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1 Introduction and Motivation

Capital Asset Pricing Model (CAPM) (Sharpe (1964), Lintner (1965), and Black (1972)) introduces a risk pricing paradigm. By incorporating factors, the model divides asset's risk into two parts: systematic risk and asset specified idiosyncratic risk. In general, the market factor captures the systematic risk, and different risk factors price the idiosyncratic risk. Researches (see Fama and French (1992), Carhart (1997), Kelly, Pruitt, and Su (2019)) have shown that, adding different risk factors into the CAPM model can enhance the ability of pricing risk. Because of this, identifying risk factors has becomes an important topic in finance. Numerous of researchers have contributed to this field, and the direct result is a explosive growth of factors. Harvey and Liu (2019) have collected over 500 factors from papers published in the top financial and economic journals, and they find the growth of new factors has speed up since 2008.

But we should notice that not all factors can pass the significance test comfortably every time like the market factor. Therefore, ? (?) a new criteria for assessing the significance of each factor, which they call it factor strength. In general, if a factor can generate loadings significantly different from zero for all assets, then we call such a factor strong factor. And the less significant loading a factor can generate, the weaker the strength it has.

In his 2011 president address Cochrane gave emphasis to the importance of finding factors which can provide independent information about average return and risk. With regard to this, a number of scholars applied various methods to find such factors. For instance, Harvey and Liu (2017) provided a bootstrap method to adjust the threshold of factor loading's significant test, trying to exclude some falsely significant factor caused by multiple-test problem. Some other scholars are using machine learning methods to reduce the potential candidates. One stream of them have used a shrinkage and subset selection method called Lasso (Tibshirani, 1996) and it's variations to find suitable factors. One example of such application is made by Rapach, Strauss, and Zhou (2013). They applied the Lasso regression, trying to find some characteristics from a large group to predict the global stock market's return.

But an additional challenge is that factors, especially in the high-dimension, are commonly correlated. Kozak, Nagel, and Santosh (2020) point out that when facing a group of correlated factors, Lasso will only pick several highly correlated factors seemly randomly, and then ignore the

other and shrink them to zero. In other words, Lasso fails to handle the issue of correlated factors appropriately.

Therefore, the main empirical question in this project is: how to select useful factors from a large group of possibly highly correlated candidates. We address this question from two different prospects.

From one side, we employ the idea of factor strength discuss above, trying to use this criteria to select those strong factors. On the other hand, we use another variable selection method called Elastic Net (Zou & Hastie, 2005) to select factors. With regard of the first approach, Bailey, Kapetanios, and Pesaran (2020) provide a consistent estimator for the factor strength, and we will use this method to examine the strength of each candidate factor, and filter out any spurious factors. Under the second approach, unlike Lasso, elastic net adds an extra penalty term into the loss function, which makes it suitable to handle the potential correlation variables. This trait makes it suitable for out purpose. We will assess and compare the methods in their selection of risk factors. Additionally, we can also use the factor strength as a standard to reduce the dimension of our candidates factor and then applied elastic net to conduct further selection.

In the rest of this thesis, we will first go through some literatures relates with CAPM model and methods about factor selection. Then in the section 3, we will provides a detailed description of the concept of factor strength and the estimation method. Also, we will introduce the elastic net. In the section 4, we set up a simple Monte Carlo simulation experiment to examine the finite sample properties of the factor strength estimator. Section 5 includes the empirical application, where we estimate the strength of potential risk factors to be included in a CAPM model, as well as apply elastic net as a method to select factors.

2   Related Literature

This project is builds on contributions to the filed of asset pricing. First formularised by Sharpe (1964), Lintner (1965), and Black (1972), the original CAPM model only contains the market factor, which is denoted by the difference between average market return and risk free return. Fama and French (1992) develop the model into three-factors, which it is then extend into four (Carhart, 1997), and five (Fama & French, 2015). Recent research created a six-factors model and claim it

outperform all other sparse factor model. (Kelly et al., 2019).

In terms of assessing the strength of risk factors, this thesis also relates with papers discussing factors that has no or weak correlation with assets' return under the paradigm of CAPM model. Kan and Zhang (1999) found that the test-statistic of FM two-stage regression (Fama & MacBeth, 1973) will inflate when incorporating factors which are independent of the cross-section return. Therefore, when factors with no pricing power was added into the model, those factors may have the chance to pass the significant test falsely. Kleibergen and Zhan (2015) found out that even when some factor-return relationship does not exist, the r-square and the t-statistic of the FM two stage regression would become in favour of the conclusion of such structure presence. Gospodinov, Kan, and Robotti (2017) show how the addition of a spurious factor will distort the statistical inference of parameters. Besides, Anatolyev and Mikusheva (2018) studied the behaviours of the model with the presence of weak factors under asymptotic settings, and they find the regression will lead to an inconsistent risk premia estimation result.

Finally, of interest in this thesis is the large dimension of potential factors. For this reasons, it borrows from researchers that identify useful factors from a group of potential factors. Harvey, Liu, and Zhu (2015) examine over 300 factors published in journals, presents that the traditional threshold for a significant test is too low for newly proposed factor, and they suggest to adjust the p-value threshold to around 3. Methods like a Bayesian procedure introduced by Barillas and Shanken (2018) were used to compare different factor models. Pukthuanthong, Roll, and Subrahmanyam (2019) defined several criteria for "genuine risk factor", and based on those criteria introduced a protocol to examine whether a factor is associated with the risk premium.

Once the factor strength are identified, the thesis will attempt to reconcile empirically the factor selection under machine learning techniques and the factor strength implied by the selection.

Gu, Kelly, and Xiu (2020) elaborate the advantages of using emerging machine learning algorithms in asset pricing. Those advantages including more accurate prediction result, and superior efficiency. Various machine learning algorithms have been adopted when selecting factors for the factor model, especially in recent years. Lettau and Pelger (2020) apply Principle Components Analysis when investigating the latent factor of model. Lasso method, since it's ability to select features, is popular in the field of the factor selection. Feng, Giglio, and Xiu (2019) used the double-selected Lasso method (Belloni, Chernozhukov, & Hansen, 2014),and a grouped lasso

method (Huang, Horowitz, & Wei, 2010) is used by Freyberger, Neuhierl, and Weber (2020) when picking factors from a group of candidates. Kozak et al. (2020) used a Bayesian-based method, combing with both Ridge and Lasso regression, arguing that the sparse factor model is ultimately futile.

3   Factor Strength

The concept of factor strength employed by this project comes from Bailey et al. (2020), and it was first introduced by Bailey, Kapetanios, and Pesaran (2016). They defined the strength of factor from prospect of the cross-section dependences of large panel and connect it to the pervasiveness of the factor, which is captured by the factor loadings. In a latter paper, Bailey et al. (2016) extended the method by loosen some restrictions, and proved that their estimation can also be applied on the residuals or regression result. Thereafter, they focusing on the case of observed factors, and proposed the method we employed in this project (Bailey et al., 2020).

3.1   Definition

Consider the following multi-factor model for n different cross-section units and T observations with k factors.

$$x_{it} = a_t + \sum_{j=1}^{k} \beta_{ij} f_{jt} + \varepsilon_{it} \tag{1}$$

In the left-hand side, we have x_{it} denotes the cross-section unit i at time t, where $i = 1, 2, 3, \cdots, n$ and $t = 1, 2, 3, \cdots T$. In the other hand, a_i is the constant term. f_{jt} of $j = 1, 2, 3 \cdots k$ is factors included in the model, and β_{ij} is the corresponding factor loading. ε_{it} is the stochastic error term.

The factor strength is relates to how many non-zero loadings correspond to a factor. More precisely, for a factor f_{jt} with n different factor loading β_j, we assume that:

$$|\beta_j| > 0 \quad i = 1, 2, \ldots, [n^{\alpha_j}]$$

$$|\beta_j| = 0 \quad i = [n^{\alpha_j}] + 1, [n^{\alpha_j}] + 2, \ldots, n$$

The α_j represents strength of factor f_{jt} and $\alpha_j \in [0,1]$. If factor has strength α_j, we will assume that the first $[n^{\alpha_j}]$ loadings are all different from zero, and here $[\cdot]$ is defined as integral operator, which will only take the integral part of inside value. The rest $n - [n^{\alpha_j}]$ terms are all equal to zero. Assume for a factor which has strength $\alpha = 1$, the factor's loadings will be non-zero for all cross-section units. We will refer such factor as strong factor. And if we have factor strength $\alpha = 0$, it means that the factor has all factor loadings equal to zero, and we will describe such factor as weak factor (Bailey et al., 2016). For any factor with strength in $[0.5, 1]$, we will refer such factor as semi-strong factor. In general term, the more non-zero loading a factor has, the stronger the factor's strength is.

3.2 Estimation Under single factor setting

To estimate the strength α_j, Bailey et al. (2020) provides following estimation.

To begin with, we consider a single-factor model with only factor named f_t. β_i is the factor loading of unit i. v_{it} is the stochastic error term.

$$x_{it} = a_i + \beta_i f_t + v_{it} \tag{2}$$

Assume we have n different units and T observations for each unit: $i = 1, 2, 3, \cdots, n$ and $t = 1, 2, 3, \cdots T$. Running the OLS regression for each $i = 1, 2, 3 \cdots, n$, we obtain:

$$x_{it} = \hat{a}_{iT} + \hat{\beta}_{iT} f_t + \hat{v}_{it}$$

For every factor loading $\hat{\beta}_{iT}$, we can examining their significance by constructing a t-test. The t-test statistic will be $t_{iT} = \frac{\hat{\beta}_{iT} - 0}{\hat{\sigma}_{iT}}$. Then the test statistic for the corresponding $\hat{\beta}_i$ will be:

$$t_{iT} = \frac{\left(\mathbf{f}'\mathbf{M}_\tau \mathbf{f}\right)^{1/2} \hat{\beta}_{iT}}{\hat{\sigma}_{iT}} = \frac{\left(\mathbf{f}'\mathbf{M}_\tau \mathbf{f}\right)^{-1/2} \left(\mathbf{f}'\mathbf{M}_\tau \mathbf{x}_i\right)}{\hat{\sigma}_{iT}} \tag{3}$$

Here, the $\mathbf{M}_\tau = \mathbf{I}_T - T^{-1}\tau\tau'$, and the τ is a $T \times 1$ vector with every elements equals to 1. $\mathbf{f}$ and $\mathbf{x_i}$ are two vectors with: $\mathbf{f} = (f_1, f_2 \cdots, f_T)'$ $\mathbf{x_i} = (x_{i1}, x_{i2}, \cdots, x_{iT})$. The denominator $\hat{\sigma}_{iT} = \frac{\sum_{i=1}^T \hat{v}_{it}^2}{T}$.

Using this test statistic, we can then define an indicator function as: $\ell_{i,n} := \mathbf{1}[|\beta_i| > 0]$. If the factor loading is none-zero, $\ell_{i,n} = 1$. In practice, we use the $\hat{\ell}_{i,nT} =:= \mathbf{1}[|t_{it}| > c_p(n)]$ Here, if the t-statistic t_{iT} is greater than critical value $c_p(n)$, $\hat{\ell}_{i,n} = 1$, otherwise $\hat{\ell}_{i,n} = 0$ In other word, we are

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counting how many $\hat{\beta}_{iT}$ are significant. With the indicator function, we then defined $\hat{\pi}_{nT}$ as the fraction of significant factor loading amount to the total factor loadings:

$$\hat{\pi}_{nT} = \frac{\sum_{i=1}^{n} \hat{\ell}_{i,nT}}{n} \tag{4}$$

In term of the critical value $c_p(n)$, rather than use the traditional critical value from student-t distribution $\Phi^{-1}(1 - \frac{P}{2})$, we use:

$$c_p(n) = \Phi^{-1}(1 - \frac{P}{2n^\delta}) \tag{5}$$

Suggested by Bailey, Pesaran, and Smith (2019), here, $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function of a standard normal distribution, P is the size of the test, and δ is a non-negative value represent the critical value exponent. This adjusted critical value, adopt helps to tackle the problem of multiple-test.

After obtain the $\hat{\pi}_{nT}$, we can use the following formula provided by Bailey et al. (2020) to estimate our strength indicator α_j:

$$\hat{\alpha} = \begin{cases} 1 + \frac{\ln(\hat{\pi}_{nT})}{\ln n} & \text{if } \hat{\pi}_{nT} > 0, \\ 0, & \text{if } \hat{\pi}_{nT} = 0. \end{cases} \tag{6}$$

Whenever we have $\hat{\pi}_{nT}$, the estimated $\hat{\alpha}$ will be equal to zero. From the estimation, we can find out that $\hat{\alpha} \in [0, 1]$

3.3 Estimation Under Multi-Factor Setting

This estimation can also be extended into a multi-factor set up. Consider the following multi-factor model:

$$x_{it} = a_i + \sum_{j=1}^{k} \beta_{ij} f_{jt} + v_{it} = a_i + \beta_i' \mathbf{f_t} + v_{it}$$

In this set up, we have $i = 1, 2, \cdots, n$ units, $t = 1, 2, \cdots, T$ time observations, and specially, $j = 1, 2, \cdots, k$ different factors. Here $\beta_i = (\beta_{i1}, \beta_{i2}, \cdots, \beta_{ij})'$ and $\mathbf{f}_t = (f_{1t}, f_{2t} \cdots, f_{jt})$. We employed the same strategy as above, after running OLS and obtain the:

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$$x_{it} = \hat{a}_{iT} + \hat{\boldsymbol{\beta}}_{\mathbf{ij}}\mathbf{f}_{jt} + \hat{v}_{it}$$

To conduct the significant test, we calculates the t-statistic: $t_{ijT} = \frac{\hat{\beta}_{ijT} - 0}{\hat{\sigma}_{ijT}}$. Empirically, the test statistic can be calculated using:

$$t_{ijT} = \frac{\left(\mathbf{f}'_{j\circ}\mathbf{M}_{F_{-j}}\mathbf{f}_{j\circ}\right)^{-1/2}\left(\mathbf{f}'_{j\circ}\mathbf{M}_{F_{-j}}\mathbf{x}_i\right)}{\hat{\sigma}_{iT}}$$

Here, $\mathbf{f}_{j\circ} = \left(f_{j1}, f_{j2}, \ldots, f_{jT}\right)'$, $\mathbf{x}_i = \left(x_{i1}, x_{i2}, \ldots, x_{iT}\right)'$, $\mathbf{M}_{F_{-j}} = \mathbf{I} - \mathbf{F}_{-j}\left(\mathbf{F}'_{-j}\mathbf{F}_{-j}\right)^{-1}\mathbf{F}'_{-j}$, and $\mathbf{F}_{-j} = \left(\mathbf{f}_{1\circ}, \ldots, \mathbf{f}_{j-1\circ}, \mathbf{f}_{j+1\circ}, \ldots, \mathbf{f}_{m\circ}\right)'$ For the denominator's $\hat{\sigma}_{iT}$, it was from $\hat{\sigma}_{iT}^2 = T^{-1}\sum_{t=1}^{T}\hat{u}_{it}^2$, the $\hat{u}_{it}$ is the residuals of the model. Then, we can use the same critical value from (5). Obtaining the correspond ratio $\hat{\pi}_{nTj}$ from (4), and after that use the function:

$$\hat{\alpha}_j = \begin{cases} 1 + \frac{\ln \hat{\pi}_{nT,j}}{\ln n}, & \text{if } \hat{\pi}_{nT,j} > 0 \\ 0, & \text{if } \hat{\pi}_{nT,j} = 0 \end{cases}$$

to estimates the factor loading.

4 Monte Carlo Design

4.1 Design

In order to study the finite sample property of factor strength $\hat{\alpha}_j$, we designed a Monte Carlo simulation. Through the simulation, we compare the property of the factor strength in different settings. We set up the experiments to reflect the CAPM model and it's extension. Consider the following data generating process (DGP):

$$r_{it} - r_{ft} = q_1(r_{mt} - r_f) + q_2\left(\sum_{j=1}^{k}\beta_{ij}f_{jt}\right) + \varepsilon_{it}$$

In the simulation, we consider a dataset has $i = 1, 2, \ldots, n$ different cross-section units, with $t = 1, 2, \ldots, T$ different observations. r_{it} is the unit's return, and r_{ft} represent the risk free rate at time t, therefore, the left hand side term $r_{it} - r_{ft}$ is the excess return of the unit i. For simplicity, we

define $x_{it} := r_{it} - r_{ft}$. f_{jt} represents different risk factors, and the corresponding β_{ij} are the factor loadings. We use $r_{mt} - r_{ft}$ to denotes the market factor, and here r_{mt} is the average market return. Also, we use the term $f_{mt} := r_{mt} - r_{ft}$ to denotes the market factor. We expect the market factor will has strength equals to one all the time, so we consider the market factor has strength $\alpha_m = 1$. ε_{it} is the stochastic error term. Therefore, the simulation model can be simplified as:

$$x_{it} = q_1(f_{mt}) + q_2\left(\sum_{j=1}^{k} \beta_{ij} f_{jt}\right) + \varepsilon_{it}$$

$q_1(\cdot)$ and $q_2(\cdot)$ are two different functions represent the unknown mechanism of market factor and other risk factors in pricing asset risk. In the classical CAPM model and it's multi-factor extensions, for example the three factor model introduced by Fama and French (1992), both q_1 and q_2 are linear.

