

Fundamental of Financial Mathematics Assignment

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1 Introduction

I am going to build two products as a bank employee, which should be attractive for both bank and clients in term of return yields. Instructions do not impose any level of bank risk aversion, therefore, I personally decided to maintain a quite low risky level and at the same time, I reasoned in order to optimize the stock features as best I can and in the most realistic way. Both products are constructed by data extracted from Yahoo Finance on the 29th of November 2019(see Appendix 4.2, Appendix 4.3, Appendix 4.4). As a free risk interest rate, I decided to use American T-Bill rates, and I came up with $i = 1.60\%$ issued by the USA government, data updated on the 29th of November 2019(see appendix 4.1). The goal is to build two products through European Options, Stocks or free risk investment, in order to satisfy the client expectations in terms of expected returns, which inevitably affects its risk aversion. Moreover, products should provide a certain margin for the bank, and should be attractive for both counterparts. Considering the few products that can be used, there is no way of portfolio diversification and thus, the amount of money invested does not affect our products. Consequently, I am going to use a general \mathbf{N} as total amount of money invested by the client. Another fixed parameter used is $\mathbf{T} = 1.2$ which approximates the time between now and the expiration of our products (15 January 2021). The stock I used is Match Group Inc. (**MTCH**) which belong to NASDAQ index and whose stock price \mathbf{S}_0 on the 29th of November was \$70.28. As we can see further, this stock its quite illiquid and options volatility is very high for the strike price closest to actual stock price (between 40% and 55%). The currency considered for every operation is the dollar. We assume that its allowed to buy and write fractions of options, so again, in this case the product is independent on the amount \mathbf{N} , which is an unrealistic condition. I tried to report accurately the main reasonings and empirical calculations, therefore, for further explanation, you can refer to the Excel file linked to this report.

2 Product 1: Partially Principal Protected Note (PPPN)

Given an investment \mathbf{N} , the goal of this product is to generate a fixed payoff of $0.9\mathbf{N}$ at maturity, plus a conditionate premium which depends on stock price value at maturity and a specific \mathbf{K} , following the relation:

- Premium = $0.25 \mathbf{N}$ if $S_T > K$
- Premium = 0 elsewhere

The client payoff is fixed, so I am going to pay more attention on bank margin assuming an initial solvency position. In order to ensure 0.9N at maturity I must put into an American T-Bill an amount such that itself discounted at time T is equal to 0.9N:

$$0.9Ne^{-iT} = 0.9Ne^{-0.016 \times 1.2} = 0.883N$$

The rest of the money consist of 0.117N, therefore, I choose to put away 0.015N as margin of the bank, which could be invested in T-Bills too, generating a 1.6% profit on that amount at maturity, therefore, I believe this amount should cover administrative expenses of the contract, and thus I will not consider the initial margin as an investment. At this point, I have 0.102N to invest in the stock market.

Our product requires a sudden growth of return depending on the stock value, usually obtained by digital options. However, we cannot use those products, but we can try to replicate a similar behavior through a “Bull Call Spread “strategy. It consists in the combination of two positions, one long and one short, of the same product but at different strikes, in order to have a constant return if the stock price goes above a certain threshold (the K the bank will propose to the client). I decide to work with Call Options, and so I buy a Call at K_1 and I write a Call at K_2 with $K_2 > K_1$. The amount of the two options must be equal, in order to hedge the risk of a strong stock bullish which would be a disaster in case of a higher weight on $EC(K_2, T)$, especially for this high volatility stock which could easily change considerably the price .

2.1 Empirically results- mid price options

I choose as K_1 and K_2 among the ones offered by the market, those which have lowest volatility(low option price, according to B&S model), which are quite liquid (otherwise I won't be able to write or buy in a real situation) and considering the bank profitability for K_1 (the lowest, the greater possibility of exercise the Call) and product attractivity for K_2 (if too high, it will not attract customers). So I decided to buy a Call with $K_1 = 63$, even though the Call with K_1 has less volatility I choose to have a greater profitability range for the bank, and I write a Call with $K_2 = 73$ forecasting a bullish of the underlying stock price.

The prices chose are mid-prices, the average between bid and ask prices.

