t)^2$$

To approach part 2, 10 grid spaces were decided to have a density or probability of 1 each, after which the probabilities were normalized. The time that the simulation was running concluded in five different snapshots over which the approaching of the Gaussian Distribution is clear. These images have been plotted in Figure 6. The snapshots taken during the duration of the simulation featured a curve which helps to determine if Gaussian would be a correct distribution to consider for this data. It was found that the simulation approaches a Normal distribution of $\sigma(t) = \sqrt{2Dt}$.

4 Mixing of Two Gases

The approach to these issues was similar to those of the last homework, such that the new values of the distribution were based on the surrounding elements of the previous array. The code was implemented to randomly select coordinates and check if there is a particle there. If there was no particle, then the program would continue running until it would find a match. Then a random number generator was used to find which way the particle would move around. These values were added to arrays to keep track of position and some of the resulting data is shown in Figure 7. This was done for data with 1000000 movements. The density of the species as more time has passed follows a gradient and approaches a gradient such that there would be little change in the local gradient, meaning the diffusion would in the real world eventually find an equilibrium. The data averages for 100 trials were determined and plotted in Figure 8. The average diffusion was done based on 1000 movements, which resulted in the apparent separation on the graph of Figure 9. If more iterations were successful, then the curves would look more like lines as diffusion would eventually stabilize or reach a steady state.

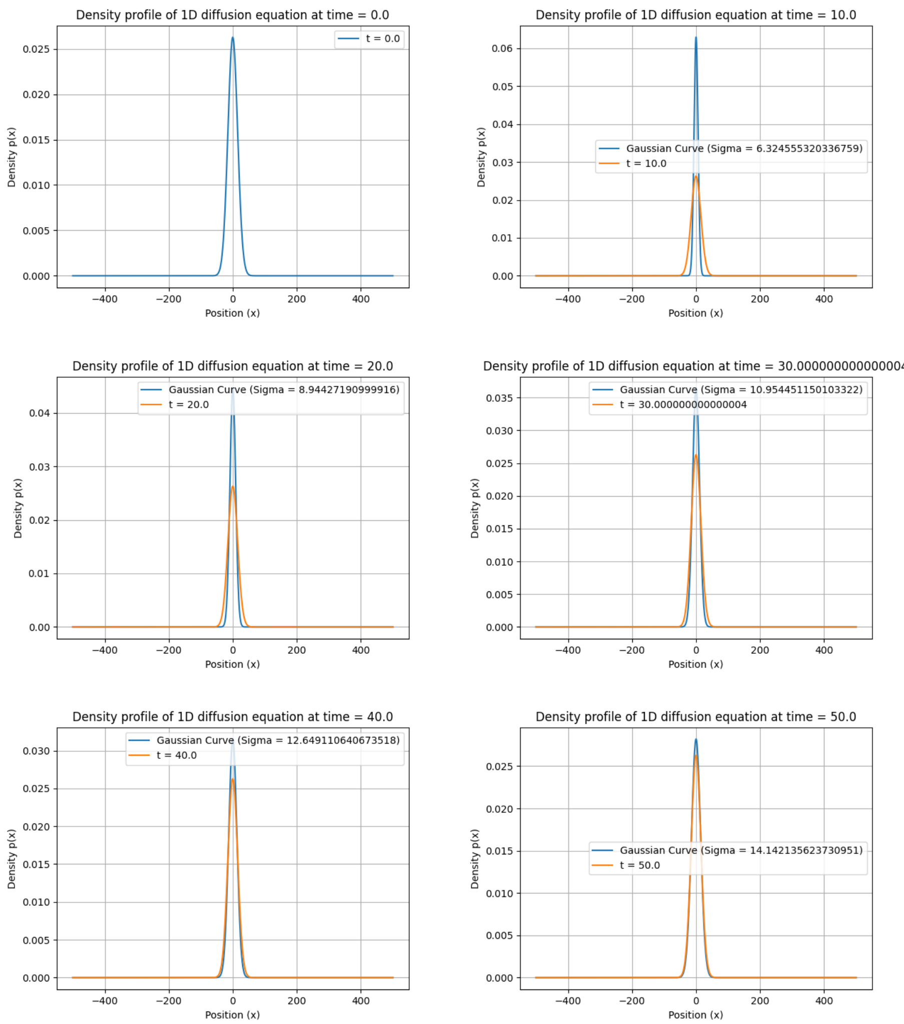

Figure 6: Plots of diffusion in 1 dimension at various time points.

