1   Factor Strength

The concept of factor strength employed by this project comes from Bailey, Kapetanios, and Pesaran (2020), and it was first introduced by Bailey, Kapetanios, and Pesaran (2016). They defined the strength of factor from prospect of the cross-section dependences of large panel and connect it to the pervasiveness of the factor, which is captured by the factor loadings. In a latter paper, ? (?) extended the method by loosen some restrictions, and proved that their estimation can also be applied on the residuals or regression result. Thereafter, they focusing on the case of observed factors, and proposed the method we employed in this project (Bailey et al., 2020).

1.1   Definition

Consider the following multi-factor model for n different cross-section units and T observations with k factors.

$$x_{it} = a_t + \sum_{j=1}^{k} \beta_{ij} f_{jt} + \varepsilon_{it} \tag{1}$$

In the left-hand side, we have x_{it} denotes the cross-section unit i at time t, where $i = 1, 2, 3, \cdots, n$ and $t = 1, 2, 3, \cdots T$. In the other hand, a_i is the constant term. f_{jt} of $j = 1, 2, 3 \cdots k$ is factors included in the model, and β_{ij} is the corresponding factor loading. ε_{it} is the stochastic error term.

The factor strength is relates to how many non-zero loadings correspond to a factor. More precisely, for a factor f_{jt} with n different factor loading β_j, we assume that:

$$|\beta_j| > 0 \quad i = 1, 2, \ldots, [n^{\alpha_j}]$$

$$|\beta_j| = 0 \quad i = [n^{\alpha_j}] + 1, [n^{\alpha_j}] + 2, \ldots, n$$

The α_j represents strength of factor f_{jt} and $\alpha_j \in [0, 1]$. If factor has strength α_j, we will assume that the first $[n^{\alpha_j}]$ loadings are all different from zero, and here $[\cdot]$ is defined as integral operator, which will only take the integral part of inside value. The rest $n - [n^{\alpha_j}]$ terms are all equal to zero. Assume for a factor which has strength $\alpha = 1$, the factor's loadings will be non-zero for all cross-section units. We will refer such factor as strong factor. And if we have factor strength $\alpha = 0$, it means

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that the factor has all factor loadings equal to zero, and we will describe such factor as weak factor (Bailey et al., 2016). For any factor with strength in $[0.5, 1]$, we will refer such factor as semi-strong factor. In general term, the more non-zero loading a factor has, the stronger the factor's strength is.

1.2 Estimation

To estimate the strength α_j, Bailey et al. (2020) provides following estimation.

To begin with, we consider a single-factor model with only factor named f_t. β_i is the factor loading of unit i. v_{it} is the stochastic error term.

$$x_{it} = a_i + \beta_i f_t + v_{it} \tag{2}$$

Assume we have n different units and T observations for each unit: $i = 1, 2, 3, \cdots, n$ and $t = 1, 2, 3, \cdots T$. Running the OLS regression for each $i = 1, 2, 3 \cdots, n$, we obtain:

$$x_{it} = \hat{a}_{iT} + \hat{\beta}_{iT} f_t + \hat{v}_{it}$$

For every factor loading $\hat{\beta}_{iT}$, we can examining their significance by constructing a t-test. The t-test statistic will be $t_{iT} = \frac{\hat{\beta}_{iT} - 0}{\hat{\sigma}_{iT}}$. Then the test statistic for the corresponding $\hat{\beta}_i$ will be:

$$t_{iT} = \frac{(\mathbf{f}'\mathbf{M}_\tau\mathbf{f})^{1/2} \hat{\beta}_{iT}}{\hat{\sigma}_{iT}} = \frac{(\mathbf{f}'\mathbf{M}_\tau\mathbf{f})^{-1/2} (\mathbf{f}'\mathbf{M}_\tau\mathbf{x}_i)}{\hat{\sigma}_{iT}} \tag{3}$$

Here, the $\mathbf{M}_\tau = \mathbf{I}_T - T^{-1}\tau\tau'$, and the τ is a $T \times 1$ vector with every elements equals to 1. $\mathbf{f}$ and $\mathbf{x_i}$ are two vectors with: $\mathbf{f} = (f_1, f_2 \cdots, f_T)'$ $\mathbf{x_i} = (x_{i1}, x_{i2}, \cdots, x_{iT})$. The denominator $\hat{\sigma}_{iT} = \frac{\sum_{i=1}^{T} \hat{v}_{it}^2}{T}$.

Using this test statistic, we can then define an indicator function as: $\ell_{i,n} := \mathbf{1}[|\beta_i| > 0]$. If the factor loading is none-zero, $\ell_{i,n} = 1$. In practice, we use the $\hat{\ell}_{i,nT} =:= \mathbf{1}[|t_{it}| > c_p(n)]$ Here, if the t-statistic t_{iT} is greater than critical value $c_p(n)$, $\hat{\ell}_{i,n} = 1$, otherwise $\hat{\ell}_{i,n} = 0$ In other word, we are counting how many $\hat{\beta}_{iT}$ are significant. With the indicator function, we then defined $\hat{\pi}_{nT}$ as the fraction of significant factor loading amount to the total factor loadings:

$$\hat{\pi}_{nT} = \frac{\sum_{i=1}^{n} \hat{\ell}_{i,nT}}{n} \tag{4}$$

In term of the critical value $c_p(n)$, rather than use the traditional critical value from student-t

distribution $\Phi^{-1}(1 - \frac{P}{2})$, we use:

$$c_p(n) = \Phi^{-1}(1 - \frac{p}{2n^\delta}) \tag{5}$$

Suggested by Bailey, Pesaran, and Smith (2019), here, $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function of a standard normal distribution, P is the size of the test, and δ is a non-negative value represent the critical value exponent. This adjusted critical value, adopt helps to tackle the problem of multiple-test.

After obtain the $\hat{\pi}_{nT}$, we can use the following formula provided by Bailey et al. (2020) to estimate our strength indicator α_j:

$$\hat{\alpha} = \begin{cases} 1 + \frac{\ln(\hat{\pi}_{nT})}{\ln n} & \text{if } \hat{\pi}_{nT} > 0, \\ 0, & \text{if } \hat{\pi}_{nT} = 0. \end{cases} \tag{6}$$

Whenever we have $\hat{\pi}_{nT}$, the estimated $\hat{\alpha}$ will be equal to zero. From the estimation, we can find out that $\hat{\alpha} \in [0,1]$

1.3 Estimation Under Multi-Factor Setting

This estimation can also be extended into a multi-factor set up. Consider the following multi-factor model:

$$x_{it} = a_i + \sum_{j=1}^{k} \beta_{ij} f_{jt} + v_{it} = a_i + \beta_{\mathbf{i}}' \mathbf{f_t} + v_{it}$$

In this set up, we have $i = 1, 2, \cdots, n$ units, $t = 1, 2, \cdots, T$ time observations, and specially, $j = 1, 2, \cdots, k$ different factors. Here $\beta_i = (\beta_{i1}, \beta_{i2}, \cdots, \beta_{ij})'$ and $\mathbf{f_t} = (f_{1t}, f_{2t} \cdots, f_{jt})$. We employed the same strategy as above, after running OLS and obtain the:

$$x_{it} = \hat{a}_{iT} + \hat{\beta}_{\mathbf{ij}} \mathbf{f}_{jt} + \hat{v}_{it}$$

To conduct the significant test, we calculates the t-statistic: $t_{ijT} = \frac{\hat{\beta}_{ijT} - 0}{\hat{\sigma}_{ijT}}$. Empirically, the test

statistic can be calculated using:

$$t_{ijT} = \frac{\left(\mathbf{f}'_{j\circ}\mathbf{M}_{F_{-j}}\mathbf{f}_{j\circ}\right)^{-1/2}\left(\mathbf{f}'_{j\circ}\mathbf{M}_{F_{-j}}\mathbf{x}_i\right)}{\hat{\sigma}_{iT}}$$

Here, $\mathbf{f}_{j\circ} = \left(f_{j1}, f_{j2}, \ldots, f_{jT}\right)'$, $\mathbf{x}_i = \left(x_{i1}, x_{i2}, \ldots, x_{iT}\right)'$, $\mathbf{M}_{F_{-j}} = \mathbf{I} - \mathbf{F}_{-j}\left(\mathbf{F}'_{-j}\mathbf{F}_{-j}\right)^{-1}\mathbf{F}'_{-j}$, and $\mathbf{F}_{-j} = \left(\mathbf{f}_{1\circ}, \ldots, \mathbf{f}_{j-1\circ}, \mathbf{f}_{j+1\circ}, \ldots, \mathbf{f}_{m\circ}\right)'$ For the denominator's $\hat{\sigma}_{iT}$, it was from $\hat{\sigma}_{iT}^2 = T^{-1}\sum_{t=1}^{T}\hat{u}_{it}^2$, the $\hat{u}_{it}$ is the residuals of the model. Then, we can use the same critical value from (5). Obtaining the correspond ratio $\hat{\pi}_{nTj}$ from (4), and after that use the function:

$$\hat{\alpha}_j = \begin{cases} 1 + \frac{\ln \hat{\pi}_{nT,j}}{\ln n}, & \text{if } \hat{\pi}_{nT,j} > 0 \\ 0, & \text{if } \hat{\pi}_{nT,j} = 0 \end{cases}$$

to estimates the factor loading.

2 Monte Carlo Design

2.1 Design

In order to study the finite sample property of factor strength $\hat{\alpha}_j$, we designed a Monte Carlo simulation. Through the simulation, we compare the property of the factor strength in different settings. We set up the experiments to reflect the CAPM model and it's extension. Consider the following data generating process (DGP):

$$r_{it} - r_{ft} = q_1(r_{mt} - r_f) + q_2\left(\sum_{j=1}^{k}\beta_{ij}f_{jt}\right) + \varepsilon_{it}$$

In the simulation, we consider a dataset has $i = 1, 2, \ldots, n$ different cross-section units, with $t = 1, 2, \ldots, T$ different observations. r_{it} is the unit's return, and r_{ft} represent the risk free rate at time t, therefore, the left hand side term $r_{it} - r_{ft}$ is the excess return of the unit i. For simplicity, we define $x_{it} := r_{it} - r_{ft}$. f_{jt} represents different risk factors, and the corresponding β_{ij} are the factor loadings. We use $r_{mt} - r_{ft}$ to denotes the market factor, and here r_{mt} is the average market return. Also, we use the term $f_{mt} := r_{mt} - r_{ft}$ to denotes the market factor. We expect the market factor

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69 will has strength equals to one all the time, so we consider the market factor has strength $\alpha_m = 1$.

70 ε_{it} is the stochastic error term. Therefore, the simulation model can be simplified as:

$$x_{it} = q_1(f_{mt}) + q_2\left(\sum_{j=1}^{k} \beta_{ij}f_{jt}\right) + \varepsilon_{it}$$

71     $q_1(\cdot)$ and $q_2(\cdot)$ are two different functions represent the unknown mechanism of market factor

72 and other risk factors in pricing asset risk. In the classical CAPM model and it's multi-factor ex-

73 tensions, for example the three factor model introduced by Fama and French (1992), both q_1 and

74 q_2 are linear.

For each factor, we assume they follow a multinomial distribution with mean zero and a $k \times k$

variance-covariance matrix Σ.

$$\mathbf{f_t} = \begin{pmatrix} f_{i,t} \\ f_{2,t} \\ \vdots \\ f_{k,t} \end{pmatrix} \sim MVN(\mathbf{0}, \Sigma) \quad \Sigma := \begin{pmatrix} \sigma_{f_1}^2, & \rho_{12}\sigma_{f1}\sigma_{f2} & \cdots & \rho_{1k}\sigma_{f1}\sigma_{fk} \\ \rho_{12}\sigma_{f2}\sigma_{f1}, & \sigma_{f2}^2 & \cdots & \rho_{2k}\sigma_{f2}\sigma_{fk} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1k}\sigma_{fk}\sigma_{f1}, & \rho_{k2}\sigma_{fk}\sigma_{f2} & \cdots & \sigma_{fk}^2 \end{pmatrix}$$

75 The diagonal of matrix Σ indicates the variance of each factor, and the rest represent the covariance

76 among all k factors.

77   2.2   Experiment Setting

Follow the general model above, we assume both $q_1(\cdot)$ and $q_2(\cdot)$ are linear function:

$$q_1(f_{mt}) = a_i + \beta_{im}f_{mt}$$

$$q_2\left(\sum_{j=1}^{k} \beta_{ij}f_{jt}\right) = \sum_{j=1}^{k} \beta_{ij}f_{jt}$$

78 To start the simulation, we consider a two factor model:

$$x_{it} = a_i + \beta_{i1}f_{1t} + \beta_{i2}f_{2t} + \varepsilon_{it} \tag{7}$$

The constant term a_i is generate from a uniform distribution, $a_{it} \sim U[-0.5, 0.5]$. For the factor loading β_{i1} and β_{i2}, we first use a uniform distribution $IIDU(\mu_\beta - 0.2, \mu_\beta + 0.2)$ to produce the values. Here we set $\mu_{beta} = 0.71$ to make sure every generated loading value is sufficiently larger than 0. Then we randomly assign $n - [n^{\alpha_1}]$ and $n - [n^{\alpha_2}]$ factor loadings as zero. α_1 and α_2 are the true factor strength of f_1 and f_2. In this simulation, we will start the factor strength from 0.7 and increase it gradually till unity with pace 0.05, say $(\alpha_1, \alpha_2) = \{0.7, 0.75, 0.8, \cdots, 1\}$. $[\cdot]$ is the integer operator defined at section (**??**). This step reflects the fact that only $[n^\alpha]$ factor loadings are non-zero. In terms of the factors, they comes from a multinomial distribution $MVN(\mathbf{0}, \Sigma)$, as we discuss before.

