1° appello 2023 (07/02/2023)

① LIS

0	1	2	3	4	5	6	7	
6	3	5	2	7	8	1	9	→ arr

$dp = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$

$prev = [-1\ -1\ -1\ -1\ -1\ -1\ -1\ -1]$

	0	1	2	3	4	5	6	7
dp =	1	1	2	1	3	4	1	5
prev =	-1	-1	1	-1	2	4	-1	5

- se arr[j] < arr[i] aggiorno
  - ▸ dp[i] = max(dp[i], dp[j]+1)
  - ▸ prev[i] = j se dp[i] è se è stato modificato

LIS_length = max(dp)

6 > 1
3 > 1
5 > 1
2 > 1
7 > 1
8 > 1

6 < 9 → max(1, 2) = 2
3 < 9 → max(2, 2) = 2
5 < 9 → max(2, 3) = 3
2 < 9 → max(3, 2) = 3
7 < 9 → max(3, 4) = 4
8 < 9 → max(4, 3) = 4
1 < 9 → max(4, 5) = 5 prev[7] = 5

6 < 8 → max(1, 2) = 2 prev[5] = 0
3 < 8 → max(2, 2) = 2
5 < 8 → max(2, 3) = 3
2 < 8 → max(3, 2) = 3
7 < 8 → max(3, 4) = 4 prev[5] = 4

6 > 2
3 > 2
5 > 2

6 > 5
3 < 5 → dp[2] = max(1, 2) = 2
prev[2] = 1

6 < 7 → dp[4] = max(dp[4], dp[0]+1) = 2
prev[4] = 0

3 < 7
5 < 7
2 < 7

dp[4] = max(dp[4], dp[1]+1) = 2
prev[4] = 1

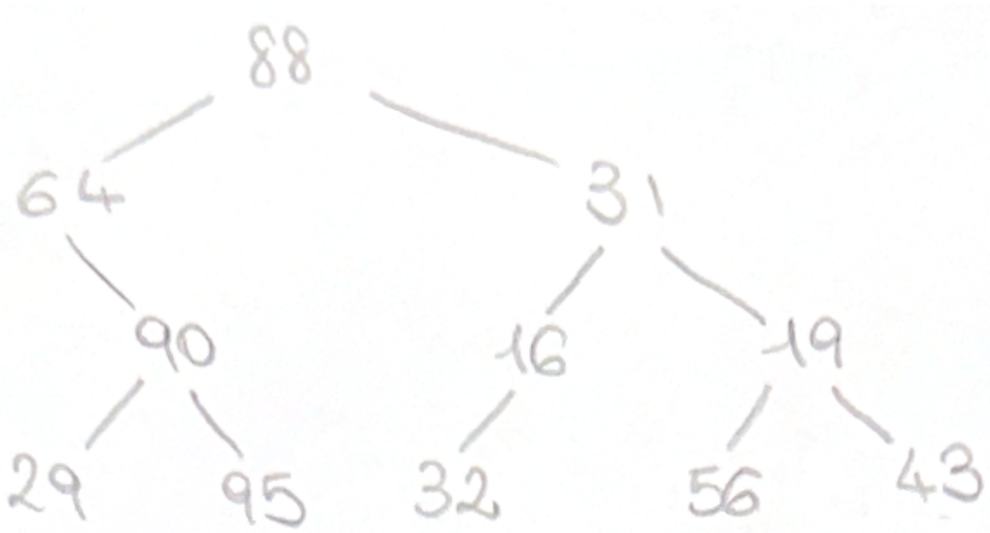
dp[4] = max(dp[4], dp[2]+1) = max(2, 3) = 3
prev[4] = 2

dp[4] = max(2, 0) = 2

max(dp) = 5, indice = 7

arr[7] = 9, prev[7] = 5
arr[5] = 8, prev[5] = 4
arr[4] = 7 prev[4] = 2
arr[2] = 5 prev[2] = 1
arr[1] = 3 prev[1] = -1

LIS = [3, 5, 7, 8, 9]

② 

IN-ORDER: 64 29 90 95 88 32 16 31 56 19 43

PRE-ORDER: 88 64 90 29 95 31 16 32 19 56 43

POST-ORDER: 29 95 90 64 32 16 56 43 19 31 88

③ Open addressing con quadratic probing

$c_1 = 1 \quad c_2 = 1$

Dim per avere fattore di carico $\alpha < 1/2$

$$\alpha = \frac{N}{M} < \frac{1}{2} \qquad \frac{9}{M} < \frac{1}{2} \qquad M > 18$$

$M = 19$ 139 47 102 59 212 27 97 21 25

0		
1	10	
2	59	11
3	212	12 25
4	97	13
5		14 21
6	139	15
7	102	16
8	27	17
9	47	18
		19