For each factor, we assume they follow a multinomial distribution with mean zero and a $k \times k$ variance-covariance matrix Σ.

$$\mathbf{f_t} = \begin{pmatrix} f_{i,t} \\ f_{2,t} \\ \vdots \\ f_{k,t} \end{pmatrix} \sim MVN(\mathbf{0}, \Sigma) \quad \Sigma := \begin{pmatrix} \sigma_{f_1}^2, & \rho_{12}\sigma_{f1}\sigma_{f2} & \cdots & \rho_{1k}\sigma_{f1}\sigma_{fk} \\ \rho_{12}\sigma_{f2}\sigma_{f1}, & \sigma_{f2}^2 & \cdots & \rho_{2k}\sigma_{f2}\sigma_{fk} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1k}\sigma_{fk}\sigma_{f1}, & \rho_{k2}\sigma_{fk}\sigma_{f2} & \cdots & \sigma_{fk}^2 \end{pmatrix}$$

The diagonal of matrix Σ indicates the variance of each factor, and the rest represent the covariance among all k factors.

4.2   Experiment Setting

Follow the general model above, we assume both $q_1(\cdot)$ and $q_2(\cdot)$ are linear function:

$$q_1(f_{mt}) = a_i + \beta_{im} f_{mt}$$

$$q_2\left(\sum_{j=1}^{k} \beta_{ij} f_{jt}\right) = \sum_{j=1}^{k} \beta_{ij} f_{jt}$$

To start the simulation, we consider a two factor model:

$$x_{it} = a_i + \beta_{i1} f_{1t} + \beta_{i2} f_{2t} + \varepsilon_{it} \tag{7}$$

The constant term a_i is generate from a uniform distribution, $a_{it} \sim U[-0.5, 0.5]$. For the factor loading β_{i1} and β_{i2}, we first use a uniform distribution $IIDU(\mu_\beta - 0.2, \mu_\beta + 0.2)$ to produce the values. Here we set $\mu_{beta} = 0.71$ to make sure every generated loading value is sufficiently larger than 0. Then we randomly assign $n - [n^{\alpha_1}]$ and $n - [n^{\alpha_2}]$ factor loadings as zero. α_1 and α_2 are the true factor strength of f_1 and f_2. In this simulation, we will start the factor strength from 0.7 and increase it gradually till unity with pace 0.05, say $(\alpha_1, \alpha_2) = \{0.7, 0.75, 0.8, \cdots, 1\}$. $[\cdot]$ is the integer operator defined at section (3.2). This step reflects the fact that only $[n^\alpha]$ factor loadings are non-zero. In terms of the factors, they comes from a multinomial distribution $MVN(\mathbf{0}, \Sigma)$, as we discuss before.

Currently, we consider three different experiments set up:

Experiment 1 (single factor, normal error, no correlation) *Set β_{i2} from (7) as 0, the error term ε_{it} and the factor f_{1t} are both standard normal.*

Experiment 2 (two factors, normal error, no correlation) *Both β_{i1} and β_{i2} are non-zero. Error term and both factors are standard normal. The correlation ρ_{12} between f_{1t} and f_{2t} is zero. The factor strength for the first factor $\alpha_1 = 1$ all the time, and α_2 various.*

Experiment 3 (two factors, normal error, weak correlation) *Both β_{i1} and β_{i2} are non-zero. Error term and both factors are standard normal. The correlation ρ_{12} between f_{1t} and f_{2t} is 0.3. The factor strength for the first factor $\alpha_1 = 1$ all the time, and α_2 various.*

The factor strength in each experiment is estimated using the method discussed in section (3.2), the size of significant test is $p = 0.05$, and the critical value exponent σ has been set as 0.5. For each of the experiment, we calculate the bias, the RMSE and the size of the test to justify the estimation performances. The bias is calculated as the difference between the true factor strength α and the estimate factor strength $\hat{\alpha}$. The Root Square Mean Error (RMSE) comes from:

$$RMSE = [\frac{1}{R} \sum_{r=1}^{R} (bias_r)^2]^{1/2}$$

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Where the R represent the total replicate times. The size of the test is under the hypothesis that $H_0 : \hat{\alpha}_j = \alpha_j$, $j = 1,2$ against the alternative hypothesis $H_1 : \hat{\alpha}_j \neq \alpha_j$, $j = 1,2$. Here we employed the following test statistic from Bailey et al. (2020).

$$z_{\hat{\alpha}_j:\alpha_j} = \frac{(\ln n)\left(\hat{\alpha}_j - \alpha_j\right) - p\left(n - n^{\hat{\alpha}_j}\right)n^{-\delta - \hat{\alpha}_j}}{\left[p\left(n - n^{\hat{\alpha}_j}\right)n^{-\delta - 2\hat{\alpha}_j}\left(1 - \frac{p}{n^\delta}\right)\right]^{1/2}} \quad j = 1,2 \tag{8}$$

Define a indicator function $\mathbf{1}(|z_{\hat{\alpha}_j:\alpha_j}| > c|H_0)$. For each replication, if this test statistic is greater than the critical value of standard normal distribution: $c = 1.96$, the indicator function will return value 1, and 0 otherwise. Therefore, we calculate the size of the test base on:

$$size = \frac{\sum_{r=1}^{R} \mathbf{1}(|z_{\hat{\alpha}_j:\alpha_j}| > 1.96|H_0)}{R} \quad j = 1,2, \tag{9}$$

In purpose of Monte Carlo Simulation, we consider the different combinations of T and n with $T = \{120, 240, 360\}$, $n = \{100, 300, 500\}$. The market factor, if included in the experiment, will have strength $\alpha_m = 1$ all the time, and the strength of the other factor will be $\alpha_x = \{0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1\}$. For every setting, we will replicate 2000 times independently, all the constant and variables will be re-generated for each replication.

4.3 Monte Carlo Results

We report the results in Table (4) , (5) and (6) in Appendix A.

Table (4) provides the results under the experiment 1. The estimation method we applied tends to over-estimate the strength slightly most of the time when the true strength is relatively weak under the single factor set up. With the strength increase, the bias will turn to negative, represents a under-estimated results. Such bias, however, vanish quickly while observation t, unit amount n, and α increase. When we increase the time spam by including more data from the time dimensions, the bias, as well as the RMSE decrease significantly. Also, when including more cross-section unit n into the simulation, the performance of the estimation improves, showing by the decrease bias and RMSE values. An impressive result is that, the gap between estimation and true strength will goes to zero when we have $\alpha = 1$, the strongest strength we can have. With the strength approaching unity, the both bias and RMSE will converge to zero. We also presents the size of the test in the table.

The size of the test will not variate too much when the strength increases, so as the unit increases, But we can observe that when observations for each unit increase, in other word, when t increases, the size will shrink dramatically. The size will become smaller than the 0.05 threshold after we extend the t to 240, or empirically speaking, when we included 20 years monthly return data into estimation. Notice that, from the equation (8), when $\hat{\alpha} = \alpha = 1$, the nominator will becomes zero. Therefore, the size will collapse into zero in all settings, so we do not report the size for $\hat{\alpha} = \alpha = 1$

For the two factors scenarios, we obtain similar conclusions in both the no correlation setting and weak correlation setting. The result of no correlation settings is shown in the table (5), and the table (6) shows the result when the correlation between two factors is 0.3. The estimation results will be improved by increasing either the observations amount t, or the cross-section units amount n. We also have the same unbiased estimation when true factor strength is unity under all unit-time combination. In some cases, even when the factor strength is relatively weak,we can have unbiased estimation if the n and t are big enough. (see table (6)). However, we should also notice that when we have a imbalanced panel data, like the scenario when t > n The results of size of the test in two factors setting are performing similar to the single factor result. The size will shrink with the observation amount t increasing, and when we have t grater than 240, the size will be smaller than 0.05 threshold in all situations.

5   Empirical Application

In this section, we introduced the data prepared for the empirical application, discuss the results of factor strength estimations. Then we apply the elastic net method introduced before,

5.1   Data

In the empirical application part, we use the monthly U.S. stock return as the assets. The companies are selected from Standard Poor (S&P) 500 index component companies.[1] We prepared three data sets for different time spams: 10 years (January 2008 to December 2017), 20 years (January 1998 to December 2017), and 30 years (January 1989 to December 2017). Because of the components companies of the index are constantly changing, bankrupt companies will be moved out, and new

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[1]The data was obtained from the Global Finance Data, Osiris, and Yahoo Finance.

companies will be added in. Also, some companies does not have enough observation. Therefore, for each of the datasets, the companies amount (n) are different, the dimensions of the data set is showing in the table (1) below.

Table 1: Data Set Dimensions

	Time Spam	Companies Amount (n)	Observations Amount (T)
10 Years	January 2008 - December 2017	419	120
20 Years	January 1998 - December 2017	342	240
30 Years	January 1988 - December 2017	242	360

For the risk free rate, we use the one-month U.S. treasury bill return.[2] For company i, we calculates the companies return at month t (r_{it}) use the following formula:

$$r_{it} = \frac{p_{it} - p_{it-1}}{p_{it-1}} \times 100$$

and calculate the excess return $x_{it} = r_{it} - r_{ft}$. Here the p_{it} and p_{it-1} are the company's close stock price at the first day of month t and t-1. The price is adjusted for the dividends and splits.[3]

With regard of the factors, we use 145 different risk factors from Feng, Giglio, and Xiu (2020). The factor set also includes one market factor, represented by the difference between the average market return and risk free return. The average market return is a weighted average return of all stocks in U.S. market, incorporated by CSRP. Each factors contains monthly value from the January 1988 to December 2017.

5.2 Factor Strength Analysis

5.2.1 Regression model setting

For the first part of the empirical application, we estimates the factor strength using the method discussed in the section 3. Precisely, we set the regression model base on the discussion of section 3.3.

$$x_{it} = a_i + \beta_{im}(r_{mt} - r_{ft}) + \beta_{ij}f_{jt} + v_{it}$$

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[2] The data was fetched from the Kenneth R. French website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/
[3] The data is adjusted base on the Central for Research in Security Price (CRSP) method.

where x_{it} is the excess return of asset i at time t, which is pre-defined in the section 5.1. $r_{mt} - r_{ft}$ represents the market factor, calculated by the difference between average market return and risk free return at the same time t. f_{jt} is the value of jth risk factor at time t. Here $j = 1, 2, 3, \cdots 145$. β_{mt} and β_{ij} are the factor loadings for market factor and risk factor, respectively.

5.2.2 Factor Strength Finding

The complete results of factor strength estimation is present in the appendix B.1 and B.2. We estimates the factors' strength using three different data set discussed in the section 5.1, and list those strength from strong to weak, alongside the market factor strength, in the table (8). For different data sets, we obtains inconsistent factor strength estimations. In general, the ten-year data set provides a significantly weaker result, compares with the other two data sets results. Except the market factor, no factors shows strength above 0.8 from the ten-years result. The strongest factor beside the market factor is the beta factor which has strength around 0.75. The strongest risk factor in the twenty-year data set is the ndp (net debt-to-price),which has strength 0.904. In the thirty-year scenario, the salecash (sales to cash) is the strongest with strength 0.857. When comparing the proportion (see table (2)), we can find that nearly 40% factors from the ten year dataset shows strength less than 0.5, which is almost three times higher than the twenty and thirty years proportion. If we use 0.8 as a threshold, we can see that there are over forty percent factors in the twenty-year results excess this threshold, and the percentage for the thirty year results is 31%.

Table 2: Proportion of Strength (Excluded Market Factor)

Strength Level	10 Year Data Proportion	20 Year Data Proportion	30 Year Data Proportion
[0.9, 1]	0%	2.07%	0%
[0.85, 0.9)	0%	24.1%	4.14%
[0.8, 0.85)	0%	16.6%	27.6%
[0.75, 0.8)	0%	8.28%	12.4%
[0.7, 0.75)	7.59%	11.7%	9.66%
[0.65, 07)	15.9%	5.52%	15.9%
[0.6, 0.65)	17.9%	8.28%	5.52%
[0.55, 0.6)	13.1%	8.97%	5.52%
[0.5, 0.55)	8.97%	2.76%	4.83%
[0, 0.5)	36.6%	11.7%	14.5%

Another important finding is that from the twenty year data set, we obtained three factors: ndq

Table 3: Selected Risk Factor with Strength

	Ten Year			Twenty Yera			Thirty Year	
Rank	Factor	Strength	Rank	Factor	Strength	Rank	Factor	Strength
1	beta	0.749	1	ndp	0.904	1	salecash	0.857
2	baspread	0.730	2	salecash	0.902	2	ndp	0.852
3	turn	0.728	3	quick	0.901	3	quick	0.851
4	zerotrade	0.725	4	dy	0.897	4	age	0.851
5	idiovol	0.723	5	lev	0.897	5	roavol	0.850
6	retvol	0.721	6	cash	0.897	6	ep	0.849
7	std_turn	0.719	7	zs	0.896	7	depr	0.848
8	HML_Devil	0.719	8	cp	0.894	8	cash	0.847
9	maret	0.715	9	roavol	0.894	9	rds	0.843
10	roavol	0.713	10	age	0.894	10	currat	0.840
20	UMD	0.678	28	HML	0.874	38	HML	0.811
24	HML	0.672	76	SMB	0.745	69	SMB	0.721
87	SMB	0.512	88	UMD	0.703	95	UMD	0.672

(Net debt-to-price, $\hat{\alpha} = 0.904$), salecash (sales to cash, $\hat{\alpha} = 0.902$), and quick (quick ratio, $\hat{\alpha} = 0.901$) has strength greater than 0.9. We would expected when applying the elastic net method with the twenty-year data set, those three factors with the market factors would be selected.

We also pay attention to some famous factors, precisely the Farma-French size factor (Small Minus Big SMB), Fama-French Value factor (High Minus Low: HML) (Fama & French, 1992) and the Momentum factor (UMD) (Carhart, 1997). It is surprise that none of these three factors enter the top ten list for each data sets. Except the HML factor from the twenty and thirty year data set has strength above 0.8, none of the other factors in any data set shows strength higher than 0.75. The value factor SMB from the ten year data set only has strength 0.512, ranked no.87.

In order to see how factor strength evolve through the time, we decompose the thirty year data set into three small subsamples. For each subsamples, it contains 242 companies (n = 242). And for each companies, we obtained 120 observations (t = 120). The results is present in the table (12) and figure (2).

In general, we can conclude that for most of the factors, their strength gradually increased from the first decade (January 1988 to December 1997) to the second decade (January 1998 to December 2007), and then decreased in the third decade (January 2008 to December 2017). This pattern can also be seen in the figure (2). The drop of factor strength in the third decades has been regards as the main reasons why the ten-years data results shows a significantly weaker results than the twenty

and thirty years data set.

5.2.3 Conclusion and Explanation

From the factor strength prospect, we would expect that for different time period, we will have different candidate factors for the CAPM model. For the ten-year data set, we would expected that only the market factor be useful, and therefore the elastic net method applied latter may only select the market factor. If we use the twenty and thirty year data, we will have a longer list for potential factors, 62 factors from the twenty-year estimation and 45 from the thirty years has strength greater than 0.8. Hence, we would expect the elastic net to select a less parsimonious model.

In terms of this discrepancy, there are several potential explanation. First if we consider the structure of our data set, we will find that the longer the time span, the less companies are included. This is because the S&P index will adjust the component, remove companies with inadequate behaviours, and add in new companies to reflect the market situation. Hence, those 242 companies in the thirty year data set can be viewed as survivals after series of financial and economic crisis.[4] We would expect those companies will have above average performances, such as better profitability and administration, comparing with other companies.

Notes: (But for how will those merits influence the factor's risk pricing ability is unclear for now)

Another possible explanation is the Global Financial Crisis in 2008. The financial market has been disturbed by this crisis, so therefore some mechanism may no longer working properly during that period.

We also need to notice that for some factors, their strength will decrease with the time. For instance, the gma (gross profitability) factor and convind (convertible debt indicator) factor (see figure 2) has consecutive strength decrease from the 1987-1997 period to 2007-2017 period. And for most of the factors, their strength will decrease significantly from the 1997-2007 period to 2007-2017 period.

