Lista 05

2a)

$$H = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}\left(1 + \varepsilon \frac{q}{q_0}\right)$$

$$H = \underbrace{\frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}}_{H_0} + \varepsilon \underbrace{\frac{m\omega^2 q^3}{2q_0}}_{H_1}$$

→ Sendo:

$$H = H_0 + \varepsilon H_1$$

→ e tendo:

$$q = \sqrt{\frac{2J}{m\omega}}\sin\phi \quad e \quad p = \sqrt{2m\omega J}\cos\phi$$

→ Em H_0:

$$H_0 = \frac{\cancel{2m}\,\omega J\cos^2\phi}{\cancel{2m}} + \frac{\cancel{m}\,\omega^{\cancel{2}}}{\cancel{2}}\frac{\cancel{2}J}{\cancel{m\omega}}\sin^2\phi$$

$$\boxed{H_0 = \omega J}$$

→ Em H_1:

$$H_1 = \frac{m\omega^2}{2q_0}\sin^3\phi\left(\frac{2J}{m\omega}\right)^{3/2} = \frac{m\omega^2}{2q_0}\sin^3\phi\, J^{3/2}\sqrt{\frac{2^3}{m^3\omega^3}}$$

$$= \sqrt{\frac{m^2\omega^4 2^3}{2^2 m^3\omega^3}}\frac{\sin^3\phi\, J^{3/2}}{q_0} = \sqrt{\frac{1\cdot 2\cdot\omega}{m}}\, J^{3/2}\sin^3\phi$$

$$\Rightarrow \boxed{H_1 = \frac{1}{q_0}\left(\frac{2\omega J^3}{m}\right)^{1/2}\sin^3\phi}$$

→ Para a perturbação

$$S(J,\phi) = J\phi + \varepsilon S_1(J_1,\phi) + \varepsilon^2 S_2(J,\phi) + \ldots$$

$$\begin{cases} K_0 = H_0 \\ K_1 = w \dfrac{\partial S_1}{\partial \theta} + H_1 \\ K_2 = w \dfrac{\partial S_2}{\partial \theta} + \Phi(\theta, J) \end{cases}$$

$$H_0 = w_0 J \quad ; \quad H_1 = \frac{1}{q}\left(\frac{2wJ^3}{m}\right)^{1/2} \sin^3\phi \quad ; \quad H_2 = 0$$

Então:

$$\phi_2(\theta, J) = \overset{\to 0}{\cancel{H_2(\theta, J)}} + \frac{\partial S_1}{\partial \theta}\frac{\partial H_1}{\partial J} + \frac{1}{2}\frac{\partial S_1}{\partial \theta}\overset{\to 0}{\cancel{\frac{\partial^2 H_0}{\partial J^2}}}\frac{\partial H_1}{\partial \theta}$$

$$\Rightarrow \phi_2(\theta, J) = \frac{\partial S_1}{\partial \phi}\frac{\partial H_1}{\partial J} = \frac{\partial S_1}{\partial \phi}\frac{1}{q_0}\frac{3}{2}\left(\frac{2w}{m}\right)^{1/2} J^{1/2} \sin^3\phi$$

$$\therefore \tilde{H}_1 = H_1 - \overset{\to 0}{\cancel{\langle H_1 \rangle}} \Rightarrow \frac{\partial}{\partial \phi}\Big[\overbrace{-\frac{1}{w}\int \tilde{H}_1 \, d\phi}^{= S_1}\Big] = -\frac{\tilde{H}_1}{w} = -\frac{1}{wq_0}\left(\frac{2wJ^3}{m}\right)^{1/2} \sin^3\phi$$

$\langle H_1 \rangle \equiv 0$ pela paridade do $\sin^3\phi$

$$\Rightarrow \text{Em } \phi_2(\theta, J) = \frac{3}{\cancel{2}q_0}\left(\frac{\cancel{2w}J}{m}\right)^{1/2}\left(\frac{-1}{\cancel{w}q_0}\right)\left(\frac{\cancel{2w}J^3}{m}\right)^{1/2} \sin^6\phi$$

$$\phi_2(\theta, J) = -\frac{3}{q_0^2}\frac{1}{m} J^2 \sin^6\phi$$

$\Rightarrow$ Em K_2

$$K_2 = w\frac{\partial S_2}{\partial \theta} + \phi_2(\theta, J) = \frac{-3J^2 \sin^6\phi}{q_0^2 m}$$

$$\langle K_2 \rangle = \frac{-3J^2}{q_0^2 m}\int_0^{2\pi} \sin^6\phi = \frac{-3J^2}{q_0^2 m} \cdot \frac{15}{48} = -\frac{45}{48}\frac{J^2}{q_0^2 m}$$

→ Então a Hamiltoniana ficará:

$$K_T = K_0 + \epsilon \langle K_1 \rangle + \epsilon^2 \langle K_2 \rangle$$

$$K_T = \omega_0 J - \epsilon^2 \frac{45}{48} \frac{J^2}{q_0^2 m}$$

$$\boxed{\frac{\partial K}{\partial J} = \omega = \omega_0 - \epsilon^2 \frac{45}{24} \frac{J}{q_0^2 m}}$$

(b) $$H = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2} + \epsilon \frac{m\omega^2 q^3}{2q_0}$$

