

1 Factor Strength

The concept of factor strength employed by this project comes from Bailey, Kapetanios, and Pesaran (2020), and it was first introduced by Bailey, Kapetanios, and Pesaran (2016). They defined the strength of factor from prospect of the cross-section dependences of large panel and connect it to the pervasiveness of the factor, which is captured by the factor loadings. In a latter paper, Bailey, Kapetanios, and Pesaran (2019) extended the method by loosen some restrictions, and proved that their estimation can also be applied on the residuals or regression result. Thereafter, they focusing on the case of observed factors, and proposed the method we employed in this project (Bailey et al., 2020).

1.1 Definition

Consider the following multi-factor model for n different cross-section units and T observations with k factors.

$$x_{it} = a_i + \sum_{j=1}^k \beta_{ij} f_{jt} + \varepsilon_{it} \quad (1)$$

In the left-hand side, we have x_{it} denotes the cross-section unit i at time t , where $i = 1, 2, 3, \dots, n$ and $t = 1, 2, 3, \dots, T$. In the other hand, a_i is the constant term. f_{jt} of $j = 1, 2, 3, \dots, k$ is factors included in the model, and β_{ij} is the corresponding factor loading. ε_{it} is the stochastic error term.

The factor strength is relates to how many non-zero loadings correspond to a factor. More precisely, for a factor f_{jt} with n different factor loading β_{ij} , we assume that:

$$|\beta_j| > 0 \quad i = 1, 2, \dots, [n^{\alpha_j}]$$

$$|\beta_j| = 0 \quad i = [n^{\alpha_j}] + 1, [n^{\alpha_j}] + 2, \dots, n$$

The α_j represents strength of factor f_{jt} and $\alpha_j \in [0, 1]$. If factor has strength α_j , we will assume that the first $[n^{\alpha_j}]$ loadings are all different from zero, and here $[\cdot]$ is defined as integral operator, which will only take the integral part of inside value. The rest $n - [n^{\alpha_j}]$ terms are all equal to zero. Assume for a factor which has strength $\alpha = 1$, the factor's loadings will be non-zero for all cross-section

units. We will refer such factor as strong factor. And if we have factor strength $\alpha = 0$, it means that the factor has all factor loadings equal to zero, and we will describe such factor as weak factor (Bailey et al., 2016). For any factor with strength in $[0.5, 1]$, we will refer such factor as semi-strong factor. In general term, the more non-zero loading a factor has, the stronger the factor's strength is.

1.2 Estimation

To estimate the strength α_j , Bailey et al. (2020) provides following estimation.

To begin with, we consider a single-factor model with only factor named f_t . β_i is the factor loading of unit i . v_{it} is the stochastic error term.

$$x_{it} = a_i + \beta_i f_t + v_{it} \quad (2)$$

Assume we have n different units and T observations for each unit: $i = 1, 2, 3, \dots, n$ and $t = 1, 2, 3, \dots, T$. Running the OLS regression for each $i = 1, 2, 3, \dots, n$, we obtain:

$$x_{it} = \hat{a}_{iT} + \hat{\beta}_{iT} f_t + \hat{v}_{it}$$

For every factor loading $\hat{\beta}_{iT}$, we can examining their significance by constructing a t-test. The t-test statistic will be $t_{iT} = \frac{\hat{\beta}_{iT} - 0}{\hat{\sigma}_{iT}}$. Then the test statistic for the corresponding $\hat{\beta}_i$ will be:

$$t_{iT} = \frac{(\mathbf{f}'\mathbf{M}_\tau\mathbf{f})^{1/2} \hat{\beta}_{iT}}{\hat{\sigma}_{iT}} = \frac{(\mathbf{f}'\mathbf{M}_\tau\mathbf{f})^{-1/2} (\mathbf{f}'\mathbf{M}_\tau\mathbf{x}_i)}{\hat{\sigma}_{iT}} \quad (3)$$

Here, the $\mathbf{M}_\tau = \mathbf{I}_T - T^{-1} \tau \tau'$, and the τ is a $T \times 1$ vector with every elements equals to 1. \mathbf{f} and \mathbf{x}_i are two vectors with: $\mathbf{f} = (f_1, f_2, \dots, f_T)'$ $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iT})'$. The denominator $\hat{\sigma}_{iT} = \frac{\sum_{t=1}^T \hat{v}_{it}^2}{T}$.