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[4]For instance, the dot-com bubble in the early 20th century, the 911 attack, and the 2008 Global Financial Crisis

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A Simulation Result

Table 4: Simulation result for single factor CAPM

	Single Factor								
	Bias × 100			RMSE × 100			Size × 100		
$\alpha_1 = 0.7$									
n\T	120	240	360	120	240	360	120	240	360
100	0.256	0.265	0.227	0.612	0.623	0.560	7.85	7.7	5.55
300	0.185	0.184	0.184	0.363	0.338	0.335	8.9	4.45	4.5
500	0.107	0.124	0.109	0.259	0.248	0.234	6.9	2.5	1.6
$\alpha_1 = 0.75$									
100	-0.178	-0.159	-0.168	0.490	0.465	0.450	2.5	0.85	0.4
300	0.154	0.156	0.143	0.281	0.258	0.234	9.4	3.7	3.35
500	0.024	0.033	0.263	0.171	0.155	0.148	7.8	2	1.25
$\alpha_1 = 0.8$									
100	-0.270	-0.265	-0.258	0.434	0.409	0.411	71.4	72.05	71.45
300	-0.052	-0.044	-0.043	0.183	0.149	0.150	10.15	2.45	2.9
500	0.045	0.068	0.067	0.136	0.126	0.121	16.6	6.4	5.9
$\alpha_1 = 0.85$									
100	0.053	0.062	0.058	0.253	0.228	0.221	6.05	2.95	2.5
300	-0.012	0.009	-0.001	0.124	0.104	0.095	10.55	1.8	1.15
500	-0.026	-0.007	-0.011	0.096	0.073	0.069	13.25	0.9	0.7
$\alpha_1 = 0.9$									
100	0.025	0.038	0.360	0.191	0.163	0.157	6.85	2	1.65
300	-0.034	-0.018	-0.020	0.099	0.069	0.068	13.2	0.8	0.9
500	-0.025	-0.001	-0.001	0.072	0.044	0.044	22.3	1.95	1.8
$\alpha_1 = 0.95$									
100	-0.099	-0.088	-0.090	0.156	0.125	0.126	5.6	0.3	0.55
300	-0.046	-0.025	-0.026	0.083	0.045	0.045	22.5	2.2	2.25
500	-0.030	-0.006	-0.006	0.061	0.026	0.025	33.1	4.4	3.8
$\alpha_1 = 1$									
100	0	0	0	0	0	0	-	-	-
300	0	0	0	0	0	0	-	-	-
500	0	0	0	0	0	0	-	-	-

Notes: This table shows the result of experiment 1. Factors and error are generate from standard normal distribution. Factor loadings come form uniform distribution $IIDU(\mu_\beta - 0.2, \mu_\beta + 0.2)$, and $\mu_\beta = 0.71$. We keep $[n^{\alpha_j}]$ amount of loadings and assign the rest as zero. For each different time-unit combinations, we replicate 2000 times. For the size of the test, we use a two-tail test, under the hypothesis of $H_0, \hat{\alpha}_j = \alpha_j \ j = 1, 2$. Cause under the scenarios of $\alpha = 1$, the size of the test will collapse, therefore the table does not report the sizes for $\alpha_1 = 1$.

Table 5: Simulation result for double factors (no correlation)

n\T	Bias × 100			RMSE × 100			Size × 100		
	Double Factor with correlation $\rho_{12} = 0$								
	120	240	360	120	240	360	120	240	360
$\alpha_1 = 1, \alpha_2 = 0.7$									
100	0.567	0.737	0.628	4.062	3.819	3.799	2.95	1.45	1.85
300	0.512	0.611	0.518	2.398	2.103	1.979	6.25	0.55	0.5
500	-0.149	0.08	-0.019	1.796	1.498	1.443	8	0.2	0.1
$\alpha_1 = 1, \alpha_2 = 0.75$									
100	-3.051	-3.02	-3.092	4.582	4.245	4.248	2.45	0.1	0.10
300	0.491	-1.035	0.640	1.843	1.460	1.576	7.6	0.8	0.55
500	-0.611	-0.372	-0.393	1.520	1.136	1.125	11.35	0.15	0.1
$\alpha_1 = 1, \alpha_2 = 0.8$									
100	-3.752	-3.630	-3.581	4.557	4.213	4.210	84.65	85.9	85.25
300	-1.218	-0.331	-1.021	1.812	0.792	1.438	9.35	0.2	0.3
500	-0.022	0.192	0.147	1.047	0.782	0.742	15.35	1.1	1.1
$\alpha_1 = 1, \alpha_2 = 0.85$									
100	-0.075	0.127	0.088	1.996	1.697	1.606	5.4	1.15	0.95
300	-0.531	-0.406	-0.351	1.097	0.613	0.777	10.8	0.15	0.2
500	-0.647	-0.391	-0.391	1.020	0.643	0.630	19.1	0.15	0
$\alpha_1 = 1, \alpha_2 = 0.9$									
100	-0.128	0.043	0.025	1.428	1.143	1.118	4.9	0.65	0.7
300	-0.651	-0.334	-0.394	1.002	0.435	0.617	17.1	0.6	0.2
500	-0.434	-0.168	-0.171	0.7435	0.367	0.368	25.2	0.4	0.3
$\alpha_1 = 1, \alpha_2 = 0.95$									
100	-1.218	-1.043	-1.036	1.603	1.222	1.212	6.65	0.25	0.05
300	-0.611	-0.344	-0.356	0.881	0.435	0.434	23.35	0.6	0.45
500	-0.415	-0.123	-0.134	0.661	0.220	0.216	36.75	1.35	1.1
$\alpha_1 = 1, \alpha_2 = 1$									
100	0	0	0	0	0	0	-	-	-
300	0	0	0	0	0	0	-	-	-
500	0	0	0	0	0	0	-	-	-

Notes: This table shows the result of experiment 2. Factors and errors are generate from standard normal distribution. Between two factors, we assume they have no correlation. Factor loadings come form uniform distribution $IIDU(\mu_\beta - 0.2, \mu_\beta + 0.2)$, and μ_β is set to 0.71. We keep $[n^{\alpha_j}]$ amount of loadings and assign the rest as zero. For each different time-unit combinations, we replicate 2000 times. For the size of the test, we use a two-tail test, under the hypothesis of $H_0, \hat{\alpha}_j = \alpha_j \ j = 1, 2$. Cause under the scenarios of $\alpha = 1$, the size of the test will collapse, therefore the table does not report the sizes for $\alpha_1 = \alpha_2 = 1$

Table 6: Simulation result for double factors (weak correlation)

	Double Factor with correlation $\rho_{12} = 0.3$								
	Bias $\times$ 100			RMSE $\times$ 100			Size $\times$ 100		
	$\alpha_1 = 1, \alpha_2 = 0.7$								
n\T	120	240	360	120	240	360	120	240	360
100	0.038	0.064	0.072	0.421	0.382	0.389	4.6	1.75	1.95
300	0.021	0.058	0.056	0.253	0.206	0.198	9.95	0.9	0.25
500	-0.032	0.006	0	0.201	0.153	0	12.20	0.1	0.05
	$\alpha_1 = 1, \alpha_2 = 0.75$								
100	-0.325	-0.313	-0.310	0.488	0.419	0.420	4.75	0.1	0
300	0.028	0.063	0.065	0.253	0.157	0.159	9.95	0.55	0.5
500	-0.082	-0.037	-0.039	0.175	0.114	0.112	19.25	0.25	0.3
	$\alpha_1 = 1, \alpha_2 = 0.8$								
100	-0.393	-0.361	-0.368	0.477	0.418	0.421	85.45	85.2	86.4
300	0.029	-0.099	-0.100	0.192	0.145	0.145	12.2	0.65	0.5
500	-0.037	-0.016	0.016	0.129	0.074	0.074	27.8	0.25	1.2
	$\alpha_1 = 1, \alpha_2 = 0.85$								
100	-0.027	0.008	0.007	0.234	0.160	0.155	9.3	0.9	0.65
300	-0.147	-0.031	-0.037	0.219	0.079	0.077	16.75	0.3	0.2
500	-0.088	-0.039	-0.039	0.136	0.063	0.062	30.6	0.15	0
	$\alpha_1 = 1, \alpha_2 = 0.9$								
100	-0.033	0.003	0.002	0.173	0.111	0.110	9.4	0.6	0.55
300	-0.087	-0.040	-0.041	0.131	0.061	0.061	27.8	0.1	0.05
500	-0.070	-0.017	-0.018	0.111	0.037	0.037	41.15	0.6	0.35
	$\alpha_1 = 1, \alpha_2 = 0.95$								
100	-0.134	-0.101	-0.104	0.185	0.122	0.122	10.15	0.1	0.15
300	-0.083	-0.034	-0.034	0.118	0.043	0.044	39.35	0.6	0.6
500	-0.062	-0.013	-0.012	0.937	0.022	0.023	51.8	1.25	2.0
	$\alpha_1 = 1, \alpha_2 = 1$								
100	0	0	0	0	0	0	-	-	-
300	0	0	0	0	0	0	-	-	-
500	0	0	0	0	0	0	-	-	-

Notes: This table shows the result of experiment 2. Factors and errors are generate from standard normal distribution. Between two factors, we assume they have correlation $\rho_{12} = 0.3$ Factor loadings come form uniform distribution $IIDU(\mu_\beta - 0.2, \mu_\beta + 0.2)$, and μ_β is set to 0.71. We keep $[n^{\alpha_j}]$ amount of loadings and assign the rest as zero. For each different time-unit combinations, we replicate 2000 times. For the size of the test, we use a two-tail test, under the hypothesis of $H_0, \hat{\alpha}_j = \alpha_j \; j = 1, 2$. Cause under the scenarios of $\alpha = 1$, the size of the test will collapse, therefore the table does not report the sizes when $\alpha_1 = \alpha_2 = 1$

B Empirical Application Result

B.1 Factor Strength Estimation Table

Table 7: Comparison table of estimated factor strength base on three different data sets, rank from strong to weak

	Ten Year Data			Twenty Year Data			Thirty Year Data		
	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength
1	beta	0.976	0.749	ndp	0.960	0.904	salecash	0.905	0.857
2	baspread	0.980	0.730	salecash	0.958	0.902	ndp	0.905	0.852
3	turn	0.983	0.728	quick	0.958	0.901	quick	0.905	0.851
4	zerotrade	0.983	0.725	dy	0.957	0.897	age	0.905	0.851
5	idiovol	0.981	0.723	lev	0.959	0.897	roavol	0.904	0.850
6	retvol	0.978	0.721	cash	0.958	0.897	ep	0.905	0.849
7	std_turn	0.983	0.719	zs	0.959	0.896	depr	0.905	0.848
8	HML_Devil	0.989	0.719	cp	0.960	0.894	cash	0.905	0.847
9	maxret	0.981	0.715	roavol	0.957	0.894	rds	0.905	0.843
10	roavol	0.985	0.713	age	0.959	0.894	currat	0.905	0.840
11	age	0.989	0.703	cfp	0.960	0.893	chcsho	0.905	0.840
12	sp	0.985	0.699	op	0.958	0.893	zs	0.903	0.839
13	ala	0.986	0.699	nop	0.958	0.893	nop	0.904	0.839
14	ndp	0.987	0.686	ebp	0.959	0.893	dy	0.905	0.838
15	orgcap	0.989	0.686	ep	0.958	0.891	lev	0.903	0.838
16	tang	0.990	0.683	rds	0.958	0.890	cfp	0.905	0.838
17	ebp	0.988	0.683	depr	0.958	0.889	stdacc	0.905	0.837
18	invest	0.986	0.683	sp	0.958	0.888	cp	0.905	0.836
19	dpia	0.986	0.681	currat	0.958	0.887	stdcf	0.905	0.836
20	UMD	0.989	0.678	kz	0.958	0.887	op	0.904	0.835
21	zs	0.986	0.675	chcsho	0.957	0.884	ebp	0.903	0.835
22	grltnoa	0.988	0.675	tang	0.960	0.884	tang	0.904	0.833
23	dy	0.988	0.672	ato	0.958	0.884	kz	0.903	0.831
24	HML	0.987	0.672	stdacc	0.958	0.883	ato	0.904	0.831
25	kz	0.986	0.669	adm	0.958	0.881	ww	0.904	0.827
26	ob_a	0.989	0.669	cashpr	0.959	0.878	std_turn	0.902	0.826
27	BAB	0.989	0.666	stdcf	0.956	0.878	adm	0.904	0.825

Table 7: Comparison table of estimated factor strength base on three different data sets, rank from strong to weak (Cont.)

| | | Ten Year Data | | | Twenty Year Data | | | Thirty Year Data | |
	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength
28	op	0.990	0.663	HML	0.958	0.874	idiovol	0.902	0.825
29	realestate_hxz	0.987	0.663	nef	0.956	0.873	maxret	0.902	0.825
30	ol	0.987	0.663	std_turn	0.956	0.870	baspread	0.902	0.820
31	adm	0.988	0.660	idiovol	0.955	0.870	IPO	0.905	0.818
32	lev	0.986	0.657	zerotrade	0.953	0.865	nef	0.902	0.818
33	nxf	0.989	0.651	turn	0.955	0.864	sp	0.903	0.817
34	nop	0.989	0.651	ww	0.959	0.863	turn	0.902	0.813
35	pm	0.986	0.648	maxret	0.956	0.863	retvol	0.902	0.813
36	pchcapx3	0.988	0.644	absacc	0.960	0.859	zerotrade	0.900	0.812
37	nef	0.988	0.644	baspread	0.955	0.854	absacc	0.905	0.812
38	cash	0.989	0.637	hire	0.959	0.851	HML	0.903	0.811
39	QMJ	0.978	0.637	IPO	0.960	0.850	lgr	0.905	0.810
40	rds	0.989	0.634	lgr	0.959	0.850	cashpr	0.903	0.808
41	LIQ_PS	0.988	0.634	nxf	0.956	0.849	dcol	0.905	0.807
42	ato	0.988	0.634	retvol	0.955	0.848	beta	0.900	0.806
43	salerec	0.992	0.630	salerec	0.957	0.847	RMW	0.904	0.806
44	currat	0.989	0.626	RMW	0.957	0.847	hire	0.905	0.805
45	acc	0.989	0.619	beta	0.954	0.846	salerec	0.905	0.803
46	stdcf	0.989	0.619	sin	0.959	0.844	nxf	0.903	0.801
47	HXZ_ROE	0.989	0.619	acc	0.960	0.843	acc	0.904	0.797
48	depr	0.988	0.615	bm_ia	0.960	0.843	dfin	0.902	0.791
49	noa	0.989	0.615	dcol	0.959	0.838	nincr	0.904	0.790
50	cashpr	0.987	0.615	dfin	0.959	0.838	noa	0.902	0.787
51	absacc	0.989	0.615	HML_Devil	0.953	0.838	HML_Devil	0.902	0.781
52	gma	0.987	0.615	HXZ_IA	0.960	0.838	HXZ_IA	0.904	0.780
53	dncl	0.986	0.611	nincr	0.959	0.834	rdm	0.904	0.778
54	ms	0.980	0.611	rna	0.958	0.826	rna	0.904	0.778
55	rna	0.989	0.611	noa	0.957	0.825	rd	0.903	0.774
56	STR	0.987	0.607	herf	0.957	0.824	bm_ia	0.904	0.772
57	rdm	0.988	0.607	rdm	0.958	0.823	sgr	0.904	0.769
58	chcsho	0.987	0.607	sgr	0.958	0.819	ps	0.904	0.769
59	sin	0.987	0.607	dnco	0.959	0.816	sin	0.904	0.769
60	salecash	0.989	0.602	ps	0.957	0.807	realestate_hxz	0.905	0.769

Table 7: Comparison table of estimated factor strength base on three different data sets, rank from strong to weak (Cont.)