- $EC(K_1, T) = \$14.55$
- $EC(K_2, T) = \$10.60$

I calculate the amount of Call I should buy and write as:

$$\frac{0.102N}{EC(K_1, T) - EC(K_2, T)} = 0.02585N$$

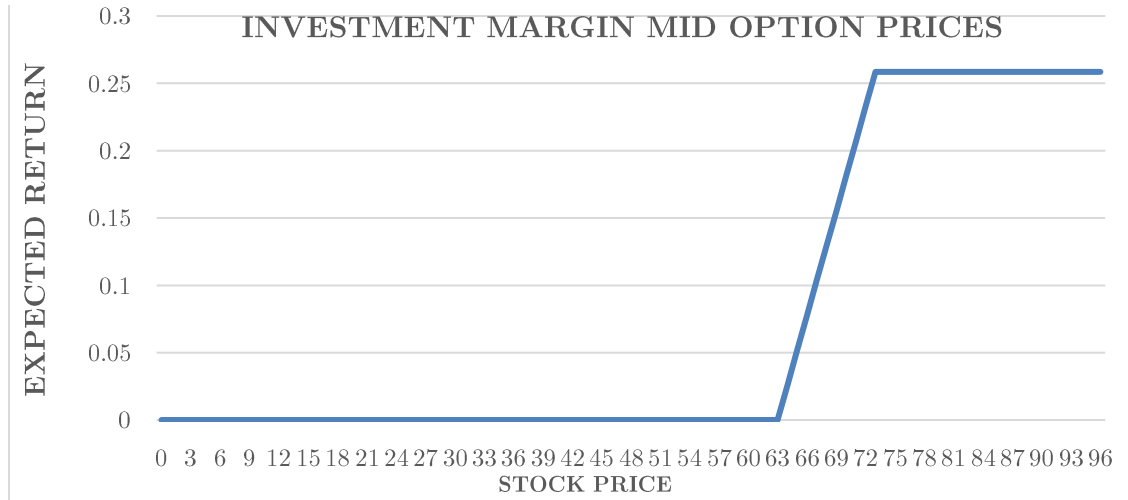
Thus, the expected return is:

$$\max(S_T - K_1, 0) - \max(S_T - K_2, 0)$$

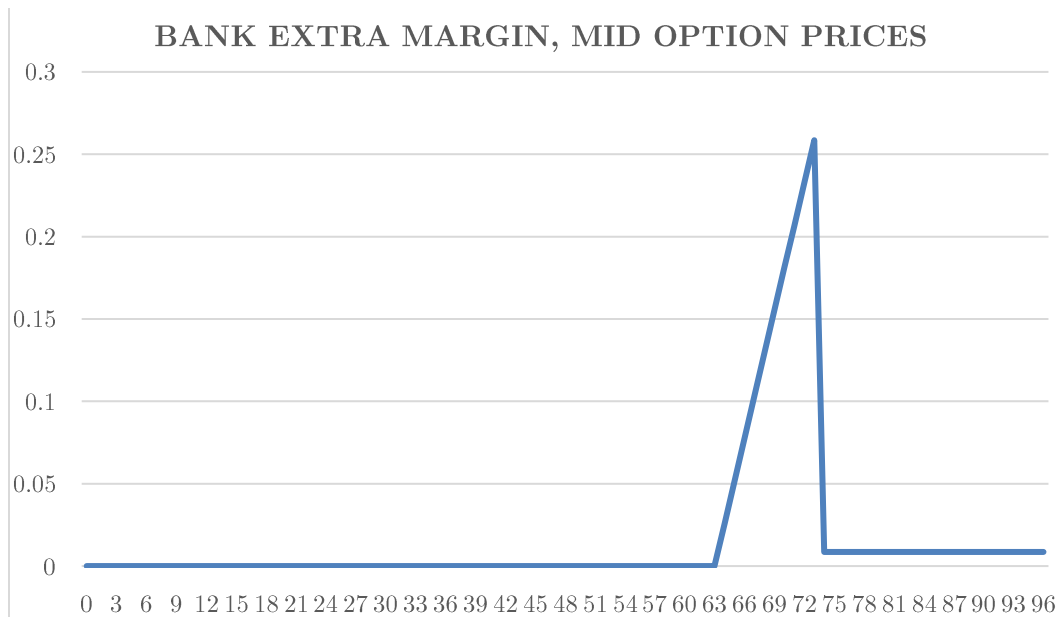
- If $S_T < K_1$ $E(R) = 0$
- If $K_1 < S_T < K_2$ $E(R) = 0.02585N (S_T - K_1)$
- If $S > K_2$ $E(R) = 0.02585N (K_2 - K_1)$

I will assume in a further analysis, how the product considerably changes considering bid and ask pricing, because of the high volatility and low liquidity of the stock that lead to a very high price for options.

The expected return of the investment in function of the stock price is shown by the following picture:



And finally, the bank extra-margin (apart from the previous 1.5% N) is:



To summarize, until K_1 the bank has not extra margin and the client will not gain extra money from the investment. Between K_1 and K_2 the bank extra-margin sharply increases until K_2 , simulating a digital option, which is the peak, and the client still has no extra gain. From a K_2 on, the client will constantly gain an extra 0.250N whereas the bank has an extra 0.85% margin, no matter what's the stock price is.

By this framework, it is likely the bank will make a relevant margin on the client investment, because apart from the case of a strong bearish or bullish of the stock price, in which the bank make respectively 1.5% and 2.35% as a total margin, the total margin will be quite high in the range $63 < S_T < 73$ with the peak of the extra margin(1.5% initial excluded) at $S_T = 73$ of approximately 26%.

