FINANCIAL MATHEMATICS

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COURSEWORK 2023/2024

Introduction

Investing in financial markets is a dynamic and complex endeavour that requires careful consideration of various factors such as expected returns, risks, and market dynamics. In this in-depth analysis, the factors needed to construct an efficient portfolio was explored by examining the daily historical prices, expected returns, volatility, correlations, and risk measures of five (5) selected stocks: Apple Inc., Amazon.com Inc., Alphabet Inc., Meta Platforms Inc., and Microsoft Corporation.

Apple Inc. designs, manufactures, and markets smartphones, personal computers, tablets, wearables, and accessories worldwide. It offers iPhone, Mac, iPad, AirPods, Apple TV, Apple Watch, Beats products, and HomePod. It also provides AppleCare support and cloud services. It distributes third-party applications for its products through the App Store. Apple Inc. with a full-time employee of 161,000 was founded in 1976 and is headquartered in Cupertino, California. Apple total asset for the quarter ending September 30, 2023, were $352.583B [1].

Amazon.com, Inc. engages in the retail sale of consumer products and subscriptions through online and physical stores in North America and internationally. It operates through three (3) segments: North America, International, and Amazon Web Services (AWS). Its products offered through its stores include merchandise and content purchased for resale, develops, and produces media content. Amazom.com, Inc. with a full-time employee of 1,500,000 was incorporated in 1994 and is headquartered in Seattle, Washington. Amazon total assets for the quarter ending September 30, 2023, were $486.883B [2].

Alphabet Inc. offers various products and platforms in the United States, Europe, the Middle East, Africa, the Asia-Pacific, Canada, and Latin America. It operates through Google Services, Google Cloud, and Other Bets segments. It is classified under the Communication Services sector within Internet Content & Information industry. Alphabet Inc. with a full-time employee of 182,381 was founded in 1998 and is headquartered in Mountain View, California. Alphabet total asset as of 2022, were $365.3B [3].

Meta Platforms, Inc. engages in the development of products that enable people to connect and share with friends and family through mobile devices, personal computers, virtual reality headsets, and wearables worldwide. It operates in two (2) segments, Family of Apps and Reality Labs. It was formerly known as Facebook, Inc. and changed its name to Meta Platforms, Inc. in October 2021.

Meta Platforms, Inc. with a full-time employee of 66,185 was incorporated in 2004 and is headquartered in Menlo Park, California. Meta total assets as of 2023, were $545B [4].

Microsoft Corporation develops and supports software, services, devices, and solutions worldwide. The Productivity and Business Processes segment offers office, exchange, SharePoint, Microsoft Teams, office 365 Security and Compliance, Microsoft viva, and Microsoft 365 copilot. It sells its products through OEMs, distributors, and resellers, and directly through digital marketplaces, online, and retail stores. Microsoft Corporation with a full-time employee of 221,000 was founded in 1975 and is headquartered in Redmond, Washington. Microsoft total assets for the quarter ending September 30, 2023, were $445.785B [5].

Expected Return and Volatility

The stock asset prices for Apple Inc., Amazon.com Inc., Alphabet Inc., Meta Platforms Inc., and Microsoft Corporation were acquired from Yahoo! Finance, UK. The dataset covers the period from 27th December 2022 to 27th December 2023, comprising 1518 data points with 252 observations and 6 attributes. In Figure 1.1, a segment is presented, displaying the adjusted daily prices of the five assets along with their corresponding daily returns. This dataset serves as a crucial foundation for the analysis, capturing the market dynamics and fluctuations in stock prices over the specified timeframe.

H3 =(B3-B4)/B4

	A	B	C	D	E	F	G	H	I	J	K	L
1		DAILY PRICES						DAILY RETURNS				
2	Date	AAPL	AMZN	GOOGL	META	MSFT		AAPL_Ret	AMZN_Ret	GOOGL_Ret	META_Ret	MSFT_Ret
3	27/12/2023	193.15	153.34	140.37	357.83	374.07		0.000517954	-0.000456346	-0.008126123	0.008454753	-0.001574753
4	26/12/2023	193.05	153.41	141.52	354.83	374.66		-0.002840924	-6.51414E-05	0.000212022	0.004074739	0.000213618
5	22/12/2023	193.60	153.42	141.49	353.39	374.58		-0.005547499	-0.002730096	0.007620047	-0.001976845	0.002784114
6	21/12/2023	194.68	153.84	140.42	354.09	373.54		-0.000769948	0.01130687	0.015035435	0.013771178	0.007878728
7	20/12/2023	194.83	152.12	138.34	349.28	370.62		-0.010713923	-0.010858951	0.012367377	-0.003082504	-0.007072858
8	19/12/2023	196.94	153.79	136.65	350.36	373.26		0.005360166	-0.001817447	0.006259138	0.016655998	0.001636968
9	18/12/2023	195.89	154.07	135.8	344.62	372.65		-0.008503355	0.027338841	0.024132706	0.028962085	0.005178925
10	15/12/2023	197.57	149.97	132.6	334.92	370.73		-0.002725728	0.017297538	0.005002304	0.005252574	0.013117312
11	14/12/2023	198.11	147.42	131.94	333.17	365.93		0.000757698	-0.009540433	-0.004752244	-0.004690139	-0.022544547
12	13/12/2023	197.96	148.84	132.57	334.74	374.37		0.016691489	0.00922159	0.000377324	0.001555828	-2.67375E-05
13	12/12/2023	194.71	147.48	132.52	334.22	374.38		0.007920147	0.010898602	-0.005776795	0.02748402	0.008295225
14	11/12/2023	193.18	145.89	133.29	325.28	371.30		-0.012927361	-0.010378504	-0.012593614	-0.022449289	-0.007829471
15	08/12/2023	195.71	147.42	134.99	332.75	374.23		0.00741238	0.003676423	-0.014167736	0.018861582	0.008842159
16	07/12/2023	194.27	146.88	136.93	326.59	370.95		0.010139335	0.016329926	0.053145584	0.028791884	0.005829783
17	06/12/2023	192.32	144.52	130.02	317.45	368.80		-0.005687059	-0.016067544	-0.007405153	-0.002639093	-0.009986044
18	05/12/2023	193.42	146.88	130.99	318.29	372.52		0.021063217	0.01408457	0.013305492	-0.00540585	0.009156347
19	04/12/2023	189.43	144.84	129.27	320.02	369.14		-0.00946461	-0.01489494	-0.019642022	-0.01477747	-0.014338722

Figure 1.1: showing daily prices and returns of 5 asset stocks.

The daily returns as depicted in figure 1.1 were computed utilizing the formula: the current price minus the previous price divided by the previous price (i.e., $(P_t - P_{t-1})/ P_{t-1}$).

In Figure 1.2 presented below, the annualized expected return and volatility of each company, along with the correlations between the asset returns, are illustrated. These metrics were derived using the Correlation method, providing insights into the performance and interrelationships of the selected assets.

=AVERAGE(H3:H253)*252

	N	O	P	Q	R	S	T	U	V	W
	Mean & SD of Returns (Annualised)					**Correlation**				
		Mean	**SD**			*AAPL_Ret*	*AMZN_Ret*	*GOOGL_Ret*	*META_Ret*	*MSFT_Ret*
	AAPL_Ret	0.424482	0.206799		AAPL_Ret	1				
	AMZN_Ret	0.670832	0.330815		AMZN_Ret	0.428296763	1			
	GOOGL_Ret	0.522532	0.304494		GOOGL_Ret	0.519401362	0.604988412	1		
	META_Ret	1.200942	0.399529		META_Ret	0.516544944	0.584702253	0.611865776	1	
	MSFT_Ret	0.499101	0.251929		MSFT_Ret	0.546040284	0.579185051	0.515337055	0.535217712	1

Figure 1.2: showing the annualised mean, standard deviation, and correlations between the asset returns.

As depicted in Figure 1.2, the mean was computed using the average formula in Excel and annualized by multiplying it by 252, representing the standard number of days in a financial year. The standard deviation was determined using the Excel formula 'STDEV' and then multiplied by the square root of 252 to annualize the result. The correlations between the asset returns were calculated using the correlation formula in the Excel Data Analysis package.