Currently, we consider three different experiments set up:

Experiment 1 (single factor, normal error, no correlation) *Set β_{i2} from (7) as 0, the error term ε_{it} and the factor f_{1t} are both standard normal.*

Experiment 2 (two factors, normal error, no correlation) *Both β_{i1} and β_{i2} are non-zero. Error term and both factors are standard normal. The correlation ρ_{12} between f_{1t} and f_{2t} is zero. The factor strength for the first factor $\alpha_1 = 1$ all the time, and α_2 various.*

Experiment 3 (two factors, normal error, weak correlation) *Both β_{i1} and β_{i2} are non-zero. Error term and both factors are standard normal. The correlation ρ_{12} between f_{1t} and f_{2t} is 0.3. The factor strength for the first factor $\alpha_1 = 1$ all the time, and α_2 various.*

The factor strength in each experiment is estimated using the method discussed in section (**??**), the size of significant test is $p = 0.05$, and the critical value exponent σ has been set as 0.5. For each of the experiment, we calculate the bias, the RMSE and the size of the test to justify the estimation performances. The bias is calculated as the difference between the true factor strength α and the estimate factor strength $\hat{\alpha}$. The Root Square Mean Error (RMSE) comes from:

$$RMSE = [\frac{1}{R}\sum_{r=1}^{R}(bias_r)^2]^{1/2}$$

Where the R represent the total replicate times. The size of the test is under the hypothesis that $H_0 : \hat{\alpha}_j = \alpha_j$, $j = 1, 2$ against the alternative hypothesis $H_1 : \hat{\alpha}_j \neq \alpha_j$, $j = 1, 2$. Here we employed the following test statistic from Bailey et al. (2020).

$$z_{\hat{\alpha}_j : \alpha_j} = \frac{(\ln n)\left(\hat{\alpha}_j - \alpha_j\right) - p\left(n - n^{\hat{\alpha}_j}\right) n^{-\delta - \hat{\alpha}_j}}{\left[p\left(n - n^{\hat{\alpha}_j}\right) n^{-\delta - 2\hat{\alpha}_j}\left(1 - \frac{p}{n^{\delta}}\right)\right]^{1/2}} \quad j = 1, 2 \tag{8}$$

Define a indicator function $\mathbf{1}(|z_{\hat{\alpha}_j : \alpha_j}| > c | H_0)$. For each replication, if this test statistic is greater than the critical value of standard normal distribution: $c = 1.96$, the indicator function will return value 1, and 0 otherwise. Therefore, we calculate the size of the test base on:

$$size = \frac{\sum_{r=1}^{R} \mathbf{1}(|z_{\hat{\alpha}_j : \alpha_j}| > 1.96 | H_0)}{R} \quad j = 1, 2, \tag{9}$$

In purpose of Monte Carlo Simulation, we consider the different combinations of T and n with $T = \{120, 240, 360\}$, $n = \{100, 300, 500\}$. The market factor, if included in the experiment, will have strength $\alpha_m = 1$ all the time, and the strength of the other factor will be $\alpha_x = \{0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1\}$. For every setting, we will replicate 2000 times independently, all the constant and variables will be re-generated for each replication.

2.3 Monte Carlo Discoveries

We report the results in Table (1) , (2) and (3) in Appendix A.

Table (1) provides the results under the experiment 1. The estimation method we applied tends to under-estimate the strength slightly most of the time when the true strength is relatively weak under the single factor set up. The bias is about 0.01 lower when the true underlying factor strength is 0.7. Such bias, however, vanish quickly while time t, unit amount n, and α increase. When we increase the time spam by including more data from the time dimensions, the bias, as well as the RMSE decrease significantly. Also, when including more cross-section unit n into the simulation, the performance of the estimation improves, showing by the decrease bias and RMSE values. An impressing result is that, the gap between estimation and true strength will goes to zero when we have $\alpha = 1$, the strongest we can have. With the strength approaching unity, the both bias and RMSE will converge to zero. Then we turn our attention to the size of the test. The size of the test will not variate too much when the strength increases, so as the unit increases, But we can observe that when observations for each unit increase, in other word, when t increases, the size will shrinkage dramatically. The size will smaller than the 0.05 threshold after we extend the t to 240, or

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empirically speaking, when we included 20 years monthly return data into estimation. Notice that, from the equation (8), when $\hat{\alpha} = \alpha = 1$, the nominator will becomes zero. Therefore, the size will collapse into zero in all settings, so we do not report the size for $\hat{\alpha} = \alpha = 1$

For the two factors scenarios, we obtain similar conclusions in both the no correlation setting and weak correlation setting. The result of no correlation settings is shown in the table (2), and the table (3) shows the result when the correlation between two factors is 0.3. Same as the single factor results, in most of the time, our estimation method will slightly under estimates the factor strength. But we can improve the estimation result by increasing either the observations amount t, or the cross-section units amount n. We also have the same unbiased estimation when true factor strength is unity under all unit-time combination. In some cases, even when the factor strength is relatively weak,we can have unbiased estimation if the n and t are big enough. (see table (3)). The results of size of the test in two factors setting are performing similar to the single factor result. The size will shrink with the observation amount t increasing, and when we have t grater than 240, the size will be smaller than 0.05 threshold in all situations.

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A   Simulation Result Table

Table 1: Simulation result for experiment 1

	Single Factor								
	Bias × 100			RMSE × 100			Size × 100		
	$\alpha_1 = 0.7$								
n\T	120	240	360	120	240	360	120	240	360
100	0.256	0.265	0.227	0.612	0.623	0.560	7.85	7.7	5.55
300	0.185	0.184	0.184	0.363	0.338	0.335	8.9	4.45	4.5
500	0.107	0.124	0.109	0.259	0.248	0.234	6.9	2.5	1.6
	$\alpha_1 = 0.75$								
100	-0.178	-0.159	-0.168	0.490	0.465	0.450	2.5	0.85	0.4
300	0.154	0.156	0.143	0.281	0.258	0.234	9.4	3.7	3.35
500	0.024	0.033	0.263	0.171	0.155	0.148	7.8	2	1.25
	$\alpha_1 = 0.8$								
100	-0.270	-0.265	-0.258	0.434	0.409	0.411	71.4	72.05	71.45
300	-0.052	-0.044	-0.043	0.183	0.149	0.150	10.15	2.45	2.9
500	0.045	0.068	0.067	0.136	0.126	0.121	16.6	6.4	5.9
	$\alpha_1 = 0.85$								
100	0.053	0.062	0.058	0.253	0.228	0.221	6.05	2.95	2.5
300	-0.012	0.009	-0.001	0.124	0.104	0.095	10.55	1.8	1.15
500	-0.026	-0.007	-0.011	0.096	0.073	0.069	13.25	0.9	0.7
	$\alpha_1 = 0.9$								
100	0.025	0.038	0.360	0.191	0.163	0.157	6.85	2	1.65
300	-0.034	-0.018	-0.020	0.099	0.069	0.068	13.2	0.8	0.9
500	-0.025	-0.001	-0.001	0.072	0.044	0.044	22.3	1.95	1.8
	$\alpha_1 = 0.95$								
100	-0.099	-0.088	-0.090	0.156	0.125	0.126	5.6	0.3	0.55
300	-0.046	-0.025	-0.026	0.083	0.045	0.045	22.5	2.2	2.25
500	-0.030	-0.006	-0.006	0.061	0.026	0.025	33.1	4.4	3.8
	$\alpha_1 = 1$								
100	0	0	0	0	0	0	-	-	-
300	0	0	0	0	0	0	-	-	-
500	0	0	0	0	0	0	-	-	-

Notes: This table shows the result of experiment 1. Factors and error are generate from standard normal distribution. Factor loadings come form uniform distribution $IIDU(\mu_\beta - 0.2, \mu_\beta + 0.2)$, and $\mu_\beta = 0.71$. We keep $[n^{\alpha_j}]$ amount of loadings and assign the rest as zero. For each different time-unit combinations, we replicate 2000 times. For the size of the test, we use a two-tail test, under the hypothesis of $H_0, \hat{\alpha}_j = \alpha_j \; j = 1,2$. Cause under the scenarios of $\alpha = 1$, the size of the test will collapse, therefore the table does not report the sizes for $\alpha_1 = 1$.

Table 2: Simulation result for experiment 2

	Double Factor with correlation $\rho_{12} = 0$								
	Bias $\times$ 100			RMSE $\times$ 100			Size $\times$ 100		
	$\alpha_1 = 1, \alpha_2 = 0.7$								
n\T	120	240	360	120	240	360	120	240	360
100	0.567	0.737	0.628	4.062	3.819	3.799	2.95	1.45	1.85
300	0.512	0.611	0.518	2.398	2.103	1.979	6.25	0.55	0.5
500	-0.149	0.08	-0.019	1.796	1.498	1.443	8	0.2	0.1
	$\alpha_1 = 1, \alpha_2 = 0.75$								
100	-3.051	-3.02	-3.092	4.582	4.245	4.248	2.45	0.1	0.10
300	0.491	-1.035	0.640	1.843	1.460	1.576	7.6	0.8	0.55
500	-0.611	-0.372	-0.393	1.520	1.136	1.125	11.35	0.15	0.1
	$\alpha_1 = 1, \alpha_2 = 0.8$								
100	-3.752	-3.630	-3.581	4.557	4.213	4.210	84.65	85.9	85.25
300	-1.218	-0.331	-1.021	1.812	0.792	1.438	9.35	0.2	0.3
500	-0.022	0.192	0.147	1.047	0.782	0.742	15.35	1.1	1.1
	$\alpha_1 = 1, \alpha_2 = 0.85$								
100	-0.075	0.127	0.088	1.996	1.697	1.606	5.4	1.15	0.95
300	-0.531	-0.406	-0.351	1.097	0.613	0.777	10.8	0.15	0.2
500	-0.647	-0.391	-0.391	1.020	0.643	0.630	19.1	0.15	0
	$\alpha_1 = 1, \alpha_2 = 0.9$								
100	-0.128	0.043	0.025	1.428	1.143	1.118	4.9	0.65	0.7
300	-0.651	-0.334	-0.394	1.002	0.435	0.617	17.1	0.6	0.2
500	-0.434	-0.168	-0.171	0.7435	0.367	0.368	25.2	0.4	0.3
	$\alpha_1 = 1, \alpha_2 = 0.95$								
100	-1.218	-1.043	-1.036	1.603	1.222	1.212	6.65	0.25	0.05
300	-0.611	-0.344	-0.356	0.881	0.435	0.434	23.35	0.6	0.45
500	-0.415	-0.123	-0.134	0.661	0.220	0.216	36.75	1.35	1.1
	$\alpha_1 = 1, \alpha_2 = 1$								
100	0	0	0	0	0	0	-	-	-
300	0	0	0	0	0	0	-	-	-
500	0	0	0	0	0	0	-	-	-

Notes: This table shows the result of experiment 2. Factors and errors are generate from standard normal distribution. Between two factors, we assume they have no correlation. Factor loadings come form uniform distribution $IIDU(\mu_\beta - 0.2, \mu_\beta + 0.2)$, and μ_β is set to 0.71. We keep $[n^{\alpha_j}]$ amount of loadings and assign the rest as zero. For each different time-unit combinations, we replicate 2000 times. For the size of the test, we use a two-tail test, under the hypothesis of $H_0, \hat{\alpha}_j = \alpha_j \ j = 1, 2$. Cause under the scenarios of $\alpha = 1$, the size of the test will collapse, therefore the table does not report the sizes for $\alpha_1 = \alpha_2 = 1$

Table 3: Simulation result for experiment 3

	Double Factor with correlation $\rho_{12} = 0.3$								
	Bias $\times$ 100			RMSE $\times$ 100			Size $\times$ 100		
	$\alpha_1 = 1, \alpha_2 = 0.7$								
n\T	120	240	360	120	240	360	120	240	360
100	0.038	0.064	0.072	0.421	0.382	0.389	4.6	1.75	1.95
300	0.021	0.058	0.056	0.253	0.206	0.198	9.95	0.9	0.25
500	-0.032	0.006	0	0.201	0.153	0	12.20	0.1	0.05
	$\alpha_1 = 1, \alpha_2 = 0.75$								
100	-0.325	-0.313	-0.310	0.488	0.419	0.420	4.75	0.1	0
300	0.028	0.063	0.065	0.253	0.157	0.159	9.95	0.55	0.5
500	-0.082	-0.037	-0.039	0.175	0.114	0.112	19.25	0.25	0.3
	$\alpha_1 = 1, \alpha_2 = 0.8$								
100	-0.393	-0.361	-0.368	0.477	0.418	0.421	85.45	85.2	86.4
300	0.029	-0.099	-0.100	0.192	0.145	0.145	12.2	0.65	0.5
500	-0.037	-0.016	0.016	0.129	0.074	0.074	27.8	0.25	1.2
	$\alpha_1 = 1, \alpha_2 = 0.85$								
100	-0.027	0.008	0.007	0.234	0.160	0.155	9.3	0.9	0.65
300	-0.147	-0.031	-0.037	0.219	0.079	0.077	16.75	0.3	0.2
500	-0.088	-0.039	-0.039	0.136	0.063	0.062	30.6	0.15	0
	$\alpha_1 = 1, \alpha_2 = 0.9$								
100	-0.033	0.003	0.002	0.173	0.111	0.110	9.4	0.6	0.55
300	-0.087	-0.040	-0.041	0.131	0.061	0.061	27.8	0.1	0.05
500	-0.070	-0.017	-0.018	0.111	0.037	0.037	41.15	0.6	0.35
	$\alpha_1 = 1, \alpha_2 = 0.95$								
100	-0.134	-0.101	-0.104	0.185	0.122	0.122	10.15	0.1	0.15
300	-0.083	-0.034	-0.034	0.118	0.043	0.044	39.35	0.6	0.6
500	-0.062	-0.013	-0.012	0.937	0.022	0.023	51.8	1.25	2.0
	$\alpha_1 = 1, \alpha_2 = 1$								
100	0	0	0	0	0	0	-	-	-
300	0	0	0	0	0	0	-	-	-
500	0	0	0	0	0	0	-	-	-

Notes: This table shows the result of experiment 2. Factors and errors are generate from standard normal distribution. Between two factors, we assume they have correlation $\rho_{12} = 0.3$ Factor loadings come form uniform distribution $IIDU(\mu_\beta - 0.2, \mu_\beta + 0.2)$, and μ_β is set to 0.71. We keep $[n^{\alpha_j}]$ amount of loadings and assign the rest as zero. For each different time-unit combinations, we replicate 2000 times. For the size of the test, we use a two-tail test, under the hypothesis of $H_0, \hat{\alpha}_j = \alpha_j \; j = 1, 2$. Cause under the scenarios of $\alpha = 1$, the size of the test will collapse, therefore the table does not report the sizes when $\alpha_1 = \alpha_2 = 1$

Table 4: Ranked Three Data Set Comparison

	Ten Year Data			Twenty Year Data			Thirty Year Data		
	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength
1	beta	0.976	0.749	ndp	0.960	0.904	salecash	0.905	0.857
2	baspread	0.980	0.730	salecash	0.958	0.902	ndp	0.905	0.852
3	turn	0.983	0.728	quick	0.958	0.901	quick	0.905	0.851
4	zerotrade	0.983	0.725	dy	0.957	0.897	age	0.905	0.851
5	idiovol	0.981	0.723	lev	0.959	0.897	roavol	0.904	0.850
6	retvol	0.978	0.721	cash	0.958	0.897	ep	0.905	0.849
7	std_turn	0.983	0.719	zs	0.959	0.896	depr	0.905	0.848
8	HML_Devil	0.989	0.719	cp	0.960	0.894	cash	0.905	0.847
9	maxret	0.981	0.715	roavol	0.957	0.894	rds	0.905	0.843
10	roavol	0.985	0.713	age	0.959	0.894	currat	0.905	0.840
11	age	0.989	0.703	cfp	0.960	0.893	chcsho	0.905	0.840
12	sp	0.985	0.699	op	0.958	0.893	zs	0.903	0.839
13	ala	0.986	0.699	nop	0.958	0.893	nop	0.904	0.839
14	ndp	0.987	0.686	ebp	0.959	0.893	dy	0.905	0.838
15	orgcap	0.989	0.686	ep	0.958	0.891	lev	0.903	0.838
16	tang	0.990	0.683	rds	0.958	0.890	cfp	0.905	0.838
17	ebp	0.988	0.683	depr	0.958	0.889	stdacc	0.905	0.837
18	invest	0.986	0.683	sp	0.958	0.888	cp	0.905	0.836
19	dpia	0.986	0.681	currat	0.958	0.887	stdcf	0.905	0.836
20	UMD	0.989	0.678	kz	0.958	0.887	op	0.904	0.835
21	zs	0.986	0.675	chcsho	0.957	0.884	ebp	0.903	0.835
22	grltnoa	0.988	0.675	tang	0.960	0.884	tang	0.904	0.833
23	dy	0.988	0.672	ato	0.958	0.884	kz	0.903	0.831
24	HML	0.987	0.672	stdacc	0.958	0.883	ato	0.904	0.831
25	kz	0.986	0.669	adm	0.958	0.881	ww	0.904	0.827
26	ob_a	0.989	0.669	cashpr	0.959	0.878	std_turn	0.902	0.826
27	BAB	0.989	0.666	stdcf	0.956	0.878	adm	0.904	0.825
28	op	0.990	0.663	HML	0.958	0.874	idiovol	0.902	0.825
29	realestate_hxz	0.987	0.663	nef	0.956	0.873	maxret	0.902	0.825
30	ol	0.987	0.663	std_turn	0.956	0.870	baspread	0.902	0.820
31	adm	0.988	0.660	idiovol	0.955	0.870	IPO	0.905	0.818
32	lev	0.986	0.657	zerotrade	0.953	0.865	nef	0.902	0.818
33	nxf	0.989	0.651	turn	0.955	0.864	sp	0.903	0.817

Table 4: Three Data Set Comparison (Cont.)