Using this test statistic, we can then define an indicator function as: $\ell_{i,n} := \mathbf{1}[|\beta_i| > 0]$. If the factor loading is none-zero, $\ell_{i,n} = 1$. In practice, we use the $\hat{\ell}_{i,nT} := \mathbf{1}[|t_{iT}| > c_p(n)]$. Here, if the t-statistic t_{iT} is greater than critical value $c_p(n)$, $\hat{\ell}_{i,n} = 1$, otherwise $\hat{\ell}_{i,n} = 0$. In other word, we are counting how many $\hat{\beta}_{iT}$ are significant. With the indicator function, we then defined $\hat{\pi}_{nT}$ as the fraction of significant factor loading amount to the total factor loadings:

$$\hat{\pi}_{nT} = \frac{\sum_{i=1}^n \hat{\ell}_{i,nT}}{n} \quad (4)$$

39 In term of the critical value $c_p(n)$, rather than use the traditional critical value from student-t
 40 distribution $\Phi^{-1}(1 - \frac{P}{2})$, we use:

$$c_p(n) = \Phi^{-1}(1 - \frac{p}{2n^\delta}) \quad (5)$$

41 Suggested by Bailey, Pesaran, and Smith (2019), here, $\Phi^{-1}(\cdot)$ is the inverse cumulative distri-
 42 bution function of a standard normal distribution, P is the size of the test, and δ is a non-negative
 43 value represent the critical value exponent. This adjusted critical value, adopt helps to tackle the
 44 problem of multiple-test.

45 After obtain the $\hat{\pi}_{nT}$, we can use the following formula provided by Bailey et al. (2020) to
 46 estimate our strength indicator α_j :

$$\hat{\alpha} = \begin{cases} 1 + \frac{\ln(\hat{\pi}_{nT})}{\ln n} & \text{if } \hat{\pi}_{nT} > 0, \\ 0, & \text{if } \hat{\pi}_{nT} = 0. \end{cases}$$

47 Whenever we have $\hat{\pi}_{nT}$, the estimated $\hat{\alpha}$ will be equal to zero. From the estimation, we can find
 48 out that $\hat{\alpha} \in [0, 1]$

49 1.3 Estimation Under Multi-Factor Setting

50 This estimation can also be extended into a multi-factor set up. Consider the following multi-factor
 51 model:

$$x_{it} = a_i + \sum_{j=1}^k \beta_{ij} f_{jt} + v_{it} = a_i + \beta_i' \mathbf{f}_t + v_{it}$$

52 In this set up, we have $i = 1, 2, \dots, n$ units, $t = 1, 2, \dots, T$ time observations, and specially,
 53 $j = 1, 2, \dots, k$ different factors. Here $\beta_i = (\beta_{i1}, \beta_{i2}, \dots, \beta_{ik})'$ and $\mathbf{f}_t = (f_{1t}, f_{2t}, \dots, f_{kt})$. We employed
 54 the same strategy as above, after running OLS and obtain the:

$$x_{it} = \hat{a}_{iT} + \hat{\beta}_{ij} \mathbf{f}_{jt} + \hat{v}_{it}$$

55 To conduct the significant test, we calculates the t-statistic: $t_{ijT} = \frac{\hat{\beta}_{ijT} - 0}{\hat{\sigma}_{ijT}}$. Empirically, the test

56 statistic can be calculated using:

$$t_{ijT} = \frac{\left(\mathbf{f}_{j\circ}' \mathbf{M}_{F-j} \mathbf{f}_{j\circ}\right)^{-1/2} \left(\mathbf{f}_{j\circ}' \mathbf{M}_{F-j} \mathbf{x}_i\right)}{\hat{\sigma}_{iT}}$$

