

# Fys 1120

## Formler og sammendrag

### I. PENSUM

**Uke 1** Coloumbs Lov, Superposisjon

**Uke 2** Skalarpotensial, gradient, romladningstetthet

**Uke 3** Gauss lov, Dielektriske medier, Gauss lov i et dielektrisk medium. Poissons likning.

**Uke 4** Poisson's likning og lyn, ideelle ledere. kapasitans.

**Uke 5** Energi i elektriske felt, strøm og strømtetthet

**Uke 6** Ohms lov, resistans, effekttap, ladningsbevaring og Kirchoffs strømlov, magnetostatikk: Biot-Savarts lov, strømelement, Lorentz kraft

**Uke 7** Hall effekt, magnetiske krefter og moment, magnetisk fluks, amperes lov.

#### Til midtveis:

Coloumbs lov, E-felt for punktladnings-, linjeladnings-, flateladnings- og romladnings-tetthet, Gauss lov, definisjon av skalarpotensial, og skalarpotensialet for punktladning, linje-, flate- og romladningstetthet, hvordan finne E fra skalarpotensialet, Gauss lov for dielektriske medier, grensebetingelser for D og E, Poissons og Laplace likning og numerisk løsning av denne, egenskaper til ideelle ledere, definisjon av kapasitans, strøm som integral over strømtetthet, ohms lov ( $J = \sigma E$ ), definisjonen av resistans, bevaringsloven for ladning på integral og differensial form, Kirchoffs strømlov, Biot-Savarts lov, Lorentz kraft på ladninger og strømelementer, Amperes lov.

## II. 1.1 ELEKTROSTATIKK

### A. Coulomb's lov

The interaction between interchanging charged point particles:

$$\mathbf{F} = \frac{qQ}{4\pi\epsilon_0} \frac{\hat{R}}{R^2} \quad (1)$$

where  $\mathbf{R} = \mathbf{r}_Q - \mathbf{r}_q$  and  $\hat{R} = \frac{\mathbf{R}}{R}$

#### 1. Charge

Charges are fundamental, quantized properties of matter which can be positive, negative or zero.

Charges are measured in units of Coulomb, C. The charge of a proton is  $e = 1.602 \times 10^{-19}$  C. Charge is conserved.  $\epsilon_0 = 8.85 \cdot 10^{-12} \text{C}^2 \text{N}^{-1} \text{m}^{-2}$  is the permittivity in vacuum, and this version of Coulomb's law is only valid in vacuum.

#### 2. Point Charges

*Point charge* is a charged body, where the dimensions of the body are much smaller than typical distances between bodies of interest.

#### 3. Sign of Coulomb's law

The direction of the force depends on both charges, and is determined by their products. Equal signs yield repulsive force, and opposite yields attractive force.

**There is no self interaction for a charged point particle**

### B. Superposition principle

The force on a point charge  $q$  at  $\mathbf{r}_q$  from point charges  $Q_1$  and  $Q_2$  at  $\mathbf{r}_{Q1}$  and  $\mathbf{r}_{Q2}$  where  $\mathbf{R}_1 = \mathbf{r}_{Q1} - \mathbf{r}_q$  and  $\mathbf{R}_2 = \mathbf{r}_{Q2} - \mathbf{r}_q$

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \frac{qQ_1}{4\pi\epsilon_0} \frac{\hat{R}_1}{R_1^2} + \frac{qQ_2}{4\pi\epsilon_0} \frac{\hat{R}_2}{R_2^2} \quad (2)$$

### III. 1.1.2 THE ELECTRIC FIELD

If there is a set of charges  $Q_i$ , we say that they set up an electric field everywhere in space

$$\mathbf{E} = \frac{\mathbf{F}_q}{q} \quad (q \rightarrow 0) \quad (3)$$

$$\mathbf{F} = q\mathbf{E} \quad (4)$$

#### 1. The Electric field from single point charge

$$\frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \hat{R} \quad (5)$$

#### 2. Superposition principle for the electric field

$$\mathbf{E} = \frac{1}{q} \sum_i \mathbf{F}_i = \sum_i \frac{\mathbf{F}_i}{q} = \sum_i \mathbf{E}_i = \sum_i \frac{1}{4\pi\epsilon_0} \frac{Q_i}{R_i^2} \hat{R}_i \quad (6)$$

**Dipol:** Består av to ladninger av samme størrelse, men forskjellig fortegn, plassert i nærheten av hverandre. Sett fra langt unna er nettoladningen 0, men det elektriske feltet er ikke 0.