		Ten Year Data			Twenty Year Data			Thirty Year Data	
	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength
34	nop	0.989	0.651	ww	0.959	0.863	turn	0.902	0.813
35	pm	0.986	0.648	maxret	0.956	0.863	retvol	0.902	0.813
36	pchcapx3	0.988	0.644	absacc	0.960	0.859	zerotrade	0.900	0.812
37	nef	0.988	0.644	baspread	0.955	0.854	absacc	0.905	0.812
38	cash	0.989	0.637	hire	0.959	0.851	HML	0.903	0.811
39	QMJ	0.978	0.637	IPO	0.960	0.850	lgr	0.905	0.810
40	rds	0.989	0.634	lgr	0.959	0.850	cashpr	0.903	0.808
41	LIQ_PS	0.988	0.634	nxf	0.956	0.849	dcol	0.905	0.807
42	ato	0.988	0.634	retvol	0.955	0.848	beta	0.900	0.806
43	salerec	0.992	0.630	salerec	0.957	0.847	RMW	0.904	0.806
44	currat	0.989	0.626	RMW	0.957	0.847	hire	0.905	0.805
45	acc	0.989	0.619	beta	0.954	0.846	salerec	0.905	0.803
46	stdcf	0.989	0.619	sin	0.959	0.844	nxf	0.903	0.801
47	HXZ_ROE	0.989	0.619	acc	0.960	0.843	acc	0.904	0.797
48	depr	0.988	0.615	bm_ia	0.960	0.843	dfin	0.902	0.791
49	noa	0.989	0.615	dcol	0.959	0.838	nincr	0.904	0.790
50	cashpr	0.987	0.615	dfin	0.959	0.838	noa	0.902	0.787
51	absacc	0.989	0.615	HML_Devil	0.953	0.838	HML_Devil	0.902	0.781
52	gma	0.987	0.615	HXZ_IA	0.960	0.838	HXZ_IA	0.904	0.780
53	dncl	0.986	0.611	nincr	0.959	0.834	rdm	0.904	0.778
54	ms	0.980	0.611	rna	0.958	0.826	rna	0.904	0.778
55	rna	0.989	0.611	noa	0.957	0.825	rd	0.903	0.774
56	STR	0.987	0.607	herf	0.957	0.824	bm_ia	0.904	0.772
57	rdm	0.988	0.607	rdm	0.958	0.823	sgr	0.904	0.769
58	chcsho	0.987	0.607	sgr	0.958	0.819	ps	0.904	0.769
59	sin	0.987	0.607	dnco	0.959	0.816	sin	0.904	0.769
60	salecash	0.989	0.602	ps	0.957	0.807	realestate_hxz	0.905	0.769
61	dnco	0.988	0.598	CMA	0.960	0.805	herf	0.902	0.766
62	quick	0.989	0.593	egr_hxz	0.958	0.803	dnco	0.904	0.761
63	stdacc	0.989	0.593	realestate_hxz	0.957	0.798	CMA	0.905	0.759
64	poa	0.988	0.593	gad	0.958	0.788	egr_hxz	0.904	0.750
65	cp	0.988	0.589	rd	0.958	0.787	ob_a	0.903	0.745
66	tb	0.988	0.589	ol	0.954	0.787	ol	0.902	0.741
67	HXZ_IA	0.987	0.584	cinvest_a	0.959	0.784	cinvest_a	0.903	0.739

Table 4: Three Data Set Comparison (Cont.)

| | | Ten Year Data | | | Twenty Year Data | | | Thirty Year Data | |
	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength
68	saleinv	0.987	0.579	dolvol	0.960	0.774	gad	0.902	0.723
69	cfp	0.988	0.579	ob_a	0.955	0.764	SMB	0.902	0.721
70	egr	0.987	0.579	ala	0.958	0.762	dolvol	0.904	0.715
71	dnca	0.986	0.579	pchdepr	0.959	0.761	gma	0.902	0.715
72	egr_hxz	0.988	0.579	BAB	0.960	0.757	ala	0.904	0.715
73	os	0.984	0.569	gma	0.955	0.756	cto	0.902	0.710
74	pps	0.983	0.563	pchcapx3	0.957	0.752	aeavol	0.905	0.710
75	cto	0.987	0.563	dnca	0.958	0.747	BAB	0.905	0.710
76	grltnoa_hxz	0.986	0.563	SMB	0.957	0.745	convind	0.904	0.710
77	cei	0.988	0.563	poa	0.957	0.739	tb	0.902	0.708
78	CMA	0.988	0.563	aeavol	0.961	0.737	QMJ	0.903	0.708
79	em	0.989	0.552	tb	0.953	0.732	pricedelay	0.904	0.701
80	ww	0.990	0.546	grltnoa_hxz	0.958	0.730	egr	0.902	0.699
81	std_dolvol	0.987	0.539	cei	0.953	0.730	orgcap	0.902	0.699
82	grcapx	0.986	0.539	indmom	0.956	0.725	pchdepr	0.903	0.696
83	pctacc	0.989	0.539	egr	0.958	0.725	indmom	0.902	0.696
84	ep	0.989	0.533	moms12m	0.957	0.725	dcoa	0.902	0.696
85	pricedelay	0.989	0.533	dsti	0.957	0.723	moms12m	0.903	0.694
86	hire	0.988	0.519	orgcap	0.956	0.715	pchcapx3	0.902	0.691
87	SMB	0.987	0.512	pchcurrat	0.958	0.710	cei	0.902	0.691
88	pchcapx_ia	0.989	0.512	UMD	0.951	0.706	roic	0.902	0.691
89	aeavol	0.988	0.512	dcoa	0.959	0.706	pm	0.903	0.691
90	moms12m	0.987	0.512	roic	0.951	0.703	dnca	0.902	0.689
91	cashdebt	0.984	0.504	QMJ	0.951	0.703	saleinv	0.903	0.686
92	lgr	0.987	0.504	cinvest	0.958	0.701	grltnoa_hxz	0.903	0.683
93	cinvest	0.988	0.496	HXZ_ROE	0.957	0.699	poa	0.903	0.681
94	herf	0.987	0.496	cto	0.955	0.694	HXZ_ROE	0.905	0.678
95	bm_ia	0.988	0.487	pctacc	0.954	0.694	UMD	0.902	0.672
96	cfp_ia	0.987	0.479	pricedelay	0.958	0.691	pctacc	0.902	0.672
97	cinvest_a	0.989	0.479	pchcapx_ia	0.957	0.681	cinvest	0.903	0.660
98	chmom	0.989	0.469	convind	0.955	0.669	dsti	0.902	0.660
99	RMW	0.987	0.469	cdi	0.958	0.654	em	0.902	0.657
100	sue	0.987	0.459	rsup	0.957	0.651	pchcurrat	0.902	0.654
101	mom36m	0.986	0.459	chtx	0.958	0.644	ms	0.902	0.648

Table 4: Three Data Set Comparison (Cont.)

		Ten Year Data			Twenty Year Data			Thirty Year Data	
	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength
102	indmom	0.987	0.459	invest	0.957	0.644	invest	0.902	0.641
103	dcoa	0.988	0.459	em	0.952	0.644	pchcapx_ia	0.902	0.630
104	etr	0.986	0.448	pm	0.957	0.641	os	0.900	0.623
105	chinv	0.988	0.448	saleinv	0.955	0.637	chtx	0.902	0.623
106	ill	0.988	0.448	ta	0.958	0.634	dpia	0.902	0.623
107	roic	0.986	0.448	dpia	0.957	0.634	cdi	0.903	0.623
108	convind	0.988	0.448	pchquick	0.957	0.626	pps	0.902	0.611
109	sgr	0.988	0.437	os	0.948	0.626	roaq	0.900	0.602
110	IPO	0.989	0.437	ms	0.950	0.619	rs	0.902	0.584
111	dolvol	0.989	0.437	roaq	0.953	0.607	rsup	0.902	0.579
112	dcol	0.987	0.425	grcapx	0.955	0.593	chinv	0.902	0.569
113	nincr	0.989	0.411	pps	0.952	0.589	cfp_ia	0.902	0.563
114	chempia	0.987	0.411	ndf	0.957	0.589	ta	0.903	0.563
115	rs	0.988	0.411	cfp_ia	0.957	0.584	cashdebt	0.900	0.557
116	pchcapx	0.988	0.411	dncl	0.957	0.584	ndf	0.902	0.557
117	chtx	0.988	0.397	pchsale_pchrect	0.955	0.574	grcapx	0.902	0.552
118	ivg	0.988	0.381	mom6m	0.958	0.569	STR	0.902	0.546
119	LTR	0.985	0.364	rs	0.955	0.563	pchcapx	0.902	0.546
120	mom6m	0.987	0.364	pchcapx	0.958	0.563	pchquick	0.902	0.539
121	cdi	0.987	0.364	cashdebt	0.951	0.557	grltnoa	0.902	0.539
122	chatoia	0.987	0.364	pchsaleinv	0.955	0.557	pchsaleinv	0.902	0.519
123	gad	0.985	0.364	chempia	0.958	0.557	dncl	0.902	0.519
124	pchcurrat	0.988	0.297	LIQ_PS	0.956	0.557	ivg	0.902	0.504
125	pchgm_pchsale	0.988	0.297	dwc	0.955	0.546	mom6m	0.902	0.496
126	rd	0.986	0.297	grltnoa	0.956	0.533	chempia	0.902	0.496
127	dsti	0.989	0.297	STR	0.956	0.526	LIQ_PS	0.902	0.496
128	dfnl	0.987	0.297	dfnl	0.955	0.519	mom36m	0.902	0.479
129	roaq	0.986	0.297	mom36m	0.957	0.496	std_dolvol	0.903	0.459
130	pchdepr	0.988	0.266	std_dolvol	0.955	0.496	pchsale_pchinvt	0.902	0.448
131	dnoa	0.988	0.230	sue	0.956	0.487	pchsale_pchxsga	0.902	0.448
132	ta	0.988	0.230	LTR	0.954	0.487	dwc	0.902	0.448
133	chpmia	0.987	0.230	chmom	0.953	0.479	dfnl	0.902	0.437
134	pchquick	0.987	0.182	pchsale_pchinvt	0.955	0.448	chmom	0.902	0.437
135	dfin	0.988	0.182	chatoia	0.957	0.437	pchsale_pchrect	0.902	0.425

Table 4: Three Data Set Comparison (Cont.)

		Ten Year Data			Twenty Year Data			Thirty Year Data	
	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength
136	rsup	0.988	0.182	pchsale_pchxsga	0.957	0.425	sue	0.902	0.397
137	pchsaleinv	0.988	0.115	lfe	0.956	0.425	LTR	0.902	0.381
138	pchsale_pchinvt	0.988	0.115	chinv	0.956	0.397	pchgm_pchsale	0.902	0.322
139	pchsale_pchrect	0.988	0.115	ivg	0.957	0.397	lfe	0.902	0.297
140	ps	0.990	0.115	pchgm_pchsale	0.957	0.381	ill	0.902	0.297
141	dwc	0.989	0.115	etr	0.955	0.344	dnoa	0.902	0.182
142	pchsale_pchxsga	0.989	0.000	chpmia	0.957	0.344	ear	0.903	0.182
143	lfe	0.988	0.000	ill	0.955	0.266	chatoia	0.902	0.182
144	ndf	0.986	0.000	dnoa	0.955	0.266	chpmia	0.902	0.182
145	ear	0.988	0.000	ear	0.958	0.266	etr	0.902	0.115

Notes: This table presents the estimation results of factors' strength, ordered by risk factor strength in decreasing sequence. For the estimation, we use the method from Section 1.3, with one market factor and one risk factor.

Table 5: Three Data Set Comparison

Factor	10 Year Strength	20 Year Strength	30 Year Strength	Mean	Standard Deviation
1 ps	0.115	0.807	0.769	0.564	0.318
2 dfin	0.182	0.838	0.791	0.604	0.299
3 ndf	0.000	0.589	0.557	0.382	0.270
4 rd	0.297	0.787	0.774	0.619	0.228
5 pchdepr	0.266	0.761	0.696	0.574	0.219
6 rsup	0.182	0.651	0.579	0.471	0.206
7 pchsale_pchxsga	0.000	0.425	0.448	0.291	0.206
8 pchsaleinv	0.115	0.557	0.519	0.397	0.200
9 pchquick	0.182	0.626	0.539	0.449	0.192
10 pchsale_pchrect	0.115	0.574	0.425	0.371	0.191
11 nincr	0.411	0.834	0.790	0.678	0.190
12 dsti	0.297	0.723	0.660	0.560	0.188
13 dcol	0.425	0.838	0.807	0.690	0.188
14 IPO	0.437	0.850	0.818	0.702	0.188
15 gad	0.364	0.788	0.723	0.625	0.187
16 dwc	0.115	0.546	0.448	0.370	0.185
17 pchcurrat	0.297	0.710	0.654	0.554	0.183
18 lfe	0.000	0.425	0.297	0.240	0.178
19 ta	0.230	0.634	0.563	0.475	0.176
20 sgr	0.437	0.819	0.769	0.675	0.170
21 RMW	0.469	0.847	0.806	0.707	0.169
22 ep	0.533	0.891	0.849	0.758	0.160
23 pchsale_pchinvt	0.115	0.448	0.448	0.337	0.157
24 lgr	0.504	0.850	0.810	0.721	0.155
25 bm_ia	0.487	0.843	0.772	0.701	0.154
26 dolvol	0.437	0.774	0.715	0.642	0.147
27 hire	0.519	0.851	0.805	0.725	0.147
28 roaq	0.297	0.607	0.602	0.502	0.145
29 herf	0.496	0.824	0.766	0.695	0.143
30 ww	0.546	0.863	0.827	0.745	0.142
31 etr	0.448	0.344	0.115	0.302	0.139
32 cfp	0.579	0.893	0.838	0.770	0.137
33 quick	0.593	0.901	0.851	0.782	0.135
34 cinvest_a	0.479	0.784	0.739	0.667	0.135
35 cp	0.589	0.894	0.836	0.773	0.133
36 salecash	0.602	0.902	0.857	0.787	0.132
37 cdi	0.364	0.654	0.623	0.547	0.130
38 stdacc	0.593	0.883	0.837	0.771	0.127
39 chcsho	0.607	0.884	0.840	0.777	0.122
40 depr	0.615	0.889	0.848	0.784	0.121
41 indmom	0.459	0.725	0.696	0.627	0.119
42 roic	0.448	0.703	0.691	0.614	0.117
43 convind	0.448	0.669	0.710	0.609	0.115
44 dcoa	0.459	0.706	0.696	0.620	0.114
45 stdcf	0.619	0.878	0.836	0.778	0.114
46 currat	0.626	0.887	0.840	0.785	0.113
47 cash	0.637	0.897	0.847	0.794	0.113
48 chtx	0.397	0.644	0.623	0.555	0.112
49 rds	0.634	0.890	0.843	0.789	0.111
50 cashpr	0.615	0.878	0.808	0.767	0.111

Table 5: Three Data Set Comparison (Cont.)