I also tried to simulate the product without taking the initial margin apart, due to the option leverage feature that amount if added to the investment would have a great positive effect on returns. For instance, the new return peak at $S_T = 73$ would be around 30% of N, and the fixed return for $S_T > 73$ would be approximately the 5% of the initial investment, which are for sure more attractive for the bank. It's a feature that can be implemented, but it is not used in my actual strategy.

2.2 Bid – Ask prices

In this section I analyzed how the product change if I consider bid and ask prices as option prices for my trades. Because of the stock illiquidity, the spread between bid and ask is very high and this definitely change the product framework. The procedure is analogue: first we put 0.883N on a free-risk investment in order to insure 0.9N at maturity. In addition, we must find a combination of Strike prices in order to reproduce a digital option with a margin for the client of 25% of the initial investment, but at different conditions.

For example, the ask price of $EC K_1 =, T = \$16.7$ which is almost 20% more expensive than the mid-price used in the previous product. The same hold for the other strike prices, hence, we have to let the product a little more risky for the bank increasing the strike K_1 , to reduce the bank initial margin, and increase also the K proposed to the client, making the product less attractive for it. I tried several combinations, excluding the $EC K = 78, T$ because too illiquid and in a real case we will not be possible to write a call at that price(there are no transactions in the last 2 months), and I come up with the optimal strategy which requires :

- Buying $EC K_1 = 75, T = \$11.3$
- Writing $EC K_2 = 90, T = \$5.4$

I confirm the bank initial margin which was 1.5% in the previous product, in order to have an initial margin for the expenses. Thus, I calculate the amount of Calls I can buy and write:

$$\frac{0.102N}{EC(K_1, T) - EC(K_2, T)} = 0.017308N$$

The amount calculated is invested in buying call with strike K_1 with expected returns in function of Stock Price:



N.B. the smooth graph behaviors at K_1 and K_2 are just a scale adaptation due to the sharp increase in returns in that points, in fact the investment return is always bigger or equal zero.

Moreover, the bank extra-margin accord to the following plot:



In this construction, the bank margin is zero until $S_T = 75$, which reduce the stock price range in which the bank can make profits than the mid-price case., when the price bear. Between K_1 and K_2 the margin sharply increases up to approximately 26% when $S_T = K_2$, because we exercise the Put we bought but

we have not to pay for the Put we wrote. Finally, if the stock price overcome K_2 , the margin falls to approximately 1%. The K proposed to the client is \$90, which is attractive in a bullish view of the stock price. Moreover, due to the choice of a K_1 higher than S_0 , the probability of earning an extra margin for the bank definitely decrease. By this example, I have shown as a high bid-ask spread on option prices inevitably affects the chance of constructing an attractive product for the client as well as interesting in term of revenue for the bank.

3 Product 2: Reverse Convertible

For this product, the assumptions on S_0 , T , N and i done previously are still valid. The goal of this financial product is to provide a semi-annual coupon C to the client, expressed as percentage per annum investment and which has value:

$$\text{Coupon premium} = \frac{C}{2} \times N$$

and a payoff at maturity T , which consist in the minimum between the initial investment and the countervalue of several predetermine underlying shares:

$$\text{Payoff} = \min(N, \frac{N}{S_0} \times S_T)$$

The framework is the following: I must ensure an attractive coupon at whatever condition the market will be. This is a problem if $S_T \approx S_0$ because the risk to pay off an amount around N without exercising any option or a stock increase is high, so I have to correctly balance investments in order to hedge this risk. I chose to go short Put and put the entire amount of the investment (N + the revenue from put writings) in a T-Bill in order to have a fixed amount as revenue, and obviously a variable amount as a payout. This strategy is quite low risk, because it gives a fixed return for the bank if $S_T > S_0$ and shrinks the range of return with a higher average than a portfolio comprising of a stock buying. In that case, the return in a bullish view would be higher, but on average riskier because the amount of the investment put on the T-Bill would be lower, and consequently a lower return in a bearish view. Moreover, the stock does not allow the “leverage” use. In addition, I decide to do not buy Call because of its very high price (high volatility and bid-ask spread, already explained in the previous product).

