



Homework 1

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15 January 2024

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Electrical and Computer Engineering
Math 220 - Mathematical Proofs
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Solutions

For Homework 1, only problems 1, 2, and 8 are worth marks.

Problem 1. (6 Points) Write the following sets by listing their elements.

1) $A_1 = \{x \in \mathbb{N} : x^2 < 5\}$

Answer: $\{1, 2\}$

2) $A_2 = \{x \in \mathbb{Z} : x^2 < 5\}$

Answer: $\{-2, -1, 0, 1, 2\}$

3) $A_3 = \{x \in \mathbb{N} : (3 \mid x) \wedge (x \mid 180)\}$

Answer: $\{3, 6, 9, 12, 15, 18, 30, 45, 60, 90, 180\}$

4) $A_4 = \{x \in \mathbb{Z} : \frac{x+2}{5} \in \mathbb{Z}\}$

Answer: $\{\dots, -17, -12, -7, -2, 3, 8, 13, 18, \dots\}$

5) $A_5 = \{a \in B : 6 \leq 4a + 1 < 17\}$

Answer: $\{2, 3\}$

6) $A_6 = \{x \in B : 50 < xd < 100 \text{ for some } d \in D\}$, where $B = \{2, 3, 5, 7, 11, 13, \dots\}$ and $D = \{5, 10\}$

Answer: $\{7, 11, 13, 17, 19\}$

Problem 2. (6 Points) Write the following sets using the set builder notation:

a) $A = \{1, 8, 15, 22, 29, \dots\}$

Answer: $A = \{x \in \mathbb{Z} : 1 \leq 7x + 1\}$

b) $B = \{16, 8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$

Answer: $B = \{x \in D : \frac{16}{x}\}$, where $D = \{1, 2, 4, 16, 31, 64, 128, 256, 512\}$

c) $C = \{\dots, -\frac{3}{10}, -\frac{2}{5}, -\frac{1}{2}, 0, \frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \dots\}$

Answer: $C = \{\frac{k}{k^2 + 1} : k \in \mathbb{Z}\}$

Problem 8. (6 Points) Consider the set

$$A = \left\{ \frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \dots \right\}.$$

Which one(s) among the following suggestions below describe(s) it correctly. (Simple answer by Yes or No - no justification required).

1. $A = \{x \in \mathbb{R} : \text{there exists } m \in \mathbb{N} \text{ satisfying } x = \frac{2m-1}{2m}\}$

Answer: YES

2. $A = \left\{ \frac{2m-1}{2m} : m \in \mathbb{N} \right\}$

Answer: YES

3. $A = \{x \in \mathbb{R} : \frac{2m-1}{2m} \text{ and } m \in \mathbb{N}\}$

Answer: NO

4. $A = \{x \in \mathbb{Q} : \frac{2m-1}{2m} \text{ and } m \in \mathbb{N}\}$

Answer: NO

5. $A = \{x \in \mathbb{Q} : \text{there exists } m \in \mathbb{N} \text{ satisfying } x = \frac{2m-1}{2m}\}$

Answer: YES

6. $A = \left\{ \frac{2m-1}{2m} : m \in \mathbb{Z} \text{ and } m \geq 1 \right\}$

Answer: YES