$97 : 2 + 1 + 1^2 = 4$

$21 : 2 + 1 + 1^2 = 4$ ×

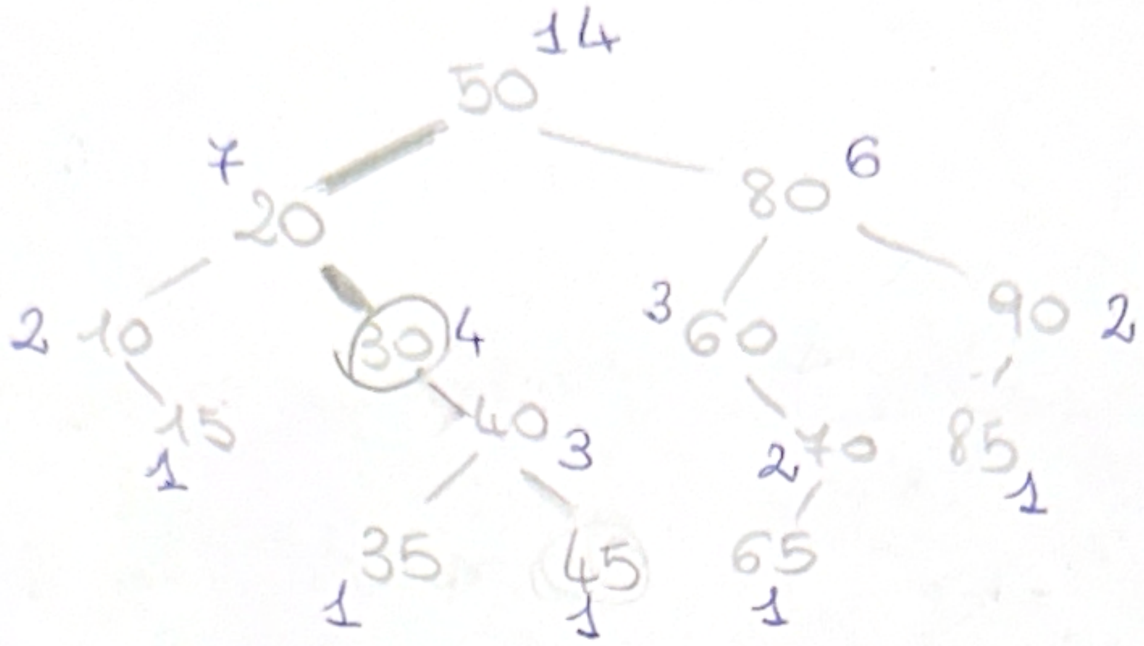
$2 + 2 + 2^2 = 8$ ×

$2 + 3 + 3^2 = 14$ ✓

$25 : 6 + 1 + 1^2 = 8$ ×

$6 + 2 + 2^2 = 12$ ✓✓

④ Chiave di rango 4 ⇒ BST select



K = 4

t = 7 > 4 ✓

K = 4

t = 2 < 4

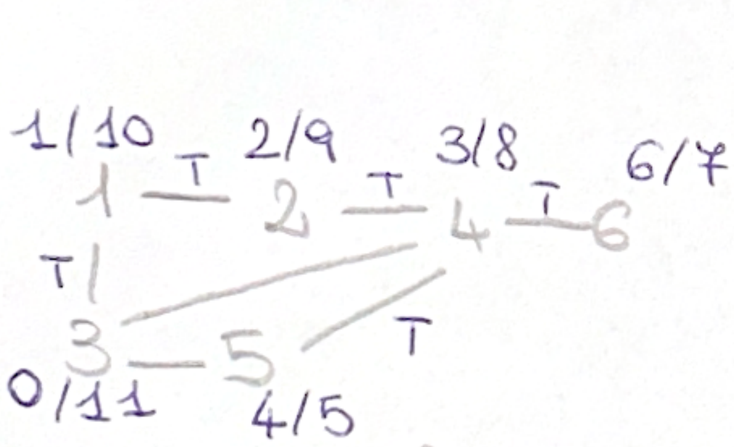
$K_{new} = 4 - 2 - 1 = 1$

K = 1

t = 0 < 1

$K_{new} = 1 - 0 - 1 = 0$

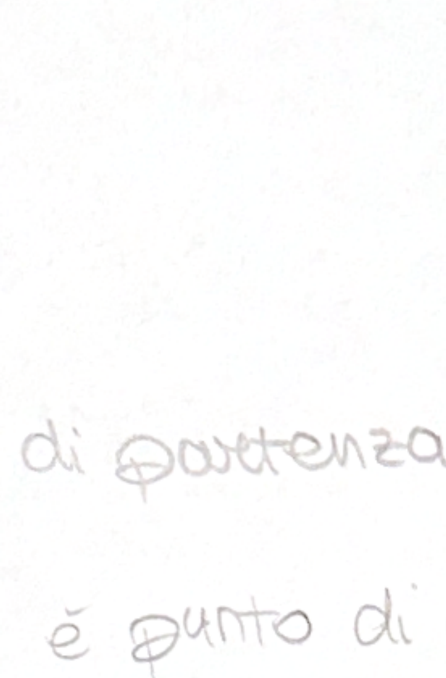
⑤



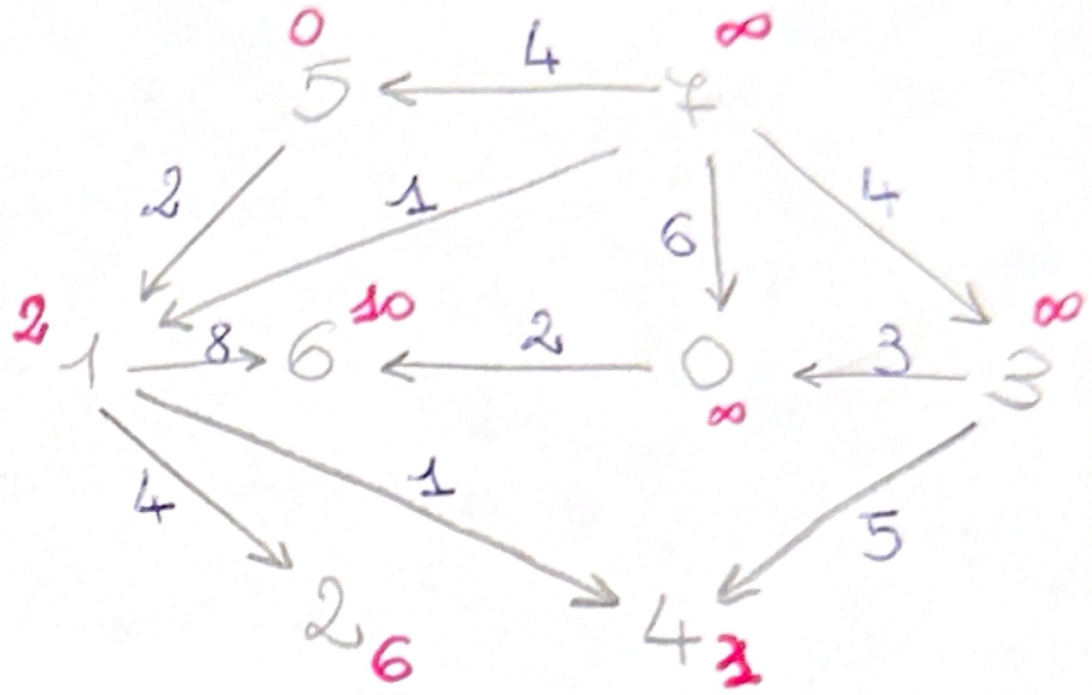
Punti di articolazione, 3 vertice di partenza

4 è punto di articolazione

3 è punto di articolazione

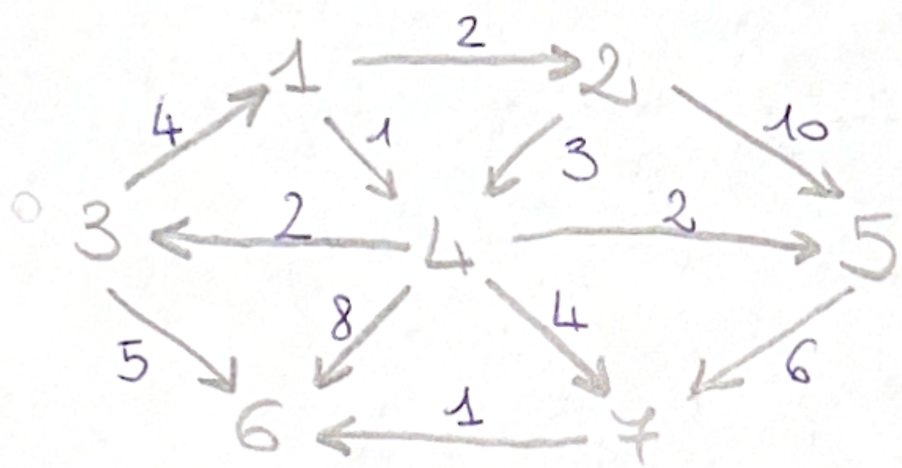


⑥ Cammini MASSIMI
A partire dal vertice 5



Es simile a quello a pag. 206
In ordine:
relaxation 5 >> 1[2]
relaxation 1 >> 6[10], 2[6], 4[3]
relaxation 2,4,6 NON hanno effetto
Ordinamento topologico
5, 1, 4, 2, 6, 0, 3, 7

⑦ Bellman-Ford, cammini minimi
A partire da 3



Ordine lessico grafico:
1-2, 1-4, 2-4, 2-5, 3-1, 3-6, ~~4-3~~, 4-5, 4-6, 4-7, 5-7, 7-6
2 1 3 10 4 5 2 2 8 4 6 1

	P_0	P_1	P_2	P_3
1	∞	4	4	4
2	∞	∞	6	6
3	0	0	0	0
4	∞	∞	5	5
5	∞	∞	7	7
6	∞	5	5	5
7	∞	∞	9	9

11/05/2023

① $T(n) = 3T\left(\frac{n}{4}\right) + 3n$ $\quad \frac{n}{4^i} = 1 \rightarrow n = 4^i \rightarrow i = \log_4 n$

$T(1) = 1$

$T\left(\frac{n}{4}\right) = 3T\left(\frac{n}{16}\right) + 3\frac{n}{4}$

$T\left(\frac{n}{16}\right) = 3T\left(\frac{n}{64}\right) + 3\frac{n}{16}$

$T(n) = 3 \cdot \left(3 \cdot \left(3\,T\left(\frac{n}{64}\right) + 3\,\frac{n}{16}\right) + 3\,\frac{n}{4}\right) + 3n =$

$= \frac{27}{16}n + \frac{9}{4}n + 3n + 27\,T\left(\frac{n}{64}\right) = 3n\left(\frac{3^2}{4^2} + \frac{3^1}{4^1} + \frac{3^0}{4^0}\right) + 27\,T\left(\frac{n}{64}\right)$

$3n \cdot \sum \left(\frac{3}{4}\right)^i \rightsquigarrow 3n\left(\frac{\frac{3}{4}^{\log_4 n + 1} - 1}{\frac{3}{4} - 1}\right) =$

$-12n\left(\frac{3}{4} \cdot \frac{3}{4}^{\log_4 n} - 1\right) = 12n - 12 \cdot \frac{3}{4}n \cdot \frac{1}{n} \cdot n^{\log_4 3} =$

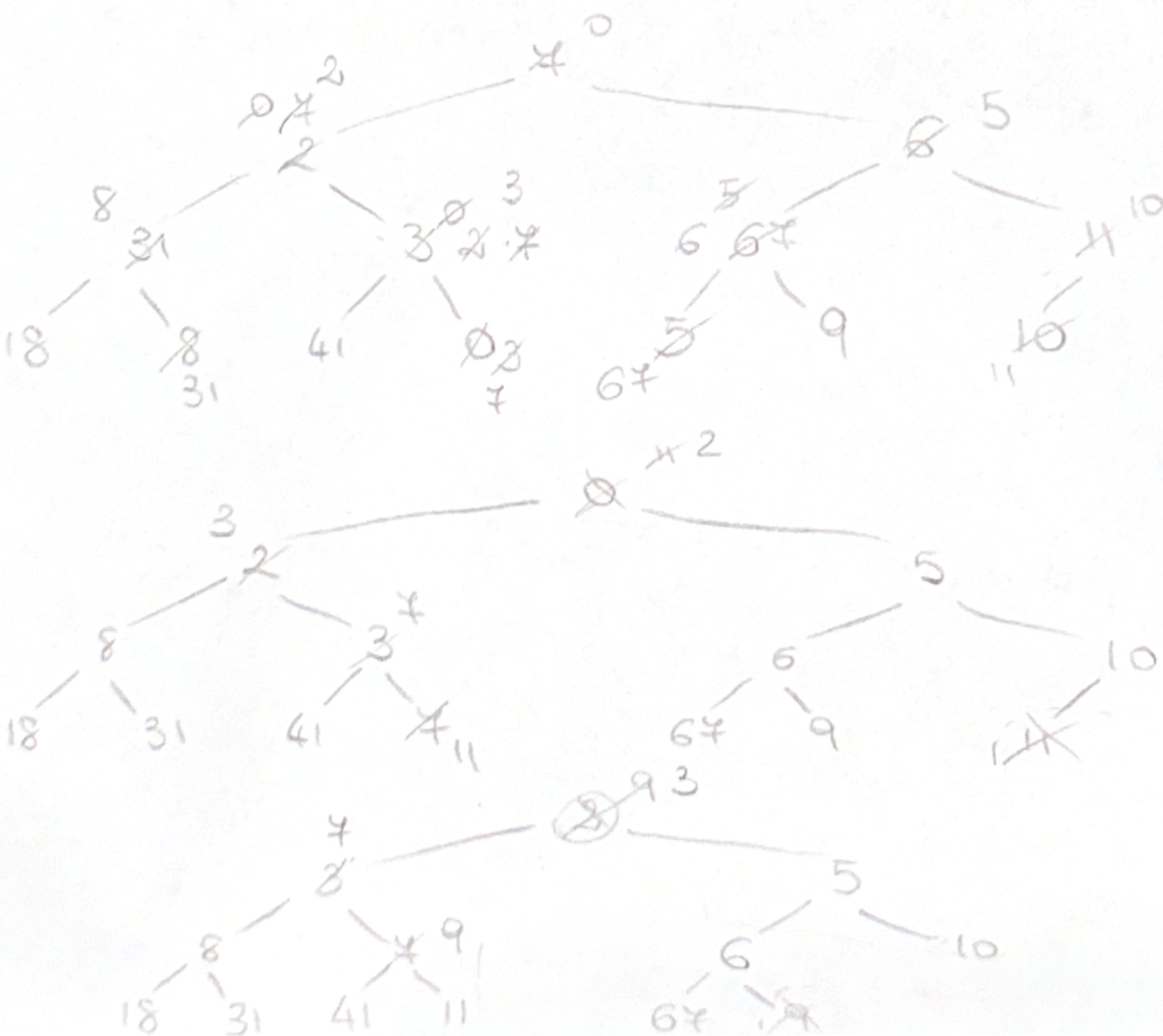
$= 12n - 9n^{\log_4 3} \rightsquigarrow \Theta(n)$