3.1 RC with mid-price option

Given these assumptions, and considering as price option the average between bid and ask, I decided to impose 1.5% fixed margin for the bank (expenses repayment), and then I found the optimal combination by:

- Writing EP $K = 63, T = 1.2 = \$10.75$, mid-price, for an equivalent quantity of 0.016279N.
- Putting $(N + 0.016279N \cdot \$10.75)$ in a T-Bill with $i = 1.6\%$
- Client Coupon of 13% per year, so $C = 13\%$, payed 6-monthly

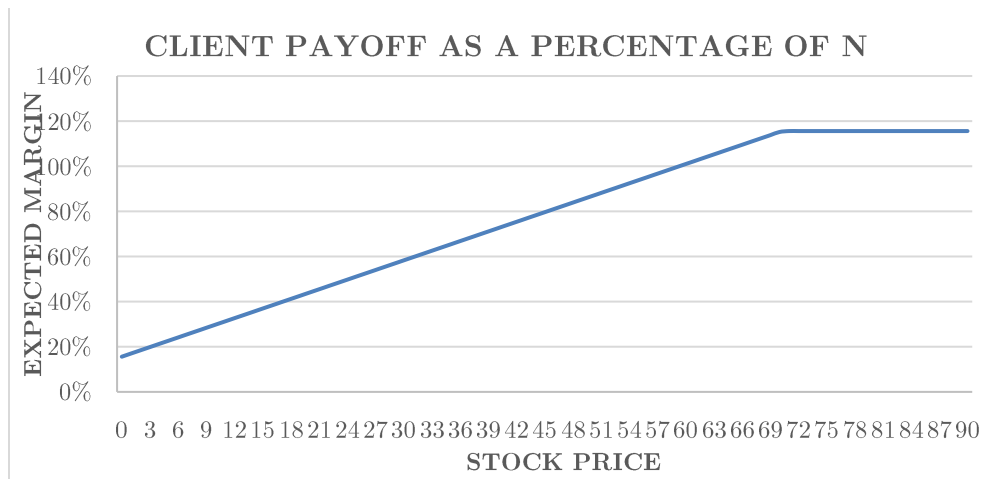
The equation that gives the amount of money invested in T-Bills for this investment, and so the money used to pay coupons, final payout, and eventually bank extra margin is:

$$\frac{0.13N}{e^{(-0.016 \cdot 0.5)}} + \frac{0.13N}{e^{(-0.016 \cdot 1)}} + \frac{0.74N}{e^{(-0.016 \cdot 1.2)}} + \frac{(0.016279N \cdot 10.75\$)}{e^{(-0.016 \cdot 1.2)}}$$

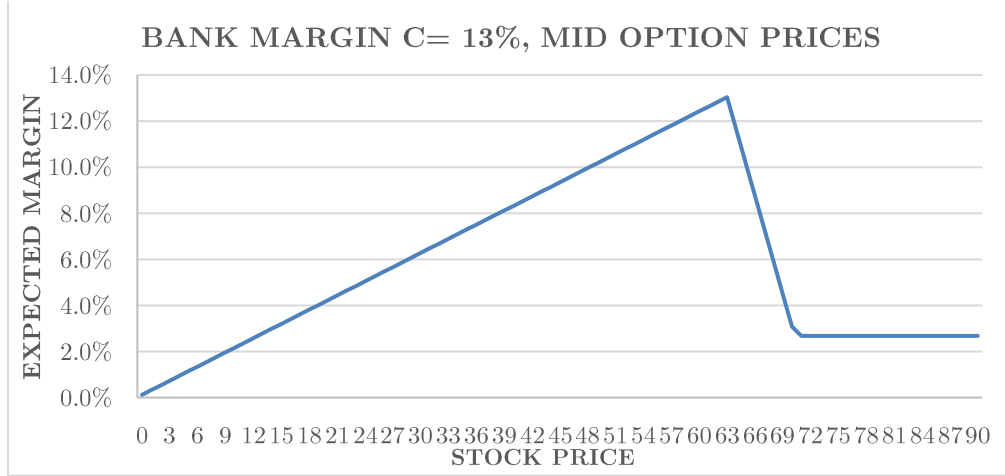
The equation concerns the two coupons payed at 6 months and 1 year, which contribute to the total bank amount of money managed just discounted for those periods. The choice of the K was taken considering volatility but also the risk bank profile I desire, which is not too high. Obviously, the higher the K the higher the writing amount initially gained, and consequently more money available to put in T-Bill but more probability to compensate the put options sold, and so the riskier would be the investment. I chose as C percentage of the coupon, the 13% of the investment, greater than the value considered an enough rate for a coupon (see Appendix 4.5), and I always consider it when I talk about client payoff. I also supposed to have the possibility of getting out from the T-Bill without expenses whenever I want.

I could have increased the value of the coupon, but consequently, it would have had a higher risk profile for the bank or an inferior bank margin on the investment.