#### 3. Continuous distributions of charge: charge density

Når det er svært mange punktladninger er det enklere å se på en ladningsfordeling, heller en mange individuelle punktladninger.

$$\rho = \rho_v = \frac{dQ}{dv} \quad \text{volume charge density } [C/m^3] \quad (7)$$

The charge in a volume  $v$  is then the sum of the charges inside the volume:

$$Q_v = \int_v \rho(\mathbf{r}) dv \quad (8)$$

Videre har vi

$$\rho_a = \sigma = \frac{dQ}{dA} \quad \text{surface charge density} \quad (9)$$

$$\rho_l = \sigma = \frac{dQ}{dl} \quad \text{line charge density} \quad (10)$$

#### 4. Elektrisk felt fra Volum ladning tetthet

$$d\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{\rho(\mathbf{r}') dv'}{R^2} \hat{R} \quad (11)$$

Trenger å summere over alle mulige  $\mathbf{r}$  verdier i volumet

$$\mathbf{E} = \int_v \frac{1}{4\pi\epsilon_0} \frac{\rho(\mathbf{r}') dv'}{R^2} \hat{R} = \int_v \frac{1}{4\pi\epsilon_0} \frac{\rho(\mathbf{r}') dv'}{(\mathbf{r}' - \mathbf{r})^2} \frac{\mathbf{r}' - \mathbf{r}}{|\mathbf{r}' - \mathbf{r}|} \quad (12)$$

Elektriske feltet fra en overflate tetthet

$$\mathbf{E} = \int_A \frac{1}{4\pi\epsilon_0} \frac{\rho(\mathbf{r}') dA'}{R^2} \hat{R} \quad (13)$$

Elektriske feltet fra en linjeladnings tetthet:

$$\mathbf{E} = \int_L \frac{1}{4\pi\epsilon_0} \frac{\rho(x') dx'}{R^2} \hat{R} \quad (14)$$

#### 5. Symmetribetraktninger

: For eksempel: Gitt en ring ladning, så vil det være umiddelbart synlig at alle komponenter i x og y retning kanselerer, og at feltet kun vil ha en komponent i z-retning. Dette er kun gyldig på z-aksen.

### IV. 1.2 ELECTRIC SCALAR POTENTIAL

The electric scalar potential is a sort of potential energy per unit charge. The work on charge  $q$  when the charge is moved along a path from A to B is

$$W_{AB} = \int_A^B \mathbf{F} \cdot d\mathbf{r} \quad (15)$$

The path integral is independent of the path, due to the fact that electric forces are conservative.

Therefore, we can introduce a potential energy for a charge  $q$  in the force field set up by  $Q$  as the work needed to move the charge from a point A to a reference point.

$$U_A = W_{A,ref} = U(r) \quad (16)$$

This is valid for any charge distribution, and subjected to the superposition principle. We define the potential energy as

$$U_A = \int_A^{ref} \mathbf{F} \cdot d\mathbf{l} = q \int_A^{ref} \mathbf{E} \cdot d\mathbf{l} \quad (17)$$

We then introduce the scalar potential  $V_A$  so that

$$U_A = W_{AB} = qV_A \quad (18)$$

The scalar potential in a point A at  $\mathbf{r}$  is

$$V_A = V_{A,1} + V_{A,2} + \dots = \sum_i V_{A,i} \quad (19)$$

It is common to set the reference point in infinity, as long as the charge distribution has a finite extent. Furthermore both the divergence and curl of  $\mathbf{E}$  is zero in electro statics.