② Heap di dati

Nella radice: val min

Primi due passi di heap sort

7 2 6 31 3 67 11 18 8 41 0 5 9 10



③ $(3\times4),(4\times2),(2\times5),(5\times3)$

$p=[\overset{0}{3},\overset{1}{4},\overset{2}{2},\overset{3}{5},\overset{4}{3}]$

Parentesizzazione ottima

$m[1,2]=p_0p_1p_2=$ 24 $K=1$

$m[2,3]=p_1p_2p_3=$ 40 $K=2$

$m[3,4]=p_2p_3p_4=$ 30 $K=3$

$m[1,3]=\cancel{m[1,1]}+m[2,3]+p_0p_1p_3=100$ $K=1$

$m[1,3]=m[1,2]+\cancel{m[3,3]}+p_0p_2p_3=$ 54 $K=2$

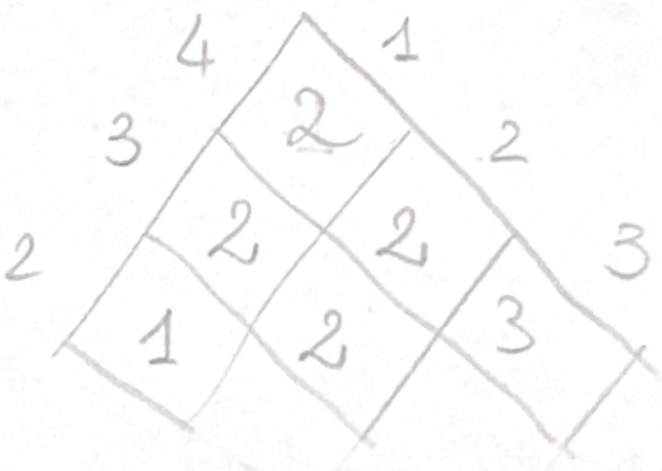
$m[2,4]=\cancel{m[2,2]}+m[3,4]+p_1p_2p_4=$ 54 $K=2$

$m[2,4]=m[2,3]+\cancel{m[4,4]}+p_1p_3p_4=120$ $K=3$

$m[1,4]=\cancel{m[1,1]}+m[2,4]+p_0p_1p_4=90$

$m[1,4]=m[1,2]+m[3,4]+p_0p_2p_4=$ 72 $K=2$

$m[1,4]=m[1,3]+\cancel{m[4,4]}+p_0p_3p_4=99$



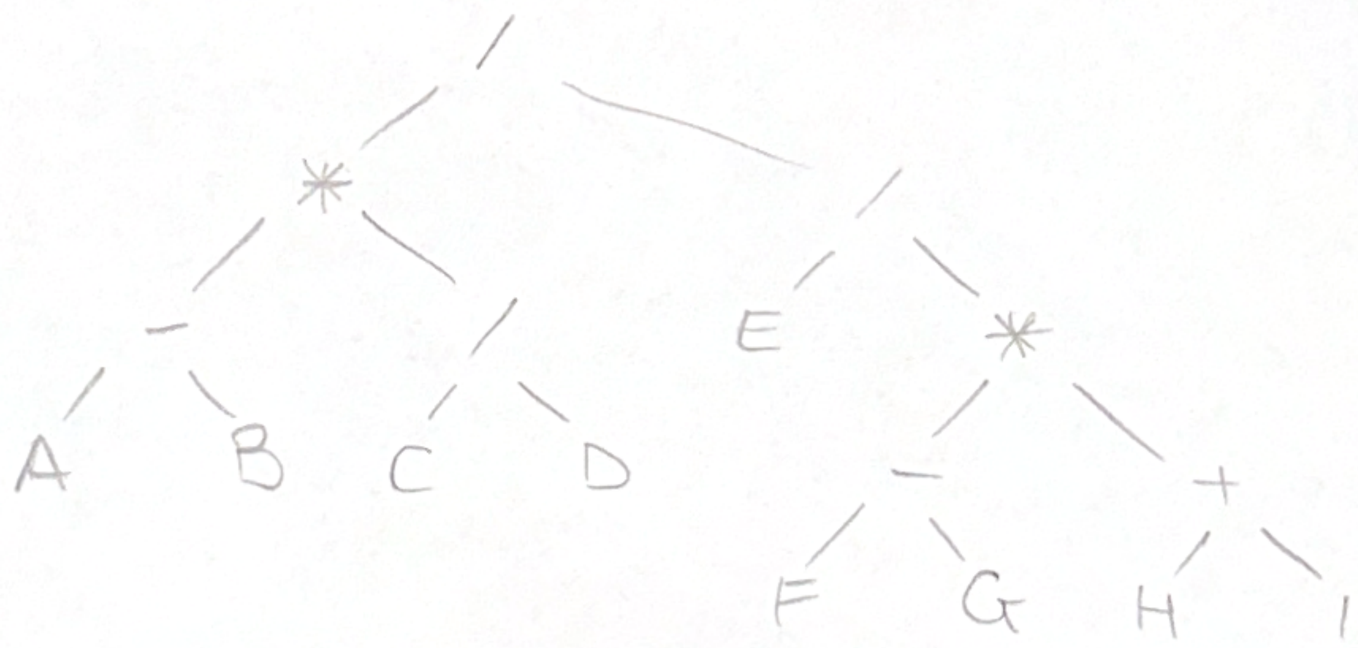
$A[1\ldots4]=[A_1\ldots A_2]\cdot[A_3\ldots4]=(A_1\cdot A_2)\cdot(A_3\cdot A_4)$

$A[1\ldots2]=A_1\cdot A_2$

$A[3\ldots4]=A_3\cdot A_4$

④ Si converta da pre-order (ABC) a post-order (BCA)

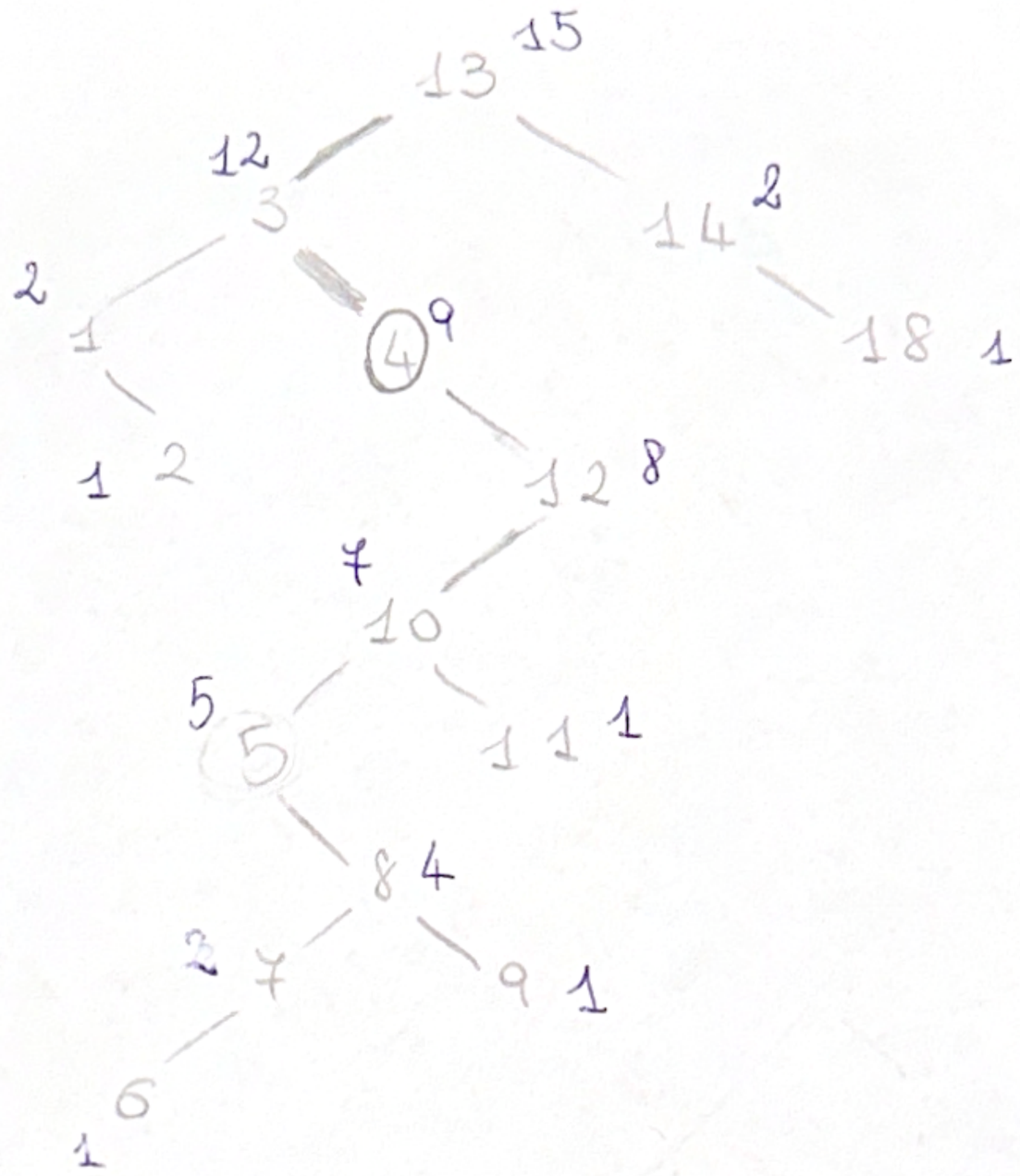
/ * - A B / C D / E * - F G + H I



Post-order:

A B - C D / * E F G - H I + * / /

⑤ Partizionamento sulla 6ª chiave più piccola k=5



k=5

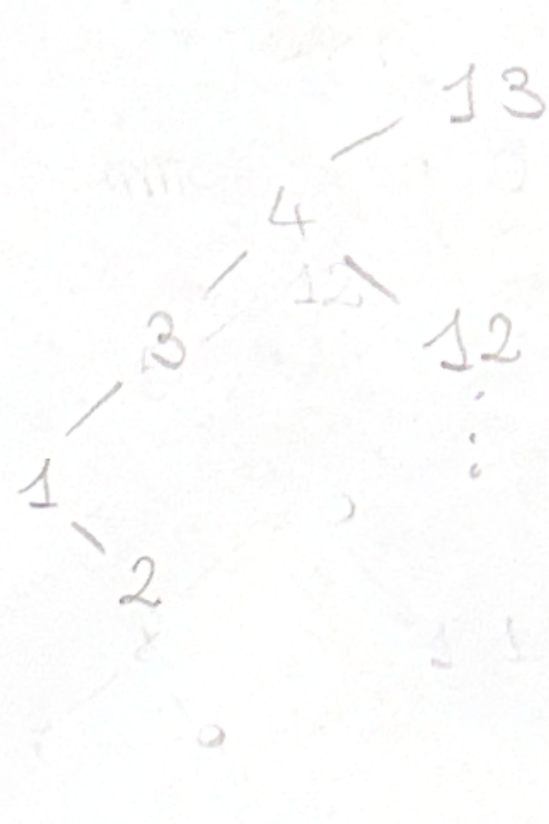
t=12>k

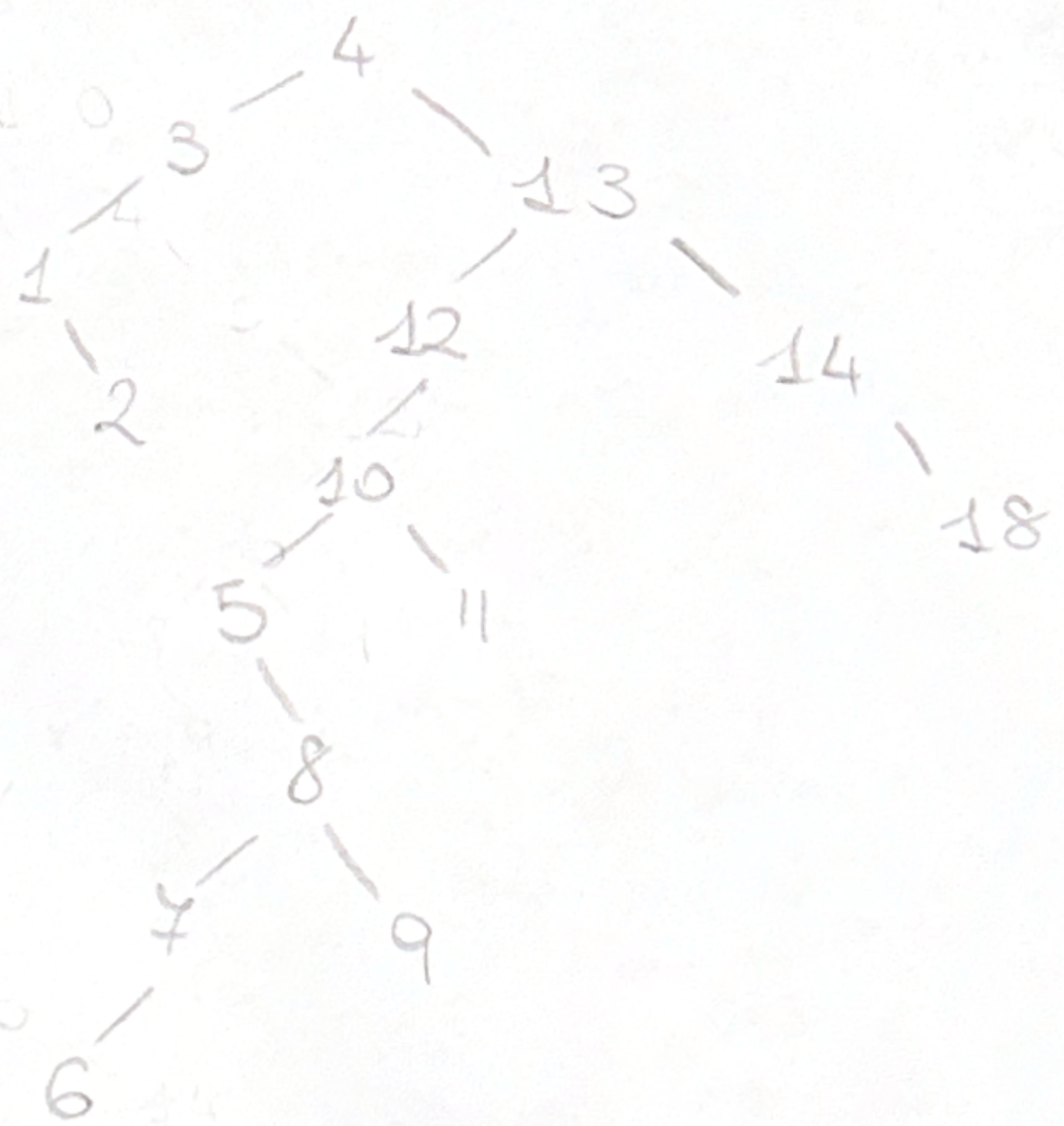
k=5

t=2<k

$k_{NEW} = 5-2-1 = 2$

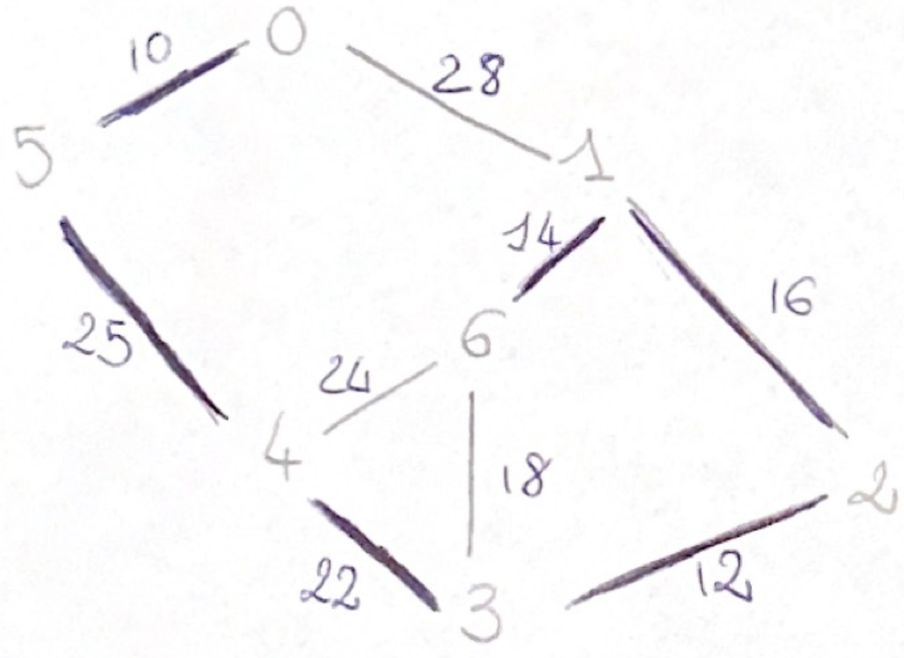
k=2





⑥ Prim a partire dal vertice 0

Val. peso minimo



Peso minimo =
10 + 25 + 22 + 12 + 16 + 14 = 99

In ordine 0-5, 5-4, 4-3, 3-2, 2-1, 1-6

III appello 2023

① 2 passi del quicksort

211 126 99 46 44 127 313 91 32 37 19 53

Pivot = 53

19 126 99 46 44 127 313 91 32 37 211 53

19 37 32 46 44 127 313 91 99 126 211 53

19 37 32 46 44 53 313 91 99 126 211 127

Pivot = 127

Pivot = 44

19 37 32 44 46 53 126 91 99 127 211 313

Dunque

1° passo

19, 37, 32, 46, 44, 53, 313, 91, 126, 211, 127

2° passo

19, 37, 32, 44, 46, 53, 126, 91, 99, 127, 211, 313

② Open addressing con linear probing

Dim M in modo da avere $\alpha < \frac{1}{2}$

211, 26, 79, 46, 154, 17, 43, 229

$N = 8 \rightarrow M \quad \frac{N}{M} < \frac{1}{2} \quad M > 2N \quad M > 16 \rightsquigarrow M = 17$