This client payoff will be constant if $S_T \geq S_0$ and linearly decrease with S_T if $S_T < S_0$. According to this strategy, I plot the bank margin as well as the client payoff:



Client payoff comprise of semi-annual coupons payments, with $C=13\%$



Bank extra-margin with $C = 13\%$

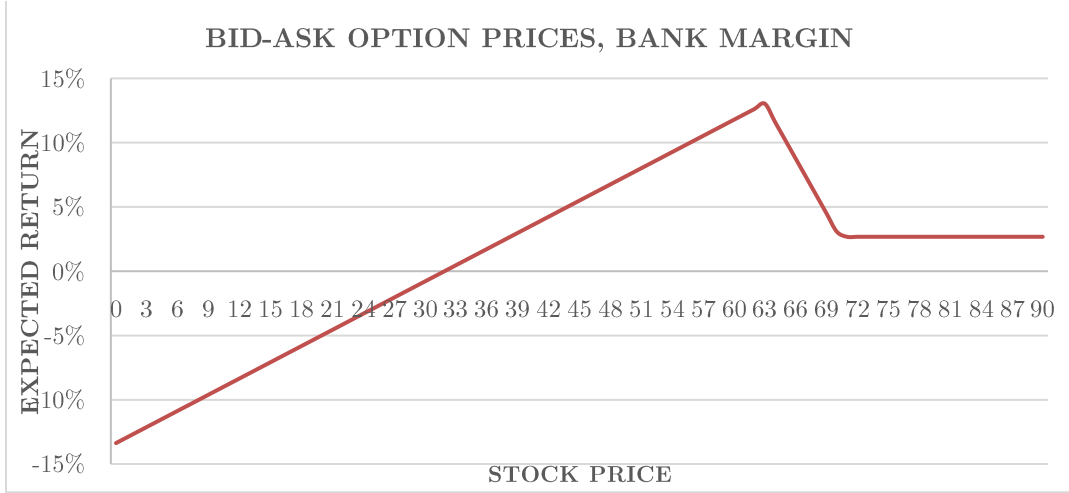
The bank extra-margin at the actual stock price and in a bullish view ($S_T > S_0$) would be around 3% of N , plus the initial 1.5%, gives approximately 4.5% N as a bank margin for the product. In a bearish view of the stock, the margin would much increase, up to 13% when $S_T = K_P$. The two ranges considered are the most likely in a time to maturity relatively short as ours. If the stock will decrease under $K_P = 63$, which would signify a 10% decrease than S_0 , the margin will decrease linearly toward zero in a very unlikely case in which the stock value is approximately zero. Therefore, there no exist a case in which the bank has less than 1.5% margin, used for contract stipulation expenses.

By this strategy, I believe to have constructed a quite attractive product for the client as well as attractive for the bank which can make a good margin on the initial client investment. It's important to say, as well as the previous product, that even a slight change in option prices or T-Bill interest rate would affect a lot our product construction.

3.2 RC with bid-ask prices

I tried also to reproduce a reverse convertible product with bid-ask option prices, and the results are very interesting.

Firstly, I suppose to construct a product with the same parameters as the previous case, but with the **bid price**: EP $K_P = 63, T = 1.2 = 9.5\%$. I want to collect the same amount of money from the option writing, so I am going to sell more contracts than the previous example. The initial bank margin is still 1.5%, coupon $C = 13\%$ and so equal client payoff, but the bank margin plot changes as follow:



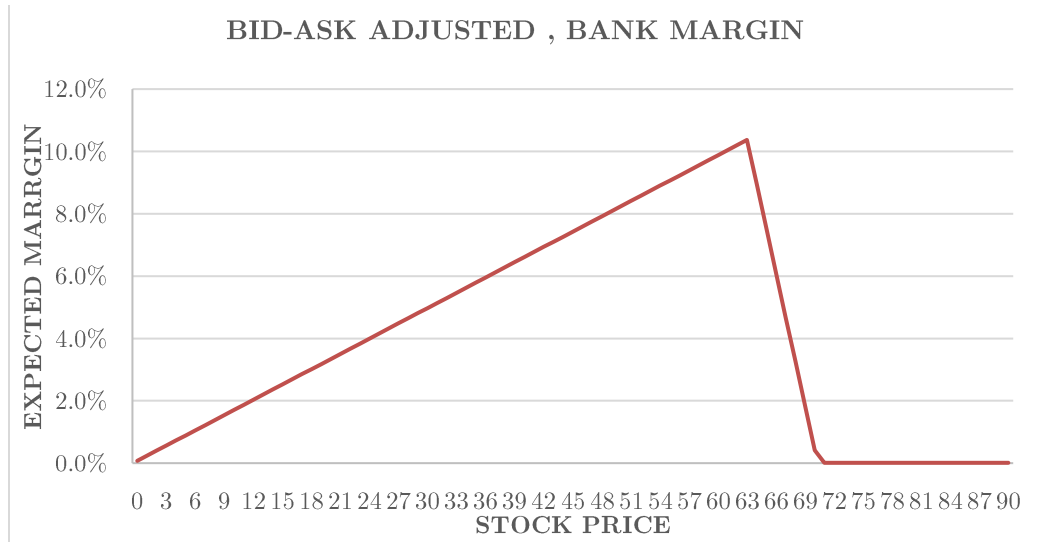
Bank Margin, BID-ASK option prices, $C = 13\%$, initial margin = 1.5%

The output is that we must sell more options to reach the same amount of money to put into the bank, thus, the margin falls in the range which requires the payment of the put options sold before. This is not obviously a product attractive for the bank, so I changed some inputs in order to achieve a discrete result also in this situation.