#### A. Kirchoff's law of voltages

The sum of potential differences along a closed loop (a circuit) is zero

$$V_{AB} + V_{BC} + V_{CA} = \oint \mathbf{E} \cdot d\mathbf{l} = 0 \quad (20)$$

#### B. Electric potential due to a given charge distribution

A set of charges:

$$V(\mathbf{r}) = \sum_i \frac{Q_i}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_i|} \quad (21)$$

A continuous distribution of charges

$$V = \int_v \frac{\rho dv}{4\pi\epsilon_0 R} \quad (22)$$

#### C. Relation between electric potential and electric field

$$V(\mathbf{r}) = \int_r^{ref} \mathbf{E} \cdot d\mathbf{l} \quad (23)$$

$$\mathbf{E} = -\nabla V \quad (24)$$

#### D. Equipotential surfaces

An equipotential surface is a surface where the potential is constant. The gradient to the potential is always normal to the equipotential surface, and points in the direction that  $V$  increases the fastest. Therefore, the electric field is normal to the equipotential surface but point in the direction that  $V$  decreases the fastest.

## V. 1.3 GAUSS' LAW

Gauss' law is well suited for systems with a high degree of symmetry.

Gauss' law states that the flux of the electric field through a closed surface  $S$  is equal to the total charge in the volume inside the surface

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q_S}{\epsilon_0} \quad (25)$$

$S$  is any closed surface and  $Q$  is the net charge in the volume.

$$Q_s = \sum_i Q_{i,in} \quad Q_s = \int_v \rho_v dv \quad (26)$$

### A. Electric flux

The electric flux through a small surface  $d\mathbf{A}$  is defined as

$$d\Phi = \mathbf{E} \cdot d\mathbf{A} = E dA \cos \theta \quad (27)$$

$d\mathbf{A}$  is the oriented surface element

$$d\mathbf{A} = \hat{n} dA \quad (28)$$

Only the field normal to the surface contributes to the flux. The surface normal points outward of the volume. Gauss law is more general than Coulomb's law, and is even valid for moving charges.

### B. Applying Gauss' law

A lot of applying Gauss' law is about finding a symmetry that makes the formula valid. First, one needs to use symmetry arguments to simplify the description of the electric field. Second, one needs to find a surface  $S$  where the electric field was constant.

#### 1. Gauss' law for a single charge

Take a single charge in the origin. We expect that the field must be the same for all possible rotations of the system. We expect the system to have spherical symmetry. Then we realize that the field only can be directed in the radial direction, or else it would break the symmetry. In addition, we realize that the field can not have an angular dependency, because this would also break the symmetry. The field cannot be larger in one direction, because the orientation of the coordinate system is arbitrary. Then we apply Gauss.

#### 2. Recipe for using Gauss law to find the electric field

- Find a set of surfaces that enclose a volume such that  $E \cdot \hat{n}$  is constant on each such surface element. (It may be zero on some of the surfaces — zero is a constant!)

- This often requires that you find a simplified description of the field in a chosen coordinate system, such as  $E = E_r(r)\hat{r}$ .
- Find the flux integral
- Use Gauss' law to find the electric field as a function of charge and position.
- Notice that the surface does not have to enclose all the charges — it is allowed and indeed often necessary to choose a surface that contains only some of the charges. However, the electric field must be a constant on the surfaces you have chosen.

### C. Gauss' law on differential form

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (29)$$

## VI. 1.4 POLARIZATION AND DIELECTRICS

What happens with the electric field inside a material, the electric field will induce local charge displacements in an insulator, which induce an electric field. In conductors, the charges become mobile. This alters the total electric field.

### A. Dielectrics

Dielectric or an insulator is a material with very little free charges (like water, ceramic, plastics or glass). We discern between two types of dielectric: polar and non-polar.

#### Polar dielectric

**Definition:** In a polar dielectric the electrons are distributed relative to the positive charges so that molecules behave as individual dipoles **External E:** If the polar dielectric is subjected to an external electric field, the dipole molecules will tend to orient in the electric field with the positive part of the dipole pointing in the direction of the local field. Without an applied electric field, the dipoles will point in random directions, with no net effect, but with an applied electric field, the dipoles will tend to align with the field. The stronger the field, the stronger will be the alignment and the stronger the net dipoles.