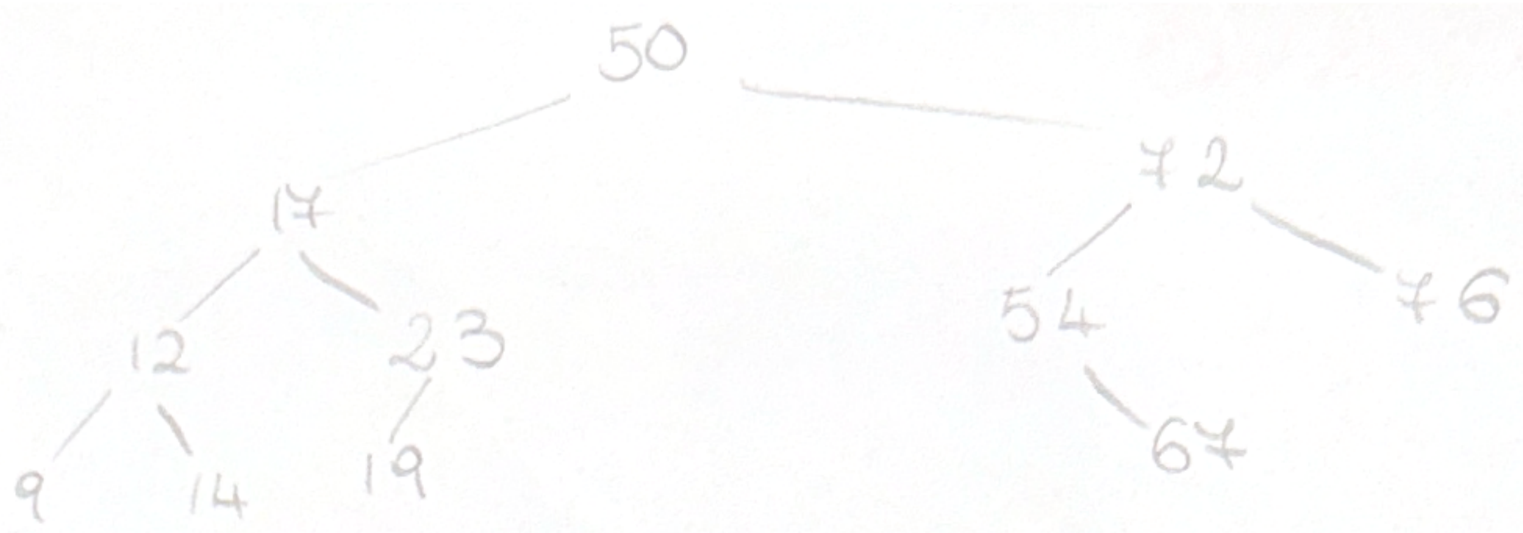
0	17
1	154
2	
3	
4	
5	
6	
7	211
8	229

9	26
10	43
11	79
12	46
13	
14	
15	
16	

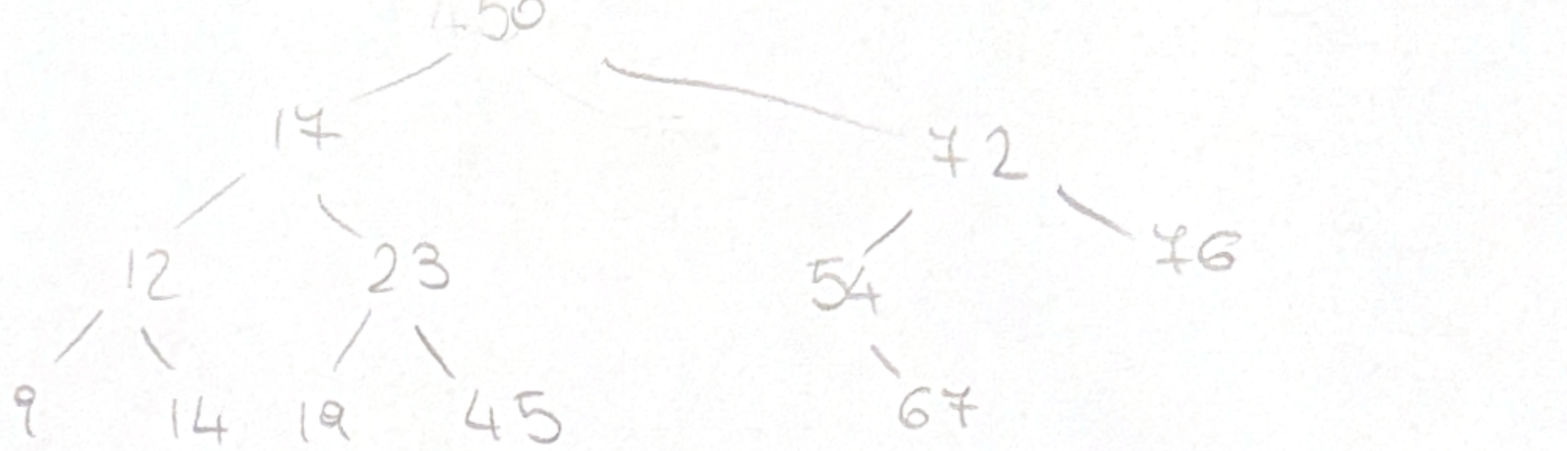
Primary cluster = sequenza continue di celle occupate

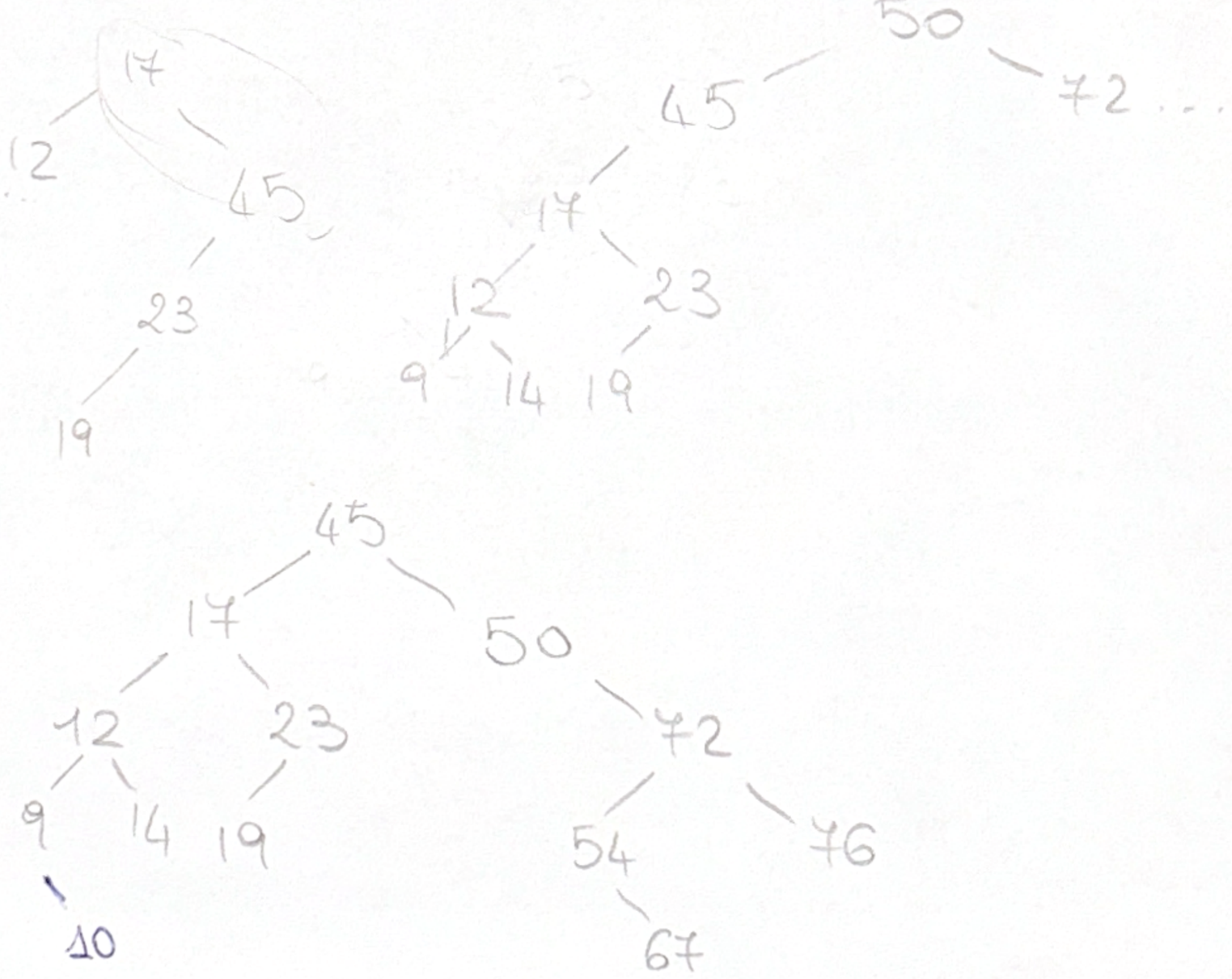
Max dim. Primary cluster = 6

(3)