The expected (mean) return signifies the average annual return of each asset over a one-year period, while the standard deviation (volatility) of returns measures the dispersion or variability of the asset's annual returns. A higher mean return indicates a greater average annual gain for the asset, while a higher standard deviation reflects increased variability in the annual returns, suggesting higher volatility and potentially higher risk.

In Figure 1.2, Meta Platforms Inc. stands out with the highest expected return of 1.2009 or 120.09%, coupled with the highest volatility of 0.3995 or 39.95%. This signifies the potential for higher returns but also comes with elevated risk. Conversely, Apple Inc. exhibits the lowest expected return of 0.4245 or 42.45% and the lowest volatility of 0.2068 or 20.68%, indicating lower potential returns but also lower risk.

Efficient Frontier Curve

The initial approach involved employing an equally weighted method to assign weights to the five assets. Given that the sum of weights must equal one (1), each asset was assigned a weight of 0.2. The expected return of the equally weighted portfolio was computed as the weighted average of individual stock returns using matrix multiplication (mmult) and transpose formulas in Excel, resulting in an expected return of 0.6636 or 66.36%. The standard deviation of the equally weighted portfolio, representing the overall risk or volatility, was calculated using mmult and transpose formulas in Excel, yielding 0.2398 or 23.98%.

Subsequently, the Sharpe ratio was calculated by dividing the excess expected return over the risk-free rate (0.015 or 1.5%) by the standard deviation. The obtained Sharpe ratio of 2.705 signifies a high risk-adjusted return for the equally weighted portfolio. In simpler terms, for each unit of risk taken (measured by standard deviation), the portfolio is expected to generate a return 2.7 times greater than the risk-free rate.

Figure 2.1 below illustrates both the equally weighted portfolio and the minimized risky (weighted) portfolio, showcasing the different weightings and risk-return profiles of each.

=SUM(O14:O18)

N	O	P	Q	R	S
Minimized risky (weighted) portfolio				**Equally weighted portfolio**	
Stocks	**Weights**			**Stocks**	**Weights**
AAPL_Ret	0.670676			AAPL_Ret	0.2
AMZN_Ret	0.034699			AMZN_Ret	0.2
GOOGL_Ret	0.048711			GOOGL_Ret	0.2
META_Ret	0			META_Ret	0.2
MSFT_Ret	0.245914			MSFT_Ret	0.2
Sum	**1**			**Sum**	**1**
Expected return	0.456156			**Expected return**	0.663578032
Std Dev	0.195282			**Std Dev**	0.23976591
Sharpe ratio	2.259073			**Sharpe ratio**	2.705046904

Figure 2.1: showing the minimized volatility and equally weighted portfolio.

After achieving a notable risk-adjusted return of 2.705 (Sharpe ratio) for the equally weighted portfolio, the Excel Solver package was employed to minimize the standard deviation (volatility) by adjusting the results of the initial equally weighted portfolio. This optimization process led to reductions in standard deviation, Sharpe ratio, and expected return. Distinct weights were assigned to each stock, with Apple receiving the highest weight of 67.07%, while Meta had a weight of 0%. It is important to note that the minimized standard deviation (volatility) method is primarily suitable for investors with a lower risk tolerance, as it aims to construct a portfolio that minimizes risk exposure.

The efficient frontier (or portfolio frontier) is an investment portfolio which occupies the efficient parts of the risk-return spectrum [6]. Various target returns were selected, each associated with different weights, while adhering to the constraint that the sum of all weights must equal 1. Figure 2.2 below illustrates the diverse weights, corresponding volatilities, and expected returns utilized in constructing the efficient frontier curve.

AAPL_w1	AMZN_w2	GOOGL_w3	META_w4	MSFT_w5	Volatility	Exp_Return
-0.6	-0.5	-0.4	-0.3	2.8	0.754682649	0.238081114
-0.5	-0.4	-0.3	-0.2	2.4	0.639145701	0.320319714
-0.4	-0.3	-0.2	-0.1	2	0.525232552	0.402558313
-0.3	-0.2	-0.1	0	1.6	0.41428483	0.484796912
-0.2	-0.1	0	0.1	1.2	0.309508139	0.567035511
-0.1	0	0.1	0.2	0.8	0.219907562	0.64927411
0	0.1	0.2	0.3	0.4	0.171242704	0.731512709
0.1	0.2	0.3	0.4	0	0.196696396	0.813751309
0.2	0.3	0.4	0.5	-0.4	0.276505987	0.895989908
0.3	0.4	0.5	0.6	-0.8	0.377653717	0.978228507
0.4	0.5	0.6	0.7	-1.2	0.487021334	1.060467106
0.5	0.6	0.7	0.8	-1.6	0.600131577	1.142705705
0.6	0.7	0.8	0.9	-2	0.715210961	1.224944305
0.7	0.8	0.9	1	-2.4	0.831442247	1.307182904
0.8	0.9	1	1.1	-2.8	0.948402014	1.389421503
0.9	1	1.1	1.2	-3.2	1.065850473	1.471660102
1	1.1	1.2	1.3	-3.6	1.18364216	1.553898701
1.1	1.2	1.3	1.4	-4	1.3016839	1.636137301

Figure 2.2: showing the different weights, volatility, and expected returns of the 5 assets.

An Efficient Frontier Curve (EFC) was subsequently plotted using the line chart feature in Excel, as depicted in Figure 2.3 below:

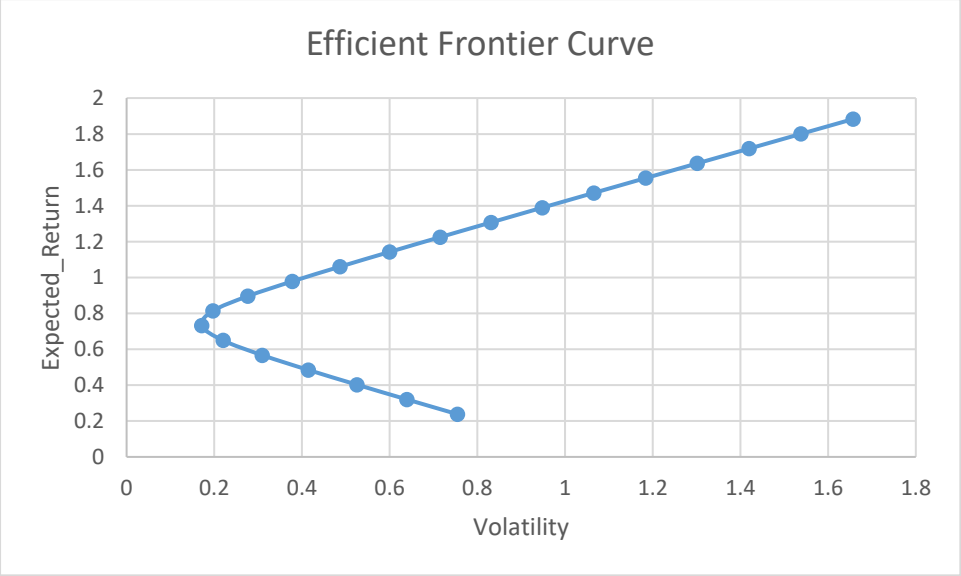

Figure 2.3: showing the efficient frontier curve of the assets return.

