(1) $$\begin{cases} x_{n+1} = x_n\cos\alpha - (y_n - x_n^2)\sin\alpha \\ y_{n+1} = x_n\sin\alpha + (y_n - x_n^2)\cos\alpha \end{cases}$$

→ Para mostrar que preserva áreas, o det. do Jacobiano tem de ser 1, logo:

$$J = \begin{bmatrix} \dfrac{\partial x_{n+1}}{\partial x_n} & \dfrac{\partial x_{n+1}}{\partial y_n} \\ \dfrac{\partial y_{n+1}}{\partial x_n} & \dfrac{\partial y_{n+1}}{\partial y_n} \end{bmatrix} =$$

$$\det J = \begin{vmatrix} \cos\alpha + 2x_n\sin\alpha & -\sin\alpha \\ \sin\alpha - 2x_n\cos\alpha & \cos\alpha \end{vmatrix} =$$

$$= \cos^2\alpha + \cancel{2x_n\sin\alpha\cos\alpha} + \sin^2\alpha - \cancel{2x_n\cos\alpha\sin\alpha}$$

$$= \cos^2\alpha + \sin^2\alpha = 1 \;//$$

→ Logo, preserva a área!!

(2) → Para encontrar o mapa inverso

$$\cos\alpha \quad x_{n+1} = [x_n\cos\alpha - (y_n - x_n^2)\sin\alpha]\cos\alpha$$

$$\sin\alpha \quad y_{n+1} = [x_n\sin\alpha + (y_n - x_n^2)\cos\alpha]\sin\alpha$$

$$\cos\alpha\, x_{n+1} = x_n\cos^2\alpha - \cancel{(y_n - x_n^2)\sin\alpha\cos\alpha}$$

$$+ \quad \sin\alpha\, y_{n+1} = x_n\sin^2\alpha + \cancel{(y_n - x_n^2)\sin\alpha\cos\alpha}$$

$$x_{n+1}\cos\alpha + y_{n+1}\sin\alpha = x_n(\cos^2\alpha + \sin^2\alpha) + 0$$

$$\therefore \boxed{x_n = \cos\alpha\, x_{n+1} + \sin\alpha\, y_{n+1}}$$

→ Para encontrar y_n

$$\sin\alpha\, x_{n+1} = [x_n\cos\alpha - (y_n - x_n^2)\sin\alpha]\sin\alpha$$

$$\cos\alpha\, y_{n+1} = [x_n\sin\alpha + (y_n - x_n^2)\cos\alpha]\cos\alpha$$

$$\sin\alpha\, x_{n+1} = x_n\cancel{\cos\alpha}\sin\alpha - (y_n - x_n^2)\sin^2\alpha$$

$$-\ \cos\alpha\, y_{n+1} = x_n\cancel{\cos\alpha}\sin\alpha + (y_n - x_n^2)\cos^2\alpha$$

$$\sin\alpha\, x_{n+1} - \cos\alpha\, y_{n+1} = -(y_n - x_n^2)\sin^2\alpha - (y_n - x_n^2)\cos^2\alpha$$

$$= -(y_n - x_n^2)(\cos^2\alpha + \sin^2\alpha)$$

$$\Rightarrow y_n - x_n^2 = \cos\alpha\, y_{n+1} - \sin\alpha\, x_{n+1}$$

$$\boxed{y_n = \cos\alpha\, y_{n+1} - \sin\alpha\, x_{n+1} + (\cos\alpha\, x_{n+1} + \sin\alpha\, y_{n+1})^2}$$

→ El Jacobiano inverso

$$J^{-1} = \begin{bmatrix} \dfrac{\partial x_n}{\partial x_{n+1}} & \dfrac{\partial x_n}{\partial y_{n+1}} \\[2ex] \dfrac{\partial y_n}{\partial x_{n+1}} & \dfrac{\partial y_n}{\partial y_{n+1}} \end{bmatrix}$$

$$(\ldots) = \cos\alpha\, x_{n+1} + \sin\alpha\, y_{n+1}$$

$$\begin{vmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha + 2(\ldots)\cos\alpha & \cos\alpha + 2(\ldots)\sin\alpha \end{vmatrix}$$

$$\cos^2\alpha + \cancel{2(\ldots)}\sin\alpha\cos\alpha + \sin^2\alpha - \cancel{2(\ldots)}\cos\alpha\sin\alpha$$

$$\cos^2\alpha + \sin^2\alpha = 1 \quad \boxed{z}$$

$\Rightarrow$ Logo, a inversa também preserva a área