		Ten Year Data			Twenty Year Data			Thirty Year Data	
	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength
61	dnco	0.988	0.598	CMA	0.960	0.805	herf	0.902	0.766
62	quick	0.989	0.593	egr_hxz	0.958	0.803	dnco	0.904	0.761
63	stdacc	0.989	0.593	realestate_hxz	0.957	0.798	CMA	0.905	0.759
64	poa	0.988	0.593	gad	0.958	0.788	egr_hxz	0.904	0.750
65	cp	0.988	0.589	rd	0.958	0.787	ob_a	0.903	0.745
66	tb	0.988	0.589	ol	0.954	0.787	ol	0.902	0.741
67	HXZ_IA	0.987	0.584	cinvest_a	0.959	0.784	cinvest_a	0.903	0.739
68	saleinv	0.987	0.579	dolvol	0.960	0.774	gad	0.902	0.723
69	cfp	0.988	0.579	ob_a	0.955	0.764	SMB	0.902	0.721
70	egr	0.987	0.579	ala	0.958	0.762	dolvol	0.904	0.715
71	dnca	0.986	0.579	pchdepr	0.959	0.761	gma	0.902	0.715
72	egr_hxz	0.988	0.579	BAB	0.960	0.757	ala	0.904	0.715
73	os	0.984	0.569	gma	0.955	0.756	cto	0.902	0.710
74	pps	0.983	0.563	pchcapx3	0.957	0.752	aeavol	0.905	0.710
75	cto	0.987	0.563	dnca	0.958	0.747	BAB	0.905	0.710
76	grltnoa_hxz	0.986	0.563	SMB	0.957	0.745	convind	0.904	0.710
77	cei	0.988	0.563	poa	0.957	0.739	tb	0.902	0.708
78	CMA	0.988	0.563	aeavol	0.961	0.737	QMJ	0.903	0.708
79	em	0.989	0.552	tb	0.953	0.732	pricedelay	0.904	0.701
80	ww	0.990	0.546	grltnoa_hxz	0.958	0.730	egr	0.902	0.699
81	std_dolvol	0.987	0.539	cei	0.953	0.730	orgcap	0.902	0.699
82	grcapx	0.986	0.539	indmom	0.956	0.725	pchdepr	0.903	0.696
83	pctacc	0.989	0.539	egr	0.958	0.725	indmom	0.902	0.696
84	ep	0.989	0.533	moms12m	0.957	0.725	dcoa	0.902	0.696
85	pricedelay	0.989	0.533	dsti	0.957	0.723	moms12m	0.903	0.694
86	hire	0.988	0.519	orgcap	0.956	0.715	pchcapx3	0.902	0.691
87	SMB	0.987	0.512	pchcurrat	0.958	0.710	cei	0.902	0.691
88	pchcapx_ia	0.989	0.512	UMD	0.951	0.706	roic	0.902	0.691
89	aeavol	0.988	0.512	dcoa	0.959	0.706	pm	0.903	0.691
90	moms12m	0.987	0.512	roic	0.951	0.703	dnca	0.902	0.689
91	cashdebt	0.984	0.504	QMJ	0.951	0.703	saleinv	0.903	0.686
92	lgr	0.987	0.504	cinvest	0.958	0.701	grltnoa_hxz	0.903	0.683
93	cinvest	0.988	0.496	HXZ_ROE	0.957	0.699	poa	0.903	0.681

Table 7: Comparison table of estimated factor strength base on three different data sets, rank from strong to weak (Cont.)

| | | Ten Year Data | | | Twenty Year Data | | | Thirty Year Data | |
	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength
94	herf	0.987	0.496	cto	0.955	0.694	HXZ_ROE	0.905	0.678
95	bm_ia	0.988	0.487	pctacc	0.954	0.694	UMD	0.902	0.672
96	cfp_ia	0.987	0.479	pricedelay	0.958	0.691	pctacc	0.902	0.672
97	cinvest_a	0.989	0.479	pchcapx_ia	0.957	0.681	cinvest	0.903	0.660
98	chmom	0.989	0.469	convind	0.955	0.669	dsti	0.902	0.660
99	RMW	0.987	0.469	cdi	0.958	0.654	em	0.902	0.657
100	sue	0.987	0.459	rsup	0.957	0.651	pchcurrat	0.902	0.654
101	mom36m	0.986	0.459	chtx	0.958	0.644	ms	0.902	0.648
102	indmom	0.987	0.459	invest	0.957	0.644	invest	0.902	0.641
103	dcoa	0.988	0.459	em	0.952	0.644	pchcapx_ia	0.902	0.630
104	etr	0.986	0.448	pm	0.957	0.641	os	0.900	0.623
105	chinv	0.988	0.448	saleinv	0.955	0.637	chtx	0.902	0.623
106	ill	0.988	0.448	ta	0.958	0.634	dpia	0.902	0.623
107	roic	0.986	0.448	dpia	0.957	0.634	cdi	0.903	0.623
108	convind	0.988	0.448	pchquick	0.957	0.626	pps	0.902	0.611
109	sgr	0.988	0.437	os	0.948	0.626	roaq	0.900	0.602
110	IPO	0.989	0.437	ms	0.950	0.619	rs	0.902	0.584
111	dolvol	0.989	0.437	roaq	0.953	0.607	rsup	0.902	0.579
112	dcol	0.987	0.425	grcapx	0.955	0.593	chinv	0.902	0.569
113	nincr	0.989	0.411	pps	0.952	0.589	cfp_ia	0.902	0.563
114	chempia	0.987	0.411	ndf	0.957	0.589	ta	0.903	0.563
115	rs	0.988	0.411	cfp_ia	0.957	0.584	cashdebt	0.900	0.557
116	pchcapx	0.988	0.411	dncl	0.957	0.584	ndf	0.902	0.557
117	chtx	0.988	0.397	pchsale_pchrect	0.955	0.574	grcapx	0.902	0.552
118	ivg	0.988	0.381	mom6m	0.958	0.569	STR	0.902	0.546
119	LTR	0.985	0.364	rs	0.955	0.563	pchcapx	0.902	0.546
120	mom6m	0.987	0.364	pchcapx	0.958	0.563	pchquick	0.902	0.539
121	cdi	0.987	0.364	cashdebt	0.951	0.557	grltnoa	0.902	0.539
122	chatoia	0.987	0.364	pchsaleinv	0.955	0.557	pchsaleinv	0.902	0.519
123	gad	0.985	0.364	chempia	0.958	0.557	dncl	0.902	0.519
124	pchcurrat	0.988	0.297	LIQ_PS	0.956	0.557	ivg	0.902	0.504
125	pchgm_pchsale	0.988	0.297	dwc	0.955	0.546	mom6m	0.902	0.496
126	rd	0.986	0.297	grltnoa	0.956	0.533	chempia	0.902	0.496

Table 7: Comparison table of estimated factor strength base on three different data sets, rank from strong to weak (Cont.)

	Ten Year Data			Twenty Year Data			Thirty Year Data		
	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength	Factor	Market Factor Strength	Risk Factor Strength
127	dsti	0.989	0.297	STR	0.956	0.526	LIQ_PS	0.902	0.496
128	dfnl	0.987	0.297	dfnl	0.955	0.519	mom36m	0.902	0.479
129	roaq	0.986	0.297	mom36m	0.957	0.496	std_dolvol	0.903	0.459
130	pchdepr	0.988	0.266	std_dolvol	0.955	0.496	pchsale_pchinvt	0.902	0.448
131	dnoa	0.988	0.230	sue	0.956	0.487	pchsale_pchxsga	0.902	0.448
132	ta	0.988	0.230	LTR	0.954	0.487	dwc	0.902	0.448
133	chpmia	0.987	0.230	chmom	0.953	0.479	dfnl	0.902	0.437
134	pchquick	0.987	0.182	pchsale_pchinvt	0.955	0.448	chmom	0.902	0.437
135	dfin	0.988	0.182	chatoia	0.957	0.437	pchsale_pchrect	0.902	0.425
136	rsup	0.988	0.182	pchsale_pchxsga	0.957	0.425	sue	0.902	0.397
137	pchsaleinv	0.988	0.115	lfe	0.956	0.425	LTR	0.902	0.381
138	pchsale_pchinvt	0.988	0.115	chinv	0.956	0.397	pchgm_pchsale	0.902	0.322
139	pchsale_pchrect	0.988	0.115	ivg	0.957	0.397	lfe	0.902	0.297
140	ps	0.990	0.115	pchgm_pchsale	0.957	0.381	ill	0.902	0.297
141	dwc	0.989	0.115	etr	0.955	0.344	dnoa	0.902	0.182
142	pchsale_pchxsga	0.989	0.000	chpmia	0.957	0.344	ear	0.903	0.182
143	lfe	0.988	0.000	ill	0.955	0.266	chatoia	0.902	0.182
144	ndf	0.986	0.000	dnoa	0.955	0.266	chpmia	0.902	0.182
145	ear	0.988	0.000	ear	0.958	0.266	etr	0.902	0.115

Notes: This table presents the estimation results of factors' strength, ordered decreasingly by risk factor strength. For the estimation, we use the method from Section 3.3, with one market factor and one risk factor. The three data set is describe in the section 5.1

Table 8: Table of estimated factor strength, base on three data set, ranked
by the standard deviation among the three results

	Factor	10 Year Strength	20 Year Strength	30 Year Strength	Mean	Standard Deviation
1	ps	0.115	0.807	0.769	0.564	0.318
2	dfin	0.182	0.838	0.791	0.604	0.299
3	ndf	0.000	0.589	0.557	0.382	0.270
4	rd	0.297	0.787	0.774	0.619	0.228
5	pchdepr	0.266	0.761	0.696	0.574	0.219
6	rsup	0.182	0.651	0.579	0.471	0.206
7	pchsale_pchxsga	0.000	0.425	0.448	0.291	0.206
8	pchsaleinv	0.115	0.557	0.519	0.397	0.200
9	pchquick	0.182	0.626	0.539	0.449	0.192
10	pchsale_pchrect	0.115	0.574	0.425	0.371	0.191
11	nincr	0.411	0.834	0.790	0.678	0.190
12	dsti	0.297	0.723	0.660	0.560	0.188
13	dcol	0.425	0.838	0.807	0.690	0.188
14	IPO	0.437	0.850	0.818	0.702	0.188
15	gad	0.364	0.788	0.723	0.625	0.187
16	dwc	0.115	0.546	0.448	0.370	0.185
17	pchcurrat	0.297	0.710	0.654	0.554	0.183
18	lfe	0.000	0.425	0.297	0.240	0.178
19	ta	0.230	0.634	0.563	0.475	0.176
20	sgr	0.437	0.819	0.769	0.675	0.170
21	RMW	0.469	0.847	0.806	0.707	0.169
22	ep	0.533	0.891	0.849	0.758	0.160
23	pchsale_pchinvt	0.115	0.448	0.448	0.337	0.157
24	lgr	0.504	0.850	0.810	0.721	0.155
25	bm_ia	0.487	0.843	0.772	0.701	0.154
26	dolvol	0.437	0.774	0.715	0.642	0.147
27	hire	0.519	0.851	0.805	0.725	0.147
28	roaq	0.297	0.607	0.602	0.502	0.145
29	herf	0.496	0.824	0.766	0.695	0.143
30	ww	0.546	0.863	0.827	0.745	0.142
31	etr	0.448	0.344	0.115	0.302	0.139
32	cfp	0.579	0.893	0.838	0.770	0.137
33	quick	0.593	0.901	0.851	0.782	0.135
34	cinvest_a	0.479	0.784	0.739	0.667	0.135
35	cp	0.589	0.894	0.836	0.773	0.133
36	salecash	0.602	0.902	0.857	0.787	0.132
37	cdi	0.364	0.654	0.623	0.547	0.130
38	stdacc	0.593	0.883	0.837	0.771	0.127
39	chcsho	0.607	0.884	0.840	0.777	0.122
40	depr	0.615	0.889	0.848	0.784	0.121
41	indmom	0.459	0.725	0.696	0.627	0.119
42	roic	0.448	0.703	0.691	0.614	0.117
43	convind	0.448	0.669	0.710	0.609	0.115
44	dcoa	0.459	0.706	0.696	0.620	0.114
45	stdcf	0.619	0.878	0.836	0.778	0.114
46	currat	0.626	0.887	0.840	0.785	0.113
47	cash	0.637	0.897	0.847	0.794	0.113
48	chtx	0.397	0.644	0.623	0.555	0.112
49	rds	0.634	0.890	0.843	0.789	0.111
50	cashpr	0.615	0.878	0.808	0.767	0.111

Table 8: Table of estimated factor strength, base on three data set, ranked
by the standard deviation among the three results (Cont.)

	Factor	10 Year Strength	20 Year Strength	30 Year Strength	Mean	Standard Deviation
51	ear	0.000	0.266	0.182	0.149	0.111
52	HXZ_IA	0.584	0.838	0.780	0.734	0.109
53	ato	0.634	0.884	0.831	0.783	0.107
54	chatoia	0.364	0.437	0.182	0.328	0.107
55	absacc	0.615	0.859	0.812	0.762	0.106
56	SMB	0.512	0.745	0.721	0.659	0.105
57	CMA	0.563	0.805	0.759	0.709	0.105
58	nop	0.651	0.893	0.839	0.794	0.104
59	lev	0.657	0.897	0.838	0.798	0.102
60	aeavol	0.512	0.737	0.710	0.653	0.101
61	sin	0.607	0.844	0.769	0.740	0.099
62	nef	0.644	0.873	0.818	0.778	0.097
63	op	0.663	0.893	0.835	0.797	0.097
64	acc	0.619	0.843	0.797	0.753	0.097
65	egr_hxz	0.579	0.803	0.750	0.711	0.096
66	dy	0.672	0.897	0.838	0.803	0.095
67	moms12m	0.512	0.725	0.694	0.644	0.094
68	adm	0.660	0.881	0.825	0.789	0.094
69	salerec	0.630	0.847	0.803	0.760	0.094
70	zs	0.675	0.896	0.839	0.803	0.094
71	rdm	0.607	0.823	0.778	0.736	0.093
72	ndp	0.686	0.904	0.852	0.814	0.093
73	dnco	0.598	0.816	0.761	0.725	0.093
74	rna	0.611	0.826	0.778	0.738	0.092
75	kz	0.669	0.887	0.831	0.796	0.092
76	dfnl	0.297	0.519	0.437	0.418	0.092
77	noa	0.615	0.825	0.787	0.742	0.091
78	cinvest	0.496	0.701	0.660	0.619	0.089
79	ebp	0.683	0.893	0.835	0.804	0.088
80	tang	0.683	0.884	0.833	0.800	0.085
81	mom6m	0.364	0.569	0.496	0.476	0.085
82	nxf	0.651	0.849	0.801	0.767	0.084
83	HML	0.672	0.874	0.811	0.786	0.084
84	age	0.703	0.894	0.851	0.816	0.082
85	ill	0.448	0.266	0.297	0.337	0.080
86	sp	0.699	0.888	0.817	0.801	0.078
87	roavol	0.713	0.894	0.850	0.819	0.077
88	pricedelay	0.533	0.691	0.701	0.642	0.077
89	rs	0.411	0.563	0.584	0.519	0.077
90	chinv	0.448	0.397	0.569	0.471	0.072
91	cei	0.563	0.730	0.691	0.661	0.071
92	pchcapx_ia	0.512	0.681	0.630	0.608	0.071
93	grltnoa_hxz	0.563	0.730	0.683	0.659	0.070
94	dnca	0.579	0.747	0.689	0.671	0.070
95	pctacc	0.539	0.694	0.672	0.635	0.068
96	chpmia	0.230	0.344	0.182	0.252	0.068
97	pchcapx	0.411	0.563	0.546	0.507	0.068
98	cto	0.563	0.694	0.710	0.656	0.066
99	grltnoa	0.675	0.533	0.539	0.582	0.066
100	egr	0.579	0.725	0.699	0.668	0.064

Table 8: Table of estimated factor strength, base on three data set, ranked
by the standard deviation among the three results (Cont.)

Factor	10 Year Strength	20 Year Strength	30 Year Strength	Mean	Standard Deviation
101 std_turn	0.719	0.870	0.826	0.805	0.064
102 maxret	0.715	0.863	0.825	0.801	0.063
103 tb	0.589	0.732	0.708	0.676	0.063
104 idiovol	0.723	0.870	0.825	0.806	0.061
105 poa	0.593	0.739	0.681	0.671	0.060
106 chempia	0.411	0.557	0.496	0.488	0.060
107 gma	0.615	0.756	0.715	0.695	0.059
108 realestate_hxz	0.663	0.798	0.769	0.743	0.058
109 zerotrade	0.725	0.865	0.812	0.801	0.058
110 turn	0.728	0.864	0.813	0.802	0.056
111 LIQ_PS	0.634	0.557	0.496	0.562	0.056
112 LTR	0.364	0.487	0.381	0.411	0.055
113 ivg	0.381	0.397	0.504	0.427	0.055
114 retvol	0.721	0.848	0.813	0.794	0.054
115 baspread	0.730	0.854	0.820	0.801	0.053
116 ol	0.663	0.787	0.741	0.731	0.051
117 HML_Devil	0.719	0.838	0.781	0.779	0.049
118 em	0.552	0.644	0.657	0.618	0.047
119 cfp_ia	0.479	0.584	0.563	0.542	0.046
120 pchcapx3	0.644	0.752	0.691	0.696	0.044
121 saleinv	0.579	0.637	0.686	0.634	0.044
122 ob_a	0.669	0.764	0.745	0.726	0.041
123 beta	0.749	0.846	0.806	0.800	0.040
124 dncl	0.611	0.584	0.519	0.571	0.038
125 sue	0.459	0.487	0.397	0.448	0.038
126 BAB	0.666	0.757	0.710	0.711	0.037
127 pchgm_pchsale	0.297	0.381	0.322	0.333	0.035
128 dnoa	0.230	0.266	0.182	0.226	0.035
129 STR	0.607	0.526	0.546	0.559	0.034
130 HXZ_ROE	0.619	0.699	0.678	0.665	0.034
131 std_dolvol	0.539	0.496	0.459	0.498	0.033
132 QMJ	0.637	0.703	0.708	0.683	0.032
133 ala	0.699	0.762	0.715	0.725	0.027
134 os	0.569	0.626	0.623	0.606	0.026
135 cashdebt	0.504	0.557	0.557	0.540	0.025
136 dpia	0.681	0.634	0.623	0.646	0.025
137 grcapx	0.539	0.593	0.552	0.561	0.023
138 pm	0.648	0.641	0.691	0.660	0.022
139 pps	0.563	0.589	0.611	0.587	0.019
140 invest	0.683	0.644	0.641	0.656	0.019
141 chmom	0.469	0.479	0.437	0.461	0.018
142 ms	0.611	0.619	0.648	0.626	0.016
143 mom36m	0.459	0.496	0.479	0.478	0.015
144 UMD	0.678	0.706	0.672	0.685	0.015
145 orgcap	0.686	0.715	0.699	0.700	0.012

Notes: This table presents the estimated factor strength, using data from three different data set. For the data description see Section 5.1. The table also presents the calculated mean and standard deviation of each factors. The table is ordered decreasingly by the standard deviation.