Sharpe Ratio

A Sharpe ratio is a mathematical expression of the insight that excess returns over a period may signify more volatility and risk, rather than investing skill. It compares the return of an investment with its risk [7]. Utilizing a risk-free rate of 1.5% or 0.015, the Sharpe ratio was computed by deducting the risk-free rate from the expected return and dividing the result by the standard deviation, as illustrated in Figure 3.1 below:

=(Z21-Q18)/Y21

T	U	V	W	X	Y	Z	AA	AB
AAPL_w1	AMZN_w2	GOOGL_w3	META_w4	MSFT_w5	Volatility	Exp_Return	Sharpe Ratio	
-0.6	-0.5	-0.4	-0.3	2.8	0.754682649	0.238081114	0.295595923	
-0.5	-0.4	-0.3	-0.2	2.4	0.639145701	0.320319714	0.477699706	
-0.4	-0.3	-0.2	-0.1	2	0.525232552	0.402558313	0.737879462	
-0.3	-0.2	-0.1	0	1.6	0.41428483	0.484796912	1.133994966	
-0.2	-0.1	0	0.1	1.2	0.309508139	0.567035511	1.783589644	
-0.1	0	0.1	0.2	0.8	0.219907562	0.64927411	2.884276034	
0	0.1	0.2	0.3	0.4	0.171242704	0.731512709	4.184194076	Opt_Portf.
0.1	0.2	0.3	0.4	0	0.196696396	0.813751309	4.06083347	
0.2	0.3	0.4	0.5	-0.4	0.276505987	0.895989908	3.186151289	
0.3	0.4	0.5	0.6	-0.8	0.377653717	0.978228507	2.550560112	
0.4	0.5	0.6	0.7	-1.2	0.487021334	1.060467106	2.146655667	
0.5	0.6	0.7	0.8	-1.6	0.600131577	1.142705705	1.879097434	
0.6	0.7	0.8	0.9	-2	0.715210961	1.224944305	1.69173065	
0.7	0.8	0.9	1	-2.4	0.831442247	1.307182904	1.554146315	
0.8	0.9	1	1.1	-2.8	0.948402014	1.389421503	1.449197157	
0.9	1	1.1	1.2	-3.2	1.065850473	1.471660102	1.366664592	
1	1.1	1.2	1.3	-3.6	1.18364216	1.553898701	1.300138465	
1.1	1.2	1.3	1.4	-4	1.3016839	1.636137301	1.245415497	

Figure 3.1: showing the weights, expected return, volatility, and sharpe ratio of the assets return.

The highlighted optimal portfolio in green, as presented in Figure 3.1 above, exhibits the highest Sharpe ratio of 4.1842 and the lowest standard deviation (volatility) of 0.1712.

Capital Market Line

The capital market line (CML) is a theoretical concept that represents all the portfolios that optimally combine the risk-free rate of return and the market portfolio of risky assets. It represents portfolios that optimally combine risk and return [8]. The Capital Market Line (CML) equation was derived by utilizing the optimal expected return, volatility, and Sharpe ratio highlighted in green in Figure 3.1. Random weights of 0, 1, 2 were assigned, and the CML was subsequently plotted on the Efficient Frontier Curve (EFC), as depicted in Figure 3.2 below:

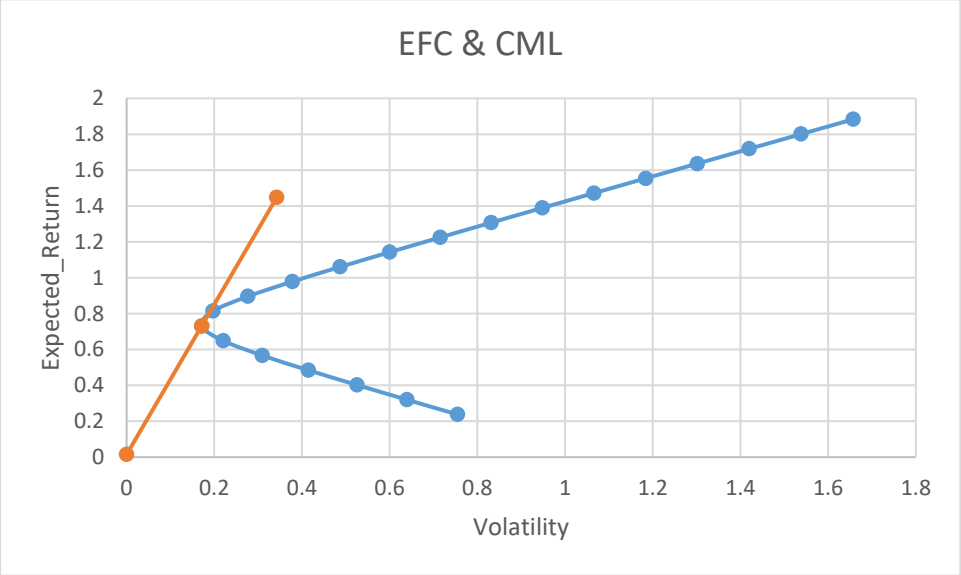

Figure 3.2: showing the EFC and CML.

The Capital Market Line (CML) holds significant importance in finance, particularly within the framework of the Capital Asset Pricing Model (CAPM). CAPM establishes a linear relationship among the expected return of an asset or portfolio, the risk-free rate, and the asset's beta (systematic rate).

The CML serves as a visual representation of the risk-return tradeoff for a portfolio of risky assets in the capital market.

Incorporating the risk-free rate as a starting point, the CML illustrates how investors can blend a risk-free asset with a portfolio of risky assets to attain varying levels of risk and return. This concept aids investors in comprehending the risk and return characteristics of the market portfolio and provides insights into the optimization of portfolios through the combination of risky assets and the risk-free rate. As a segment of the Efficient Frontier Curve (EFC), the CML offers a framework that includes both risky assets and the risk-free asset.

Investors can leverage the CML to make informed decisions regarding portfolio allocation. By selecting a combination along the CML that involves the risk-free asset and the market portfolio, investors can optimize their portfolios based on individual risk preferences and overarching investment objectives.

Beta

Beta coefficient is a measure of the volatility, or systematic risk, of an asset or a portfolio in comparison to the market as a whole. A beta of 1 indicates that the asset's price will move with the market. A beta of less than 1 means that the asset will be less volatile than the market. A beta of greater than 1 indicates that the asset's price will be more volatile than the market [9]. The formula for beta in a simple linear regression involves calculating the covariance between the asset's returns and market returns (R_{asset} , R_{market}) and dividing it by the variance of the market returns. To incorporate the market return, represented by the S&P 500 market index, it was downloaded for the same duration and added to the dataset. The returns of the market index were also computed, as illustrated in Figure 4.1 below:

N3 =(G3-G4)/G4

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1		DAILY STOCK PRICES							DAILY RETURNS					
2	Date	AAPL	AMZN	GOOGL	META	MSFT	S&P500		AAPL_Ret	AMZN_Ret	GOOGL_R	META_Ret	MSFT_Ret	Market_Ret
3	27/12/2023	193.15	153.34	140.37	357.83	374.07	4781.58		0.000518	-0.0004563	-0.00813	0.008455	-0.00157	0.00143
4	26/12/2023	193.05	153.41	141.52	354.83	374.66	4774.75		-0.00284	-6.514E-05	0.000212	0.004075	0.000214	0.004232
5	22/12/2023	193.60	153.42	141.49	353.39	374.58	4754.63		-0.00555	-0.0027301	0.00762	-0.00198	0.002784	0.00166
6	21/12/2023	194.68	153.84	140.42	354.09	373.54	4746.75		-0.00077	0.0113069	0.015035	0.013771	0.007879	0.010301
7	20/12/2023	194.83	152.12	138.34	349.28	370.62	4698.35		-0.01071	-0.010859	0.012367	-0.00308	-0.00707	-0.01468
8	19/12/2023	196.94	153.79	136.65	350.36	373.26	4768.37		0.00536	-0.0018174	0.006259	0.016656	0.001637	0.005866
9	18/12/2023	195.89	154.07	135.8	344.62	372.65	4740.56		-0.0085	0.0273388	0.024133	0.028962	0.005179	0.004528
10	15/12/2023	197.57	149.97	132.6	334.92	370.73	4719.19		-0.00273	0.0172975	0.005002	0.005253	0.013117	-7.6E-05
11	14/12/2023	198.11	147.42	131.94	333.17	365.93	4719.55		0.000758	-0.0095404	-0.00475	-0.00469	-0.02254	0.002647
12	13/12/2023	197.96	148.84	132.57	334.74	374.37	4707.09		0.016691	0.0092216	0.000377	0.001556	-2.7E-05	0.013651
13	12/12/2023	194.71	147.48	132.52	334.22	374.38	4643.7		0.00792	0.0108986	-0.00578	0.027484	0.008295	0.004599
14	11/12/2023	193.18	145.89	133.29	325.28	371.30	4622.44		-0.01293	-0.0103785	-0.01259	-0.02245	-0.00783	0.003925
15	08/12/2023	195.71	147.42	134.99	332.75	374.23	4604.37		0.007412	0.0036764	-0.01417	0.018862	0.008842	0.004095
16	07/12/2023	194.27	146.88	136.93	326.59	370.95	4585.59		0.010139	0.0163299	0.053146	0.028792	0.00583	0.007968
17	06/12/2023	192.32	144.52	130.02	317.45	368.80	4549.34		-0.00569	-0.0160675	-0.00741	-0.00264	-0.00999	-0.00391
18	05/12/2023	193.42	146.88	130.99	318.29	372.52	4567.18		0.021063	0.0140846	0.013305	-0.00541	0.009156	-0.00057
19	04/12/2023	189.43	144.84	129.27	320.02	369.14	4569.78		-0.00946	-0.0148949	-0.01964	-0.01478	-0.01434	-0.00541

Figure 4.1: showing the daily stock prices and its returns.