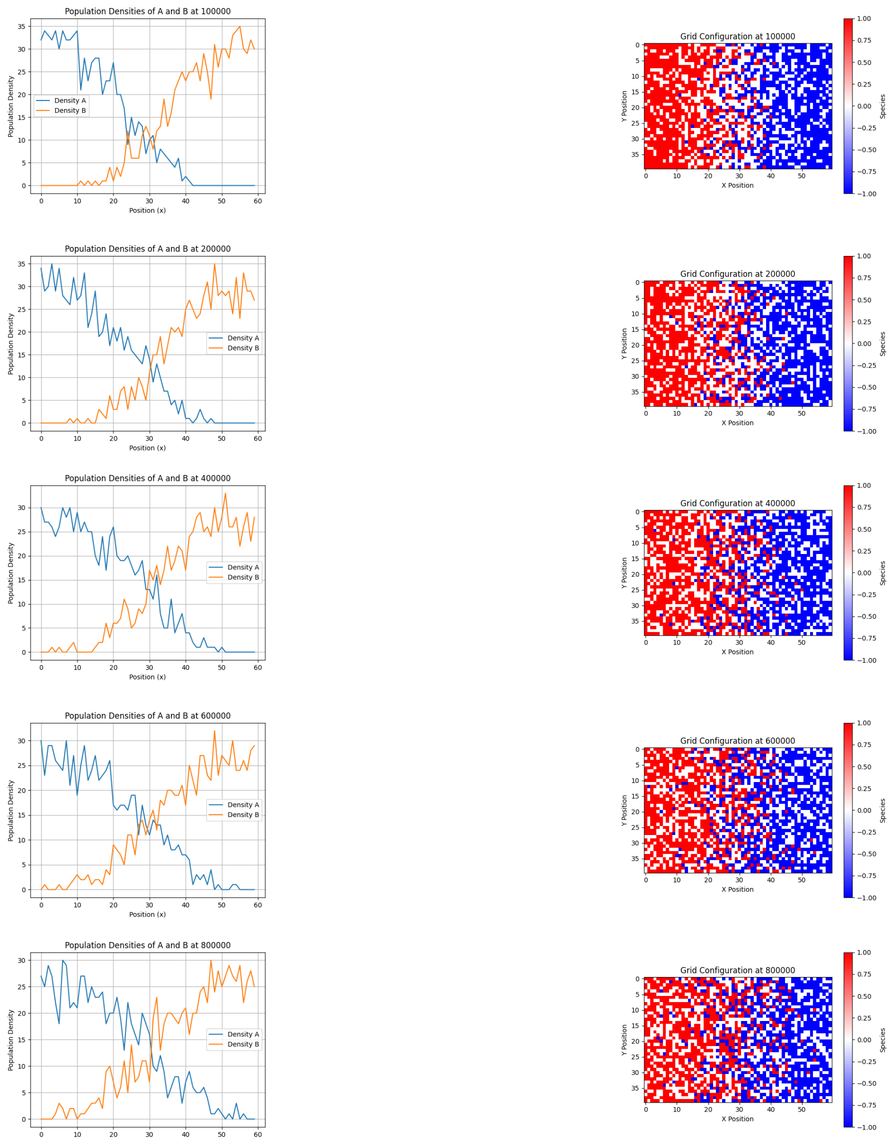

Figure 7: Single trial of diffusion of two gases.

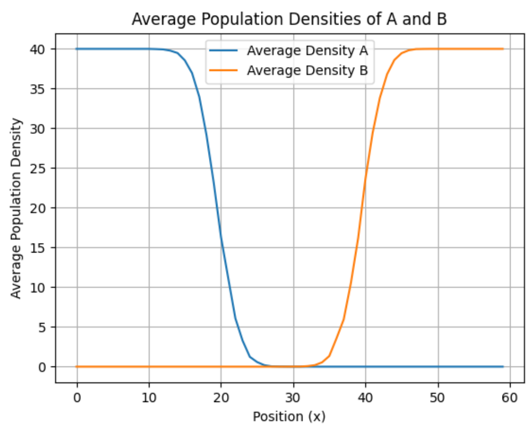

Figure 8: Plots of average diffusion in 1 dimension using 1000 steps from 100 samples.