Table 9: Comparing the ten year estimation strength and twenty year estimation strength, ranking base on the strength differences, from big to small

	Factor	10 Year Strength	20 Year Strength	Difference
1	ps	0.115	0.807	0.692
2	dfin	0.182	0.838	0.656
3	ndf	0.000	0.589	0.589
4	pchdepr	0.266	0.761	0.494
5	rd	0.297	0.787	0.490
6	rsup	0.182	0.651	0.469
7	pchsale_pchrect	0.115	0.574	0.459
8	pchquick	0.182	0.626	0.445
9	pchsaleinv	0.115	0.557	0.443
10	dwc	0.115	0.546	0.431
11	dsti	0.297	0.723	0.427
12	pchsale_pchxsga	0.000	0.425	0.425
13	lfe	0.000	0.425	0.425
14	gad	0.364	0.788	0.425
15	nincr	0.411	0.834	0.423
16	pchcurrat	0.297	0.710	0.414
17	dcol	0.425	0.838	0.414
18	IPO	0.437	0.850	0.413
19	ta	0.230	0.634	0.404
20	sgr	0.437	0.819	0.382
21	RMW	0.469	0.847	0.378
22	ep	0.533	0.891	0.358
23	bm_ia	0.487	0.843	0.356
24	lgr	0.504	0.850	0.346
25	dolvol	0.437	0.774	0.337
26	pchsale_pchinvt	0.115	0.448	0.334
27	hire	0.519	0.851	0.332
28	herf	0.496	0.824	0.328
29	ww	0.546	0.863	0.318
30	cfp	0.579	0.893	0.314
31	roaq	0.297	0.607	0.310
32	quick	0.593	0.901	0.308
33	cp	0.589	0.894	0.306
34	cinvest_a	0.479	0.784	0.306
35	salecash	0.602	0.902	0.300
36	cdi	0.364	0.654	0.290
37	stdacc	0.593	0.883	0.290
38	chcsho	0.607	0.884	0.278
39	depr	0.615	0.889	0.274
40	indmom	0.459	0.725	0.266
41	ear	0.000	0.266	0.266
42	cashpr	0.615	0.878	0.263
43	currat	0.626	0.887	0.260
44	cash	0.637	0.897	0.260
45	stdcf	0.619	0.878	0.259
46	rds	0.634	0.890	0.256
47	roic	0.448	0.703	0.255
48	HXZ_IA	0.584	0.838	0.254
49	ato	0.634	0.884	0.250

Table 9: Comparing the ten year estimation strength and twenty year estimation strength, ranking base on the strength differences, from big to small (Cont.)

Factor	10 Year Strength	20 Year Strength	Difference
50 chtx	0.397	0.644	0.247
51 dcoa	0.459	0.706	0.247
52 absacc	0.615	0.859	0.244
53 nop	0.651	0.893	0.242
54 CMA	0.563	0.805	0.241
55 lev	0.657	0.897	0.240
56 sin	0.607	0.844	0.238
57 SMB	0.512	0.745	0.233
58 op	0.663	0.893	0.230
59 nef	0.644	0.873	0.229
60 aeavol	0.512	0.737	0.226
61 dy	0.672	0.897	0.225
62 acc	0.619	0.843	0.225
63 egr_hxz	0.579	0.803	0.224
64 dfnl	0.297	0.519	0.222
65 convind	0.448	0.669	0.221
66 zs	0.675	0.896	0.221
67 adm	0.660	0.881	0.221
68 ndp	0.686	0.904	0.218
69 dnco	0.598	0.816	0.218
70 kz	0.669	0.887	0.217
71 salerec	0.630	0.847	0.217
72 rdm	0.607	0.823	0.216
73 rna	0.611	0.826	0.215
74 moms12m	0.512	0.725	0.214
75 noa	0.615	0.825	0.210
76 ebp	0.683	0.893	0.210
77 cinvest	0.496	0.701	0.205
78 mom6m	0.364	0.569	0.205
79 HML	0.672	0.874	0.202
80 tang	0.683	0.884	0.200
81 nxf	0.651	0.849	0.198
82 age	0.703	0.894	0.191
83 sp	0.699	0.888	0.189
84 roavol	0.713	0.894	0.182
85 ill	0.448	0.266	0.182
86 pchcapx_ia	0.512	0.681	0.169
87 dnca	0.579	0.747	0.168
88 grltnoa_hxz	0.563	0.730	0.166
89 cei	0.563	0.730	0.166
90 pricedelay	0.533	0.691	0.158
91 pctacc	0.539	0.694	0.154
92 rs	0.411	0.563	0.152
93 pchcapx	0.411	0.563	0.152
94 std_turn	0.719	0.870	0.151
95 maxret	0.715	0.863	0.149
96 idiovol	0.723	0.870	0.147
97 egr	0.579	0.725	0.147
98 poa	0.593	0.739	0.146

Table 9: Comparing the ten year estimation strength and twenty year estimation strength, ranking base on the strength differences, from big to small (Cont.)

Factor	10 Year Strength	20 Year Strength	Difference
99 chempia	0.411	0.557	0.146
100 tb	0.589	0.732	0.143
101 grltnoa	0.675	0.533	0.142
102 gma	0.615	0.756	0.141
103 zerotrade	0.725	0.865	0.140
104 turn	0.728	0.864	0.137
105 realestate_hxz	0.663	0.798	0.135
106 cto	0.563	0.694	0.131
107 retvol	0.721	0.848	0.127
108 baspread	0.730	0.854	0.125
109 LTR	0.364	0.487	0.124
110 ol	0.663	0.787	0.124
111 HML_Devil	0.719	0.838	0.119
112 chpmia	0.230	0.344	0.115
113 pchcapx3	0.644	0.752	0.108
114 cfp_ia	0.479	0.584	0.105
115 etr	0.448	0.344	0.104
116 beta	0.749	0.846	0.098
117 ob_a	0.669	0.764	0.095
118 em	0.552	0.644	0.093
119 BAB	0.666	0.757	0.091
120 pchgm_pchsale	0.297	0.381	0.085
121 STR	0.607	0.526	0.080
122 HXZ_ROE	0.619	0.699	0.080
123 LIQ_PS	0.634	0.557	0.076
124 chatoia	0.364	0.437	0.073
125 QMJ	0.637	0.703	0.066
126 ala	0.699	0.762	0.064
127 saleinv	0.579	0.637	0.059
128 os	0.569	0.626	0.058
129 grcapx	0.539	0.593	0.054
130 cashdebt	0.504	0.557	0.053
131 chinv	0.448	0.397	0.051
132 dpia	0.681	0.634	0.047
133 std_dolvol	0.539	0.496	0.043
134 invest	0.683	0.644	0.039
135 mom36m	0.459	0.496	0.037
136 dnoa	0.230	0.266	0.037
137 orgcap	0.686	0.715	0.029
138 sue	0.459	0.487	0.028
139 UMD	0.678	0.706	0.028
140 dncl	0.611	0.584	0.027
141 pps	0.563	0.589	0.026
142 ivg	0.381	0.397	0.016
143 chmom	0.469	0.479	0.009
144 ms	0.611	0.619	0.008
145 pm	0.648	0.641	0.007

Notes: This table presents the estimated factor strength, using data from the ten year data and twenty year data. For the data description see Section 5.1. The table also presents the difference between the two estimated strengths. The table is ordered decreasingly by the difference.

Table 10: Comparing the ten year estimation strength and thirty year estimation strength, ranking base on the strength differences, from big to small

	Factor	10 Year Strength	30 Year Strength	Difference
1	ps	0.115	0.769	0.654
2	dfin	0.182	0.791	0.609
3	ndf	0.000	0.557	0.557
4	rd	0.297	0.774	0.477
5	pchsale_pchxsga	0.000	0.448	0.448
6	pchdepr	0.266	0.696	0.430
7	pchsaleinv	0.115	0.519	0.404
8	rsup	0.182	0.579	0.397
9	dcol	0.425	0.807	0.382
10	IPO	0.437	0.818	0.381
11	nincr	0.411	0.790	0.378
12	dsti	0.297	0.660	0.364
13	gad	0.364	0.723	0.360
14	pchquick	0.182	0.539	0.358
15	pchcurrat	0.297	0.654	0.358
16	RMW	0.469	0.806	0.337
17	pchsale_pchinvt	0.115	0.448	0.334
18	etr	0.448	0.115	0.334
19	dwc	0.115	0.448	0.334
20	ta	0.230	0.563	0.334
21	sgr	0.437	0.769	0.332
22	ep	0.533	0.849	0.316
23	pchsale_pchrect	0.115	0.425	0.310
24	roaq	0.297	0.602	0.306
25	lgr	0.504	0.810	0.306
26	lfe	0.000	0.297	0.297
27	hire	0.519	0.805	0.285
28	bm_ia	0.487	0.772	0.285
29	ww	0.546	0.827	0.282
30	dolvol	0.437	0.715	0.278
31	herf	0.496	0.766	0.270
32	convind	0.448	0.710	0.262
33	cinvest_a	0.479	0.739	0.261
34	cfp	0.579	0.838	0.259
35	cdi	0.364	0.623	0.259
36	quick	0.593	0.851	0.258
37	salecash	0.602	0.857	0.255
38	cp	0.589	0.836	0.247
39	stdacc	0.593	0.837	0.244
40	roic	0.448	0.691	0.243
41	indmom	0.459	0.696	0.237
42	dcoa	0.459	0.696	0.237
43	chcsho	0.607	0.840	0.234
44	depr	0.615	0.848	0.233
45	chtx	0.397	0.623	0.226
46	stdcf	0.619	0.836	0.217
47	currat	0.626	0.840	0.214
48	cash	0.637	0.847	0.210
49	SMB	0.512	0.721	0.210

Table 10: Comparing the ten year estimation strength and thirty year estimation strength, ranking base on the strength differences, from big to small(Cont.)

Factor	10 Year Strength	30 Year Strength	Difference
50 rds	0.634	0.843	0.209
51 aeavol	0.512	0.710	0.199
52 absacc	0.615	0.812	0.197
53 ato	0.634	0.831	0.197
54 CMA	0.563	0.759	0.196
55 HXZ_IA	0.584	0.780	0.196
56 cashpr	0.615	0.808	0.194
57 nop	0.651	0.839	0.188
58 moms12m	0.512	0.694	0.182
59 chatoia	0.364	0.182	0.182
60 ear	0.000	0.182	0.182
61 lev	0.657	0.838	0.181
62 acc	0.619	0.797	0.178
63 nef	0.644	0.818	0.174
64 salerec	0.630	0.803	0.173
65 rs	0.411	0.584	0.172
66 noa	0.615	0.787	0.172
67 rdm	0.607	0.778	0.172
68 op	0.663	0.835	0.172
69 egr_hxz	0.579	0.750	0.172
70 pricedelay	0.533	0.701	0.168
71 rna	0.611	0.778	0.167
72 ndp	0.686	0.852	0.166
73 dy	0.672	0.838	0.166
74 adm	0.660	0.825	0.165
75 cinvest	0.496	0.660	0.164
76 zs	0.675	0.839	0.164
77 dnco	0.598	0.761	0.163
78 sin	0.607	0.769	0.162
79 kz	0.669	0.831	0.161
80 ill	0.448	0.297	0.152
81 ebp	0.683	0.835	0.152
82 nxf	0.651	0.801	0.150
83 tang	0.683	0.833	0.150
84 age	0.703	0.851	0.148
85 cto	0.563	0.710	0.147
86 dfnl	0.297	0.437	0.140
87 HML	0.672	0.811	0.139
88 LIQ_PS	0.634	0.496	0.138
89 roavol	0.713	0.850	0.138
90 grltnoa	0.675	0.539	0.136
91 pchcapx	0.411	0.546	0.134
92 pctacc	0.539	0.672	0.133
93 mom6m	0.364	0.496	0.132
94 cei	0.563	0.691	0.128
95 ivg	0.381	0.504	0.123
96 chinv	0.448	0.569	0.120
97 grltnoa_hxz	0.563	0.683	0.120
98 egr	0.579	0.699	0.120

Table 10: Comparing the ten year estimation strength and thirty year estimation strength, ranking base on the strength differences, from big to small(Cont.)

Factor	10 Year Strength	30 Year Strength	Difference
99 tb	0.589	0.708	0.119
100 pchcapx_ia	0.512	0.630	0.118
101 sp	0.699	0.817	0.118
102 maxret	0.715	0.825	0.110
103 dnca	0.579	0.689	0.110
104 saleinv	0.579	0.686	0.107
105 std_turn	0.719	0.826	0.107
106 em	0.552	0.657	0.106
107 realestate_hxz	0.663	0.769	0.105
108 idiovol	0.723	0.825	0.102
109 gma	0.615	0.715	0.100
110 retvol	0.721	0.813	0.092
111 dncl	0.611	0.519	0.092
112 baspread	0.730	0.820	0.091
113 poa	0.593	0.681	0.087
114 zerotrade	0.725	0.812	0.087
115 turn	0.728	0.813	0.086
116 cfp_ia	0.479	0.563	0.085
117 chempia	0.411	0.496	0.085
118 std_dolvol	0.539	0.459	0.080
119 ol	0.663	0.741	0.078
120 ob_a	0.669	0.745	0.076
121 QMJ	0.637	0.708	0.071
122 sue	0.459	0.397	0.062
123 HML_Devil	0.719	0.781	0.062
124 STR	0.607	0.546	0.061
125 HXZ_ROE	0.619	0.678	0.059
126 dpia	0.681	0.623	0.058
127 beta	0.749	0.806	0.057
128 os	0.569	0.623	0.054
129 cashdebt	0.504	0.557	0.053
130 dnoa	0.230	0.182	0.048
131 chpmia	0.230	0.182	0.048
132 pps	0.563	0.611	0.048
133 pchcapx3	0.644	0.691	0.047
134 BAB	0.666	0.710	0.044
135 pm	0.648	0.691	0.043
136 invest	0.683	0.641	0.042
137 ms	0.611	0.648	0.037
138 chmom	0.469	0.437	0.032
139 pchgm_pchsale	0.297	0.322	0.026
140 mom36m	0.459	0.479	0.019
141 LTR	0.364	0.381	0.017
142 ala	0.699	0.715	0.016
143 orgcap	0.686	0.699	0.013
144 grcapx	0.539	0.552	0.012
145 UMD	0.678	0.672	0.006

Notes: This table presents the estimated factor strength, using data from the ten year data and thirty year data. For the data description see Section 5.1. The table also presents the difference between the two estimated strengths. The table is ordered decreasingly by the difference.

Table 11: Comparing the twenty year estimation strength and thirty year estimation strength, ranking base on the strength differences, from big to small

Factor	20 Year Strength	30 Year Strength	Difference
1 chatoia	0.437	0.182	0.255
2 etr	0.344	0.115	0.230
3 chinv	0.397	0.569	0.172
4 chpmia	0.344	0.182	0.162
5 pchsale_pchrect	0.574	0.425	0.149
6 lfe	0.425	0.297	0.128
7 ivg	0.397	0.504	0.107
8 LTR	0.487	0.381	0.106
9 dwc	0.546	0.448	0.097
10 sue	0.487	0.397	0.090
11 pchquick	0.626	0.539	0.087
12 dnoa	0.266	0.182	0.085
13 ear	0.266	0.182	0.085
14 dfnl	0.519	0.437	0.082
15 sin	0.844	0.769	0.075
16 mom6m	0.569	0.496	0.073
17 rsup	0.651	0.579	0.072
18 bm_ia	0.843	0.772	0.071
19 ta	0.634	0.563	0.071
20 sp	0.888	0.817	0.071
21 cashpr	0.878	0.808	0.070
22 gad	0.788	0.723	0.065
23 dncl	0.584	0.519	0.065
24 pchdepr	0.761	0.696	0.065
25 dsti	0.723	0.660	0.063
26 HML	0.874	0.811	0.063
27 chempia	0.557	0.496	0.062
28 LIQ_PS	0.557	0.496	0.062
29 pchcapx3	0.752	0.691	0.061
30 dy	0.897	0.838	0.059
31 lev	0.897	0.838	0.059
32 pchgm_pchsale	0.381	0.322	0.059
33 dolvol	0.774	0.715	0.059
34 poa	0.739	0.681	0.059
35 HXZ_IA	0.838	0.780	0.058
36 cp	0.894	0.836	0.058
37 dnca	0.747	0.689	0.058
38 herf	0.824	0.766	0.058
39 op	0.893	0.835	0.058
40 ebp	0.893	0.835	0.058
41 HML_Devil	0.838	0.781	0.057
42 zs	0.896	0.839	0.057
43 adm	0.881	0.825	0.056
44 kz	0.887	0.831	0.056
45 pchcurrat	0.710	0.654	0.056
46 dnco	0.816	0.761	0.055
47 nef	0.873	0.818	0.055
48 cfp	0.893	0.838	0.055
49 nop	0.893	0.839	0.054

Table 11: Comparing the twenty year estimation strength and thirty year estimation strength, ranking base on the strength differences, from big to small(Cont.)