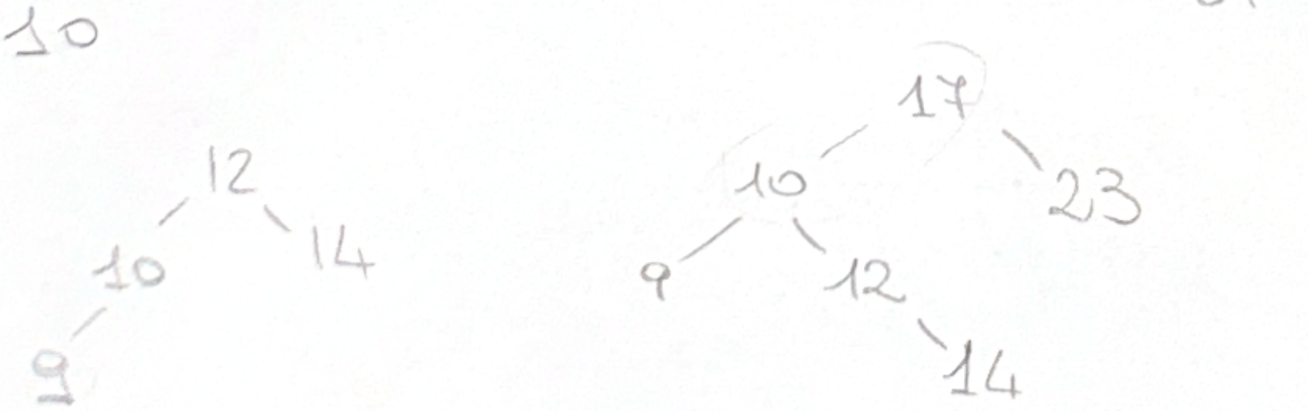


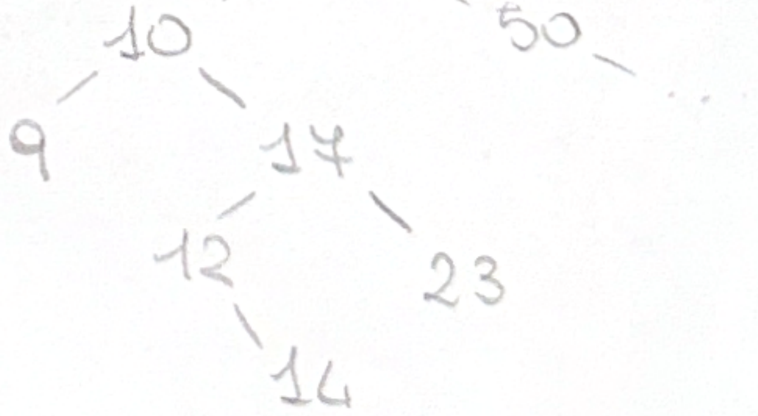
Dopo l'inserzione 45 e 10 in radice
cancellazione 50