Here, $\mathbf{f}_{j\circ} = (f_{j1}, f_{j2}, \dots, f_{jT})'$, $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{iT})'$, $\mathbf{M}_{F-j} = \mathbf{I} - \mathbf{F}_{-j} (\mathbf{F}_{-j}' \mathbf{F}_{-j})^{-1} \mathbf{F}_{-j}'$, and $\mathbf{F}_{-j} = (\mathbf{f}_{1\circ}, \dots, \mathbf{f}_{j-1\circ}, \mathbf{f}_{j+1\circ}, \dots, \mathbf{f}_{m\circ})'$. For the denominator's $\hat{\sigma}_{iT}$, it was from $\hat{\sigma}_{iT}^2 = T^{-1} \sum_{t=1}^T \hat{u}_{it}^2$, the \hat{u}_{it} is the residuals of the model. Then, we can use the same critical value from (5). Obtaining the correspond ratio $\hat{\pi}_{nTj}$ from (4), and after that use the function:

$$\hat{\alpha}_j = \begin{cases} 1 + \frac{\ln \hat{\pi}_{nT,j}}{\ln n}, & \text{if } \hat{\pi}_{nT,j} > 0 \\ 0, & \text{if } \hat{\pi}_{nT,j} = 0 \end{cases}$$

57 to estimates the factor loading.

58 2 Monte Carlo Design

59 2.1 Design

60 In order to study the finite sample property of factor strength $\hat{\alpha}_j$, we designed a Monte Carlo simu-
 61 lation. Through the simulation, we compare the property of the factor strength in different settings.
 62 We set up the experiments to reflect the CAPM model and it's extension. Consider the following
 63 data generating process (DGP):

$$r_{it} - r_{ft} = q_1(r_{mt} - r_{ft}) + q_2\left(\sum_{j=1}^k \beta_{ij} f_{jt}\right) + \varepsilon_{it}$$

64 In the simulation, we consider a dataset has $i = 1, 2, \dots, n$ different cross-section units, with
 65 $t = 1, 2, \dots, T$ different observations. r_{it} is the unit's return, and r_{ft} represent the risk free rate at
 66 time t, therefore, the left hand side term $r_{it} - r_{ft}$ is the excess return of the unit i. For simplicity, we
 67 define $x_{it} := r_{it} - r_{ft}$. f_{jt} represents different risk factors, and the corresponding β_{ij} are the factor
 68 loadings. We use $r_{mt} - r_{ft}$ to denotes the market factor, and here r_{mt} is the average market return.
 69 Also, we use the term $f_{mt} := r_{mt} - r_{ft}$ to denotes the market factor. We expect the market factor

will has strength equals to one all the time, so we consider the market factor has strength $\alpha_m = 1$.
 ε_{it} is the stochastic error term. Therefore, the simulation model can be simplified as:

$$x_{it} = q_1(f_{mt}) + q_2\left(\sum_{j=1}^k \beta_{ij} f_{jt}\right) + \varepsilon_{it}$$

$q_1(\cdot)$ and $q_2(\cdot)$ are two different functions represent the unknown mechanism of market factor and other risk factors in pricing asset risk. In the classical CAPM model and it's multi-factor extensions, for example the three factor model introduced by Fama and French (1992), both q_1 and q_2 are linear.