Linear regression analysis was employed to compute the beta for each asset in the portfolio, as demonstrated in Figure 4.1.1 below:

=COVARIANCE.S(I3:I253, N3:N253)/VAR(N3:N253)

P	Q
Simple Linear Regression	
AAPL	1.133953889
AMZN	1.528503307
GOOGL	1.385510277
META	1.76371359
MSFT	1.18388423

Figure 4.1.1: showing the beta values of the assets using the simple linear regression formula.

As depicted in Figure 4.1.1 above, both Apple (AAPL) and Microsoft (MSFT) exhibit beta values of 1.13 and 1.18, respectively, which are closer to 1. This proximity suggests that their stock movements are anticipated to be relatively in line with the overall market. In contrast, Amazon (AMZN), Alphabet (GOOGL), and Meta (META) display higher beta values of 1.53, 1.39, and 1.76, respectively, indicating an expectation of greater volatility compared to the market as a whole.

Additionally, the regression function in the data analysis package in Excel was utilized to confirm the beta values. Samples of the output for Amazon (AMZN) and Microsoft (MSFT) are illustrated in Figure 4.1.2 and 4.1.3, respectively.

SUMMARY OUTPUT AMZN

Regression Statistics	
Multiple R	0.611502325
R Square	0.373935093
Adjusted R Squa	0.371420776
Standard Error	0.016522113
Observations	251

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.040598257	0.040598257	148.7223405	3.91982E-27
Residual	249	0.067972074	0.00027298		
Total	250	0.108570331			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.001256044	0.00104922	1.197121785	0.232397544	-0.00081043	0.003322521	-0.000810433	0.00332252
Market_Ret	1.528503307	0.125336706	12.19517693	3.91982E-27	1.281648043	1.77535857	1.281648043	1.77535857

Figure 4.1.2: showing the summary output of AMZN using regression function.

SUMMARY OUTPUT MSFT								
Regression Statistics								
Multiple R	0.621940934							
R Square	0.386810526							
Adjusted R Squa	0.384347918							
Standard Error	0.012452171							
Observations	251							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	0.024355278	0.024355278	157.0735067	2.90063E-28			
Residual	249	0.038609084	0.000155057					
Total	250	0.062964363						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.000891567	0.000790762	1.127477943	0.260625741	-0.00066587	0.002449003	-0.000665869	0.002449
Market_Ret	1.18388423	0.094462137	12.53289698	2.90063E-28	0.997837569	1.369930891	0.997837569	1.36993089

Figure 4.1.3: showing the summary output of MSFT using the regression function.

Value at Risk (VaR)

Value at Risk (VaR) is an estimate, with a given degree of confidence, of how much one can lose from one's portfolio over a given time horizon [10]. An estimate of the 5% Value at Risk (VaR) was provided, indicating a 95% confidence level. The asset values were obtained from Yahoo! Finance for each asset in dollars ($) and converted to pounds (£) using the Xe currency converter. Figure 4.2.1 below illustrates the VaR for Apple.

=Q161*T154

P	Q	R	S	T
Calculating VaR (AAPL)				£(B)
Expected Return	42%		**Asset Value**	278.298
Expected Volatility	21%			
Time(days)	1			
Confidence Level	0.95			
Stress Event (Z(1-alpha))	-1.644854			
		£(B)		
VaR_{AAPL}	8.43285331%	23.4684621		

Figure 4.2.1: showing the value at risk for Apple.

As depicted in Figure 4.2.1 above, the contribution of AAPL to the estimated 5% Value at Risk (VaR) is approximately £23.468 billion. This amount represents 8.43% of its asset value, which is £278.298 billion. This contribution signifies the potential loss in AAPL's value under normal market conditions with a 5% probability.

=Q172*T165

P	Q	R	S	T
Calculating VaR (AMZN)				£(B)
Expected Return	67%		**Asset Value**	383.85
Expected Volatility	33%			
Time(days)	1			
Confidence Level	0.95			
Stress Event (Z(1-alpha))	-1.644854			
		£(B)		
VaR_{AMZN}	12.66890783%	48.62960272		

Figure 4.2.2: showing the value at risk for Amazon.

As illustrated in Figure 4.2.2 above, the contribution of AMZN to the estimated 5% Value at Risk (VaR) is approximately £48.63 billion. This amount accounts for 12.67% of its asset value, which is £383.85 billion. It signifies the potential loss in AMZN's value under normal market conditions with a 5% probability.

=NORM.S.INV(1-Q179)

P	Q	R	S	T
Calculating VaR (GOOGL)				£(B)
Expected Return	52%		**Asset Value**	288.17
Expected Volatility	30%			
Time(days)	1			
Confidence Level	0.95			
Stress Event (Z(1-alpha))	-1.644854			
		£(B)		
VaR_{GOOGL}	2.16846298%	6.248859765		

Figure 4.2.3: showing the value at risk for Alphabet.

Illustrated in Figure 4.2.3 above, the contribution of GOOGL to the estimated 5% Value at Risk (VaR) is approximately £6.25 billion. This contribution represents 2.17% of its asset value, which amounts to £288.17 billion. It indicates the potential loss in GOOGL's value under normal market conditions with a 5% probability.

=NORM.S.INV(1-Q190)

P	Q	R	S	T
Calculating VaR (META)				£(B)
Expected Return	120%		**Asset Value**	429.96
Expected Volatility	40%			
Time(days)	1			
Confidence Level	0.95			
Stress Event (Z(1-alpha))	-1.644854			
		£(B)		
VaR_{META}	54.37751812%	233.8015769		

Figure 4.2.4: showing the value at risk for Meta.

As depicted in Figure 4.2.4 above, the contribution of META to the estimated 5% Value at Risk (VaR) is approximately £233.802 billion. This amount constitutes 54.38% of its asset value, which is £429.96 billion. It signifies the potential loss in META's value under normal market conditions with a 5% probability.

=Q198+Q202*Q199

P	Q	R	S	T
Calculating VaR (MSFT)				£(B)
Expected Return	50%		**Asset Value**	351.59
Expected Volatility	25%			
Time(days)	1			
Confidence Level	0.95			
Stress Event (Z(1-alpha))	-1.644854			
		£(B)		
VaR_{MSFT}	8.47149733%	29.78493747		

Figure 4.2.5: showing the value at risk for Microsoft.

As illustrated in Figure 4.2.5 above, the contribution of MSFT to the estimated 5% Value at Risk (VaR) is approximately £29.785 billion. This contribution accounts for 8.47% of its asset value, which is £351.59 billion. It signifies the potential loss in MSFT's value under normal market conditions with a 5% probability.

ARCH/GARCH

The ARCH (Autoregressive Conditional Heteroscedastic) model consists of the regression fit and an additional equation which models the conditional variance as a linear function of squared returns, increasing the estimation possibilities of the linear regression fit and the GARCH (Generalized Autoregressive Conditional Heteroscedastic) have become very in Finance [11].

Figure 5.1 below depicts the RStudio environment, encompassing the codes, outputs and plots related to Microsoft prices and its returns spanning from 27/12/2022 to 27/12/2023.