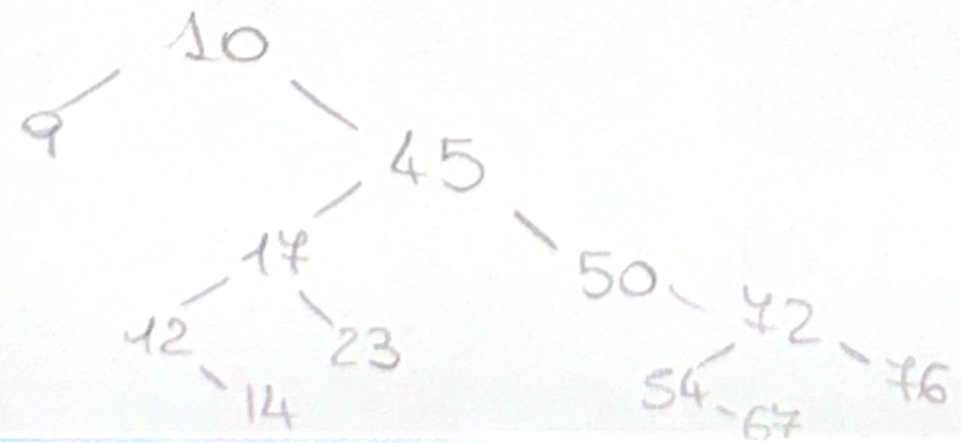




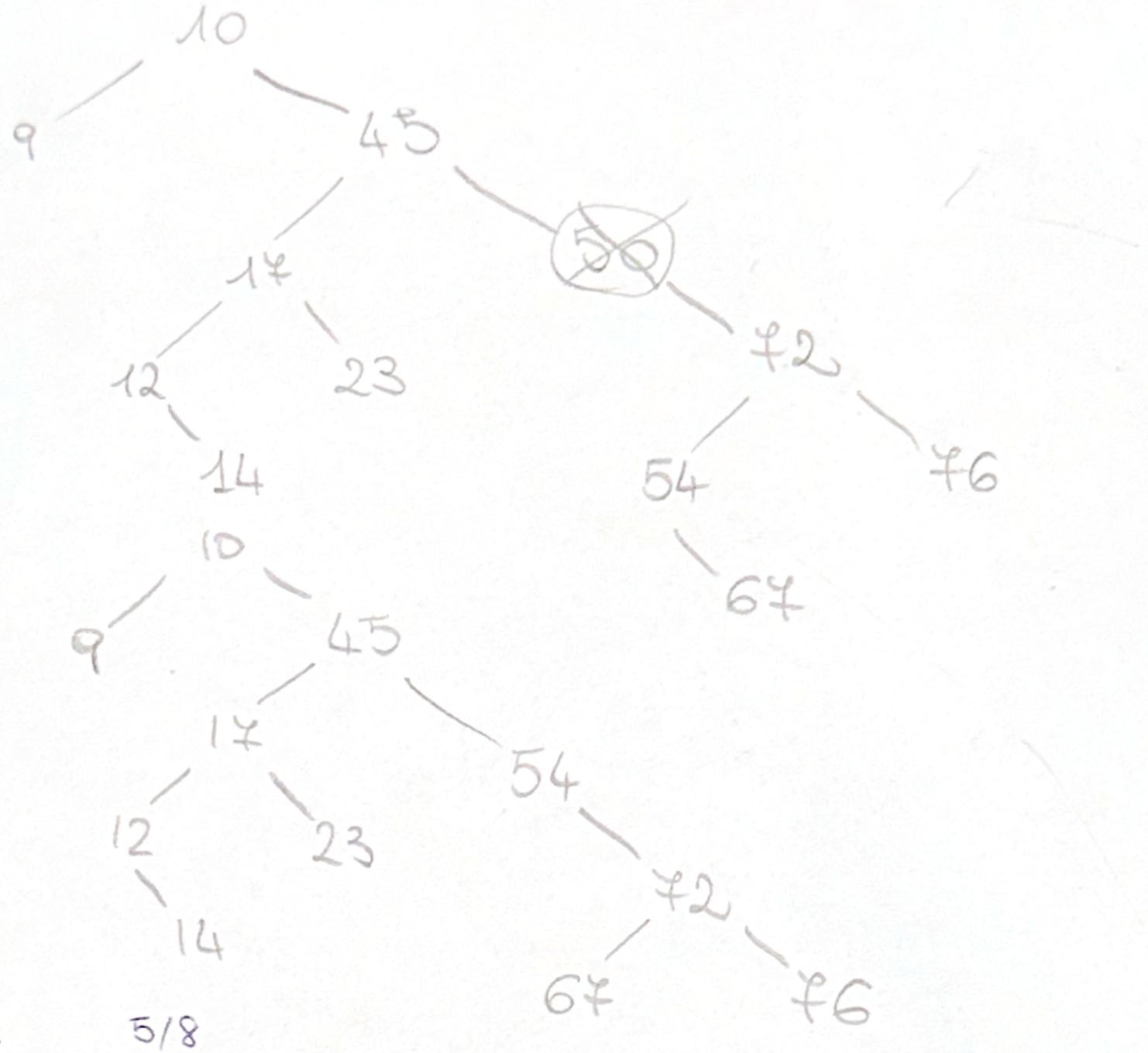
+10



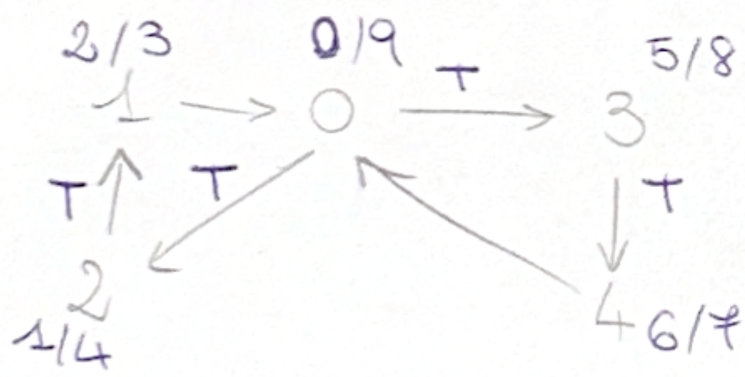




-50



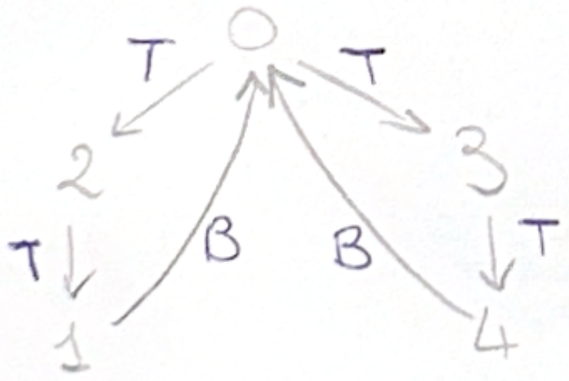
④



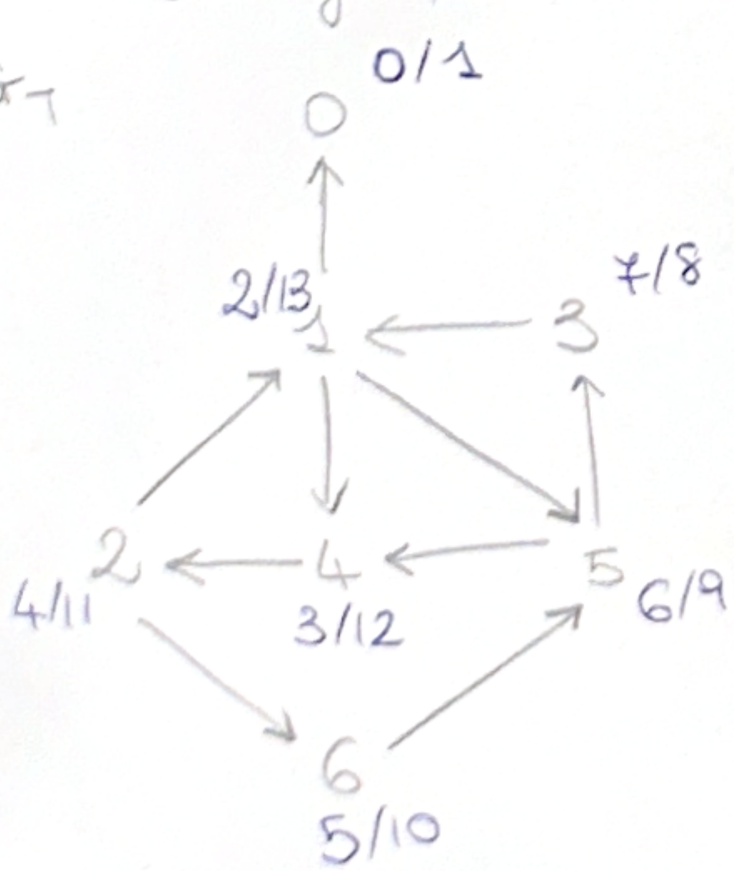
Tempi di inizio e fine visita in profondità

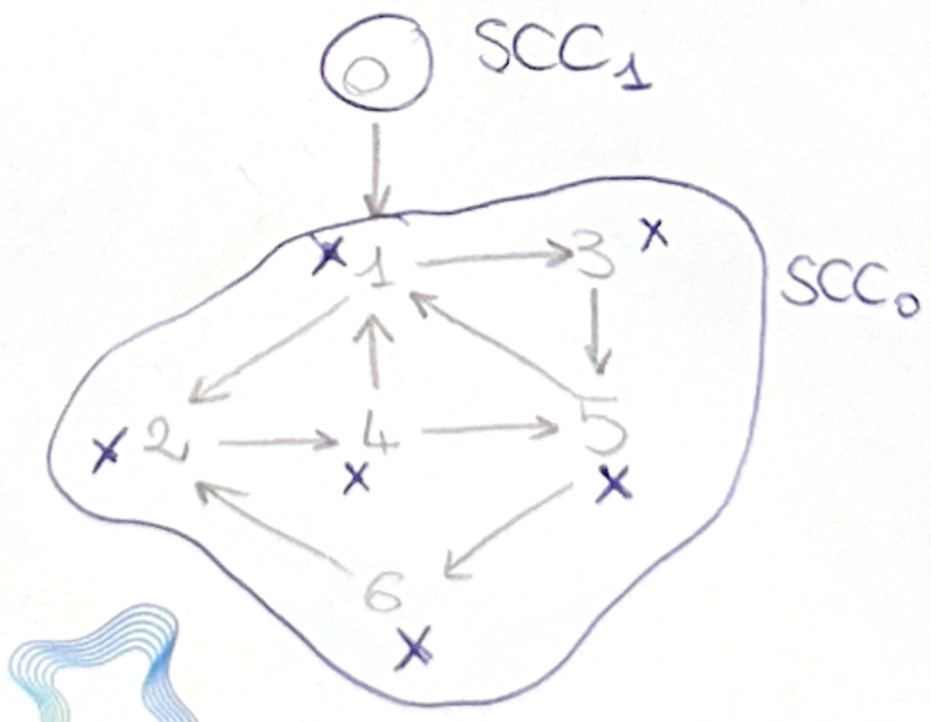
Etichettatura archi (T, B, F, C)

A partire da 0



⑤ Kosaraju, 0 vertice di partenza





TXT

IV appello 2023

① Unfolding

$T(n) = T(n-3) + T(n-2) + T(n-1) + 1$

$T(0) = 0$

$T(1) = 1$

$T(2) = 2$

$T(n) = 2 \cdot T(n-1) + 1$

$T(n-1) = 2 \cdot T(n-2) + 1$

$T(n-2) = 2 \cdot T(n-3) + 1$

$T(n) = 1 + 2 \cdot (2 \cdot (2 \cdot T(n-3) + 1) + 1) + 1 =$

$= 4 + 2 + 1 + 8\,T(n-3)$

Terminazione per:

$n - i = 1$

$i = n - 1$

$\sum_{i=0}^{n-1} 2^i \rightsquigarrow \frac{2^{(n-1)+1} - 1}{2 - 1} = 2^n - 1$

$\Downarrow$

$\Theta(2^n)$

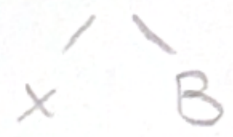
② Huffman

0 a sx 1 a dx

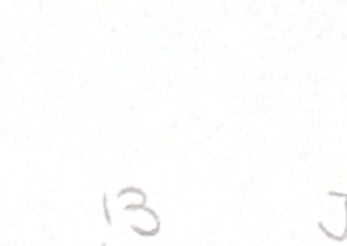
B:4 C:10 D:12 F:11 J:14 L:15 R:8 X:3 Z:6