For each factor, we assume they follow a multinomial distribution with mean zero and a $k \times k$ variance-covariance matrix Σ .

$$\mathbf{f}_t = \begin{pmatrix} f_{1,t} \\ f_{2,t} \\ \vdots \\ f_{k,t} \end{pmatrix} \sim MVN(\mathbf{0}, \Sigma) \quad \Sigma := \begin{pmatrix} \sigma_{f1}^2 & \rho_{12}\sigma_{f1}\sigma_{f2} & \cdots & \rho_{1k}\sigma_{f1}\sigma_{fk} \\ \rho_{12}\sigma_{f2}\sigma_{f1} & \sigma_{f2}^2 & \cdots & \rho_{2k}\sigma_{f2}\sigma_{fk} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1k}\sigma_{fk}\sigma_{f1} & \rho_{k2}\sigma_{fk}\sigma_{f2} & \cdots & \sigma_{fk}^2 \end{pmatrix}$$

The diagonal of matrix Σ indicates the variance of each factor, and the rest represent the covariance among all k factors.

2.2 Experiment Setting

Follow the general model above, we assume both $q_1(\cdot)$ and $q_2(\cdot)$ are linear function:

$$q_1(f_{mt}) = a_i + \beta_{im} f_{mt}$$

$$q_2\left(\sum_{j=1}^k \beta_{ij} f_{jt}\right) = \sum_{j=1}^k \beta_{ij} f_{jt}$$

To start the simulation, we consider a two factor model:

$$x_{it} = a_i + \beta_{i1} f_{1t} + \beta_{i2} f_{2t} + \varepsilon_{it} \tag{6}$$

80 The constant term a_i is generate from a uniform distribution, $a_{it} \sim U[-0.5, 0.5]$. For the factor
81 loading β_{i1} and β_{i2} , we first use a uniform distribution $IIDU(\mu_\beta - 0.2, \mu_\beta + 0.2)$ to produce the
82 values. Here we set $\mu_{beta} = 0.71$ to make sure every generated loading value is sufficiently larger
83 than 0. Then we randomly assign $n - [n^{\alpha_1}]$ and $n - [n^{\alpha_2}]$ factor loadings as zero. α_1 and α_2 are
84 the true factor strength of f_1 and f_2 . In this simulation, we will start the factor strength from 0.7
85 and increase it gradually till unity with pace 0.05, say $(\alpha_1, \alpha_2) = \{0.7, 0.75, 0.8, \dots, 1\}$. $[\cdot]$ is the
86 integer operator defined at section (1.2). This step reflects the fact that only $[n^\alpha]$ factor loadings are
87 non-zero.

88 In terms of the factors, they comes from a multinomial distribution $MVN(\mathbf{0}, \Sigma)$, as we discuss
89 before.

90 Currently, we consider three different experiments set up:

91 **Experiment 1 (single factor, normal error, no correlation)** Set β_{i2} from (6) as 0, the error term
92 ε_{it} and the factor f_{1t} are both standard normal.

93 **Experiment 2 (two factors, normal error, no correlation)** Both β_{i1} and β_{i2} are non-zero. Error
94 term and both factors are standard normal. The correlation ρ_{12} between f_{1t} and f_{2t} is zero. The
95 factor strength for the first factor $\alpha_1 = 1$ all the time, and α_2 various.

96 **Experiment 3 (two factors, normal error, weak correlation)** Both β_{i1} and β_{i2} are non-zero. Er-
97 ror term and both factors are standard normal. The correlation ρ_{12} between f_{1t} and f_{2t} is 0.3. The
98 factor strength for the first factor $\alpha_1 = 1$ all the time, and α_2 various.

99 The factor strength in each experiment is estimated using the method discussed in section (1.2),
100 the size of significant test is $p = 0.05$, and the critical value exponent σ has been set as 0.5. For each
101 of the experiment, we calculate the bias, the RMSE and the size of the test to justify the estimation
102 performances. The bias is calculated as the difference between the true factor strength α and the
103 estimate factor strength $\hat{\alpha}$. The Root Square Mean Error (RMSE) comes from:

$$RMSE = [\frac{1}{R} \sum_{r=1}^R (bias_r)^2]^{1/2}$$

104 Where the R represent the total replicate times. The size of the test is under the hypothesis that

105 $H_0 : \hat{\alpha}_j = \alpha_j, j = 1, 2$ against the alternative hypothesis $H_1 : \hat{\alpha}_j \neq \alpha_j, j = 1, 2$. Here we employed
 106 the following test statistic from Bailey et al. (2020).