Factor	20 Year Strength	30 Year Strength	Difference
50 zerotrade	0.865	0.812	0.053
51 ato	0.884	0.831	0.053
52 egr_hxz	0.803	0.750	0.053
53 ndp	0.904	0.852	0.052
54 turn	0.864	0.813	0.051
55 tang	0.884	0.833	0.051
56 sgr	0.819	0.769	0.050
57 pchcapx_ia	0.681	0.630	0.050
58 pm	0.641	0.691	0.050
59 cash	0.897	0.847	0.050
60 quick	0.901	0.851	0.050
61 nxf	0.849	0.801	0.049
62 saleinv	0.637	0.686	0.049
63 rna	0.826	0.778	0.048
64 ala	0.762	0.715	0.048
65 BAB	0.757	0.710	0.047
66 dfin	0.838	0.791	0.047
67 absacc	0.859	0.812	0.047
68 hire	0.851	0.805	0.047
69 acc	0.843	0.797	0.047
70 rds	0.890	0.843	0.047
71 currat	0.887	0.840	0.047
72 grltnoa_hxz	0.730	0.683	0.046
73 stdacc	0.883	0.837	0.046
74 ol	0.787	0.741	0.046
75 CMA	0.805	0.759	0.046
76 idiovol	0.870	0.825	0.045
77 salecash	0.902	0.857	0.045
78 cinvest_a	0.784	0.739	0.045
79 rdm	0.823	0.778	0.045
80 chcsho	0.884	0.840	0.044
81 std_turn	0.870	0.826	0.044
82 roavol	0.894	0.850	0.044
83 nincr	0.834	0.790	0.044
84 salerec	0.847	0.803	0.044
85 age	0.894	0.851	0.043
86 stdcf	0.878	0.836	0.042
87 chmom	0.479	0.437	0.042
88 grcapx	0.593	0.552	0.042
89 RMW	0.847	0.806	0.041
90 ep	0.891	0.849	0.041
91 convind	0.669	0.710	0.041
92 gma	0.756	0.715	0.041
93 depr	0.889	0.848	0.041
94 lgr	0.850	0.810	0.041
95 cinvest	0.701	0.660	0.041
96 beta	0.846	0.806	0.040
97 cei	0.730	0.691	0.038
98 pchsaleinv	0.557	0.519	0.038

Table 11: Comparing the twenty year estimation strength and thirty year estimation strength, ranking base on the strength differences, from big to small(Cont.)

Factor	20 Year Strength	30 Year Strength	Difference
99 maxret	0.863	0.825	0.038
100 ps	0.807	0.769	0.038
101 noa	0.825	0.787	0.038
102 std_dolvol	0.496	0.459	0.037
103 ww	0.863	0.827	0.036
104 retvol	0.848	0.813	0.035
105 baspread	0.854	0.820	0.034
106 UMD	0.706	0.672	0.033
107 IPO	0.850	0.818	0.032
108 moms12m	0.725	0.694	0.032
109 cdi	0.654	0.623	0.031
110 ndf	0.589	0.557	0.031
111 dcol	0.838	0.807	0.031
112 ill	0.266	0.297	0.030
113 indmom	0.725	0.696	0.029
114 realestate_hxz	0.798	0.769	0.029
115 ms	0.619	0.648	0.029
116 aeavol	0.737	0.710	0.027
117 egr	0.725	0.699	0.027
118 SMB	0.745	0.721	0.024
119 pchsale_pchxsga	0.425	0.448	0.024
120 tb	0.732	0.708	0.024
121 pps	0.589	0.611	0.022
122 chtx	0.644	0.623	0.022
123 pctacc	0.694	0.672	0.021
124 cfp_ia	0.584	0.563	0.021
125 rs	0.563	0.584	0.021
126 HXZ_ROE	0.699	0.678	0.021
127 STR	0.526	0.546	0.019
128 ob_a	0.764	0.745	0.019
129 pchcapx	0.563	0.546	0.017
130 mom36m	0.496	0.479	0.017
131 cto	0.694	0.710	0.017
132 orgcap	0.715	0.699	0.016
133 rd	0.787	0.774	0.013
134 em	0.644	0.657	0.013
135 roic	0.703	0.691	0.012
136 dpia	0.634	0.623	0.011
137 pricedelay	0.691	0.701	0.010
138 dcoa	0.706	0.696	0.010
139 grltnoa	0.533	0.539	0.006
140 QMJ	0.703	0.708	0.005
141 roaq	0.607	0.602	0.004
142 os	0.626	0.623	0.004
143 invest	0.644	0.641	0.003
144 cashdebt	0.557	0.557	0.000
145 pchsale_pchinvt	0.448	0.448	0.000

Notes: This table presents the estimated factor strength, using data from the twenty year data and thirty year data. For the data description see Section 5.1. The table also presents the difference between the two estimated strengths. The table is ordered decreasingly by the difference.

Table 12: Decompose the thirty year data into three ten year subset, estimated the factor strength base on those three data set separately. Rank the result base on the factor strength, from strong to weak.

	January 1988 to December 1997		January 1998 to December 2007		January 2008 to December 2017	
Rank	Factor	Strength	Factor	Strength	Factor	Strength
1	herf	0.69	ep	0.83	beta	0.70
2	turn	0.67	roavol	0.83	baspread	0.66
3	saleinv	0.66	nop	0.82	turn	0.66
4	beta	0.66	salecash	0.82	zerotrade	0.66
5	cto	0.66	ndp	0.82	retvol	0.66
6	nef	0.66	dy	0.82	std_turn	0.66
7	zerotrade	0.65	depr	0.82	idiovol	0.66
8	ala	0.65	cp	0.82	roavol	0.65
9	idiovol	0.64	quick	0.82	maxret	0.65
10	ol	0.64	op	0.82	sp	0.63
11	depr	0.64	cash	0.82	age	0.63
12	std_turn	0.64	lev	0.82	dy	0.63
13	gma	0.64	age	0.82	tang	0.63
14	dy	0.63	cfp	0.82	ol	0.62
15	retvol	0.63	ebp	0.82	HML_Devil	0.62
16	baspread	0.63	HML	0.81	op	0.61
17	currat	0.63	zs	0.81	ala	0.61
18	op	0.63	kz	0.81	realestate_hxz	0.61
19	nxf	0.62	currat	0.81	dpia	0.61
20	tang	0.61	rds	0.81	invest	0.61
21	nop	0.61	baspread	0.81	orgcap	0.61
22	maxret	0.61	chcsho	0.81	pm	0.60
23	pm	0.61	beta	0.81	ob_a	0.60
24	orgcap	0.61	sp	0.80	nop	0.60
25	quick	0.61	stdcf	0.80	BAB	0.60
26	SMB	0.61	zerotrade	0.80	grltnoa	0.59
27	sp	0.59	stdacc	0.80	HML	0.59
28	roavol	0.59	maxret	0.80	nef	0.59
29	pricedelay	0.59	retvol	0.80	nxf	0.58
30	aeavol	0.59	std_turn	0.80	UMD	0.58
31	convind	0.59	idiovol	0.80	absacc	0.58
32	bm_ia	0.58	nef	0.80	currat	0.57
33	cash	0.58	turn	0.80	acc	0.57
34	ndp	0.57	ato	0.80	pchcapx3	0.57
35	ivg	0.57	tang	0.79	dncl	0.56
36	hire	0.57	ww	0.79	salerec	0.56
37	egr_hxz	0.56	cashpr	0.79	ndp	0.56
38	roic	0.56	adm	0.79	ebp	0.56
39	HXZ_IA	0.56	IPO	0.78	adm	0.55
40	salerec	0.56	HML_Devil	0.77	LIQ_PS	0.55
41	cp	0.56	RMW	0.77	cash	0.55
42	chinv	0.56	dfin	0.77	QMJ	0.55
43	dcoa	0.56	sin	0.77	rds	0.55
44	dnco	0.56	acc	0.76	stdcf	0.55
45	ebp	0.56	nxf	0.76	lev	0.54
46	cashpr	0.56	bm_ia	0.76	zs	0.54
47	invest	0.56	absacc	0.76	kz	0.54

Table 12: Decompose the thirty year data into three ten year subset, esti-
mated the factor strength base on those three data set separately(cont.)

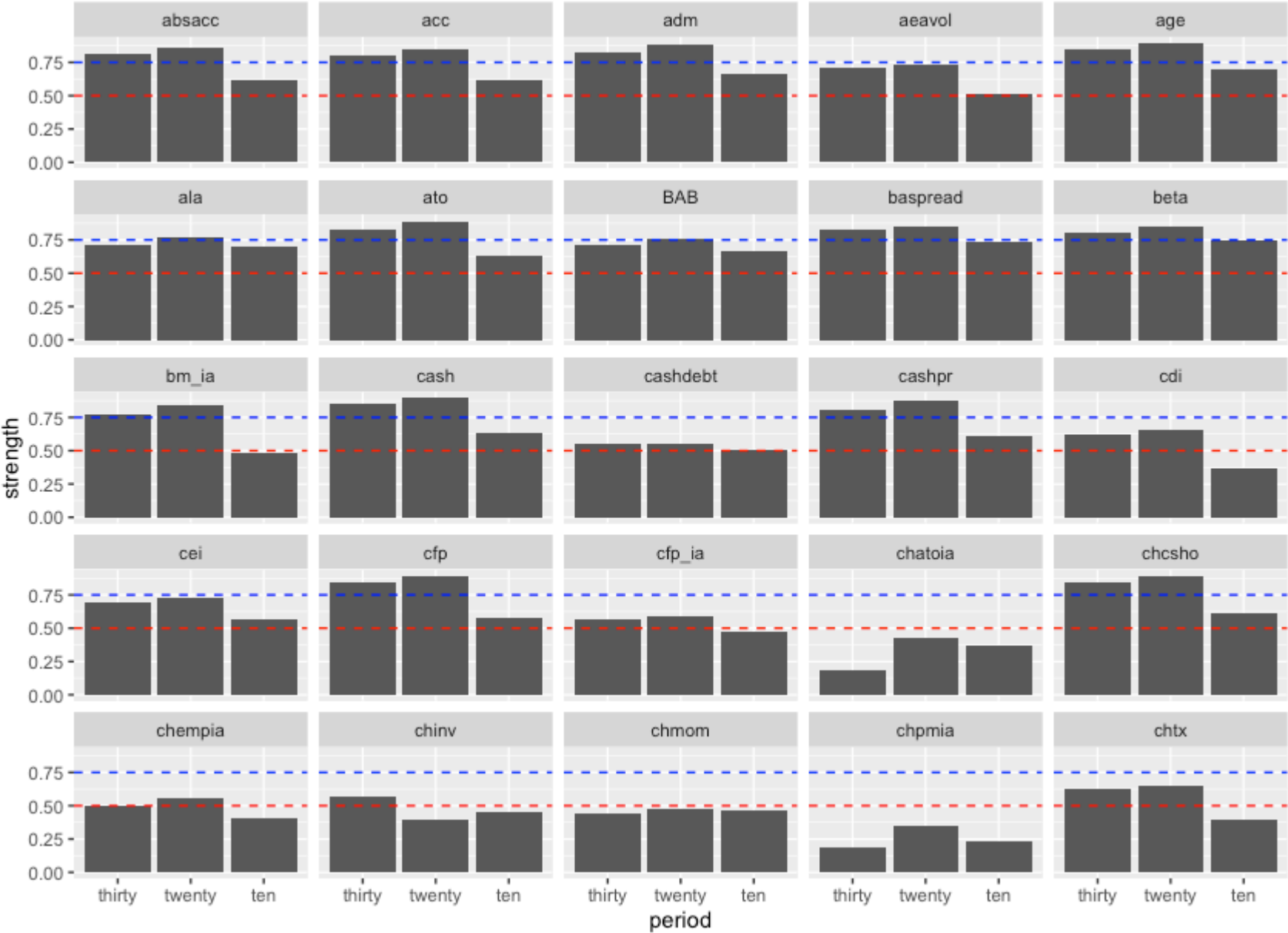|	January 1988 to December 1997		January 1998 to December 2007		January 2008 to December 2017	
Rank	Factor	Strength	Factor	Strength	Factor	Strength
48	HML_Devil	0.56	salerec	0.75	saleinv	0.53
49	poa	0.55	lgr	0.75	depr	0.53
50	sgr	0.55	dcol	0.75	STR	0.53
51	age	0.55	hire	0.75	ato	0.53
52	dpia	0.55	noa	0.74	poa	0.53
53	cdi	0.55	nincr	0.74	chcsho	0.53
54	em	0.55	dnco	0.74	stdacc	0.53
55	QMJ	0.55	herf	0.73	salecash	0.52
56	salecash	0.54	ala	0.73	cto	0.52
57	HML	0.54	rdm	0.73	noa	0.52
58	egr	0.54	ps	0.71	rna	0.52
59	pchcapx3	0.54	rd	0.71	cashpr	0.52
60	dcol	0.53	HXZ_IA	0.71	gma	0.52
61	acc	0.52	cinvest_a	0.71	HXZ_IA	0.52
62	zs	0.52	BAB	0.71	HXZ_ROE	0.52
63	nincr	0.52	sgr	0.70	quick	0.51
64	rdm	0.52	dolvol	0.70	cp	0.51
65	kz	0.52	SMB	0.70	rdm	0.51
66	rsup	0.52	realestate_hxz	0.70	dnco	0.51
67	pps	0.51	rna	0.69	egr_hxz	0.51
68	grltnoa_hxz	0.51	ndf	0.68	ms	0.51
69	cfp	0.51	CMA	0.68	pps	0.50
70	pctacc	0.51	pchdepr	0.67	egr	0.50
71	ep	0.50	egr_hxz	0.67	ww	0.50
72	lev	0.50	cei	0.67	grltnoa_hxz	0.49
73	chcsho	0.50	poa	0.66	cfp	0.49
74	UMD	0.50	QMJ	0.66	dnca	0.49
75	lgr	0.50	moms12m	0.66	os	0.47
76	CMA	0.50	ol	0.66	sin	0.47
77	dnca	0.49	tb	0.65	pctacc	0.47
78	chmom	0.49	roic	0.64	tb	0.45
79	cei	0.49	ob_a	0.64	grcapx	0.45
80	indmom	0.48	indmom	0.63	cei	0.45
81	dwc	0.48	HXZ_ROE	0.62	hire	0.45
82	realestate_hxz	0.48	gma	0.62	CMA	0.45
83	STR	0.47	aeavol	0.61	SMB	0.44
84	absacc	0.47	orgcap	0.61	pchcapx_ia	0.44
85	RMW	0.45	rsup	0.61	std_dolvol	0.44
86	os	0.44	UMD	0.61	ep	0.42
87	chempia	0.44	grltnoa_hxz	0.60	cfp_ia	0.42
88	rds	0.44	cinvest	0.60	aeavol	0.42
89	ob_a	0.44	pchcapx3	0.60	RMW	0.42
90	ms	0.44	pchcurrat	0.59	pricedelay	0.41
91	ps	0.42	pctacc	0.59	dcoa	0.41
92	grltnoa	0.42	pchcapx_ia	0.59	herf	0.41
93	roaq	0.42	dfnl	0.59	em	0.41
94	BAB	0.42	dnca	0.58	dolvol	0.40
95	grcapx	0.41	gad	0.58	chinv	0.40
96	rs	0.41	mom36m	0.57	cinvest	0.40

Table 12: Decompose the thirty year data into three ten year subset, estimated the factor strength base on those three data set separately(cont.)