X:3 B:4 Z:6 R:8 C:10 F:11 D:12 J:14 L:15

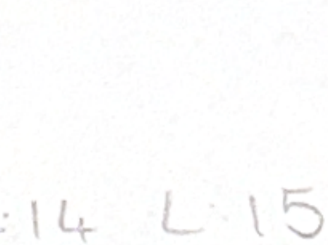
Z:6 Y R:8 C:10 F:11 D:12 J:14 L:15



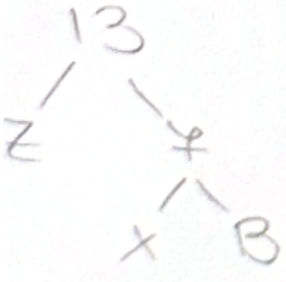
R:8 C:10 F:11 D:12 13 J:14 L:15

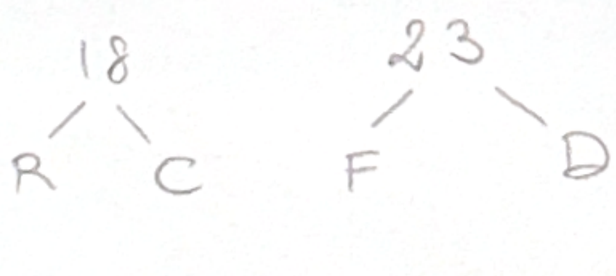


F:11 D:12 13 J:14 L:15 18

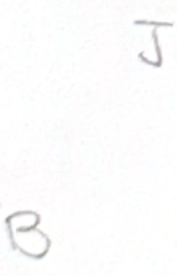


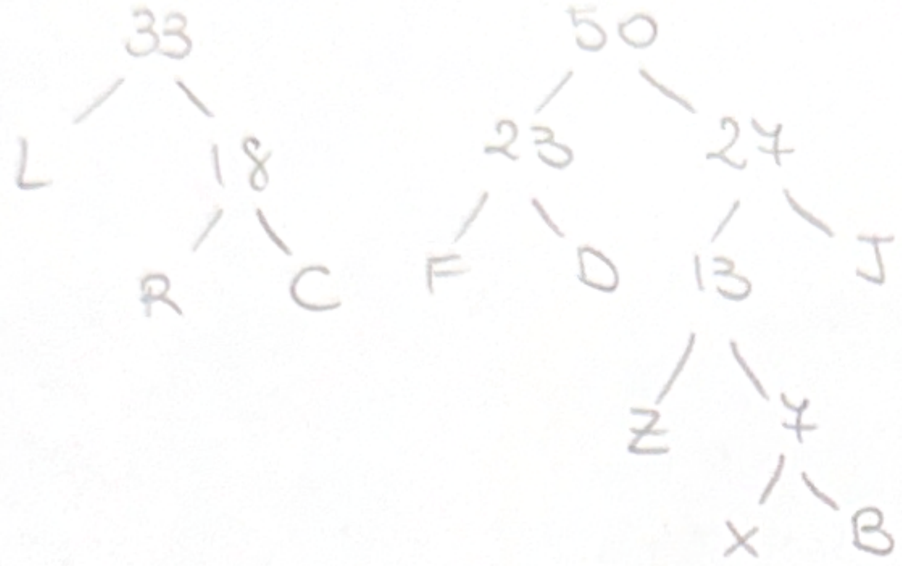
13 J:14 L:15 18 23

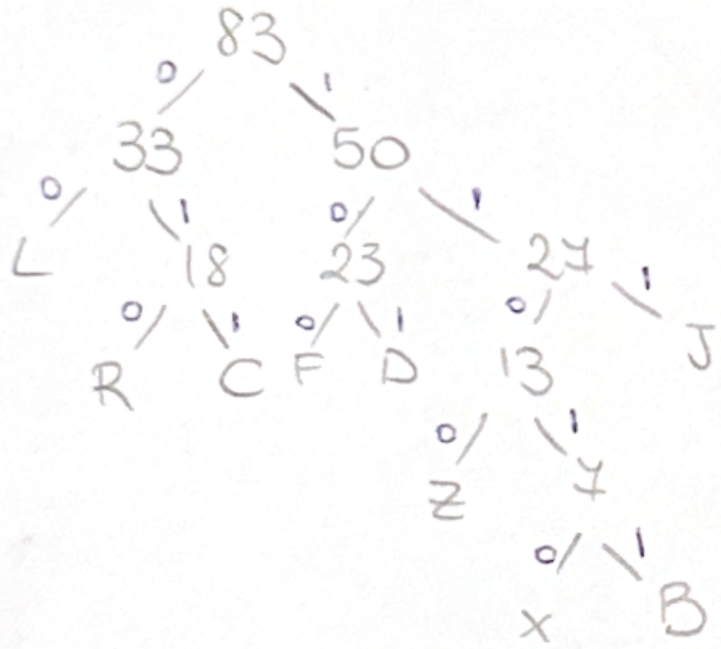




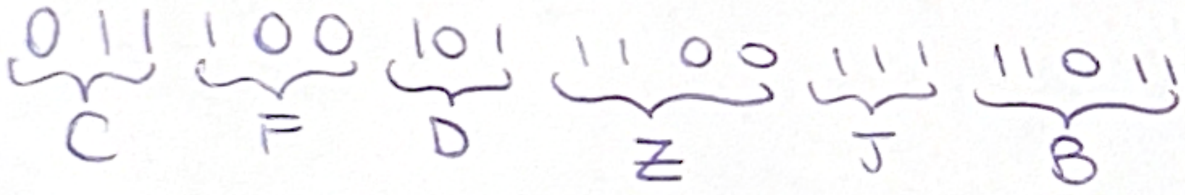
L:15 18 23 27 33



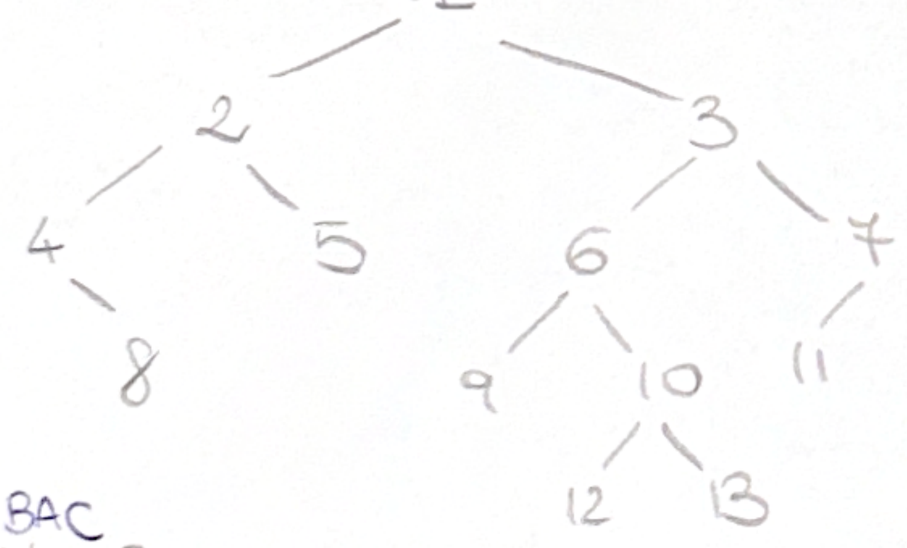




X: 11010



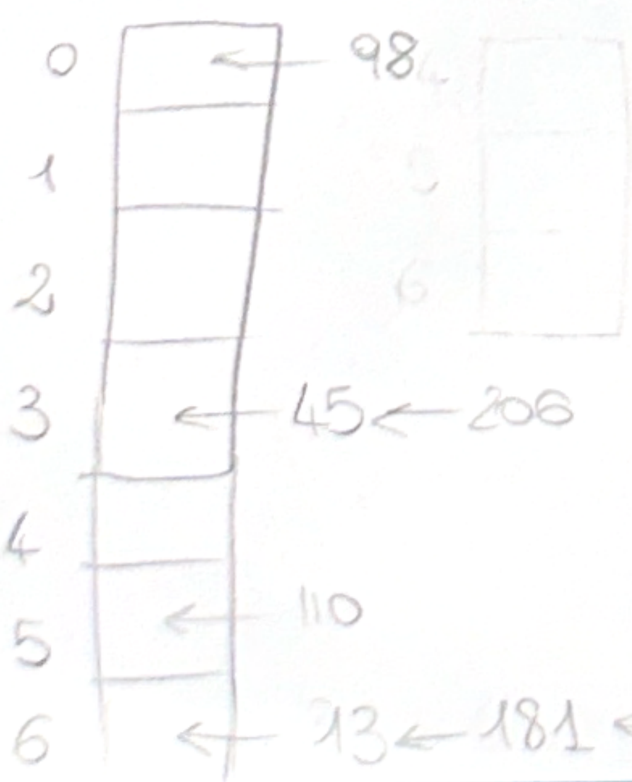
(3)



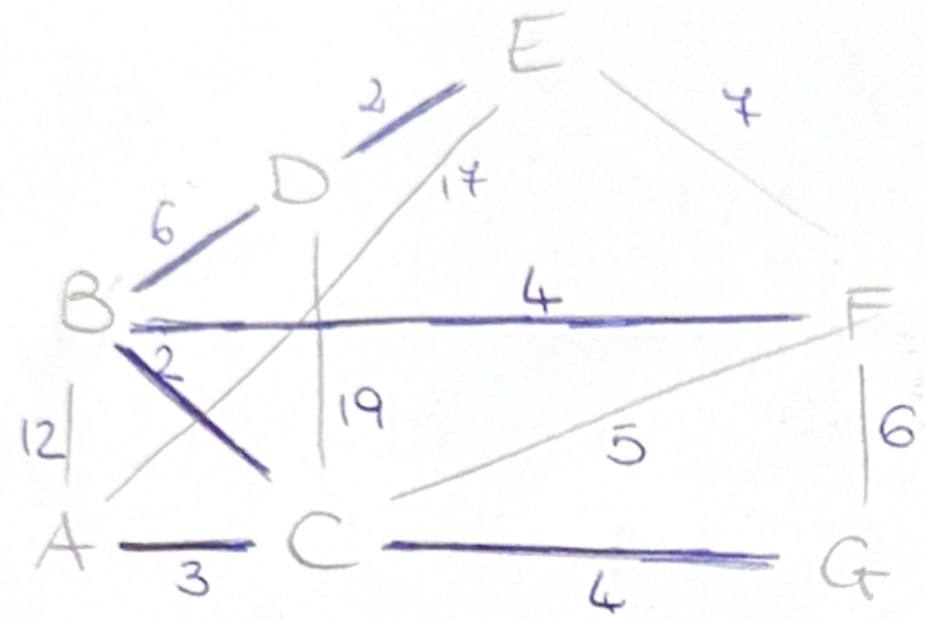
BAC
IN-ORDER: 4 8 2 5 1 9 6 12 10 13 3 11 7
ABC
PRE-ORDER: 1 2 4 8 5 3 6 9 10 12 13 7 11
POST-ORDER: 8 4 5 2 6 12 13 10 6 11 7 3 1
BCA

(4) Linear chaining

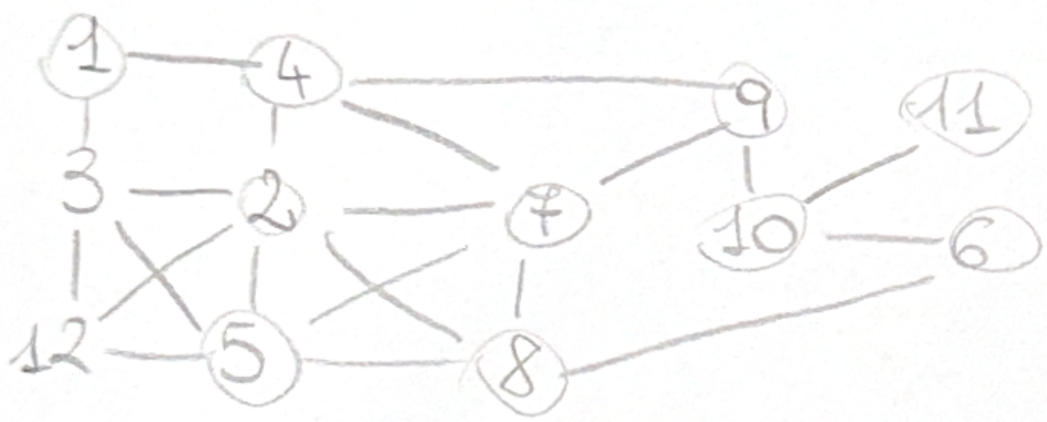
13 181 98 110 45 206 167 202 M=7 →



⑤ Kruskal



⑥ Visita in ampiezza a partire da 11



11	0
10	1
6 9	2
4 7 8	3
1 2 5	4
3 12	5