$$z_{\hat{\alpha}_j:\alpha_j} = \frac{(\ln n) (\hat{\alpha}_j - \alpha_j) - p (n - n^{\hat{\alpha}_j}) n^{-\delta - \hat{\alpha}_j}}{\left[p (n - n^{\hat{\alpha}_j}) n^{-\delta - 2\hat{\alpha}_j} \left(1 - \frac{p}{n^\delta} \right) \right]^{1/2}} \quad j = 1, 2$$

107 Here we set a indicator function $\mathbf{1}(|z_{\hat{\alpha}_j:\alpha_j}| > c|H_0)$. For each replication, if this test statistic is
 108 greater than the critical value of standard normal distribution: $c = 1.96$, the indicator function will
 109 return value 1, and 0 otherwise. Therefore, we calculate the size of the test base on:

$$size = \frac{\sum_{r=1}^R \mathbf{1}(|z_{\hat{\alpha}_j:\alpha_j}| > 1.96|H_0)}{R} \quad j = 1, 2$$

110 In purpose of Monte Carlo Simulation, we consider the different combinations of T and n with
 111 $T = \{120, 240, 360\}, n = \{100, 300, 500\}$. The market factor, if included in the experiment, will
 112 have strength $\alpha_m = 1$ all the time, and the strength of the other factor will be $\alpha_x = \{0.7, 0.75, 0.8, 0.85,$
 113 $0.9, 0.95, 1\}$. For every setting, we will replicate 2000 times independently, all the constant and
 114 variables will be re-generated for each replication.

115 2.3 Monte Carlo Discoveries

116 We report the results in Table (1) , (2) and (3) in Appendix A.

117 Table (1) provides the results under the experiment 1. The estimation method we applied tends to
 118 under-estimate the strength slightly when the true strength is relatively weak under the single factor
 119 set up. The bias is about 0.01 lower when the true underlying factor strength is 0.7. This indicates
 120 that the estimated strength $\hat{\alpha}$ is around 0.69 to 0.68. Such bias, however, decrease gradually while
 121 time t, and α increase. When we increase the time spam by including more data from the time
 122 dimensions, the bias, as well as the RMSE decrease significantly. By contrast, when including
 123 more cross-section unit n into the simulation, the results does not shows significant improvements.
 124 Another more interesting result is that, the gap between estimation and true strength will vanish
 125 when we have $\alpha = 1$, the strongest we can have. With the strength approaching unity, the both bias
 126 and RMSE will converge to zero. After turn our attention to the size of the test, under the single
 127 factor setting. The size of the test will not various too much when the strength increases, so as the

unit increases, But we can observe that the when observations for each unit increase, the size will shrinkage dramatically, although overall the size is still larger than the usual threshold 0.05.

Fro the two factors scenarios, both the correlation zero and correlation point three, we have similar but better conclusions. The result of no correlation settings is shown in the table (2), and the table (3) shows the result when the correlation between two factors is point three. Same as the single factor results, in most of the time, our estimation method will slightly under estimates the factor strength. But for when the we increase either the observations amount t , or the cross-section units amount n , the improvement of estimation is significant. Even when the factor strength is relatively weak, when the n and t are large enough, we can have unbiased estimation (see table (3)). Also, when same as the experiment one, when the true factor strength goes to one, we will consistently obtain unbiased estimation for every time-unit combinations. The results of the size of the test in two factors setting are performing better than the single factor result. Although the size is excess the 0.05 threshold when observation amount is small, it will decrease quickly and dramatically when the observations amount increase.