Rank	January 1988 to December 1997		January 1998 to December 2007		January 2008 to December 2017	
	Factor	Strength	Factor	Strength	Factor	Strength
97	moms12m	0.41	pchquick	0.57	lgr	0.40
98	chatoia	0.41	cfp_ia	0.57	mom36m	0.38
99	mom36m	0.40	pricedelay	0.56	rs	0.38
100	dolvol	0.40	egr	0.56	moms12m	0.38
101	cinvest	0.40	dsti	0.56	LTR	0.36
102	ww	0.40	convind	0.56	IPO	0.36
103	std_dolvol	0.38	ms	0.56	nincr	0.36
104	stdcf	0.38	cdi	0.55	indmom	0.36
105	chtx	0.36	dcoa	0.55	bm_ia	0.36
106	rd	0.36	dncl	0.55	chempia	0.36
107	ato	0.36	cto	0.54	cinvest_a	0.36
108	HXZ_ROE	0.36	cashdebt	0.53	roic	0.36
109	cashdebt	0.34	ta	0.53	cashdebt	0.34
110	ta	0.34	roaq	0.53	sgr	0.34
111	dfnl	0.34	pchsaleinv	0.52	ill	0.32
112	pchcapx	0.32	pchsale_pchrect	0.52	chmom	0.32
113	stdacc	0.32	chtx	0.50	etr	0.30
114	adm	0.30	em	0.50	ivg	0.30
115	noa	0.30	pchsale_pchinvt	0.50	convind	0.30
116	pchsale_pchinvt	0.27	mom6m	0.48	pchgm_pchsale	0.27
117	cfp_ia	0.27	pm	0.48	chtx	0.27
118	cinvest_a	0.27	pchsale_pchxsga	0.47	dcol	0.27
119	dfin	0.27	os	0.47	cdi	0.27
120	rna	0.27	pchcapx	0.45	sue	0.23
121	sue	0.23	chempia	0.44	mom6m	0.23
122	IPO	0.23	dwc	0.44	dfnl	0.23
123	pchsale_pchxsga	0.23	saleinv	0.42	pchcapx	0.23
124	etr	0.23	LIQ_PS	0.40	chatoia	0.23
125	lfe	0.23	pchgm_pchsale	0.38	pchcurrat	0.18
126	ndf	0.23	pps	0.34	pchdepr	0.18
127	LTR	0.18	rs	0.34	dwc	0.18
128	pchsaleinv	0.18	lfe	0.32	chpmia	0.18
129	mom6m	0.18	LTR	0.30	gad	0.18
130	ill	0.18	std_dolvol	0.30	rd	0.11
131	sin	0.18	chinv	0.27	dnoa	0.11
132	pchsale_pchrect	0.11	chmom	0.27	dfin	0.11
133	LIQ_PS	0.11	grcapx	0.27	dsti	0.11
134	tb	0.11	STR	0.23	rsup	0.11
135	dncl	0.11	ill	0.23	roaq	0.11
136	dsti	0.11	chpmia	0.23	pchquick	0.00
137	ear	0.11	sue	0.18	pchsaleinv	0.00
138	pchcurrat	0.00	grltnoa	0.18	pchsale_pchinvt	0.00
139	pchquick	0.00	dnoa	0.18	pchsale_pchrect	0.00
140	pchdepr	0.00	ear	0.18	pchsale_pchxsga	0.00
141	pchgm_pchsale	0.00	chatoia	0.18	lfe	0.00
142	pchcapx_ia	0.00	etr	0.11	ps	0.00
143	dnoa	0.00	ivg	0.11	ta	0.00
144	chpmia	0.00	dpia	0.00	ndf	0.00

Table 12: Decompose the thirty year data into three ten year subset, esti-
mated the factor strength base on those three data set separately(cont.)

	January 1988 to December 1997		January 1998 to December 2007		January 2008 to December 2017	
Rank	Factor	Strength	Factor	Strength	Factor	Strength
145	gad	0.00	invest	0.00	ear	0.00

Notes: This table presents the estimated factor strength, using the decomposed thirty years data. The thirty year data set is decomposed into three subsets: January 1988 to December 1997, January 1998 to December 2007, and January 2008 to December 2017. For each data set, it contains 120 observations (t = 120), and 242 units (n = 242) The table also contains the full sample estimation results of factor strength, and the standard deviation among the three sub samples results. The table is ordered decreasingly base on the full sample factor strength.