Factor	10 Year Strength	20 Year Strength	30 Year Strength	Mean	Standard Deviation
51 ear	0.000	0.266	0.182	0.149	0.111
52 HXZ_IA	0.584	0.838	0.780	0.734	0.109
53 ato	0.634	0.884	0.831	0.783	0.107
54 chatoia	0.364	0.437	0.182	0.328	0.107
55 absacc	0.615	0.859	0.812	0.762	0.106
56 SMB	0.512	0.745	0.721	0.659	0.105
57 CMA	0.563	0.805	0.759	0.709	0.105
58 nop	0.651	0.893	0.839	0.794	0.104
59 lev	0.657	0.897	0.838	0.798	0.102
60 aeavol	0.512	0.737	0.710	0.653	0.101
61 sin	0.607	0.844	0.769	0.740	0.099
62 nef	0.644	0.873	0.818	0.778	0.097
63 op	0.663	0.893	0.835	0.797	0.097
64 acc	0.619	0.843	0.797	0.753	0.097
65 egr_hxz	0.579	0.803	0.750	0.711	0.096
66 dy	0.672	0.897	0.838	0.803	0.095
67 moms12m	0.512	0.725	0.694	0.644	0.094
68 adm	0.660	0.881	0.825	0.789	0.094
69 salerec	0.630	0.847	0.803	0.760	0.094
70 zs	0.675	0.896	0.839	0.803	0.094
71 rdm	0.607	0.823	0.778	0.736	0.093
72 ndp	0.686	0.904	0.852	0.814	0.093
73 dnco	0.598	0.816	0.761	0.725	0.093
74 rna	0.611	0.826	0.778	0.738	0.092
75 kz	0.669	0.887	0.831	0.796	0.092
76 dfnl	0.297	0.519	0.437	0.418	0.092
77 noa	0.615	0.825	0.787	0.742	0.091
78 cinvest	0.496	0.701	0.660	0.619	0.089
79 ebp	0.683	0.893	0.835	0.804	0.088
80 tang	0.683	0.884	0.833	0.800	0.085
81 mom6m	0.364	0.569	0.496	0.476	0.085
82 nxf	0.651	0.849	0.801	0.767	0.084
83 HML	0.672	0.874	0.811	0.786	0.084
84 age	0.703	0.894	0.851	0.816	0.082
85 ill	0.448	0.266	0.297	0.337	0.080
86 sp	0.699	0.888	0.817	0.801	0.078
87 roavol	0.713	0.894	0.850	0.819	0.077
88 pricedelay	0.533	0.691	0.701	0.642	0.077
89 rs	0.411	0.563	0.584	0.519	0.077
90 chinv	0.448	0.397	0.569	0.471	0.072
91 cei	0.563	0.730	0.691	0.661	0.071
92 pchcapx_ia	0.512	0.681	0.630	0.608	0.071
93 grltnoa_hxz	0.563	0.730	0.683	0.659	0.070
94 dnca	0.579	0.747	0.689	0.671	0.070
95 pctacc	0.539	0.694	0.672	0.635	0.068
96 chpmia	0.230	0.344	0.182	0.252	0.068
97 pchcapx	0.411	0.563	0.546	0.507	0.068
98 cto	0.563	0.694	0.710	0.656	0.066
99 grltnoa	0.675	0.533	0.539	0.582	0.066
100 egr	0.579	0.725	0.699	0.668	0.064
101 std_turn	0.719	0.870	0.826	0.805	0.064

Table 5: Three Data Set Comparison (Cont.)

Factor	10 Year Strength	20 Year Strength	30 Year Strength	Mean	Standard Deviation
102 maxret	0.715	0.863	0.825	0.801	0.063
103 tb	0.589	0.732	0.708	0.676	0.063
104 idiovol	0.723	0.870	0.825	0.806	0.061
105 poa	0.593	0.739	0.681	0.671	0.060
106 chempia	0.411	0.557	0.496	0.488	0.060
107 gma	0.615	0.756	0.715	0.695	0.059
108 realestate_hxz	0.663	0.798	0.769	0.743	0.058
109 zerotrade	0.725	0.865	0.812	0.801	0.058
110 turn	0.728	0.864	0.813	0.802	0.056
111 LIQ_PS	0.634	0.557	0.496	0.562	0.056
112 LTR	0.364	0.487	0.381	0.411	0.055
113 ivg	0.381	0.397	0.504	0.427	0.055
114 retvol	0.721	0.848	0.813	0.794	0.054
115 baspread	0.730	0.854	0.820	0.801	0.053
116 ol	0.663	0.787	0.741	0.731	0.051
117 HML_Devil	0.719	0.838	0.781	0.779	0.049
118 em	0.552	0.644	0.657	0.618	0.047
119 cfp_ia	0.479	0.584	0.563	0.542	0.046
120 pchcapx3	0.644	0.752	0.691	0.696	0.044
121 saleinv	0.579	0.637	0.686	0.634	0.044
122 ob_a	0.669	0.764	0.745	0.726	0.041
123 beta	0.749	0.846	0.806	0.800	0.040
124 dncl	0.611	0.584	0.519	0.571	0.038
125 sue	0.459	0.487	0.397	0.448	0.038
126 BAB	0.666	0.757	0.710	0.711	0.037
127 pchgm_pchsale	0.297	0.381	0.322	0.333	0.035
128 dnoa	0.230	0.266	0.182	0.226	0.035
129 STR	0.607	0.526	0.546	0.559	0.034
130 HXZ_ROE	0.619	0.699	0.678	0.665	0.034
131 std_dolvol	0.539	0.496	0.459	0.498	0.033
132 QMJ	0.637	0.703	0.708	0.683	0.032
133 ala	0.699	0.762	0.715	0.725	0.027
134 os	0.569	0.626	0.623	0.606	0.026
135 cashdebt	0.504	0.557	0.557	0.540	0.025
136 dpia	0.681	0.634	0.623	0.646	0.025
137 grcapx	0.539	0.593	0.552	0.561	0.023
138 pm	0.648	0.641	0.691	0.660	0.022
139 pps	0.563	0.589	0.611	0.587	0.019
140 invest	0.683	0.644	0.641	0.656	0.019
141 chmom	0.469	0.479	0.437	0.461	0.018
142 ms	0.611	0.619	0.648	0.626	0.016
143 mom36m	0.459	0.496	0.479	0.478	0.015
144 UMD	0.678	0.706	0.672	0.685	0.015
145 orgcap	0.686	0.715	0.699	0.700	0.012

[158] **Notes:** This table shows the factor strength for three data sets, ranked by the standard deviation among the three data sets in decreasing sequence.

Table 6: Ten and Twenty Comparison

	Factor	10 Year Strength	20 Year Strength	Difference
1	ps	0.115	0.807	0.692
2	dfin	0.182	0.838	0.656
3	ndf	0.000	0.589	0.589
4	pchdepr	0.266	0.761	0.494
5	rd	0.297	0.787	0.490
6	rsup	0.182	0.651	0.469
7	pchsale_pchrect	0.115	0.574	0.459
8	pchquick	0.182	0.626	0.445
9	pchsaleinv	0.115	0.557	0.443
10	dwc	0.115	0.546	0.431
11	dsti	0.297	0.723	0.427
12	pchsale_pchxsga	0.000	0.425	0.425
13	lfe	0.000	0.425	0.425
14	gad	0.364	0.788	0.425
15	nincr	0.411	0.834	0.423
16	pchcurrat	0.297	0.710	0.414
17	dcol	0.425	0.838	0.414
18	IPO	0.437	0.850	0.413
19	ta	0.230	0.634	0.404
20	sgr	0.437	0.819	0.382
21	RMW	0.469	0.847	0.378
22	ep	0.533	0.891	0.358
23	bm_ia	0.487	0.843	0.356
24	lgr	0.504	0.850	0.346
25	dolvol	0.437	0.774	0.337
26	pchsale_pchinvt	0.115	0.448	0.334
27	hire	0.519	0.851	0.332
28	herf	0.496	0.824	0.328
29	ww	0.546	0.863	0.318
30	cfp	0.579	0.893	0.314
31	roaq	0.297	0.607	0.310
32	quick	0.593	0.901	0.308
33	cp	0.589	0.894	0.306
34	cinvest_a	0.479	0.784	0.306
35	salecash	0.602	0.902	0.300
36	cdi	0.364	0.654	0.290
37	stdacc	0.593	0.883	0.290
38	chcsho	0.607	0.884	0.278
39	depr	0.615	0.889	0.274
40	indmom	0.459	0.725	0.266
41	ear	0.000	0.266	0.266
42	cashpr	0.615	0.878	0.263
43	currat	0.626	0.887	0.260
44	cash	0.637	0.897	0.260
45	stdcf	0.619	0.878	0.259
46	rds	0.634	0.890	0.256
47	roic	0.448	0.703	0.255
48	HXZ_IA	0.584	0.838	0.254
49	ato	0.634	0.884	0.250
50	chtx	0.397	0.644	0.247

Table 6: Ten and Twenty Comparison (Cont.)

	Factor	10 Year Strength	20 Year Strength	Difference
51	dcoa	0.459	0.706	0.247
52	absacc	0.615	0.859	0.244
53	nop	0.651	0.893	0.242
54	CMA	0.563	0.805	0.241
55	lev	0.657	0.897	0.240
56	sin	0.607	0.844	0.238
57	SMB	0.512	0.745	0.233
58	op	0.663	0.893	0.230
59	nef	0.644	0.873	0.229
60	aeavol	0.512	0.737	0.226
61	dy	0.672	0.897	0.225
62	acc	0.619	0.843	0.225
63	egr_hxz	0.579	0.803	0.224
64	dfnl	0.297	0.519	0.222
65	convind	0.448	0.669	0.221
66	zs	0.675	0.896	0.221
67	adm	0.660	0.881	0.221
68	ndp	0.686	0.904	0.218
69	dnco	0.598	0.816	0.218
70	kz	0.669	0.887	0.217
71	salerec	0.630	0.847	0.217
72	rdm	0.607	0.823	0.216
73	rna	0.611	0.826	0.215
74	moms12m	0.512	0.725	0.214
75	noa	0.615	0.825	0.210
76	ebp	0.683	0.893	0.210
77	cinvest	0.496	0.701	0.205
78	mom6m	0.364	0.569	0.205
79	HML	0.672	0.874	0.202
80	tang	0.683	0.884	0.200
81	nxf	0.651	0.849	0.198
82	age	0.703	0.894	0.191
83	sp	0.699	0.888	0.189
84	roavol	0.713	0.894	0.182
85	ill	0.448	0.266	0.182
86	pchcapx_ia	0.512	0.681	0.169
87	dnca	0.579	0.747	0.168
88	grltnoa_hxz	0.563	0.730	0.166
89	cei	0.563	0.730	0.166
90	pricedelay	0.533	0.691	0.158
91	pctacc	0.539	0.694	0.154
92	rs	0.411	0.563	0.152
93	pchcapx	0.411	0.563	0.152
94	std_turn	0.719	0.870	0.151
95	maxret	0.715	0.863	0.149
96	idiovol	0.723	0.870	0.147
97	egr	0.579	0.725	0.147
98	poa	0.593	0.739	0.146
99	chempia	0.411	0.557	0.146
100	tb	0.589	0.732	0.143
101	grltnoa	0.675	0.533	0.142

Factor	10 Year Strength	20 Year Strength	Difference
102 gma	0.615	0.756	0.141
103 zerotrade	0.725	0.865	0.140
104 turn	0.728	0.864	0.137
105 realestate_hxz	0.663	0.798	0.135
106 cto	0.563	0.694	0.131
107 retvol	0.721	0.848	0.127
108 baspread	0.730	0.854	0.125
109 LTR	0.364	0.487	0.124
110 ol	0.663	0.787	0.124
111 HML_Devil	0.719	0.838	0.119
112 chpmia	0.230	0.344	0.115
113 pchcapx3	0.644	0.752	0.108
114 cfp_ia	0.479	0.584	0.105
115 etr	0.448	0.344	0.104
116 beta	0.749	0.846	0.098
117 ob_a	0.669	0.764	0.095
118 em	0.552	0.644	0.093
119 BAB	0.666	0.757	0.091
120 pchgm_pchsale	0.297	0.381	0.085
121 STR	0.607	0.526	0.080
122 HXZ_ROE	0.619	0.699	0.080
123 LIQ_PS	0.634	0.557	0.076
124 chatoia	0.364	0.437	0.073
125 QMJ	0.637	0.703	0.066
126 ala	0.699	0.762	0.064
127 saleinv	0.579	0.637	0.059
128 os	0.569	0.626	0.058
129 grcapx	0.539	0.593	0.054
130 cashdebt	0.504	0.557	0.053
131 chinv	0.448	0.397	0.051
132 dpia	0.681	0.634	0.047
133 std_dolvol	0.539	0.496	0.043
134 invest	0.683	0.644	0.039
135 mom36m	0.459	0.496	0.037
136 dnoa	0.230	0.266	0.037
137 orgcap	0.686	0.715	0.029
138 sue	0.459	0.487	0.028
139 UMD	0.678	0.706	0.028
140 dncl	0.611	0.584	0.027
141 pps	0.563	0.589	0.026
142 ivg	0.381	0.397	0.016
143 chmom	0.469	0.479	0.009
144 ms	0.611	0.619	0.008
145 pm	0.648	0.641	0.007

Notes: This table compare the factor strength between the ten year data set and twenty year data set, ranked by the difference between the strength in decreasing sequence.