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A Simulation Result Table

Table 1: Simulation result for experiment 1

	Single Factor								
	Bias $\times 100$			RMSE $\times 100$			Size $\times 100$		
$\alpha_1 = 0.7$									
n\T	120	240	360	120	240	360	120	240	360
100	0.256	0.265	0.227	0.612	0.623	0.560	7.85	7.7	5.55
300	0.185	0.184	0.184	0.363	0.338	0.335	8.9	4.45	4.5
500	0.107	0.124	0.109	0.259	0.248	0.234	6.9	2.5	1.6
$\alpha_1 = 0.75$									
100	-0.178	-0.159	-0.168	0.490	0.465	0.450	2.5	0.85	0.4
300	0.154	0.156	0.143	0.281	0.258	0.234	9.4	3.7	3.35
500	0.024	0.033	0.263	0.171	0.155	0.148	7.8	2	1.25
$\alpha_1 = 0.8$									
100	-0.270	-0.265	-0.258	0.434	0.409	0.411	71.4	72.05	71.45
300	-0.052	-0.044	-0.043	0.183	0.149	0.150	10.15	2.45	2.9
500	0.045	0.068	0.067	0.136	0.126	0.121	16.6	6.4	5.9
$\alpha_1 = 0.85$									
100	0.053	0.062	0.058	0.253	0.228	0.221	6.05	2.95	2.5
300	-0.012	0.009	-0.001	0.124	0.104	0.095	10.55	1.8	1.15
500	-0.026	-0.007	-0.011	0.096	0.073	0.069	13.25	0.9	0.7
$\alpha_1 = 0.9$									
100	0.025	0.038	0.360	0.191	0.163	0.157	6.85	2	1.65
300	-0.034	-0.018	-0.020	0.099	0.069	0.068	13.2	0.8	0.9
500	-0.025	-0.001	-0.001	0.072	0.044	0.044	22.3	1.95	1.8
$\alpha_1 = 0.95$									
100	-0.099	-0.088	-0.090	0.156	0.125	0.126	5.6	0.3	0.55
300	-0.046	-0.025	-0.026	0.083	0.045	0.045	22.5	2.2	2.25
500	-0.030	-0.006	-0.006	0.061	0.026	0.025	33.1	4.4	3.8
$\alpha_1 = 1$									
100	0	0	0	0	0	0	-	-	-
300	0	0	0	0	0	0	-	-	-
500	0	0	0	0	0	0	-	-	-

This table shows the result of one risk factor model. In this experiment, we have standard normal errors and factors. Factor loadings come from uniform distribution $IIDU(\mu_\beta - 0.2, \mu_\beta + 0.2)$.

$[n^{\alpha_j}]$

Table 2: Simulation result for experiment 2

	Double Factor with correlation $\rho_{12} = 0$								
	Bias $\times 100$			RMSE $\times 100$			Size $\times 100$		
$\alpha_1 = 1, \alpha_2 = 0.7$									
n\T	120	240	360	120	240	360	120	240	360
100	0.567	0.737	0.628	4.062	3.819	3.799	2.95	1.45	1.85
300	0.512	0.611	0.518	2.398	2.103	1.979	6.25	0.55	0.5
500	-0.149	0.08	-0.019	1.796	1.498	1.443	8	0.2	0.1
$\alpha_1 = 1, \alpha_2 = 0.75$									
100	-3.051	-3.02	-3.092	4.582	4.245	4.248	2.45	0.1	0.10
300	0.491	-1.035	0.640	1.843	1.460	1.576	7.6	0.8	0.55
500	-0.611	-0.372	-0.393	1.520	1.136	1.125	11.35	0.15	0.1
$\alpha_1 = 1, \alpha_2 = 0.8$									
100	-3.752	-3.630	-3.581	4.557	4.213	4.210	84.65	85.9	85.25
300	-1.218	-0.331	-1.021	1.812	0.792	1.438	9.35	0.2	0.3
500	-0.022	0.192	0.147	1.047	0.782	0.742	15.35	1.1	1.1
$\alpha_1 = 1, \alpha_2 = 0.85$									
100	-0.075	0.127	0.088	1.996	1.697	1.606	5.4	1.15	0.95
300	-0.531	-0.406	-0.351	1.097	0.613	0.777	10.8	0.15	0.2
500	-0.647	-0.391	-0.391	1.020	0.643	0.630	19.1	0.15	0
$\alpha_1 = 1, \alpha_2 = 0.9$									
100	-0.128	0.043	0.025	1.428	1.143	1.118	4.9	0.65	0.7
300	-0.651	-0.334	-0.394	1.002	0.435	0.617	17.1	0.6	0.2
500	-0.434	-0.168	-0.171	0.7435	0.367	0.368	25.2	0.4	0.3
$\alpha_1 = 1, \alpha_2 = 0.95$									
100	-1.218	-1.043	-1.036	1.603	1.222	1.212	6.65	0.25	0.05
300	-0.611	-0.344	-0.356	0.881	0.435	0.434	23.35	0.6	0.45
500	-0.415	-0.123	-0.134	0.661	0.220	0.216	36.75	1.35	1.1
$\alpha_1 = 1, \alpha_2 = 1$									
100	0	0	0	0	0	0	-	-	-
300	0	0	0	0	0	0	-	-	-
500	0	0	0	0	0	0	-	-	-

