Homework 2 Fixed Income: Curve calibration and option pricing

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1 Details

Second homework assignment Fixed Income. Due before 23.59 on Thursday 21 Feb 2019. Both report and computer code used to generate results are required.

2 Linear products

2.1 single curve

In this exercise we are looking at a simplified example of discount curve calibration. We assume all interest rates are driven by a single discount curve $Z(0,T)$.

1. calibrate the discount curve $Z(0,T)$ for all necessary T using the swap quotes in Table 1.
2. interpolate the discount curve $Z(0,T)$ using log-linear interpolation
3. plot the discount curve, $Z(0,T)$
4. plot the yield curve, $y(0,T)$
5. plot the 1d forward rates, e.g. $f(t,t+1) = 365 \times (Z(0,t)/Z(0,t+1) - 1)$ Using the calibrated discount curve price the following products
6. fixed-floating swap with maturity 10 years and fixed rate equal to [2%, 2.5%, 3%]. You can ignore date rule and daycount fractions.
7. A pension fund has the liability pattern given in Table 2. They would like to hedge their position against parallel interest rate moves using only the 10, 20, and 30 year interest rate swaps. Provide them with an appropriate hedging portfolio. There is no unique answer, so you need to clarify your choice.

maturity	quote
1Y	0.1%
2Y	0.15%
3Y	0.23%
4Y	0.32%
5Y	0.4%
6Y	0.5%
7Y	0.7%
8Y	0.8%
9Y	0.9%
10Y	1.0%
11Y	1.2%
12Y	1.3%
15Y	1.5%
20Y	1.8%
25Y	1.9%
30Y	2.1%

Table 1: quotes for annual OIS swaps.

maturity	liability
1Y	1m
2Y	2m
3Y	4m
4Y	5m
5Y	8m
6Y	10m
7Y	12m
8Y	15m
9Y	18m
10Y	25m
11Y	23m
12Y	20m
15Y	15m
20Y	9m
25Y	6m
30Y	2m

Table 2: Liability profile of a small pension fund.

maturity	quote	maturity	quote
1Y	0.3%	16Y	2.4%
2Y	0.4%	17Y	2.4%
3Y	0.6%	18Y	2.4%
4Y	0.9%	19Y	2.4%
5Y	1.1%	20Y	2.4%
6Y	1.3%	21Y	2.4%
7Y	1.5%	22Y	2.4%
8Y	1.7%	23Y	2.4%
9Y	1.8%	24Y	2.4%
10Y	1.9%	25Y	2.4%
11Y	2.0%	26Y	2.4%
12Y	2.1%	27Y	2.4%
13Y	2.2%	28Y	2.5%
14Y	2.2%	29Y	2.5%
15Y	2.3%	30Y	2.5%

Table 3: quotes for annual fixed-floating swaps.

2.2 multi-curve

In this exercise the setting becomes more realistic and we use a multi-curve calibration. We assume a model that has 3 discount curve $Z(0,T)$, $Z_{3M}(0,T)$, and $Z_{6M}(0,T)$. The quotes for fixed-floating swaps are given in Table 3 and the quotes for tenor basis swaps are given in Table 4.

1. Compute the swap rate of a fixed-floating swap with 10y maturity, annual fixed payments and quarterly 3m libor payments. All daycount fractions are simple (e.g. 1 for annual, 0.25 for 3m).
2. Compute the swap rate of a fixed-floating swap with 10y maturity, annual fixed payments and semi-annual 6m libor payments. All daycount fractions are simple (e.g. 1 for annual, 0.5 for 6m).
3. Compute the value of a floating-floating swap with 5y maturity, semi-annual 6m libor payments and quarterly 3m libor payments plus a spread of 130 bp. All daycount fractions are simple (e.g. 0.5 for 6m, 0.25 for 3m)

3 Swaption pricing

3.1 xABR models

After calibrating the discounting curves in the previous exercise, we are now in a position to calibrate the volatility market.

1. implement the Hagan expansion of the SABR formula as in class

maturity	quote
1Y	29bp
2Y	29bp
3Y	28bp
4Y	28bp
5Y	27bp
6Y	26bp
7Y	25bp
8Y	24bp
9Y	23bp
10Y	22bp
11Y	22bp
12Y	21bp
15Y	20bp
20Y	19bp
25Y	18bp
30Y	18bp

Table 4: quotes for spreads of tenor-basis swaps.

strike	quote
3.0%	25.7%
3.5%	22.2%
4.0%	19.9%
4.5%	18.9%
5.0%	20.9%

Table 5: Black implied volatility quotes.

2. implement the Normal version of the ZABR model with SABR specification.
3. implement the Lognormal version of the ZABR model with SABR specification.
4. plot the implied volatility for various strikes (i.e. the volatility smile) for the following parameters $\sigma(s) = 0.085 \times s^{\beta}, \beta = 0.5, \varepsilon = 0.44, \rho = -0.46, T = 10, z_0 = 1, S_t = 4\%$
5. for all these models, make a plot of the density of the underlying swap rate. Report on any undesirable behaviour.
6. calibrate the model (pick any formula) to the volatility quotes in Table 5. Set $\beta = 0.6$ and $S = 4.0\%$.

3.2 Linear Gaussian Model option valuation

Compute the price of a payer and receiver swaption in the linear Gaussian / Hull-White model.

Parameters given are: $\kappa = 0.05, \sigma = 0.01$.

Use the discount curve you have computed in the first exercise.

1. T swaption payoff at maturity is given by:

$$V^{pay}(5) = \sum_{j=6}^{10} Z(5,j) \left[S_5(5) - 1\%\right]_+ \tag{1}$$

$$V^{rec}(5) = \sum_{j=6}^{10} Z(5,j) \left[1\% - S_5(5)\right]_+ , \tag{2}$$

   where $S_5(5)$ denotes the 5y swaprate in 5 years from today.

   We ignore daycount fractions and coupons are annual on both fixed as floating leg.

2. Compute the value of the above options using Monte-Carlo simulation using $40,000$ samples. Also report a 95% confidence interval. For the determination of the confidence interval, you need to use the Central Limit Theorem.