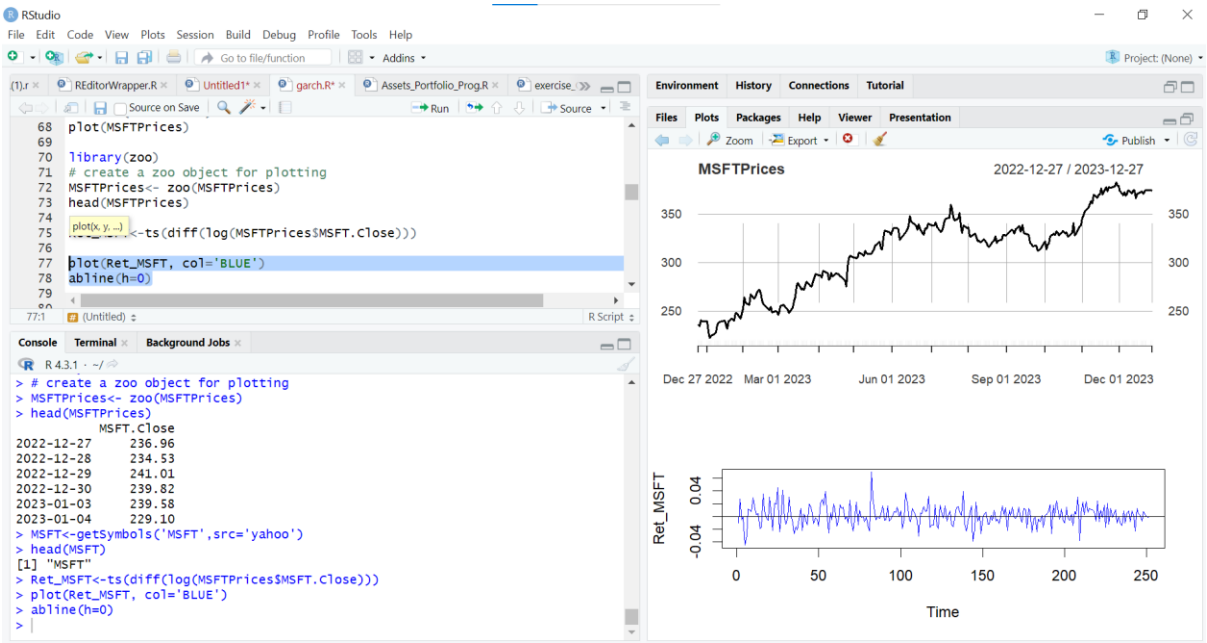

Figure 5.1: showing the rstudio environment codes, outputs, and plots of Microsoft prices and its returns.

Various models were employed to capture the time-varying volatility of Microsoft's stock returns. The model with the lowest AIC (Akaike Information Criterion) of -1357.802 was selected as the optimal model for Microsoft returns, as illustrated in Figure 5.2 below.

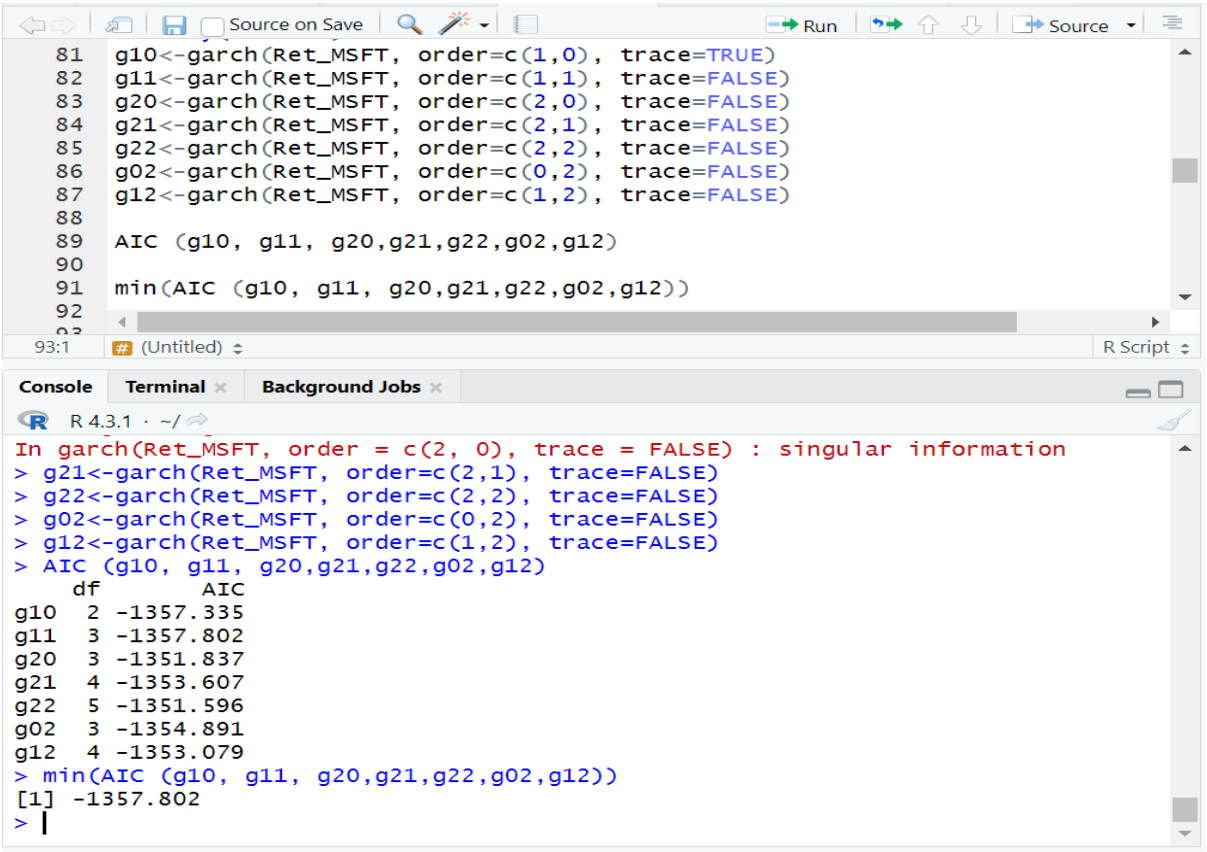

Figure 5.2: showing codes and output of different models for Microsoft.

The summary of the optimal model result, fitting a GARCH (1,1) model to the MSFT stock returns, is presented in figure 5.3 below:

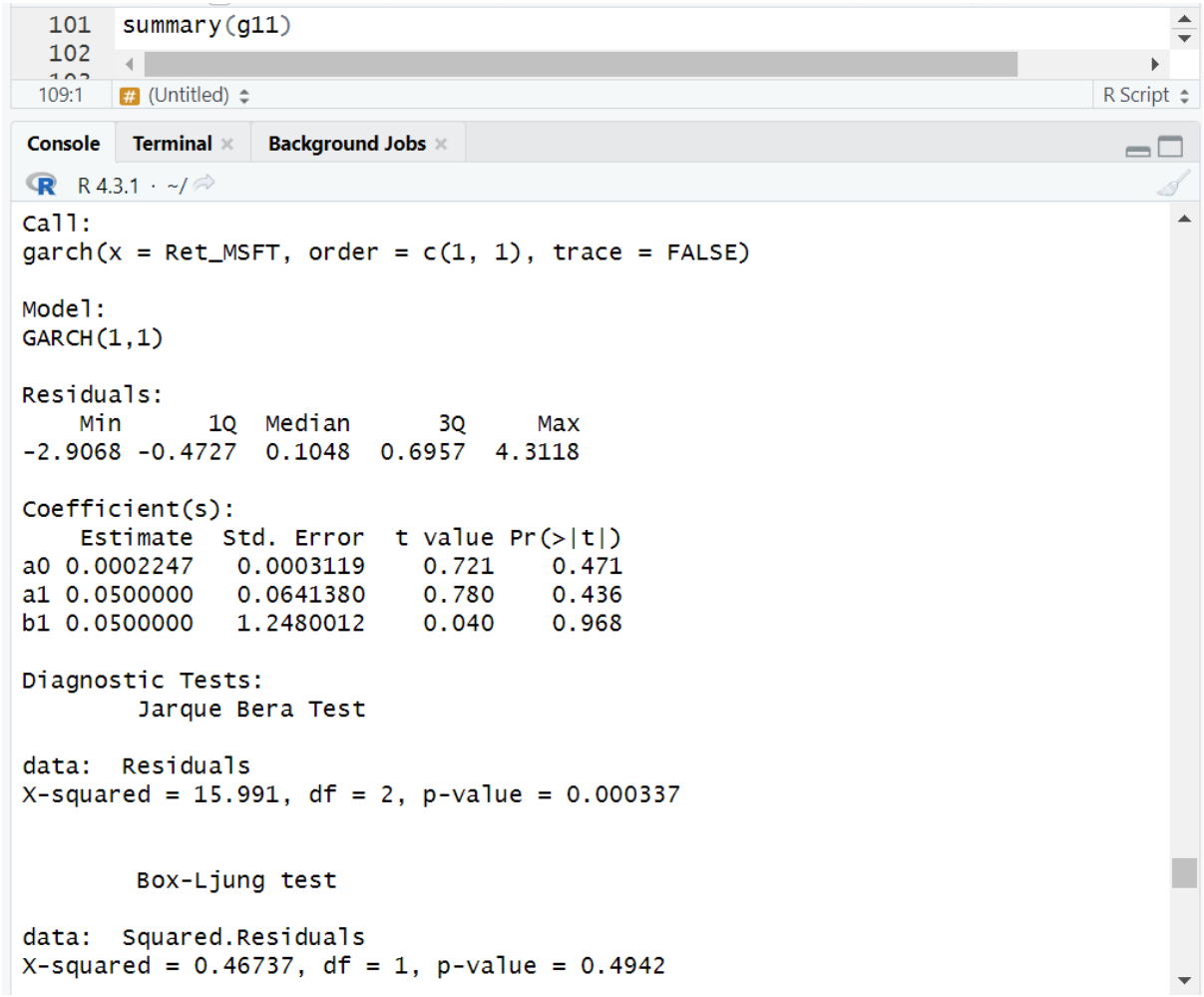

Figure 5.3: showing the summary of the best fitted model.

The model assumes that the conditional variance (volatility) at time t is a function of the past squared returns and past conditional variances. The AIC serves as a measure of the model's goodness of fit, striking a balance between the fit of the model and its complexity. The GARCH (1,1) model with an AIC of -1357.802 (g11), the smallest among other considered models, is deemed the most suitable based on the AIC criterion. The estimated coefficients (a0, a1, and b1) determine the impact of these terms on the volatility. Diagnostic tests provide insights into the model's adequacy. While the Jarque Bera Test suggests non-normality in the residuals, the Box-Ljung Test indicates no significant autocorrelation in the squared residuals.

Summary

Navigating the realm of investments involves finding a delicate balance between anticipated returns and associated risks. From the insights garnered, Meta Platforms Inc. (META) emerges with the highest expected return at 120.09%, promising significant gains but also accompanied by the highest volatility of 39.95%, indicative of elevated risk. On the contrary, Apple Inc. (AAPL) offers a more stable investment landscape with a lower expected return of 42.45% and the lowest volatility at 20.68%. An equally weighted portfolio, distributing 20% to each asset, yields an expected return of 66.36% and a volatility of 23.98%. This well-diversified portfolio boasts a Sharpe ratio of 2.705,

indicating a favourable risk-adjusted return – essentially, a promising return relative to the undertaken risk.

Optimizing to minimize volatility results in Apple claiming the highest weight at 67.07%, consequently reducing the overall portfolio risk to 19.53% with a slightly lower Sharpe ratio of 2.259. Investors with a lower risk tolerance may find this portfolio more aligned with their preferences. Notably, Apple and Microsoft exhibit beta values close to 1, suggesting their movements align closely with the market. On the flip side, Amazon, Alphabet, and Meta boast higher beta values, indicating a propensity for increased volatility compared to the overall market.

The Value at Risk (VaR) at a 5% probability reveals that Meta Platforms Inc. faces the highest potential loss, standing at 54.38% of its net worth, highlighting the elevated risk associated with this investment. In contrast, Apple Inc. presents a lower potential loss at 8.43% of its net worth, reflecting a comparatively lower risk.

Potential investors should weigh these factors carefully. A comprehensive evaluation should consider diversification, risk-adjusted returns, beta values, market sensitivity, volatility, and value at risk collectively to make well-informed decisions in the dynamic landscape of investments.

Conclusion

Selecting the most efficient portfolio hinges on an investor's unique blend of risk tolerance, investment objectives, and personal preferences. Savvy investors, aiming for well-informed decisions, carefully weigh the equilibrium between anticipated returns and associated risks. Diversifying assets and comprehending their market sensitivity becomes paramount in this decision-making process.

The optimized portfolio, tailored to minimize risk, emerges as an apt choice for risk-averse investors prioritizing capital preservation. In contrast, the equally weighted portfolio offers a more balanced approach, distributing risks across various assets. Ultimately, the optimal choice depends on aligning the investment strategy with individual risk appetite and financial goals.

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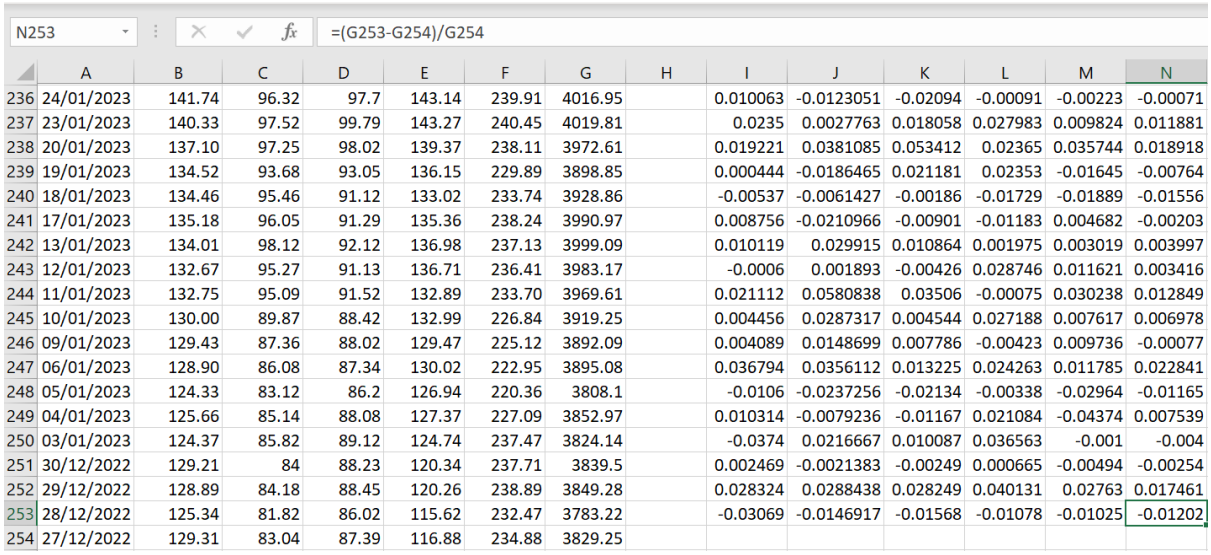
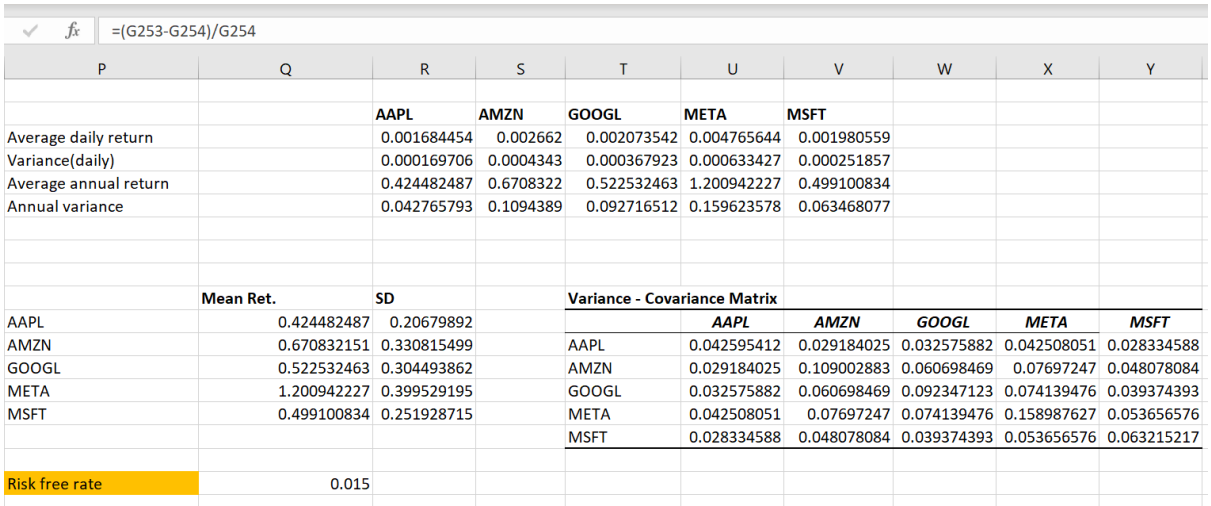
Appendix