Table 7: Ten and Thirty Comparison

Factor	10 Year Strength	30 Year Strength	Difference
1 ps	0.115	0.769	0.654
2 dfin	0.182	0.791	0.609
3 ndf	0.000	0.557	0.557
4 rd	0.297	0.774	0.477
5 pchsale_pchxsga	0.000	0.448	0.448
6 pchdepr	0.266	0.696	0.430
7 pchsaleinv	0.115	0.519	0.404
8 rsup	0.182	0.579	0.397
9 dcol	0.425	0.807	0.382
10 IPO	0.437	0.818	0.381
11 nincr	0.411	0.790	0.378
12 dsti	0.297	0.660	0.364
13 gad	0.364	0.723	0.360
14 pchquick	0.182	0.539	0.358
15 pchcurrat	0.297	0.654	0.358
16 RMW	0.469	0.806	0.337
17 pchsale_pchinvt	0.115	0.448	0.334
18 etr	0.448	0.115	0.334
19 dwc	0.115	0.448	0.334
20 ta	0.230	0.563	0.334
21 sgr	0.437	0.769	0.332
22 ep	0.533	0.849	0.316
23 pchsale_pchrect	0.115	0.425	0.310
24 roaq	0.297	0.602	0.306
25 lgr	0.504	0.810	0.306
26 lfe	0.000	0.297	0.297
27 hire	0.519	0.805	0.285
28 bm_ia	0.487	0.772	0.285
29 ww	0.546	0.827	0.282
30 dolvol	0.437	0.715	0.278
31 herf	0.496	0.766	0.270
32 convind	0.448	0.710	0.262
33 cinvest_a	0.479	0.739	0.261
34 cfp	0.579	0.838	0.259
35 cdi	0.364	0.623	0.259
36 quick	0.593	0.851	0.258
37 salecash	0.602	0.857	0.255
38 cp	0.589	0.836	0.247
39 stdacc	0.593	0.837	0.244
40 roic	0.448	0.691	0.243
41 indmom	0.459	0.696	0.237
42 dcoa	0.459	0.696	0.237
43 chcsho	0.607	0.840	0.234
44 depr	0.615	0.848	0.233
45 chtx	0.397	0.623	0.226
46 stdcf	0.619	0.836	0.217
47 currat	0.626	0.840	0.214
48 cash	0.637	0.847	0.210
49 SMB	0.512	0.721	0.210
50 rds	0.634	0.843	0.209

Table 7: Ten and Thirty Comparison (Cont.)

Factor	10 Year Strength	30 Year Strength	Difference
51 aeavol	0.512	0.710	0.199
52 absacc	0.615	0.812	0.197
53 ato	0.634	0.831	0.197
54 CMA	0.563	0.759	0.196
55 HXZ_IA	0.584	0.780	0.196
56 cashpr	0.615	0.808	0.194
57 nop	0.651	0.839	0.188
58 moms12m	0.512	0.694	0.182
59 chatoia	0.364	0.182	0.182
60 ear	0.000	0.182	0.182
61 lev	0.657	0.838	0.181
62 acc	0.619	0.797	0.178
63 nef	0.644	0.818	0.174
64 salerec	0.630	0.803	0.173
65 rs	0.411	0.584	0.172
66 noa	0.615	0.787	0.172
67 rdm	0.607	0.778	0.172
68 op	0.663	0.835	0.172
69 egr_hxz	0.579	0.750	0.172
70 pricedelay	0.533	0.701	0.168
71 rna	0.611	0.778	0.167
72 ndp	0.686	0.852	0.166
73 dy	0.672	0.838	0.166
74 adm	0.660	0.825	0.165
75 cinvest	0.496	0.660	0.164
76 zs	0.675	0.839	0.164
77 dnco	0.598	0.761	0.163
78 sin	0.607	0.769	0.162
79 kz	0.669	0.831	0.161
80 ill	0.448	0.297	0.152
81 ebp	0.683	0.835	0.152
82 nxf	0.651	0.801	0.150
83 tang	0.683	0.833	0.150
84 age	0.703	0.851	0.148
85 cto	0.563	0.710	0.147
86 dfnl	0.297	0.437	0.140
87 HML	0.672	0.811	0.139
88 LIQ_PS	0.634	0.496	0.138
89 roavol	0.713	0.850	0.138
90 grltnoa	0.675	0.539	0.136
91 pchcapx	0.411	0.546	0.134
92 pctacc	0.539	0.672	0.133
93 mom6m	0.364	0.496	0.132
94 cei	0.563	0.691	0.128
95 ivg	0.381	0.504	0.123
96 chinv	0.448	0.569	0.120
97 grltnoa_hxz	0.563	0.683	0.120
98 egr	0.579	0.699	0.120
99 tb	0.589	0.708	0.119
100 pchcapx_ia	0.512	0.630	0.118
101 sp	0.699	0.817	0.118

Table 7: Ten and Thirty Comparison (Cont.)

	Factor	10 Year Strength	30 Year Strength	Difference
102	maxret	0.715	0.825	0.110
103	dnca	0.579	0.689	0.110
104	saleinv	0.579	0.686	0.107
105	std_turn	0.719	0.826	0.107
106	em	0.552	0.657	0.106
107	realestate_hxz	0.663	0.769	0.105
108	idiovol	0.723	0.825	0.102
109	gma	0.615	0.715	0.100
110	retvol	0.721	0.813	0.092
111	dncl	0.611	0.519	0.092
112	baspread	0.730	0.820	0.091
113	poa	0.593	0.681	0.087
114	zerotrade	0.725	0.812	0.087
115	turn	0.728	0.813	0.086
116	cfp_ia	0.479	0.563	0.085
117	chempia	0.411	0.496	0.085
118	std_dolvol	0.539	0.459	0.080
119	ol	0.663	0.741	0.078
120	ob_a	0.669	0.745	0.076
121	QMJ	0.637	0.708	0.071
122	sue	0.459	0.397	0.062
123	HML_Devil	0.719	0.781	0.062
124	STR	0.607	0.546	0.061
125	HXZ_ROE	0.619	0.678	0.059
126	dpia	0.681	0.623	0.058
127	beta	0.749	0.806	0.057
128	os	0.569	0.623	0.054
129	cashdebt	0.504	0.557	0.053
130	dnoa	0.230	0.182	0.048
131	chpmia	0.230	0.182	0.048
132	pps	0.563	0.611	0.048
133	pchcapx3	0.644	0.691	0.047
134	BAB	0.666	0.710	0.044
135	pm	0.648	0.691	0.043
136	invest	0.683	0.641	0.042
137	ms	0.611	0.648	0.037
138	chmom	0.469	0.437	0.032
139	pchgm_pchsale	0.297	0.322	0.026
140	mom36m	0.459	0.479	0.019
141	LTR	0.364	0.381	0.017
142	ala	0.699	0.715	0.016
143	orgcap	0.686	0.699	0.013
144	grcapx	0.539	0.552	0.012
145	UMD	0.678	0.672	0.006

Notes: This table compare the factor strength between the ten year data set and thirty year data set, ranked by the difference between the strength in decreasing sequence.

Table 8: Twenty and Thirty Comparison

	Factor	20 Year Strength	30 Year Strength	Difference
1	chatoia	0.437	0.182	0.255
2	etr	0.344	0.115	0.230
3	chinv	0.397	0.569	0.172
4	chpmia	0.344	0.182	0.162
5	pchsale_pchrect	0.574	0.425	0.149
6	lfe	0.425	0.297	0.128
7	ivg	0.397	0.504	0.107
8	LTR	0.487	0.381	0.106
9	dwc	0.546	0.448	0.097
10	sue	0.487	0.397	0.090
11	pchquick	0.626	0.539	0.087
12	dnoa	0.266	0.182	0.085
13	ear	0.266	0.182	0.085
14	dfnl	0.519	0.437	0.082
15	sin	0.844	0.769	0.075
16	mom6m	0.569	0.496	0.073
17	rsup	0.651	0.579	0.072
18	bm_ia	0.843	0.772	0.071
19	ta	0.634	0.563	0.071
20	sp	0.888	0.817	0.071
21	cashpr	0.878	0.808	0.070
22	gad	0.788	0.723	0.065
23	dncl	0.584	0.519	0.065
24	pchdepr	0.761	0.696	0.065
25	dsti	0.723	0.660	0.063
26	HML	0.874	0.811	0.063
27	chempia	0.557	0.496	0.062
28	LIQ_PS	0.557	0.496	0.062
29	pchcapx3	0.752	0.691	0.061
30	dy	0.897	0.838	0.059
31	lev	0.897	0.838	0.059
32	pchgm_pchsale	0.381	0.322	0.059
33	dolvol	0.774	0.715	0.059
34	poa	0.739	0.681	0.059
35	HXZ_IA	0.838	0.780	0.058
36	cp	0.894	0.836	0.058
37	dnca	0.747	0.689	0.058
38	herf	0.824	0.766	0.058
39	op	0.893	0.835	0.058
40	ebp	0.893	0.835	0.058
41	HML_Devil	0.838	0.781	0.057
42	zs	0.896	0.839	0.057
43	adm	0.881	0.825	0.056
44	kz	0.887	0.831	0.056
45	pchcurrat	0.710	0.654	0.056
46	dnco	0.816	0.761	0.055
47	nef	0.873	0.818	0.055
48	cfp	0.893	0.838	0.055
49	nop	0.893	0.839	0.054
50	zerotrade	0.865	0.812	0.053

Table 8: Twenty and Thirty Comparison (Cont.)

	Factor	20 Year Strength	30 Year Strength	Difference
51	ato	0.884	0.831	0.053
52	egr_hxz	0.803	0.750	0.053
53	ndp	0.904	0.852	0.052
54	turn	0.864	0.813	0.051
55	tang	0.884	0.833	0.051
56	sgr	0.819	0.769	0.050
57	pchcapx_ia	0.681	0.630	0.050
58	pm	0.641	0.691	0.050
59	cash	0.897	0.847	0.050
60	quick	0.901	0.851	0.050
61	nxf	0.849	0.801	0.049
62	saleinv	0.637	0.686	0.049
63	rna	0.826	0.778	0.048
64	ala	0.762	0.715	0.048
65	BAB	0.757	0.710	0.047
66	dfin	0.838	0.791	0.047
67	absacc	0.859	0.812	0.047
68	hire	0.851	0.805	0.047
69	acc	0.843	0.797	0.047
70	rds	0.890	0.843	0.047
71	currat	0.887	0.840	0.047
72	grltnoa_hxz	0.730	0.683	0.046
73	stdacc	0.883	0.837	0.046
74	ol	0.787	0.741	0.046
75	CMA	0.805	0.759	0.046
76	idiovol	0.870	0.825	0.045
77	salecash	0.902	0.857	0.045
78	cinvest_a	0.784	0.739	0.045
79	rdm	0.823	0.778	0.045
80	chcsho	0.884	0.840	0.044
81	std_turn	0.870	0.826	0.044
82	roavol	0.894	0.850	0.044
83	nincr	0.834	0.790	0.044
84	salerec	0.847	0.803	0.044
85	age	0.894	0.851	0.043
86	stdcf	0.878	0.836	0.042
87	chmom	0.479	0.437	0.042
88	grcapx	0.593	0.552	0.042
89	RMW	0.847	0.806	0.041
90	ep	0.891	0.849	0.041
91	convind	0.669	0.710	0.041
92	gma	0.756	0.715	0.041
93	depr	0.889	0.848	0.041
94	lgr	0.850	0.810	0.041
95	cinvest	0.701	0.660	0.041
96	beta	0.846	0.806	0.040
97	cei	0.730	0.691	0.038
98	pchsaleinv	0.557	0.519	0.038
99	maxret	0.863	0.825	0.038
100	ps	0.807	0.769	0.038
101	noa	0.825	0.787	0.038

Table 8: Twenty and Thirty Comparison (Cont.)

Factor	20 Year Strength	30 Year Strength	Difference
102 std_dolvol	0.496	0.459	0.037
103 ww	0.863	0.827	0.036
104 retvol	0.848	0.813	0.035
105 baspread	0.854	0.820	0.034
106 UMD	0.706	0.672	0.033
107 IPO	0.850	0.818	0.032
108 moms12m	0.725	0.694	0.032
109 cdi	0.654	0.623	0.031
110 ndf	0.589	0.557	0.031
111 dcol	0.838	0.807	0.031
112 ill	0.266	0.297	0.030
113 indmom	0.725	0.696	0.029
114 realestate_hxz	0.798	0.769	0.029
115 ms	0.619	0.648	0.029
116 aeavol	0.737	0.710	0.027
117 egr	0.725	0.699	0.027
118 SMB	0.745	0.721	0.024
119 pchsale_pchxsga	0.425	0.448	0.024
120 tb	0.732	0.708	0.024
121 pps	0.589	0.611	0.022
122 chtx	0.644	0.623	0.022
123 pctacc	0.694	0.672	0.021
124 cfp_ia	0.584	0.563	0.021
125 rs	0.563	0.584	0.021
126 HXZ_ROE	0.699	0.678	0.021
127 STR	0.526	0.546	0.019
128 ob_a	0.764	0.745	0.019
129 pchcapx	0.563	0.546	0.017
130 mom36m	0.496	0.479	0.017
131 cto	0.694	0.710	0.017
132 orgcap	0.715	0.699	0.016
133 rd	0.787	0.774	0.013
134 em	0.644	0.657	0.013
135 roic	0.703	0.691	0.012
136 dpia	0.634	0.623	0.011
137 pricedelay	0.691	0.701	0.010
138 dcoa	0.706	0.696	0.010
139 grltnoa	0.533	0.539	0.006
140 QMJ	0.703	0.708	0.005
141 roaq	0.607	0.602	0.004
142 os	0.626	0.623	0.004
143 invest	0.644	0.641	0.003
144 cashdebt	0.557	0.557	0.000
145 pchsale_pchinvt	0.448	0.448	0.000

Notes: This table compare the factor strength between the twenty year data set and thirty year data set, ranked by the difference between the strength in decreasing sequence.