3.3 RC with bid-ask “adjusted”

I built a product which I called “adjusted”, which has a positive margin for the bank maintaining the same parameters of the previous product, for the client. For this “adjusted” version I rearranged a little bit the strategy by:

- Maintaining initial bank margin at 1.5%
- Decreasing the number of put options traded as well as the total amount gained from the put option writing, and so I write: $EP K_P = 63, T = 1.2 = 9.5\$$ for an amount of 0.0158632N.
- Putting $(N + 0.0158632N \cdot 9.5\$)$ on a T-Bill with interest rate $i = 1.6\%$



Bank Margin, BID-ASK option prices, $C = 13\%$, initial margin = 1.5%

By this construction the client payoff is still the same, but the bank margin has relevantly decreased. The path of the margin plot is pretty the same, but now the yield when $S_T > S_0$ is close to 0%, the peak of the margin, around 10.5%, is still reached at $S_T = K_P = 63$ and the rate linearly decrease toward 0.

By reducing the number of options sold, I decrease the amount of money to pay in case of a stock market bearish and so having a positive margin for the bank everywhere. In the end the bank would achieve a lower margin in all the situations, therefore, the 1.5% N initial margin is still insured as well as the client coupon with $C = 13\%$.

4 Appendix

https://www.treasury.gov/resource-center/data-chart-center/interest-rates/

11/12/19	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.57
11/13/19	1.53	1.56	1.54	1.57	1.54	1.57	1.55	1.59	1.53		
11/14/19	1.56	1.59	1.54	1.57	1.54	1.57	1.54	1.58	1.51		1.55
11/15/19	1.56	1.59	1.53	1.56	1.54	1.57	1.55	1.59	1.50		1.54
11/18/19	1.56	1.59	1.54	1.57	1.54	1.57	1.54	1.58	1.50		1.54
11/19/19	1.55	1.58	1.54	1.57	1.54	1.57	1.54	1.58	1.50		1.54
11/20/19	1.54	1.57	1.53	1.56	1.54	1.57	1.54	1.58	1.50		1.54
11/21/19	1.54	1.57	1.54	1.57	1.55	1.58	1.55	1.59	1.51		1.55
11/22/19	1.55	1.58	1.56	1.59	1.55	1.58	1.55	1.59	1.52		1.56
11/25/19	1.57	1.60	1.57	1.60	1.58	1.61	1.57	1.61	1.54		1.58
11/26/19	1.60	1.63	1.58	1.61	1.57	1.60	1.58	1.62	1.55		1.59
11/27/19	1.62	1.65	1.57	1.60	1.59	1.62	1.59	1.63	1.56		1.60
11/29/19	1.59	1.62	1.57	1.60	1.56	1.59	1.59	1.63	1.56		1.60
12/02/19	1.58	1.61	1.55	1.58	1.57	1.60	1.58	1.62	1.56		1.60
12/03/19	1.54	1.57	1.51	1.54	1.54	1.57	1.53	1.57	1.53		1.57
12/04/19	1.58	1.61	1.51	1.54	1.52	1.55	1.52	1.56	1.52		1.56
12/05/19	1.49	1.52	1.53	1.56	1.51	1.54	1.51	1.55	1.52		1.56
12/06/19	1.49	1.52	1.52	1.55	1.50	1.53	1.52	1.56	1.53		1.57
12/09/19	1.51	1.54	1.51	1.54	1.51	1.54	1.54	1.58	1.52		1.56
12/10/19	1.50	1.53	1.52	1.55	1.53	1.56	1.53	1.57	1.52		1.56
12/11/19	1.51	1.54	1.53	1.56	1.54	1.57	1.54	1.58	1.51		1.55
12/12/19	1.54	1.57	1.54	1.57	1.53	1.56	1.53	1.57	1.51		1.55

Thursday Dec 12, 2019

Daily Treasury Bill Rates: These rates are the daily secondary market quotation on the most recently auctioned Treasury Bills for each maturity tranche (4-week, 8-week, 13-week, 26-week, and 52-week) for which Treasury currently issues new Bills. Market quotations are obtained at approximately 3:30 PM each business day by the Federal Reserve Bank of New York. The Bank Discount rate is the rate at which a Bill is quoted in the secondary market and is based on the par value, amount of the discount and a 360-day year. The Coupon Equivalent, also called the Bond Equivalent, or the Investment Yield, is the bill's yield based on the purchase price, discount, and a 365- or 366-day year. The Coupon Equivalent can be used to compare the yield on a discount bill to the yield on a nominal coupon bond that pays semiannual interest.