N253 =(G253-G254)/G254

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
236	24/01/2023	141.74	96.32	97.7	143.14	239.91	4016.95		0.010063	-0.0123051	-0.02094	-0.00091	-0.00223	-0.00071
237	23/01/2023	140.33	97.52	99.79	143.27	240.45	4019.81		0.0235	0.0027763	0.018058	0.027983	0.009824	0.011881
238	20/01/2023	137.10	97.25	98.02	139.37	238.11	3972.61		0.019221	0.0381085	0.053412	0.02365	0.035744	0.018918
239	19/01/2023	134.52	93.68	93.05	136.15	229.89	3898.85		0.000444	-0.0186465	0.021181	0.02353	-0.01645	-0.00764
240	18/01/2023	134.46	95.46	91.12	133.02	233.74	3928.86		-0.00537	-0.0061427	-0.00186	-0.01729	-0.01889	-0.01556
241	17/01/2023	135.18	96.05	91.29	135.36	238.24	3990.97		0.008756	-0.0210966	-0.00901	-0.01183	0.004682	-0.00203
242	13/01/2023	134.01	98.12	92.12	136.98	237.13	3999.09		0.010119	0.029915	0.010864	0.001975	0.003019	0.003997
243	12/01/2023	132.67	95.27	91.13	136.71	236.41	3983.17		-0.0006	0.001893	-0.00426	0.028746	0.011621	0.003416
244	11/01/2023	132.75	95.09	91.52	132.89	233.70	3969.61		0.021112	0.0580838	0.03506	-0.00075	0.030238	0.012849
245	10/01/2023	130.00	89.87	88.42	132.99	226.84	3919.25		0.004456	0.0287317	0.004544	0.027188	0.007617	0.006978
246	09/01/2023	129.43	87.36	88.02	129.47	225.12	3892.09		0.004089	0.0148699	0.007786	-0.00423	0.009736	-0.00077
247	06/01/2023	128.90	86.08	87.34	130.02	222.95	3895.08		0.036794	0.0356112	0.013225	0.024263	0.011785	0.022841
248	05/01/2023	124.33	83.12	86.2	126.94	220.36	3808.1		-0.0106	-0.0237256	-0.02134	-0.00338	-0.02964	-0.01165
249	04/01/2023	125.66	85.14	88.08	127.37	227.09	3852.97		0.010314	-0.0079236	-0.01167	0.021084	-0.04374	0.007539
250	03/01/2023	124.37	85.82	89.12	124.74	237.47	3824.14		-0.0374	0.0216667	0.010087	0.036563	-0.001	-0.004
251	30/12/2022	129.21	84	88.23	120.34	237.71	3839.5		0.002469	-0.0021383	-0.00249	0.000665	-0.00494	-0.00254
252	29/12/2022	128.89	84.18	88.45	120.26	238.89	3849.28		0.028324	0.0288438	0.028249	0.040131	0.02763	0.017461
253	28/12/2022	125.34	81.82	86.02	115.62	232.47	3783.22		-0.03069	-0.0146917	-0.01568	-0.01078	-0.01025	-0.01202
254	27/12/2022	129.31	83.04	87.39	116.88	234.88	3829.25							



=(G253-G254)/G254

	P	Q	R	S	T	U	V	W	X	Y
			AAPL	**AMZN**	**GOOGL**	**META**	**MSFT**			
	Average daily return		0.001684454	0.002662	0.002073542	0.004765644	0.001980559			
	Variance(daily)		0.000169706	0.0004343	0.000367923	0.000633427	0.000251857			
	Average annual return		0.424482487	0.6708322	0.522532463	1.200942227	0.499100834			
	Annual variance		0.042765793	0.1094389	0.092716512	0.159623578	0.063468077			
		Mean Ret.	**SD**		**Variance - Covariance Matrix**					
	AAPL	0.424482487	0.20679892			***AAPL***	***AMZN***	***GOOGL***	***META***	***MSFT***
	AMZN	0.670832151	0.330815499		AAPL	0.042595412	0.029184025	0.032575882	0.042508051	0.028334588
	GOOGL	0.522532463	0.304493862		AMZN	0.029184025	0.109002883	0.060698469	0.07697247	0.048078084
	META	1.200942227	0.399529195		GOOGL	0.032575882	0.060698469	0.092347123	0.074139476	0.039374393
	MSFT	0.499100834	0.251928715		META	0.042508051	0.07697247	0.074139476	0.158987627	0.053656576
					MSFT	0.028334588	0.048078084	0.039374393	0.053656576	0.063215217
	Risk free rate	0.015								

SUMMARY OUTPUT AAPL

Regression Statistics	
Multiple R	0.725712672
R Square	0.526658882
Adjusted R Squ	0.524757914
Standard Error	0.008980602
Observations	251

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.022344231	0.022344231	277.0476862	2.51356E-42
Residual	249	0.020082151	8.06512E-05		
Total	250	0.042426382			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.000641391	0.000570304	1.124647429	0.261821432	-0.00048184	0.001764626	-0.000481844	0.00176463
Market_Ret	1.133953889	0.068126822	16.64474951	2.51356E-42	0.999775601	1.268132177	0.999775601	1.26813218

SUMMARY OUTPUT GOOGL

Regression Statistics	
Multiple R	0.602211143
R Square	0.36265826
Adjusted R Squ	0.360098655
Standard Error	0.015343866
Observations	251

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.033357549	0.033357549	141.6852234	3.67303E-26
Residual	249	0.058623118	0.000235434		
Total	250	0.091980667			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.000799085	0.000974397	0.820081787	0.412953815	-0.00112003	0.002718195	-0.001120025	0.00271819
Market_Ret	1.385510277	0.116398524	11.90316023	3.67303E-26	1.156259093	1.61476146	1.156259093	1.61476146