Table 9: Thirty Year Decompose

	Factor Strength $\hat{\alpha}$				
Factor	Full Sample	January 1988 to December 1997	January 1998 to December 2007	January 2008 to December 2017	Standard Deviation of Three sub-samples
1 ps	0.769	0.425	0.715	0.000	0.294
2 ndf	0.557	0.230	0.678	0.000	0.281
3 pchdepr	0.696	0.000	0.666	0.182	0.281
4 dfin	0.791	0.266	0.772	0.115	0.281
5 invest	0.641	0.557	0.000	0.607	0.275
6 dpia	0.623	0.546	0.000	0.607	0.273
7 pchquick	0.539	0.000	0.569	0.000	0.268
8 pchcapx_ia	0.630	0.000	0.589	0.437	0.250
9 pchcurrat	0.654	0.000	0.593	0.182	0.248
10 rd	0.774	0.364	0.715	0.115	0.246
11 gad	0.723	0.000	0.584	0.182	0.244
12 sin	0.769	0.182	0.770	0.469	0.240
13 IPO	0.818	0.230	0.777	0.364	0.233
14 pchsale_pchrect	0.425	0.115	0.519	0.000	0.223
15 tb	0.708	0.115	0.651	0.448	0.221
16 ta	0.563	0.344	0.533	0.000	0.221
17 rsup	0.579	0.519	0.611	0.115	0.215
18 pchsaleinv	0.519	0.182	0.519	0.000	0.215
19 dsti	0.660	0.115	0.563	0.115	0.211
20 dncl	0.519	0.115	0.546	0.563	0.207
21 pchsale_pchinvt	0.448	0.266	0.496	0.000	0.203
22 adm	0.825	0.297	0.786	0.552	0.200
23 dcol	0.807	0.526	0.752	0.266	0.198
24 stdacc	0.837	0.322	0.803	0.526	0.197
25 pchsale_pchxsga	0.448	0.230	0.469	0.000	0.192
26 cinvest_a	0.739	0.266	0.708	0.364	0.189
27 ivg	0.504	0.569	0.115	0.297	0.186
28 noa	0.787	0.297	0.741	0.519	0.182
29 LIQ_PS	0.496	0.115	0.397	0.552	0.181
30 ato	0.831	0.364	0.797	0.533	0.178
31 roaq	0.602	0.425	0.533	0.115	0.177
32 stdcf	0.836	0.381	0.805	0.546	0.174
33 ep	0.849	0.504	0.826	0.425	0.174
34 rna	0.778	0.266	0.686	0.519	0.172
35 grltnoa	0.539	0.425	0.182	0.593	0.169
36 ww	0.827	0.397	0.791	0.496	0.167
37 dwc	0.448	0.479	0.437	0.115	0.163
38 bm_ia	0.772	0.584	0.756	0.364	0.160
39 pchgm_pchsale	0.322	0.000	0.381	0.266	0.160
40 RMW	0.806	0.448	0.774	0.425	0.159
41 rds	0.843	0.437	0.810	0.546	0.156
42 nincr	0.790	0.519	0.736	0.364	0.152
43 dfnl	0.437	0.344	0.589	0.230	0.150
44 lgr	0.810	0.496	0.752	0.397	0.150
45 cfp	0.838	0.512	0.816	0.487	0.149
46 sgr	0.769	0.546	0.703	0.344	0.147
47 dolvol	0.715	0.397	0.703	0.397	0.144
48 herf	0.766	0.689	0.728	0.411	0.141

Table 9: Thirty Year Decompose (Cont.)

| Factor | Factor Strength $\hat{\alpha}$ | | | | Standard Deviation of Three sub-samples |
	Full Sample	January 1988 to December 1997	January 1998 to December 2007	January 2008 to December 2017	
49 lev	0.838	0.504	0.817	0.539	0.140
50 salecash	0.857	0.539	0.823	0.519	0.139
51 chcsho	0.840	0.504	0.808	0.526	0.139
52 cp	0.836	0.557	0.820	0.512	0.136
53 lfe	0.297	0.230	0.322	0.000	0.135
54 zs	0.839	0.519	0.813	0.539	0.134
55 kz	0.831	0.519	0.813	0.539	0.134
56 cdi	0.623	0.546	0.552	0.266	0.133
57 convind	0.710	0.589	0.563	0.297	0.132
58 STR	0.546	0.469	0.230	0.533	0.131
59 mom6m	0.496	0.182	0.479	0.230	0.130
60 quick	0.851	0.607	0.819	0.512	0.129
61 moms12m	0.694	0.411	0.657	0.381	0.124
62 hire	0.805	0.569	0.749	0.448	0.123
63 cfp_ia	0.563	0.266	0.569	0.425	0.123
64 ebp	0.835	0.557	0.816	0.557	0.122
65 ndp	0.852	0.574	0.823	0.557	0.121
66 HML	0.811	0.539	0.815	0.589	0.120
67 cashpr	0.808	0.557	0.790	0.519	0.120
68 cash	0.847	0.584	0.819	0.552	0.119
69 chinv	0.569	0.557	0.266	0.397	0.119
70 depr	0.848	0.637	0.820	0.533	0.119
71 absacc	0.812	0.469	0.756	0.579	0.118
72 roic	0.691	0.563	0.641	0.364	0.117
73 BAB	0.710	0.425	0.706	0.598	0.116
74 age	0.851	0.546	0.817	0.630	0.113
75 indmom	0.696	0.479	0.630	0.364	0.109
76 SMB	0.721	0.607	0.699	0.437	0.108
77 HXZ_ROE	0.678	0.364	0.623	0.519	0.106
78 acc	0.797	0.519	0.764	0.574	0.105
79 nop	0.839	0.615	0.824	0.598	0.103
80 currat	0.840	0.626	0.810	0.574	0.101
81 rdm	0.778	0.519	0.725	0.512	0.099
82 chatoia	0.182	0.411	0.182	0.230	0.099
83 CMA	0.759	0.496	0.678	0.448	0.099
84 chpmia	0.182	0.000	0.230	0.182	0.099
85 roavol	0.850	0.593	0.826	0.654	0.099
86 saleinv	0.686	0.663	0.425	0.533	0.098
87 chtx	0.623	0.364	0.504	0.266	0.098
88 cinvest	0.660	0.397	0.602	0.397	0.097
89 dnco	0.761	0.557	0.736	0.512	0.097
90 cei	0.691	0.487	0.666	0.448	0.095
91 chmom	0.437	0.487	0.266	0.322	0.094
92 op	0.835	0.626	0.819	0.615	0.094
93 salerec	0.803	0.557	0.754	0.557	0.093
94 sp	0.817	0.593	0.805	0.630	0.092
95 HML_Devil	0.781	0.557	0.774	0.619	0.091
96 dy	0.838	0.634	0.820	0.626	0.090
97 pchcapx	0.546	0.322	0.448	0.230	0.090

Table 9: Thirty Year Decompose (Cont.)

| Factor | Factor Strength $\hat{\alpha}$ | | | | Standard Deviation of Three sub-samples |
	Full Sample	January 1988 to December 1997	January 1998 to December 2007	January 2008 to December 2017	
98 realestate_hxz	0.769	0.479	0.696	0.611	0.090
99 cashdebt	0.557	0.344	0.533	0.344	0.089
100 nef	0.818	0.657	0.799	0.589	0.088
101 mom36m	0.479	0.397	0.574	0.381	0.087
102 ob_a	0.745	0.437	0.637	0.598	0.087
103 aeavol	0.710	0.589	0.615	0.425	0.084
104 HXZ_IA	0.780	0.563	0.713	0.519	0.083
105 tang	0.833	0.615	0.793	0.626	0.081
106 maxret	0.825	0.615	0.803	0.654	0.081
107 pricedelay	0.701	0.593	0.563	0.411	0.080
108 grcapx	0.552	0.411	0.266	0.448	0.078
109 baspread	0.820	0.630	0.808	0.663	0.077
110 nxf	0.801	0.623	0.762	0.584	0.077
111 pps	0.611	0.512	0.344	0.496	0.076
112 LTR	0.381	0.182	0.297	0.364	0.075
113 etr	0.115	0.230	0.115	0.297	0.075
114 dnoa	0.182	0.000	0.182	0.115	0.075
115 ear	0.182	0.115	0.182	0.000	0.075
116 retvol	0.813	0.634	0.801	0.660	0.073
117 std_turn	0.826	0.637	0.799	0.657	0.072
118 idiovol	0.825	0.644	0.799	0.657	0.070
119 zerotrade	0.812	0.651	0.803	0.663	0.069
120 dcoa	0.696	0.557	0.546	0.411	0.066
121 egr_hxz	0.750	0.563	0.666	0.512	0.064
122 turn	0.813	0.666	0.798	0.663	0.063
123 beta	0.806	0.660	0.807	0.696	0.062
124 cto	0.710	0.657	0.539	0.519	0.061
125 pm	0.691	0.611	0.479	0.602	0.060
126 ill	0.297	0.182	0.230	0.322	0.058
127 std_dolvol	0.459	0.381	0.297	0.437	0.058
128 poa	0.681	0.552	0.660	0.533	0.056
129 em	0.657	0.546	0.504	0.411	0.056
130 QMJ	0.708	0.546	0.660	0.552	0.053
131 gma	0.715	0.637	0.619	0.519	0.052
132 pctacc	0.672	0.512	0.593	0.469	0.052
133 ms	0.648	0.437	0.557	0.512	0.050
134 grltnoa_hxz	0.683	0.512	0.602	0.487	0.049
135 ala	0.715	0.651	0.728	0.615	0.047
136 UMD	0.672	0.496	0.607	0.579	0.047
137 dnca	0.689	0.487	0.584	0.487	0.045
138 chempia	0.496	0.437	0.437	0.364	0.034
139 egr	0.699	0.539	0.563	0.496	0.028
140 rs	0.584	0.411	0.344	0.381	0.027
141 pchcapx3	0.691	0.539	0.598	0.574	0.024
142 sue	0.397	0.230	0.182	0.230	0.022
143 ol	0.741	0.644	0.657	0.619	0.016
144 os	0.623	0.437	0.469	0.469	0.015
145 orgcap	0.699	0.611	0.615	0.607	0.003

Notes: This table decompose the thirty year data set into three sub sample set, each contain ten years data. The table is ranked by the standard deviation among three sub samples, in decreasing sequence

Table 10: Twenty Year Decompose

| | | Factor Strength $\hat{\alpha}$ | | |
Factor	Full Sample	January 1998 to December 2007	January 2008 to December 2017	Difference between Sub samples
1 ndf	0.589	0.721	0.000	0.721
2 invest	0.644	0.000	0.648	0.648
3 dpia	0.634	0.000	0.644	0.644
4 ps	0.807	0.752	0.115	0.637
5 dfin	0.838	0.810	0.182	0.628
6 rsup	0.651	0.663	0.115	0.549
7 pchsale_pchxsga	0.425	0.512	0.000	0.512
8 rd	0.787	0.756	0.266	0.489
9 pchquick	0.626	0.602	0.115	0.487
10 pchsale_pchrect	0.574	0.584	0.115	0.469
11 pchsaleinv	0.557	0.574	0.115	0.459
12 pchcurrat	0.710	0.634	0.182	0.452
13 pchdepr	0.761	0.696	0.266	0.430
14 pchsale_pchinvt	0.448	0.539	0.115	0.425
15 dcol	0.838	0.786	0.364	0.422
16 nincr	0.834	0.777	0.364	0.413
17 ta	0.634	0.593	0.182	0.411
18 IPO	0.850	0.802	0.397	0.405
19 grltnoa	0.533	0.230	0.634	0.404
20 dwc	0.546	0.519	0.115	0.404
21 dfnl	0.519	0.648	0.266	0.381
22 lfe	0.425	0.364	0.000	0.364
23 RMW	0.847	0.808	0.448	0.360
24 gad	0.788	0.626	0.266	0.360
25 lgr	0.850	0.787	0.437	0.350
26 sgr	0.819	0.741	0.397	0.344
27 ep	0.891	0.862	0.519	0.342
28 dsti	0.723	0.626	0.297	0.330
29 bm_ia	0.843	0.793	0.469	0.324
30 roaq	0.607	0.584	0.266	0.317
31 dolvol	0.774	0.752	0.437	0.315
32 cinvest_a	0.784	0.737	0.425	0.313
33 ww	0.863	0.822	0.512	0.310
34 etr	0.344	0.115	0.411	0.297
35 cfp	0.893	0.853	0.557	0.296
36 herf	0.824	0.778	0.487	0.291
37 mom6m	0.569	0.519	0.230	0.290
38 hire	0.851	0.783	0.496	0.287
39 cp	0.894	0.858	0.574	0.284
40 salecash	0.902	0.861	0.584	0.277
41 quick	0.901	0.852	0.579	0.273
42 chcsho	0.884	0.843	0.574	0.270
43 roic	0.703	0.694	0.425	0.269
44 stdacc	0.883	0.842	0.574	0.269
45 STR	0.526	0.297	0.563	0.266
46 ear	0.266	0.266	0.000	0.266
47 depr	0.889	0.854	0.589	0.266
48 SMB	0.745	0.736	0.487	0.248

Table 10: Thirty Year Decompose (Cont.)

Factor	Factor Strength $\hat{\alpha}$			
	Full Sample	January 1998 to December 2007	January 2008 to December 2017	Difference between Sub samples
49 cdi	0.654	0.569	0.322	0.246
50 stdcf	0.878	0.839	0.593	0.246
51 rds	0.890	0.852	0.607	0.246
52 indmom	0.725	0.669	0.425	0.245
53 cash	0.897	0.856	0.615	0.241
54 cei	0.730	0.719	0.479	0.241
55 lev	0.897	0.861	0.626	0.234
56 dnco	0.816	0.770	0.539	0.231
57 sin	0.844	0.810	0.579	0.231
58 nop	0.893	0.857	0.626	0.230
59 currat	0.887	0.844	0.615	0.230
60 convind	0.669	0.611	0.381	0.230
61 HML	0.874	0.853	0.626	0.227
62 kz	0.887	0.860	0.634	0.226
63 cashpr	0.878	0.832	0.607	0.225
64 zs	0.896	0.858	0.641	0.217
65 mom36m	0.496	0.626	0.411	0.215
66 moms12m	0.725	0.683	0.469	0.214
67 nef	0.873	0.833	0.619	0.214
68 ato	0.884	0.827	0.619	0.209
69 ndp	0.904	0.862	0.654	0.208
70 op	0.893	0.852	0.644	0.208
71 dy	0.897	0.853	0.648	0.205
72 ebp	0.893	0.859	0.654	0.205
73 noa	0.825	0.787	0.584	0.203
74 CMA	0.805	0.699	0.496	0.203
75 acc	0.843	0.799	0.602	0.197
76 adm	0.881	0.833	0.637	0.195
77 salerec	0.847	0.801	0.615	0.186
78 sue	0.487	0.230	0.411	0.182
79 absacc	0.859	0.788	0.607	0.182
80 ivg	0.397	0.182	0.364	0.182
81 HXZ_IA	0.838	0.739	0.557	0.182
82 pchgm_pchsale	0.381	0.448	0.266	0.182
83 rdm	0.823	0.764	0.584	0.180
84 aeavol	0.737	0.657	0.479	0.179
85 age	0.894	0.847	0.669	0.178
86 LIQ_PS	0.557	0.425	0.598	0.173
87 cinvest	0.701	0.626	0.459	0.167
88 nxf	0.849	0.790	0.623	0.167
89 roavol	0.894	0.858	0.691	0.167
90 sp	0.888	0.847	0.681	0.167
91 tang	0.884	0.830	0.669	0.160
92 pchcapx_ia	0.681	0.651	0.496	0.155
93 egr_hxz	0.803	0.686	0.533	0.153
94 maxret	0.863	0.838	0.686	0.152
95 grcapx	0.593	0.344	0.496	0.152
96 tb	0.732	0.703	0.552	0.152
97 zerotrade	0.865	0.835	0.691	0.144

Table 10: Thirty Year Decompose (Cont.)