For more information regarding these statistics contact the Office of Debt Management by email at debt.management@do.treas.gov

4.1 Free risk account rate on 29th November, Treasury USA Government

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Match Group, Inc. (MTCH)
NasdaqGS - NasdaqGS Real Time Price. Currency in USD ★ Add to watchlist Quote Lookup

70.28 +0.13 (+0.19%)
As of 11:36AM EST. Market open. Buy Sell

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KONICA MINOLTA ZOEKT NIEUWE PARTNERS IN OFFICE EN IT Ontdek de voordelen

January 15, 2021 In The Money Show: List Straddle Option Lookup

Calls for January 15, 2021

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
MTCH210115C00018000	2019-11-07 3:02PM EST	18.00	54.00	51.80	56.50	0.00	-	1	0	119.48%
MTCH210115C00018000	2019-06-09 11:14PM EST	18.00	35.61	52.00	55.00	0.00	-	1	4	93.12%
MTCH210115C00020000	2018-11-30 1:40PM EST	20.00	23.30	0.00	0.00	0.00	-	8	0	0.00%
MTCH210115C00021000	2019-06-25 2:16PM EST	21.00	47.80	55.50	59.80	0.00	-	6	5	170.02%
MTCH210115C00023000	2019-06-07 2:04PM EST	23.00	48.40	48.50	52.80	0.00	-	1	5	103.91%
MTCH210115C00025000	2019-06-09 11:14PM EST	25.00	24.08	45.10	48.80	0.00	-	10	10	78.32%
MTCH210115C00028000	2019-10-17 9:22AM EST	28.00	48.14	41.00	43.10	0.00	-	8	0	59.03%
MTCH210115C00030000	2019-11-12 11:05AM EST	30.00	37.40	38.20	42.80	0.00	-	5	0	74.98%
MTCH210115C00033000	2019-11-18 9:46AM EST	33.00	35.50	34.70	39.50	0.00	-	1	1	65.19%
MTCH210115C00035000	2019-11-19 2:59PM EST	35.00	36.40	32.70	37.50	0.00	-	1	0	61.01%