→ Vamos encontrar a eq. de movimento

ECH: $$\begin{cases} \dot{q} = \dfrac{\partial H}{\partial p} \Rightarrow \dot{q} = p/m \\ \dot{p} = -\dfrac{\partial H}{\partial q} \Rightarrow \dot{p} = -m\omega q - \epsilon \dfrac{3}{2} \dfrac{m\omega^2 q^2}{q_0} \end{cases}$$

$$m\ddot{q} = \dot{p} = -m\omega q - \epsilon \frac{3}{2} \frac{m\omega^2 q^2}{q_0}$$

$$m\ddot{q} + m\omega^2 q + \epsilon \frac{3}{2} \frac{m\omega^2 q^2}{q_0} = 0$$

$$\boxed{\ddot{q} + w^2\left(q + \varepsilon \frac{3}{2}\frac{q^2}{q_0}\right) = 0}$$ → Usado para plotagem

→ Aumento da energia, a amplitude vai aumentando

→ Numericamente da para notar esse aspecto

→ Com o aumento da energia, a frequência diminui

(C) $$H = \underbrace{\frac{p^2}{2m} + \frac{m w^2 q^2}{2}}_{H_0} + \underbrace{\epsilon \frac{m w^2 q^3}{2 q_0}}_{\Delta H}$$

$$K(C,D,t) = H + \frac{\partial S}{\partial t} = H_0 + \frac{\partial S}{\partial t} + \Delta H$$

$$\dot{C} = \frac{\partial \Delta H}{\partial D} \quad ; \quad \dot{D} = -\frac{\partial \Delta H}{\partial C}$$

$$q = \sqrt{\frac{2J}{mw}}\sin\phi \quad ; \quad p = \sqrt{2mJw}\cos\phi$$

→ Já sabendo da solução de HJ:

$$p = \sqrt{2m W_0\left(C_0 - \frac{w_0 q^2}{2}\right)}$$

→ Logo:

$$S(q, C_0, t) = \int p\,dq - \underbrace{\int H dt}_{w_0 J} = \int \sqrt{2m w_0\left(C_0 - \frac{w_0 q^2}{2}\right)}\,dq - w_0 C_0 t$$

$$D = \frac{\partial S}{\partial C_0} = \int 2m w_0\left[2m w_0\left(C_0 - \frac{w_0 q^2}{2}\right)\right]^{-1/2} dq - w_0 t$$

→ Subs q e dq na integral

$$q = \frac{\sqrt{2mW_0C_0}}{mW_0}\sin\phi \;;\; dq = \frac{\sqrt{2mW_0C_0}}{mW_0}\cos\phi\, d\phi$$

$$D_0 = \cancel{mW_0} \int \frac{\cancel{\sqrt{2mW_0C_0}}\cos\phi\, d\phi}{\cancel{mW_0}\cancel{\sqrt{2mW_0C_0}}\sqrt{1-\sin^2\phi}} - W_0 t$$

$$D_0 \int \frac{\cos\phi\, d\phi}{\cos\phi} - W_0 t \Rightarrow D_0 = \phi - W_0 t$$

$$\phi = D_0 + W_0 t$$

$$\Delta H = \epsilon \frac{mW_0^2}{2q_0}\sqrt{\left(\frac{2C(t)}{mW_0}\right)^3}\sin^3(D(t) + W_0 t)$$

$$\therefore \dot{D}_1(t) = \frac{\partial H}{\partial C} = \frac{\epsilon m W_0^2}{2q_0}\sqrt{\frac{8}{m^3W_0^3}}\,\frac{3}{2}C^{1/2}\sin^3(D + W_0 t)$$

$$= \frac{\epsilon}{q_0}\frac{3}{2}C^{1/2}\sqrt{\frac{4W_0}{m}}\sin^3(D + W_0 t)$$

integrando:

$$\Rightarrow D_1(t) = D_0 + \varepsilon\frac{3}{q_0}\sqrt{\frac{CW_0}{m}}\left[\frac{\cos^3(D + W_0 t)}{3} - \cos(D + W_0 t)\right]$$

→ Para $C_1(t)$, logo $\lambda = \frac{3}{2}\frac{\varepsilon}{q_0}$ $\quad \eta = \frac{mW_0^2}{2}$

$$\dot{C}_1 = -\frac{\partial \Delta H}{\partial D} = -\epsilon\, 3W_0 \frac{mW_0^2}{2}\frac{3}{2q_0}\left(\frac{2C_0}{m}\right)^{3/2}\frac{1}{W_0}\sin^2(D_0 + W_0 t)\cos(D_0 + W_0 t)$$

integrando

$$C(t) = \varepsilon \frac{3}{2q_0^3} \frac{m w_0^2}{2} \left(\frac{2C_0}{m}\right)^{3/2} \frac{1}{w_0} \left(\frac{\sin^3(D_0 + w_0 t)}{3} \right) + C_0$$

$$\Rightarrow c(t) = C_0 + \epsilon\, 3 m w_0 \left(\frac{C_0}{m}\right)^{3/2} \sin^3(D_0 + wt)$$

→ logo:

$$\begin{cases} q_0 = (C_0, D_0, t) = \sqrt{\dfrac{2C_0}{m w_0^2}} \sin(D_0 + w_0 t) \\ q_1 = (C_1, D_1, t) = \sqrt{\dfrac{2C_1}{m w_0^2}} \sin(D_1 + w_0 t) \end{cases}$$

Usado para o plot!

→ Para efeitos de comparação, nos dois casos quando w é diminuído, a amplitude aumenta, quando há o aumento da energia a frequência w diminui!!