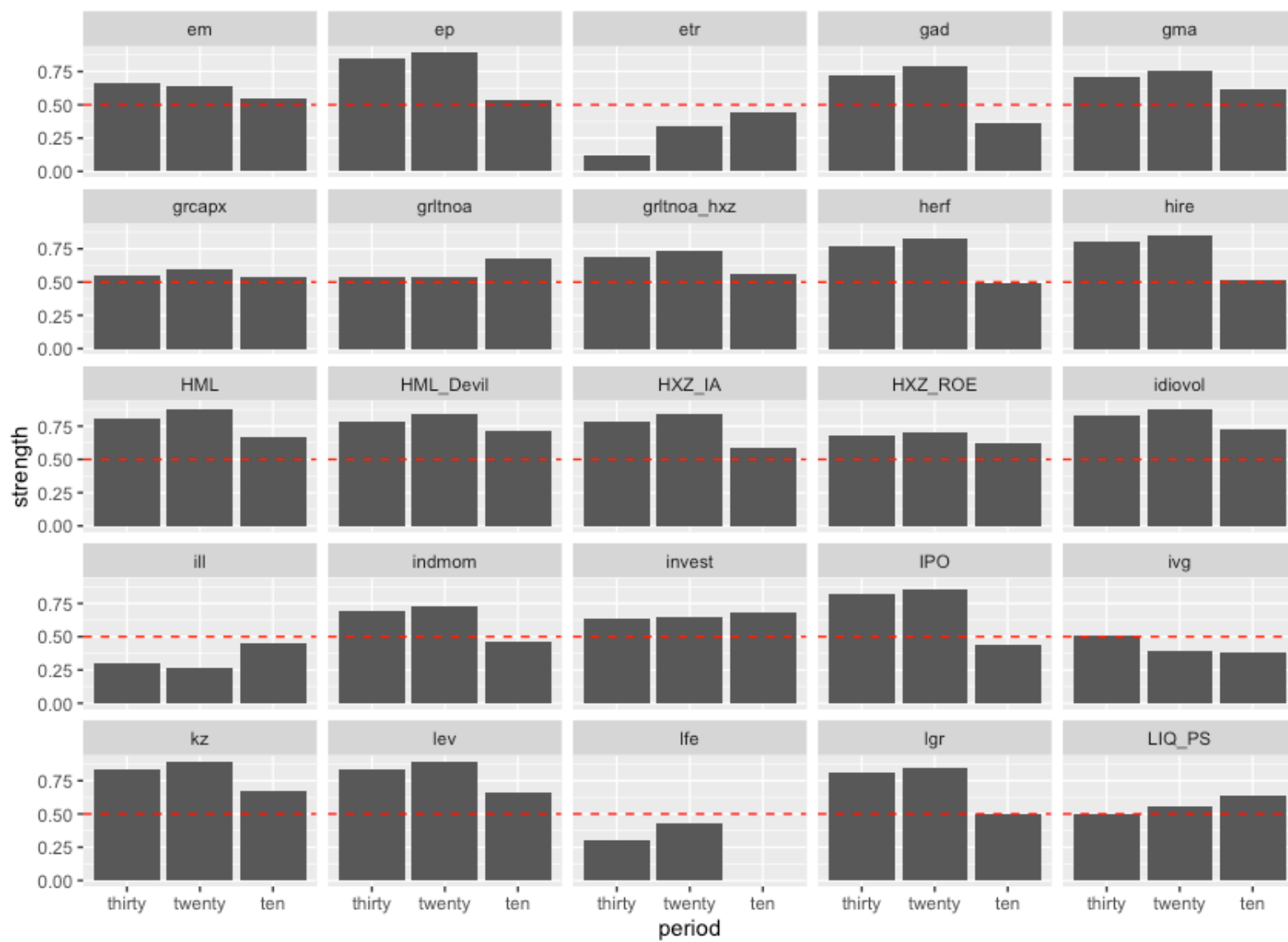| Factor | Factor Strength $\hat{\alpha}$ | | | |
	Full Sample	January 1998 to December 2007	January 2008 to December 2017	Difference between Sub samples
98 rna	0.826	0.730	0.589	0.141
99 retvol	0.848	0.834	0.694	0.140
100 std_turn	0.870	0.832	0.694	0.138
101 idiovol	0.870	0.832	0.694	0.138
102 chtx	0.644	0.519	0.381	0.138
103 baspread	0.854	0.840	0.703	0.137
104 pchcapx	0.563	0.459	0.322	0.137
105 turn	0.864	0.830	0.696	0.133
106 HML_Devil	0.838	0.818	0.686	0.132
107 cfp_ia	0.584	0.589	0.459	0.130
108 poa	0.739	0.696	0.574	0.122
109 pm	0.641	0.512	0.634	0.122
110 pctacc	0.694	0.641	0.519	0.122
111 chinv	0.397	0.297	0.411	0.115
112 dcoa	0.706	0.563	0.448	0.115
113 chmom	0.479	0.322	0.437	0.115
114 chpmia	0.344	0.344	0.230	0.115
115 BAB	0.757	0.749	0.637	0.111
116 beta	0.846	0.838	0.728	0.111
117 HXZ_ROE	0.699	0.672	0.563	0.109
118 realestate_hxz	0.798	0.752	0.648	0.105
119 grltnoa_hxz	0.730	0.623	0.519	0.104
120 std_dolvol	0.496	0.411	0.512	0.100
121 ala	0.762	0.766	0.666	0.099
122 pps	0.589	0.448	0.546	0.097
123 QMJ	0.703	0.701	0.607	0.094
124 cashdebt	0.557	0.563	0.469	0.094
125 dnca	0.747	0.611	0.519	0.092
126 pricedelay	0.691	0.598	0.512	0.086
127 dnoa	0.266	0.266	0.182	0.085
128 saleinv	0.637	0.469	0.552	0.083
129 gma	0.756	0.660	0.579	0.082
130 chempia	0.557	0.487	0.411	0.076
131 egr	0.725	0.593	0.533	0.060
132 ol	0.787	0.703	0.644	0.059
133 cto	0.694	0.602	0.552	0.051
134 LTR	0.487	0.411	0.364	0.048
135 ill	0.266	0.381	0.425	0.043
136 ob_a	0.764	0.694	0.651	0.043
137 rs	0.563	0.364	0.397	0.033
138 dncl	0.584	0.615	0.584	0.031
139 pchcapx3	0.752	0.637	0.607	0.031
140 em	0.644	0.546	0.519	0.027
141 chatoia	0.437	0.344	0.322	0.022
142 ms	0.619	0.563	0.584	0.021
143 os	0.626	0.533	0.546	0.013
144 orgcap	0.715	0.657	0.666	0.009
145 UMD	0.706	0.641	0.637	0.003

Notes: This table decompose the twenty year data set into two sub sample set, each contain ten years data. The table is ranked by the difference between sub samples, in decreasing sequence