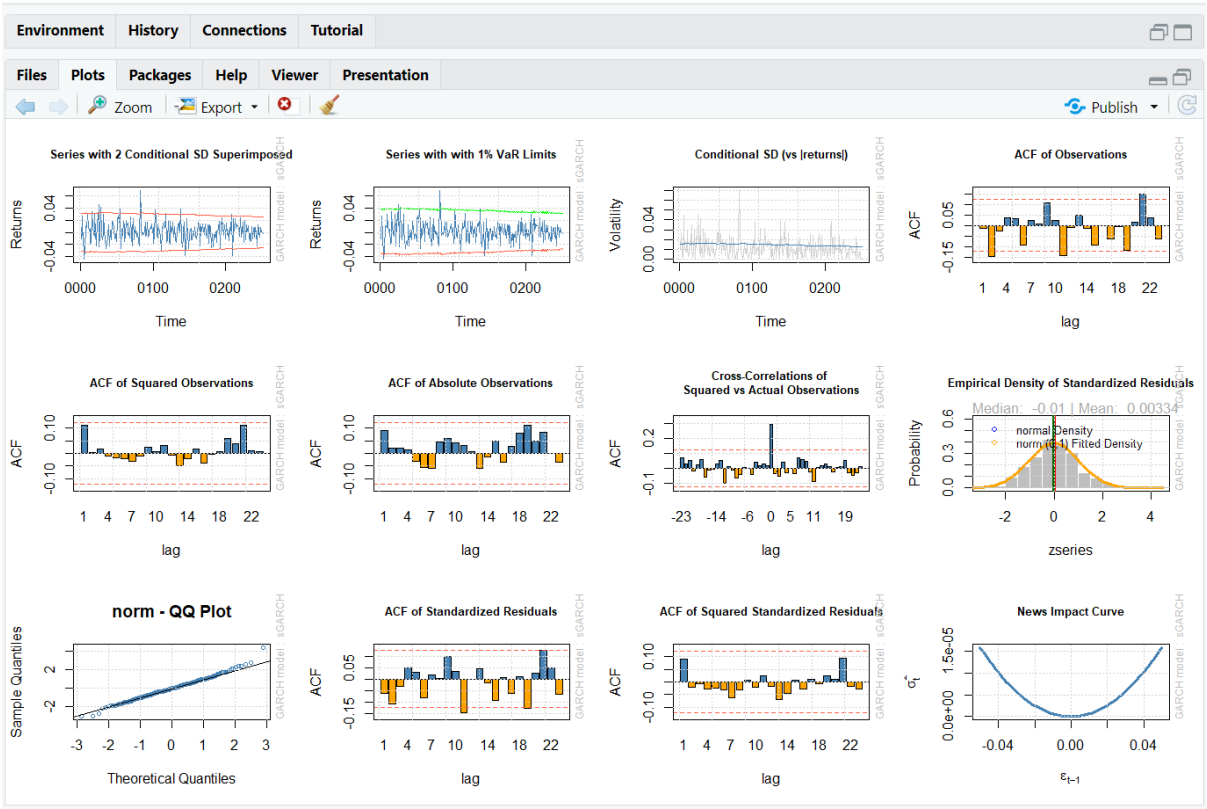
SUMMARY OUTPUT META

Regression Statistics	
Multiple R	0.584247844
R Square	0.341345543
Adjusted R Squ	0.338700345
Standard Error	0.02046668
Observations	251

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.054054362	0.054054362	129.0434452	2.27198E-24
Residual	249	0.104302362	0.000418885		
Total	250	0.158356724			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.003143298	0.001299716	2.418450632	0.016304895	0.00058346	0.005703136	0.00058346	0.00570314
Market_Ret	1.76371359	0.15526018	11.3597291	2.27198E-24	1.45792294	2.069504239	1.45792294	2.06950424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ACF of Squared Observations
ACF of Absolute Observations
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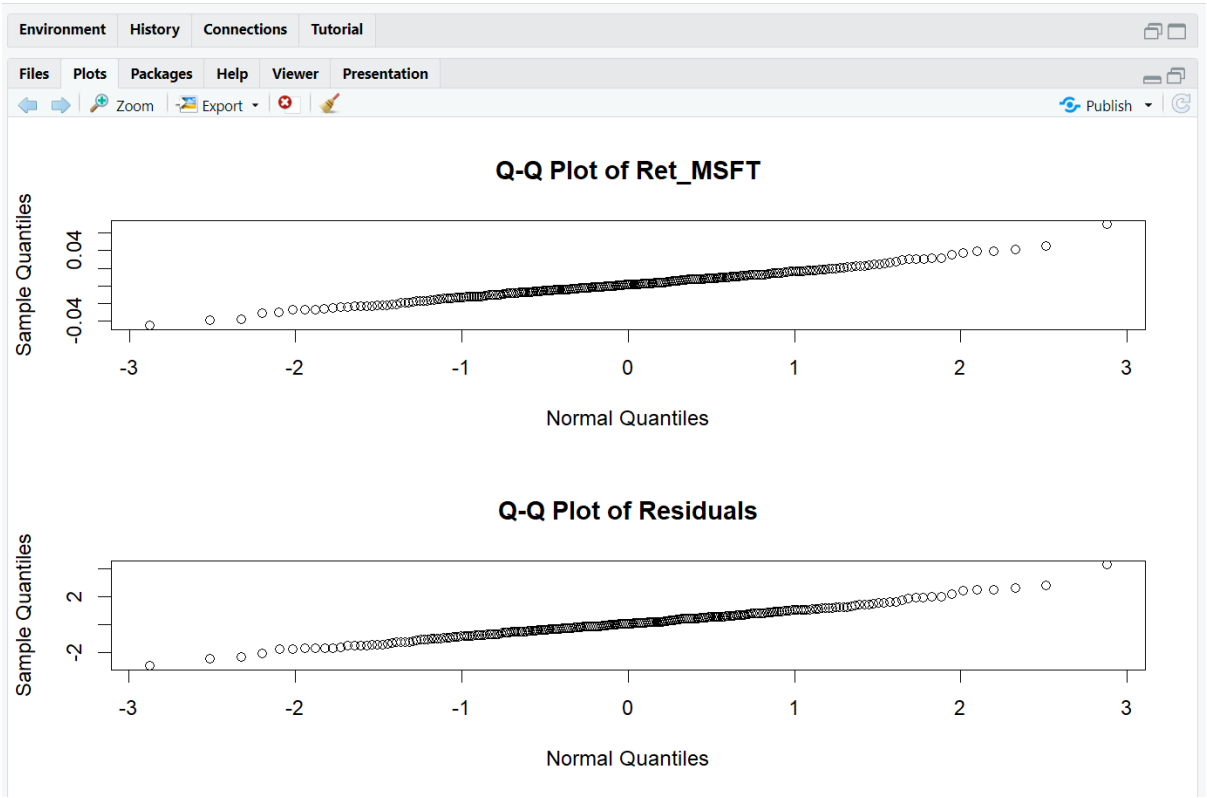
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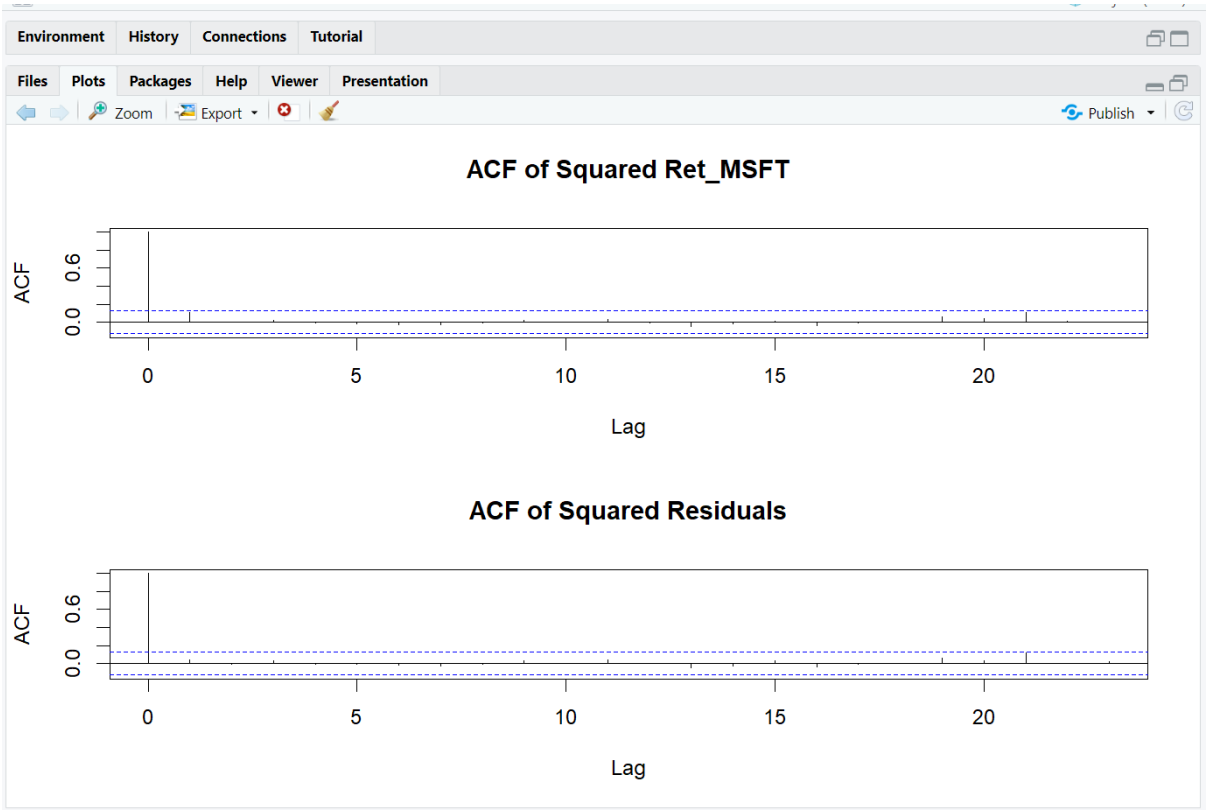
Sample Quantiles
Normal Quantiles
Q-Q Plot of Residuals
Sample Quantiles
Normal Quantiles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ACF
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Lag

ACF of Squared Residuals
ACF
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