Zero Trade-Offs **Fidelity**

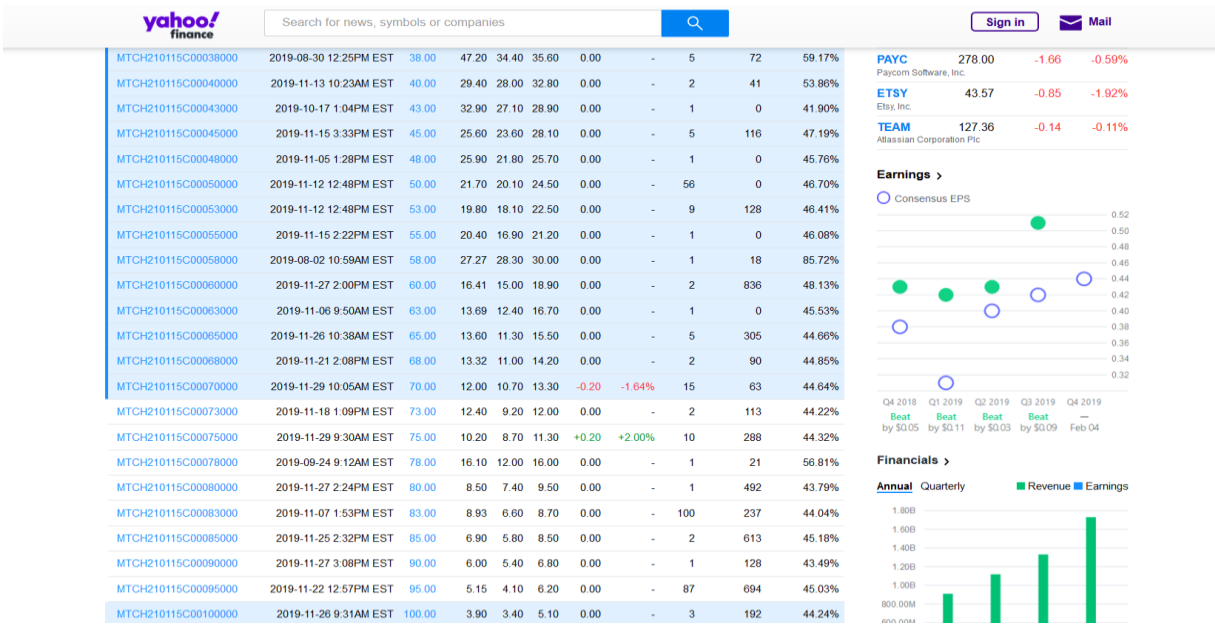
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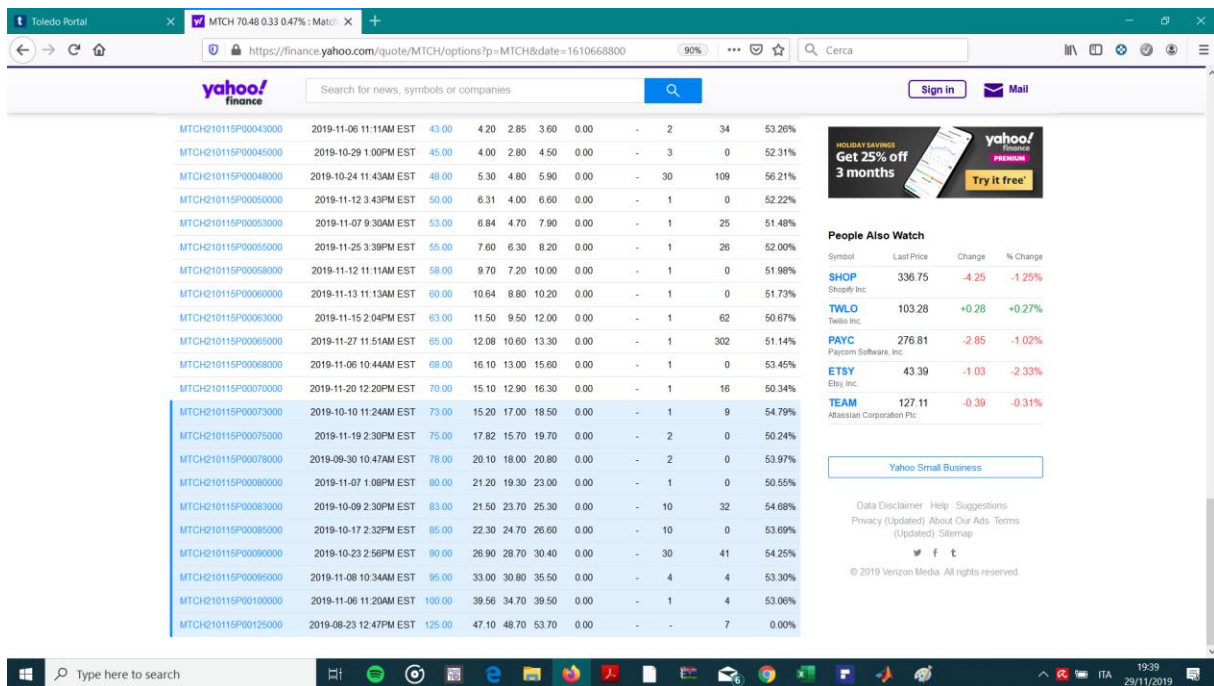
Symbol	Last Price	Change	% Change
SHOP	338.81	-2.19	-0.64%
Shopify Inc.			
TWLO	103.14	+0.14	+0.14%
Twilio Inc.			

search ITA 17:42 29/11/2019

4.2 Stock Price, Call Options MTCH on 29th November, source Yahoo Finance.



4.3 Call Options MTCH on 29th November, Source Yahoo Finance.



4.4 Put Options MTCH on 29th November, Source Yahoo Finance.

Reverse Convertible SECURITIES

Definition:

Short-term investors seeking a high coupon rate. Risk that may be rewarded with a dependable income stream and high interest rates.

A Reverse Convertible Security is a short-term investment tied to one or more underlying stocks. It provides an income stream with return of principal based on the performance of the underlying stock. Reverse convertible securities can provide a short-term steady income stream, however overall returns may be lower based on call features and other factors. Your income stream may be less than stated if the security is called and you may not be able to reinvest the funds at similar rates.

IS IT RIGHT FOR YOU?

- Are you looking for a dependable stream of income generated by a high interest rate?
- Are you seeking to diversify your investment portfolio?
- Are you knowledgeable concerning how options work and comfortable with investing in securities incorporating option features?
- Are you willing to accept the risk of possibly owning at maturity an underlying stock whose value is less than your initial investment?

Checklist of Features

- | | |
|---|--|
| <input checked="" type="checkbox"/> Low minimum investment. \$10,000 minimum initial purchase; \$1,000 increments thereafter. | <input checked="" type="checkbox"/> Short-term maturity. 3 month, 6 month or one year. |
| <input checked="" type="checkbox"/> High coupon. Typically in the range of 8% - 30% per year. | <input checked="" type="checkbox"/> High credit quality. Typical issues are from banks with a credit rating that is investment grade or better. |
| <input checked="" type="checkbox"/> Steady income. Regardless of the performance of the underlying stock, the stated amount of interest is paid, usually on a monthly basis. | <input checked="" type="checkbox"/> Not principal-protected. Principal may be reduced depending on performance of the underlying stock(s). |
| | <input checked="" type="checkbox"/> Contingent downside protection. A "knock-in" feature provides a set amount of downside protection. |

4.5 High typical coupon range, Source StockCross.com