Figure 1: Strength Comparison
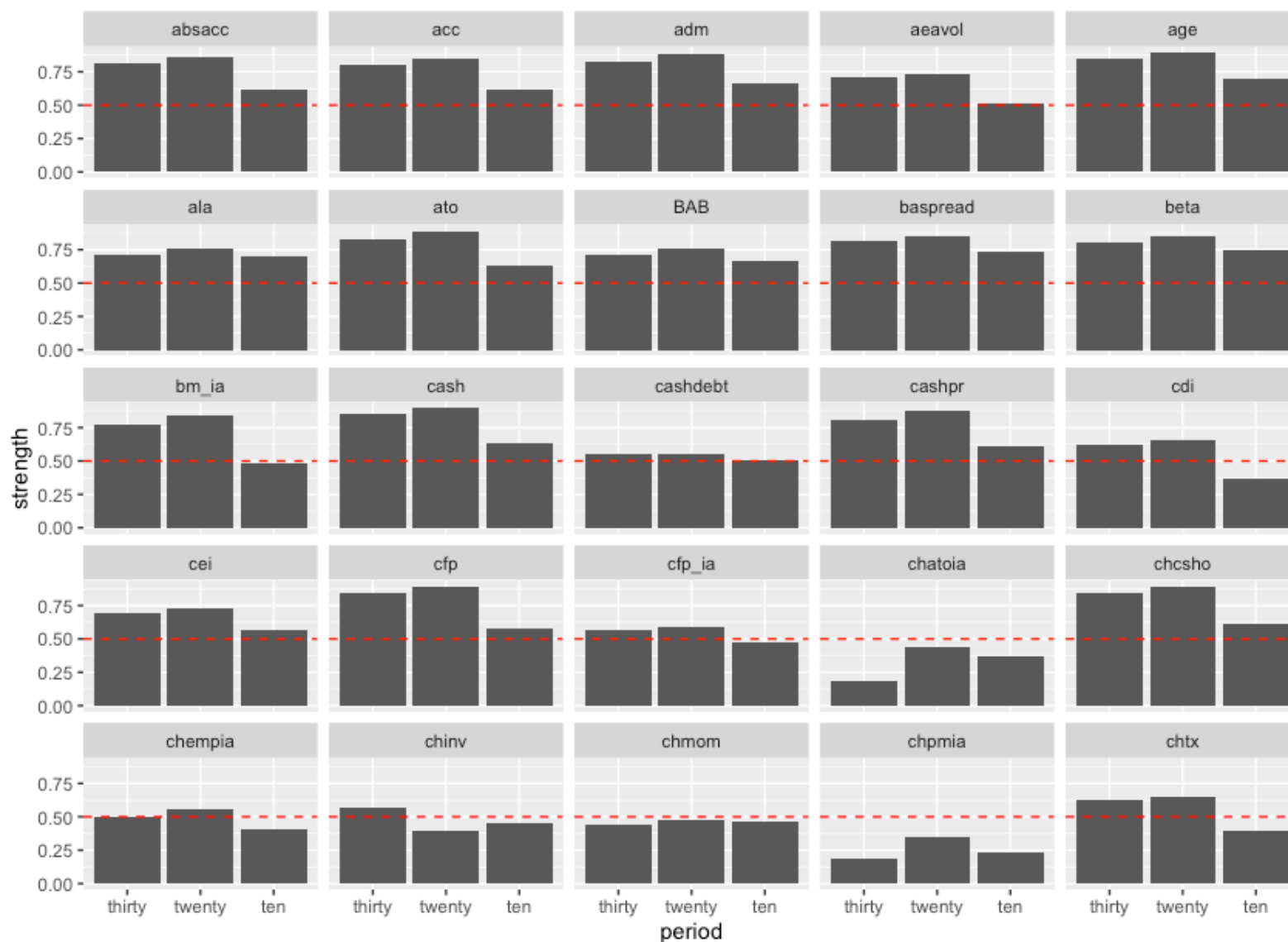
Notes: The figure compare the strength of every factor's strength in different data set. The x-axis indicates the data set: thirty is thirty years data set (January 1987 to December 2017), twenty is twenty year data set (January 1997 to December 2017), and ten is ten year data set (January 2007 to December 2017). The red dash line draws from the 0,5 as the 0.5 threshold for weak and strong factor.
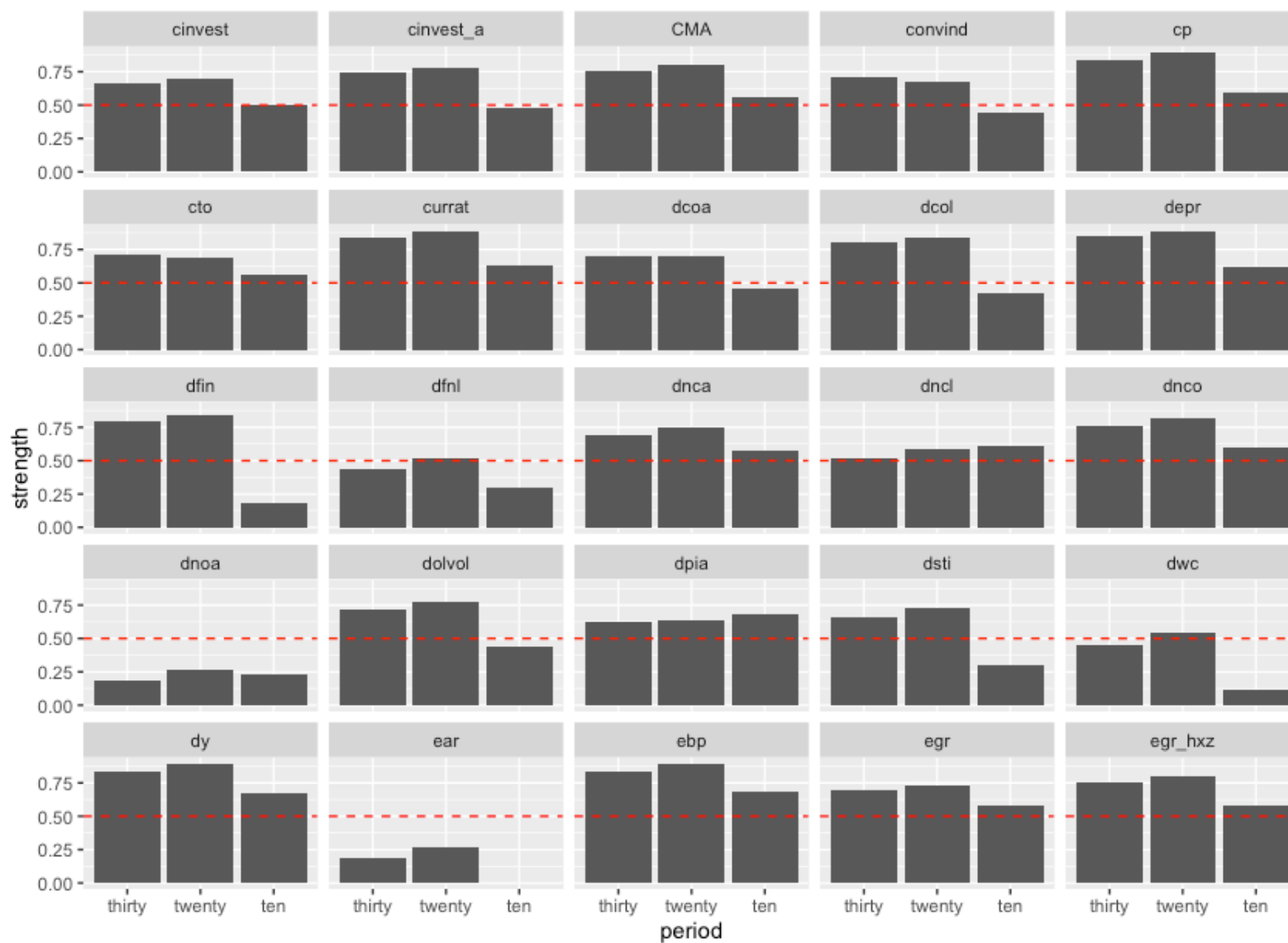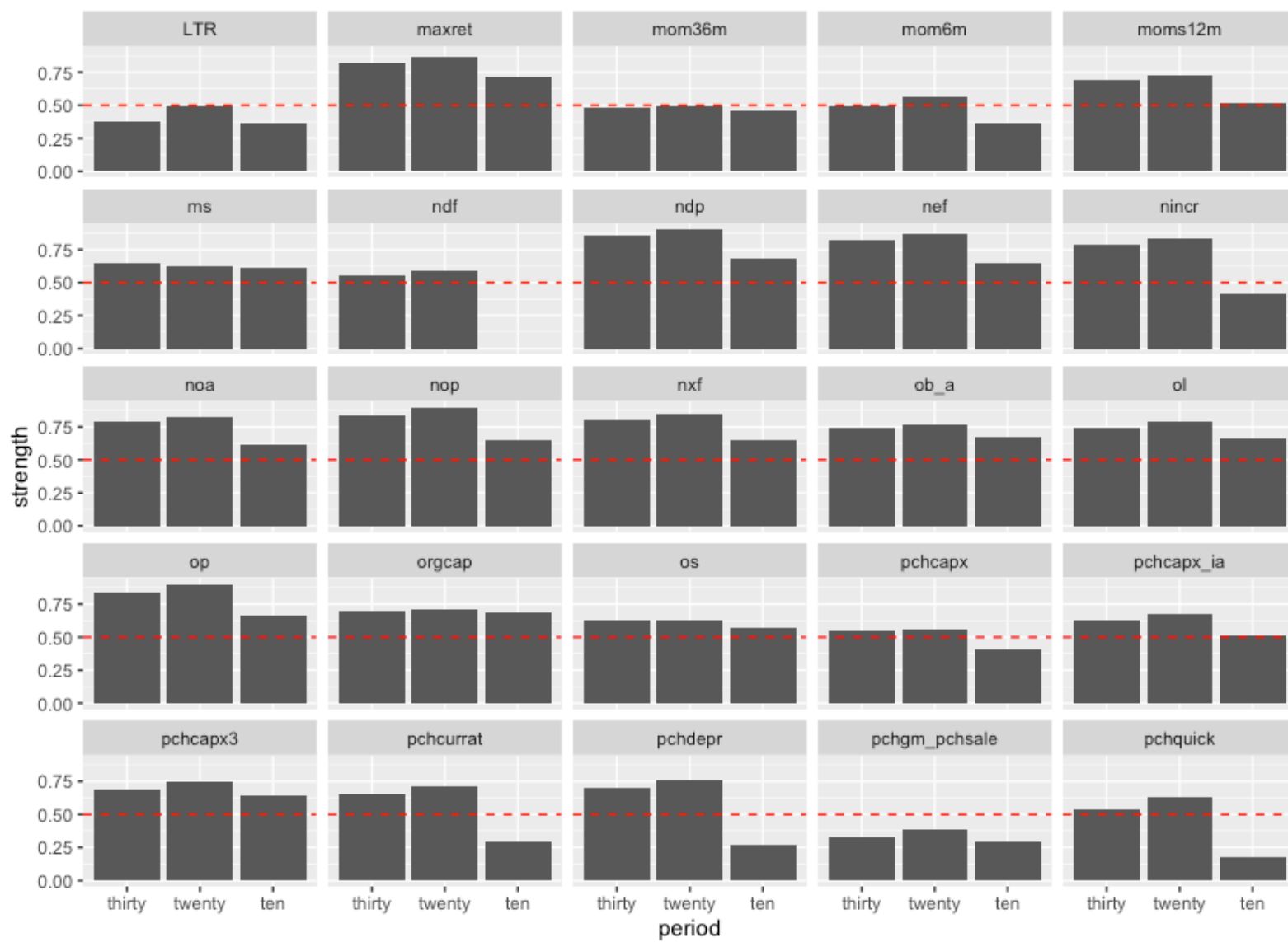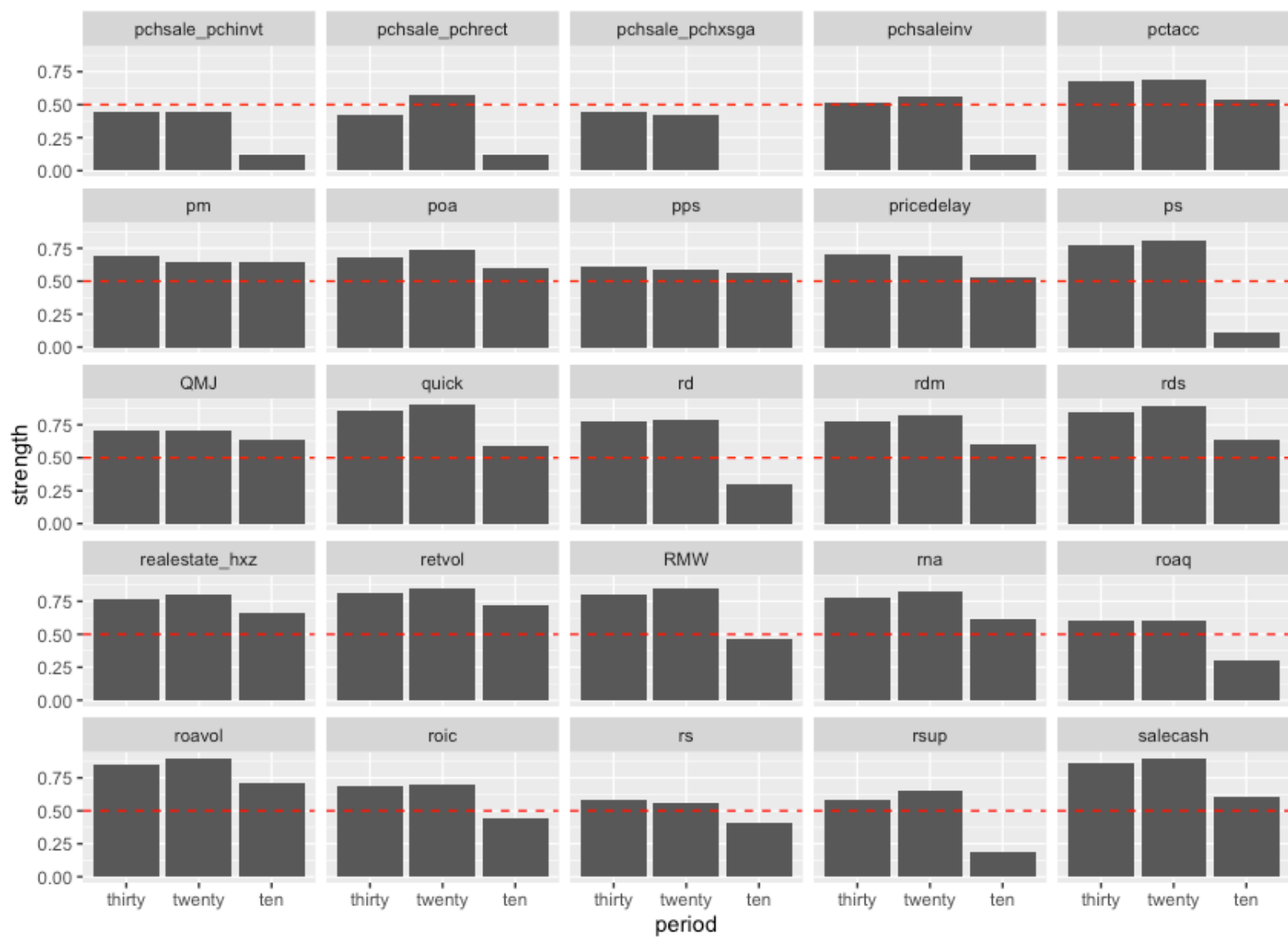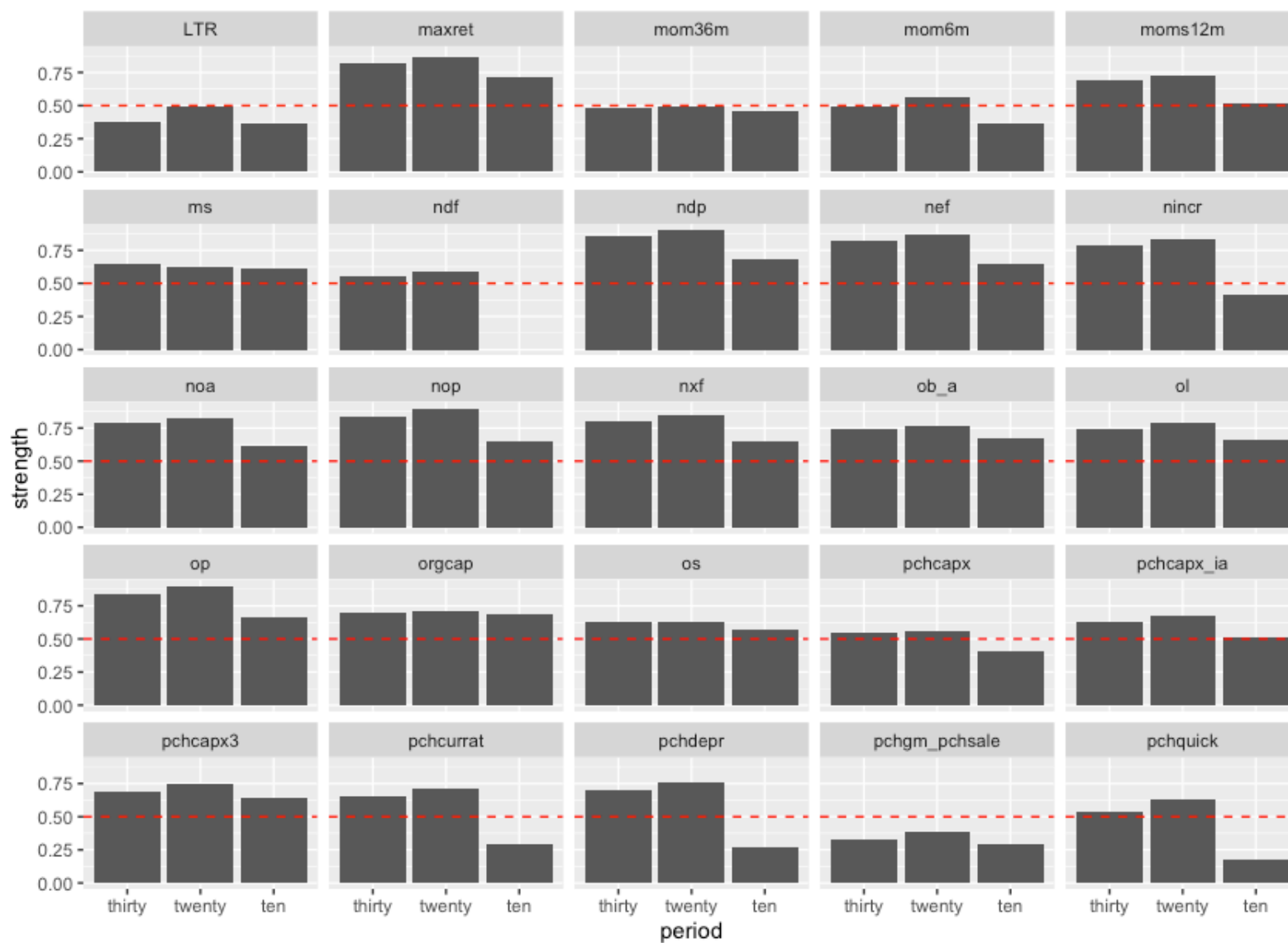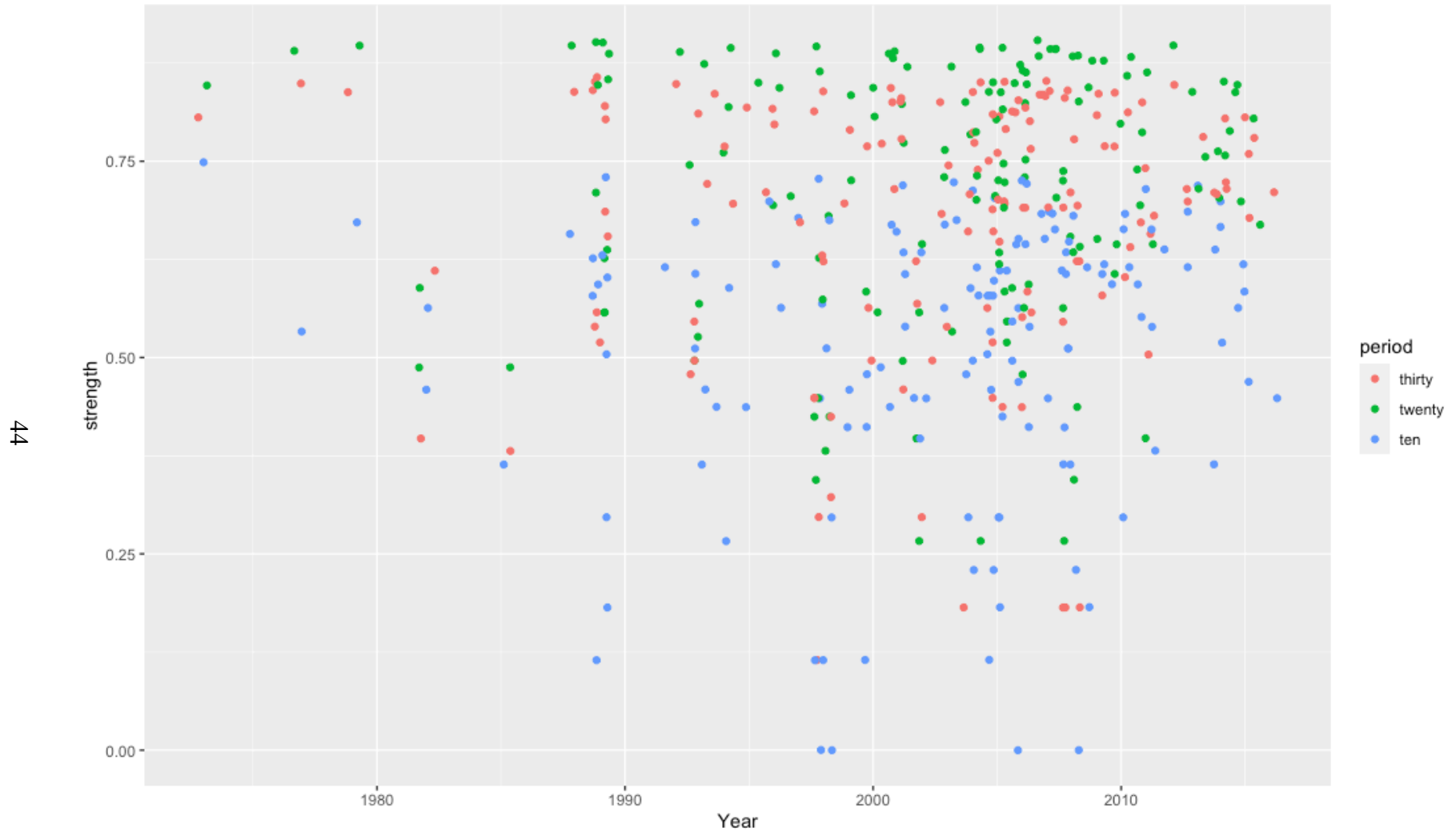
Figure 2: The scatter Plot for three period

Notes: This scatter plot illustrates the relationship between the factor strength and the factor's publish year. The x-axis represent the year, and the y-axis represent the factor strength. For each factor, we estimate it's strength base on three different data sets, and we use different colour to indicates the three different data set.