This table shows the result of two factor model, with one market factor and one risk factor. We simulated scenarios of factor strength equals to 0.5, 0.7, 0.9, and 1 with different time, assets size combination. The replication times is 500

Table 3: Simulation result for experiment 3

	Double Factor with correlation $\rho_{12} = 0.3$								
	Bias $\times 100$			RMSE $\times 100$			Size $\times 100$		
$\alpha_1 = 1, \alpha_2 = 0.7$									
n\T	120	240	360	120	240	360	120	240	360
100	0.038	0.064	0.072	0.421	0.382	0.389	4.6	1.75	1.95
300	0.021	0.058	0.056	0.253	0.206	0.198	9.95	0.9	0.25
500	-0.032	0.006	0	0.201	0.153	0	12.20	0.1	0.05
$\alpha_1 = 1, \alpha_2 = 0.75$									
100	-0.325	-0.313	-0.310	0.488	0.419	0.420	4.75	0.1	0
300	0.028	0.063	0.065	0.253	0.157	0.159	9.95	0.55	0.5
500	-0.082	-0.037	-0.039	0.175	0.114	0.112	19.25	0.25	0.3
$\alpha_1 = 1, \alpha_2 = 0.8$									
100	-0.393	-0.361	-0.368	0.477	0.418	0.421	85.45	85.2	86.4
300	0.029	-0.099	-0.100	0.192	0.145	0.145	12.2	0.65	0.5
500	-0.037	-0.016	0.016	0.129	0.074	0.074	27.8	0.25	1.2
$\alpha_1 = 1, \alpha_2 = 0.85$									
100	-0.027	0.008	0.007	0.234	0.160	0.155	9.3	0.9	0.65
300	-0.147	-0.031	-0.037	0.219	0.079	0.077	16.75	0.3	0.2
500	-0.088	-0.039	-0.039	0.136	0.063	0.062	30.6	0.15	0
$\alpha_1 = 1, \alpha_2 = 0.9$									
100	-0.033	0.003	0.002	0.173	0.111	0.110	9.4	0.6	0.55
300	-0.087	-0.040	-0.041	0.131	0.061	0.061	27.8	0.1	0.05
500	-0.070	-0.017	-0.018	0.111	0.037	0.037	41.15	0.6	0.35
$\alpha_1 = 1, \alpha_2 = 0.95$									
100	-0.134	-0.101	-0.104	0.185	0.122	0.122	10.15	0.1	0.15
300	-0.083	-0.034	-0.034	0.118	0.043	0.044	39.35	0.6	0.6
500	-0.062	-0.013	-0.012	0.937	0.022	0.023	51.8	1.25	2.0
$\alpha_1 = 1, \alpha_2 = 1$									
100	0	0	0	0	0	0	-	-	-
300	0	0	0	0	0	0	-	-	-
500	0	0	0	0	0	0	-	-	-