Building a model for the solar system using ordinary differential equations

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In this project code is developed for simulating the solar system using the Verlet Algorithm

I. INTRODUCTION

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TABLE I: *The mass of the Sun and the masses of all relevant planets and their distances from the sun listed in units of kg and AU,* $1\ AU = 1.5 \times 10^{11}\ m$.

Planet	Mass [kg]	Distance to sun [AU]
Earth	$M_{\text{Earth}} = 6 \times 10^{24}$	1
Jupiter	$M_{\text{Jupiter}} = 1.9 \times 10^{27}$	5.20
Mars	$M_{\text{Mars}} = 6.6 \times 10^{23}$	1.52
Venus	$M_{\text{Venus}} = 4.9 \times 10^{24}$	0.72
Saturn	$M_{\text{Saturn}} = 5.5 \times 10^{26}$	9.54
Mercury	$M_{\text{Mercury}} = 3.3 \times 10^{23}$	0.39
Uranus	$M_{\text{Uranus}} = 8.8 \times 10^{25}$	19.19
Neptun	$M_{\text{Neptun}} = 1.03 \times 10^{26}$	30.06
Sun	$M_{\text{sun}} = M_{\odot} = 2 \times 10^{30}$	-

II. THEORY

A. Newton's law of gravitation

Newton's law of gravitation is given by a force F_G

$$F_G = \frac{GM_{\odot}M_{\text{Earth}}}{r^2} = \frac{M_{\text{Earth}}v^2}{r} \tag{1}$$

where $M_{\odot}$ is the mass of the Sun and M_{Earth} is the mass of the Earth. G is the gravitational constant, r is the distance between the Sun and the Earth and v is the velocity of Earth. This implies that

$$v^2 r = GM_{\odot} = 4\pi^2 \frac{(\text{AU})^3}{(\text{yr})^2} \tag{2}$$

B. Co-planar motion

If it is assumed that the orbit of the Sun and the Earth is co-planer in the xy-plane, one can employ Newton's law of motion to derive the following equations

$$\frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_{\text{Earth}}} \tag{3}$$

$$\frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_{\text{Earth}}} \tag{4}$$

where $F_{G,x}$ and $F_{G,y}$ are the x and y components of the gravitational force.

C. Planetary sizes

III. NUMERICAL ALGORITHMS

Euler's forward algorithm states that

$$\frac{du(x)}{dt} = \frac{u(x+h) - u(x)}{h} + \mathcal{O}() \tag{5}$$

IV. METHOD

It is assumed that the Sun has a mass which is much larger than the mass of the Earth, so that the motion of the Sun is neglected. In order to discretize equation 2 a discrete time axis $t \in [t_0, t_f]$ of N points is defined, where $t_i = t_0 + ih$ for $i \in [0, N]$. This implies that $h = (t_f - t_0)/N$. The discretized positions and velocities are then functions of the discretized time $x_i = x(t_i)$ and $v_i = v(t_i)$, and equivalently for y. The acceleration is then $a_{xi} = a_x(x_i, y_i)$.
The differential equation 2 is then solved by Euler's forward algorithm and the velocity Verlet method.

Compute the motion of the earth using different methods for solving odrinary differential equations

1. *Initial conditions*

The NASA jet propulsion laboratory's HORIZONS Web-Interface is used to generate a cartesian state vector table of any object with respect to any major body, to extract initial conditions for each planet.

V. RESULTS

VI. DISCUSSION

VII. CONCLUSION

VIII. REFERENCES

IX. APPENDIX