**Example:** water

## Non-polar dielectric

**Definition:** A non-polar dielectric has electrons distributed symmetrically around the positive charges. There is no net dipoles in the system when there is no applied electric field. However if we apply an external electric field, the electron clouds around atoms or molecules tend to be displaced: The electrons will be displaced in a direction opposite the electric field, whereas the nucleus is much less displaced. Each atom becomes a small dipole. **Example:**

### 1. Polar and non-polar induce dipoles

Formation of a net set of dipoles. Average dipole moment:

$$P_{av} = \frac{\sum_i \mathbf{P}_{iindv}}{N_{indv}} \quad (30)$$

The polarization vector  $\mathbf{P}$  is the dipole moment per unit volume. The polarization vector depends on total field.

### 2. Linearr dielectrics

For a linear dielectric the polarization is proportional to the total electric field

$$\mathbf{P} = \chi_e \epsilon_0 \mathbf{E} \quad (31)$$

$\chi_e$  is called the electric susceptibility, it has no unit.

Te the effect of an applied field is only a reorientation of dipoles (for a polar dielectric), or a local displacement of the electron cloud (non-polar dielectric), there is no change in the net charge in a volume.

polarization due to small dipoles with dipole moment

$$\mathbf{p} = Q\mathbf{d} \quad (32)$$

## B. Gauss' law and total charge in a volume

Includes both the free charges, and the bound charges that are the result of the total electric field.

$$\epsilon_0 \oint_S \mathbf{E} \cdot d\mathbf{S} = Q_f + Q_b \quad (33)$$

$$\epsilon_0 \oint_S \mathbf{E} \cdot d\mathbf{S} = Q_f - \oint_S \mathbf{P} \cdot d\mathbf{S} \quad (34)$$

$$\oint_S (\epsilon_0 \mathbf{E} + \mathbf{P}) \cdot d\mathbf{S} = Q_f \quad (35)$$



### 1. Displacement field

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad (36)$$

### 2. Gauss law

$$\epsilon_0 \oint_S \mathbf{D} \cdot d\mathbf{S} = Q_{\text{free in } S} \quad (37)$$

### 3. Linearly polarized media

$$\mathbf{D} = \epsilon \mathbf{E} \quad (38)$$

$\epsilon_r$  is the relative permittivity, and  $\epsilon$  is the absolute permittivity.

### C. Gauss' law on differential form

$$\nabla \cdot \mathbf{D} = \rho_{\text{free}} \quad (39)$$

Materials are dielectric only up to a given field strength. If  $E$  becomes too large, the material ionizes, and we get formation of free charges, and the material becomes a conductor. This limit is called the dielectric strength.

### D. Surface charges

$$\rho_S = \mathbf{P} \cdot \hat{n} \quad (40)$$

### E. Boundary conditions for $\mathbf{E}$ and $\mathbf{D}$

On the interface of two dielectric media, like plastic and water. For instance, we know the electric field  $\mathbf{E}$  and displacement  $\mathbf{D}$  on one side of the interface, what is it on the other?

### 1. Tangential boundary condition

Use that the line integral of  $\mathbf{E}$  over a closed loop is zero.

$$\mathbf{E}_1 \cdot d\mathbf{l} + \mathbf{E}_2 \cdot (-d\mathbf{l}) = 0 \quad (41)$$

$$E_{1t} = E_{2t} \quad (42)$$

### 2. Normal boundary condition

Use Gauss' law to relate the normal component of the field on each side

$$D_{1n} - D_{2n} = \sigma_s \quad (43)$$

The normal components of  $\mathbf{D}$  is related to the surface charge density.

### 3. Two dielectric media

If both media are dielectric, there are no free surface charges, and then

$$D_{1n} = D_{2n} \quad (44)$$

## VII. 1.5 LAPLACE EQUATION

stuff

## VIII. 1.6 CONDUCTORS

## IX. EXAMPLES

- Electric field from two charges (FN 1)
- Electric field from a dipole
- Visualizing a dipole field
- Electric Field from a ring charge