## B.2   Strength Comparison Figures

Figure 1: Strength Comparison
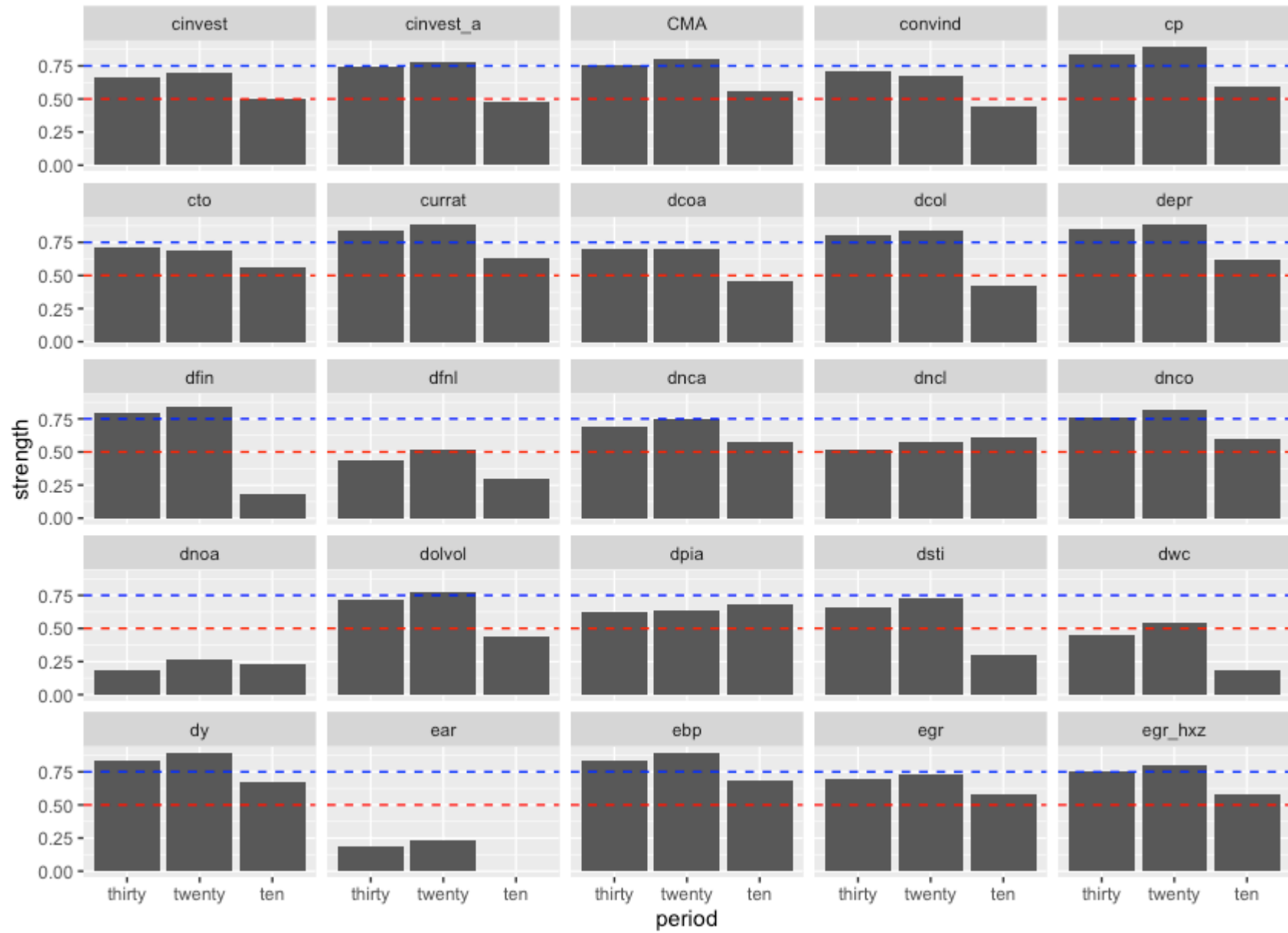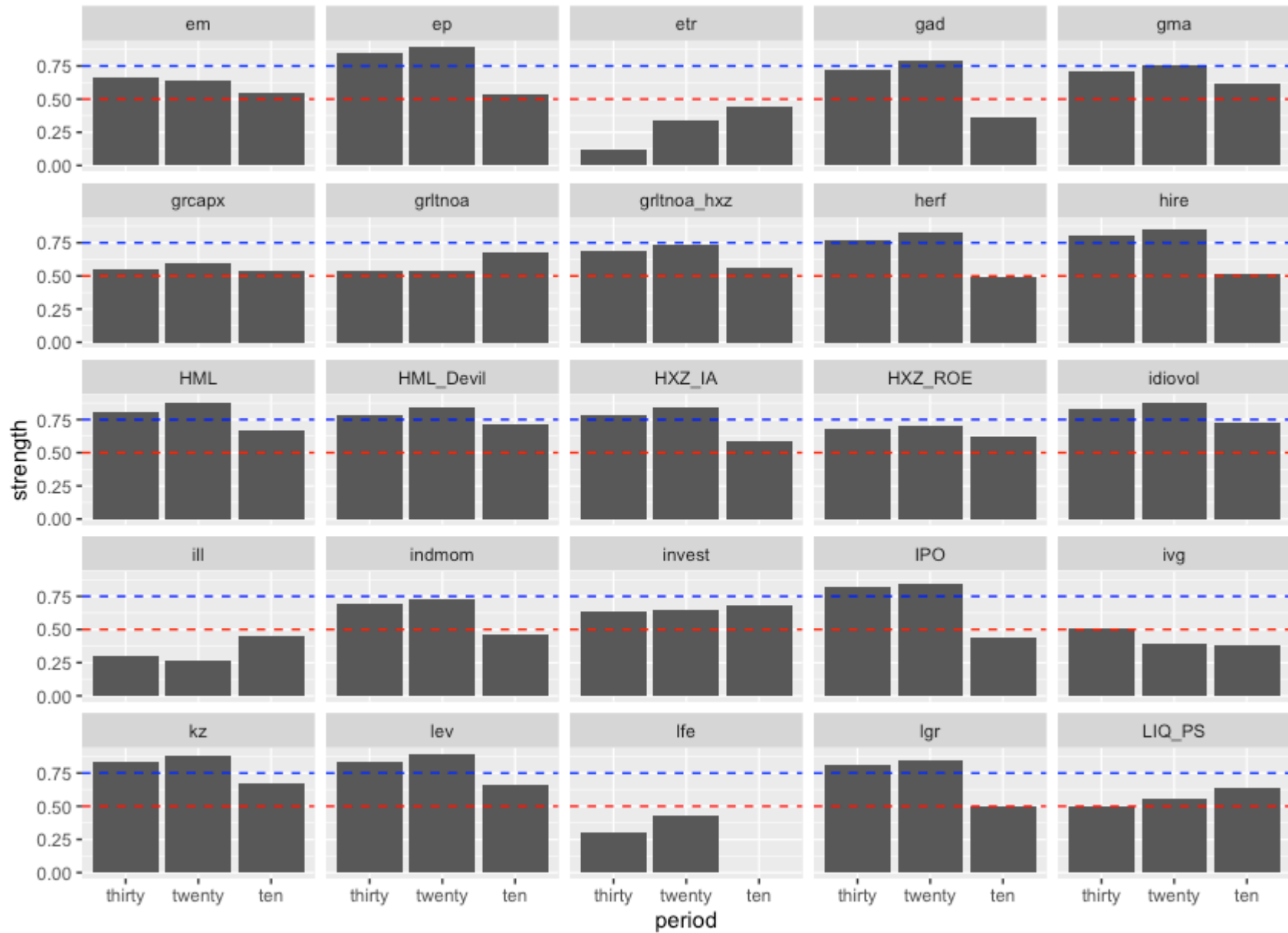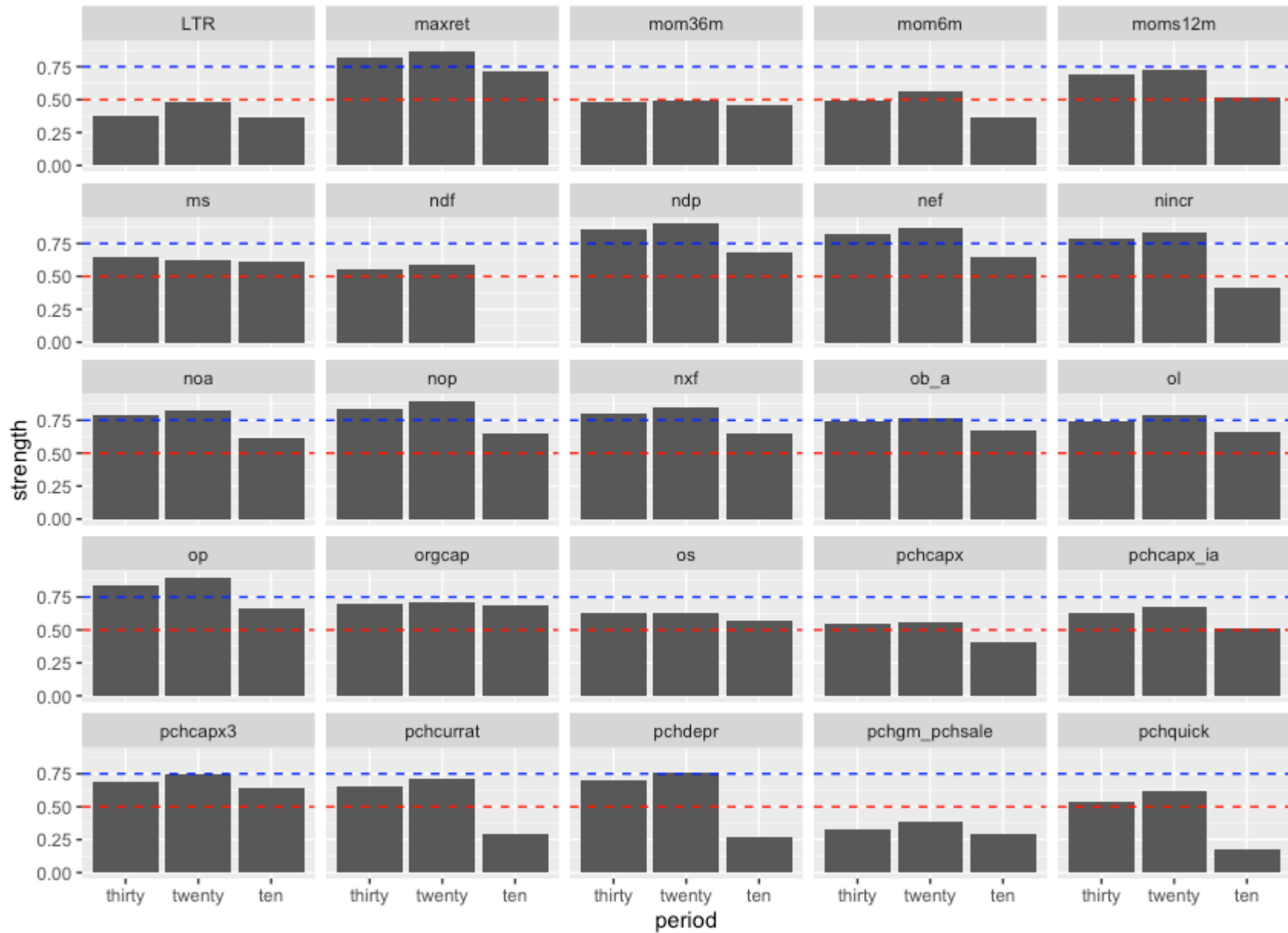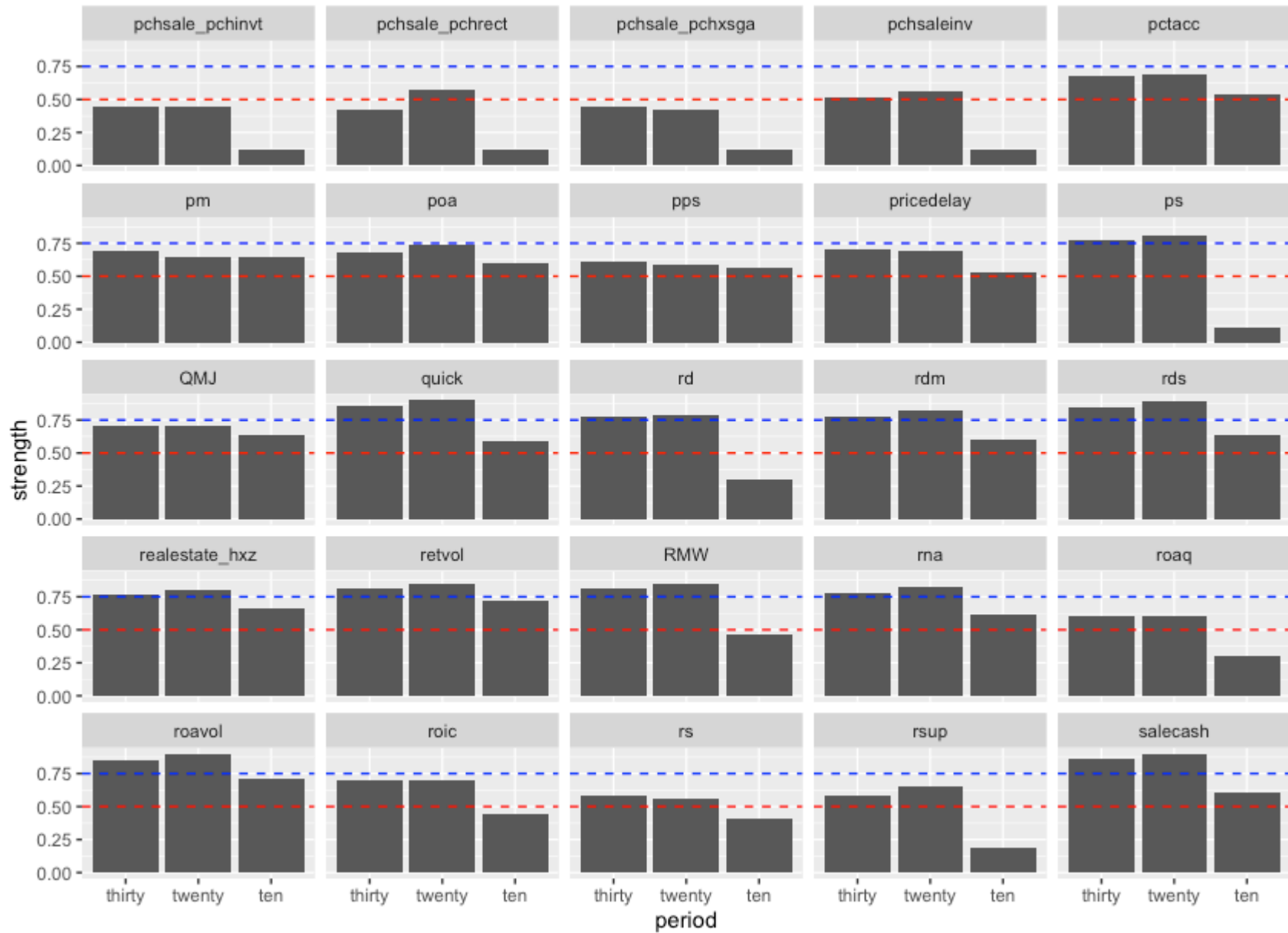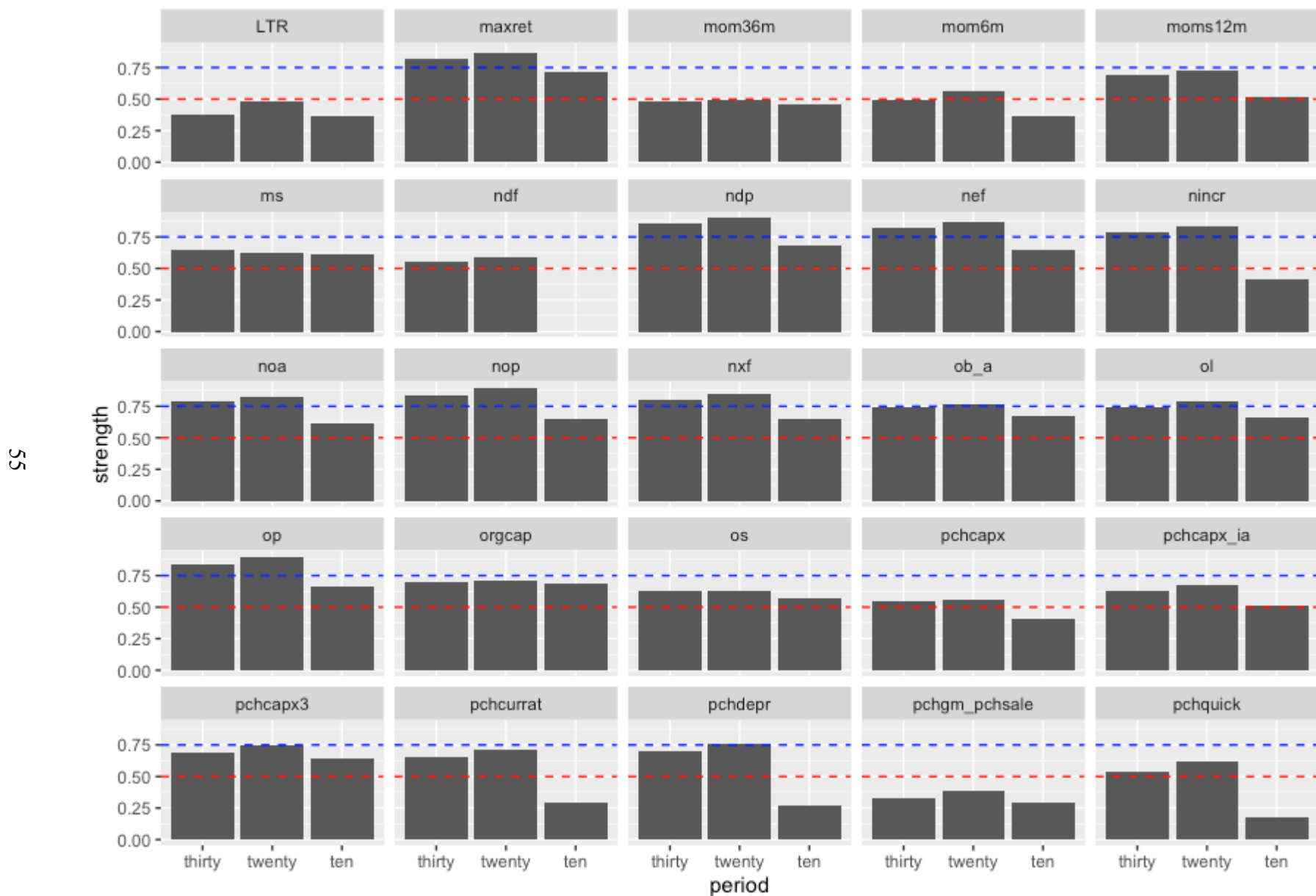
Notes: The figure compare the strength of every factor's strength in different data set. The x-axis indicates the data set: thirty is thirty years data set (January 1987 to December 2017), twenty is twenty year data set (January 1997 to December 2017), and ten is ten year data set (January 2007 to December 2017). The red dash line and blue dash line represent 0.5 and 0.75 threshold value respectively.
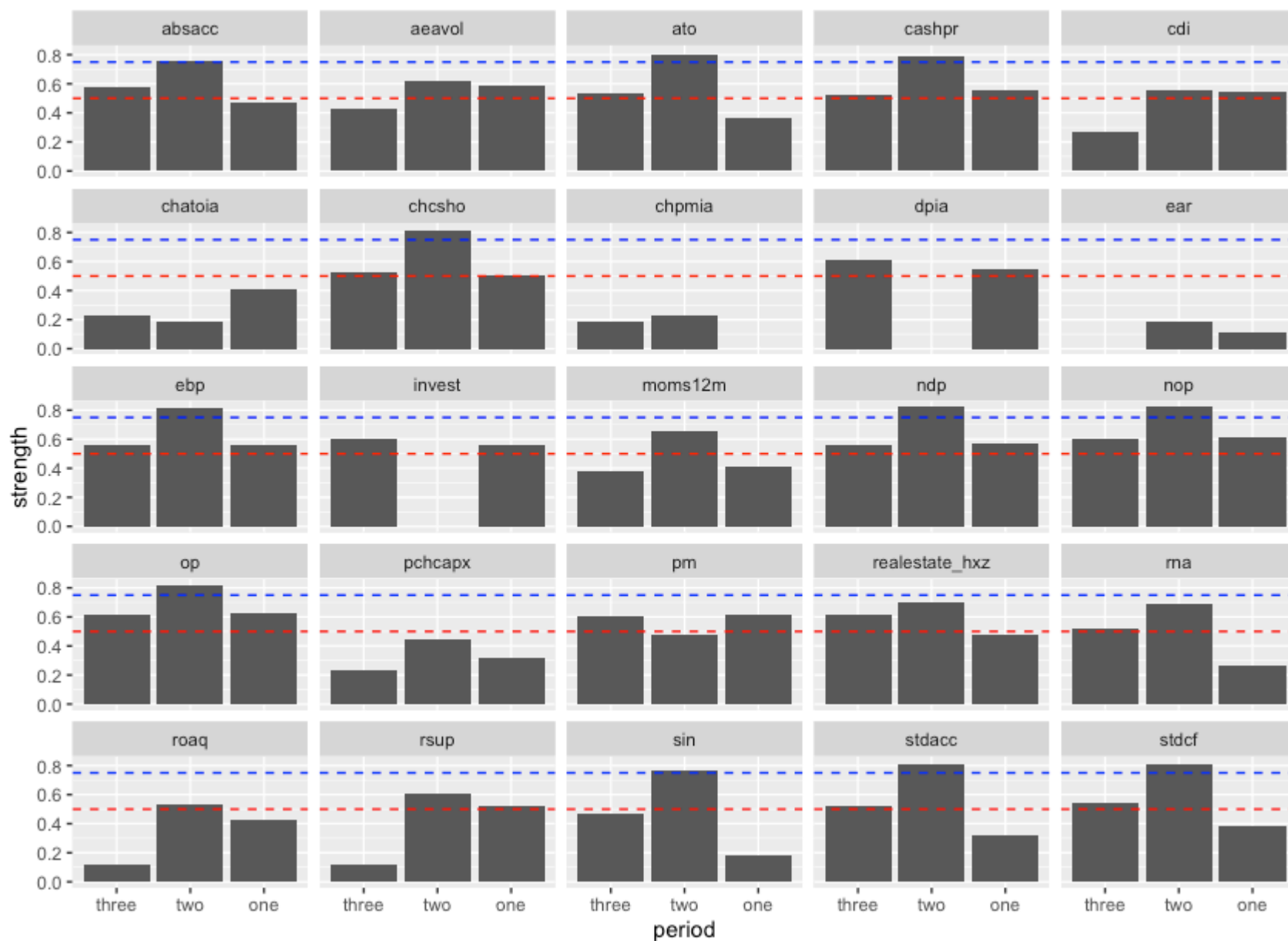
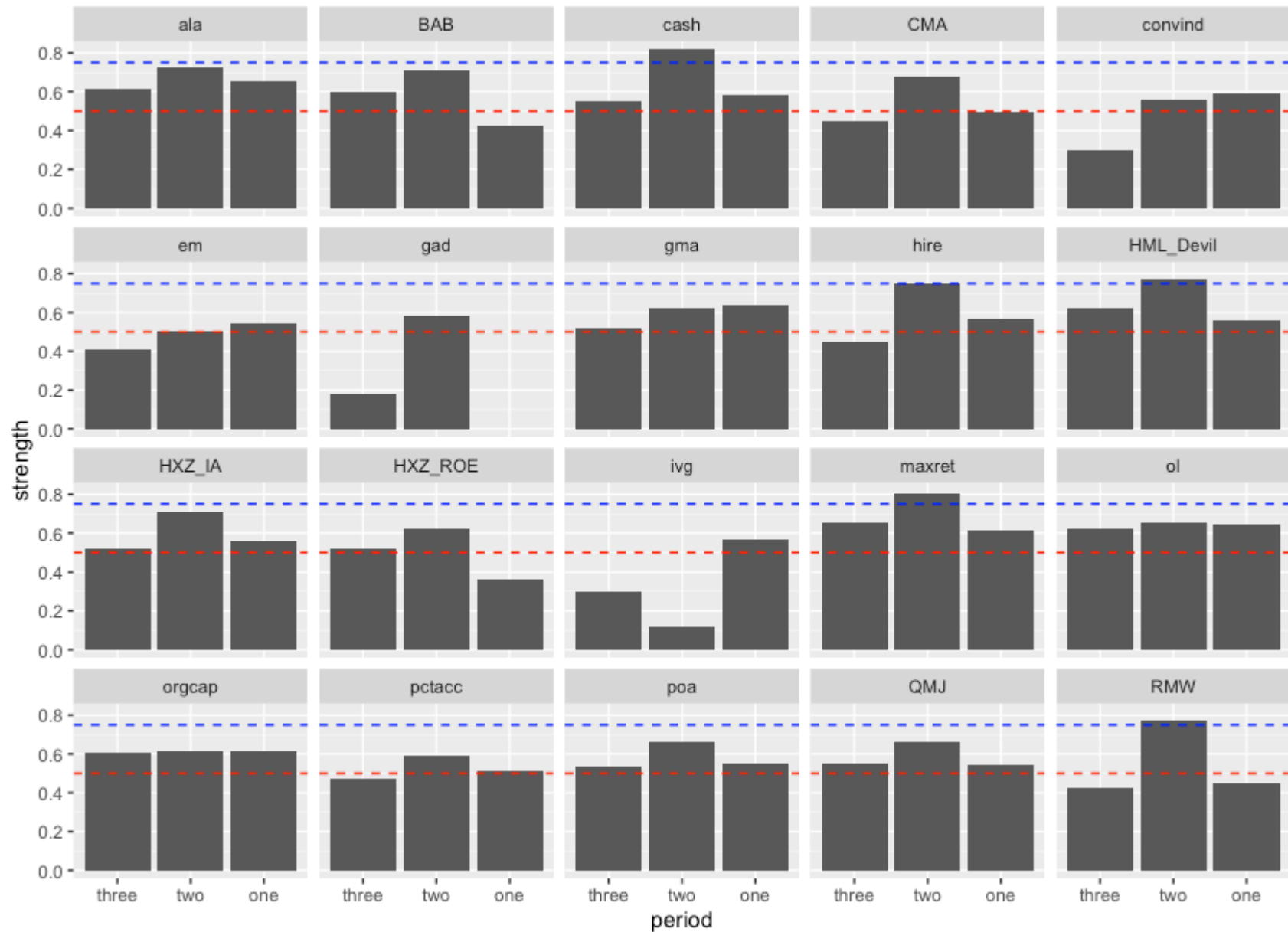Figure 2: Thirty Year Decompose Comparison
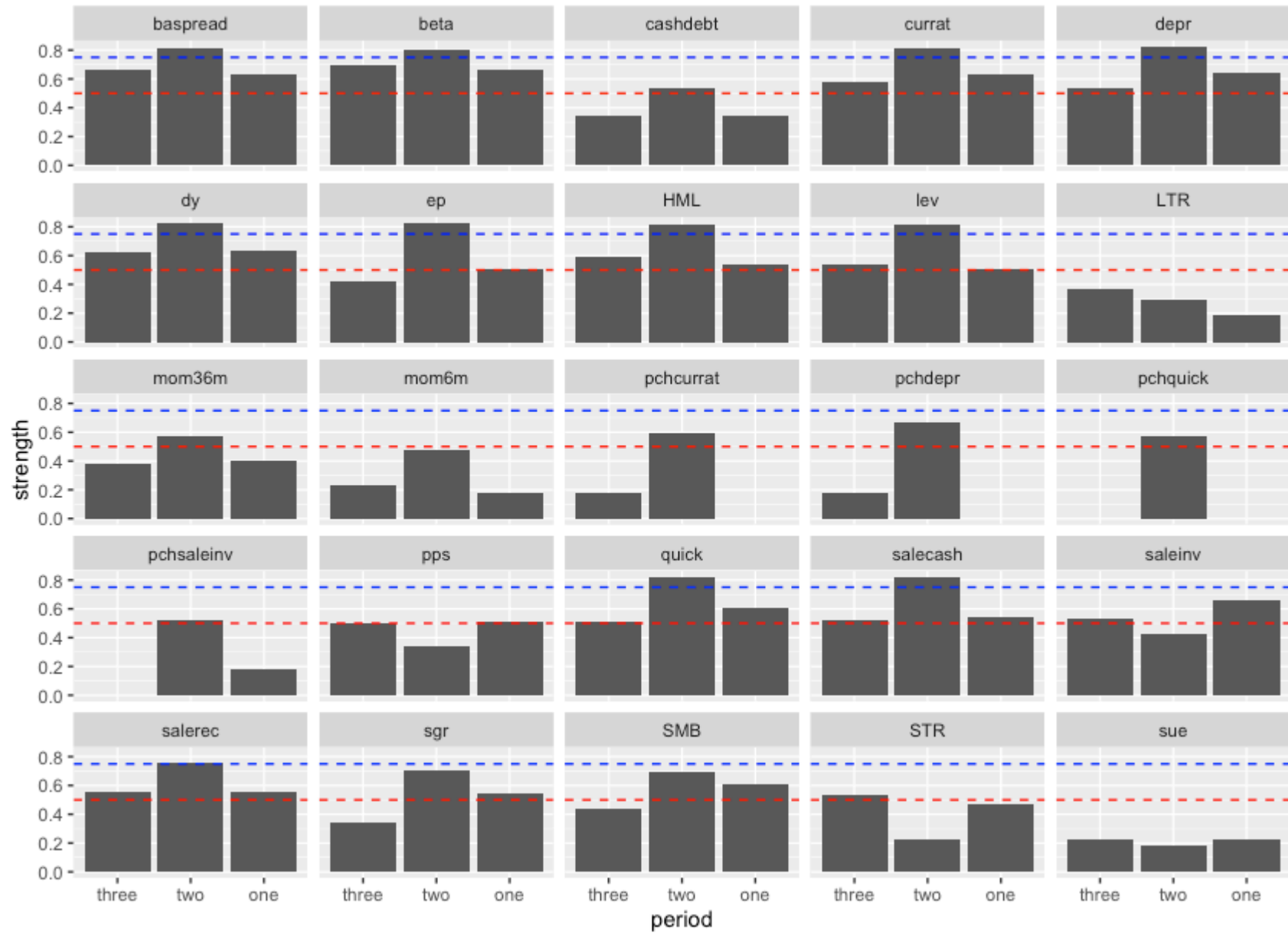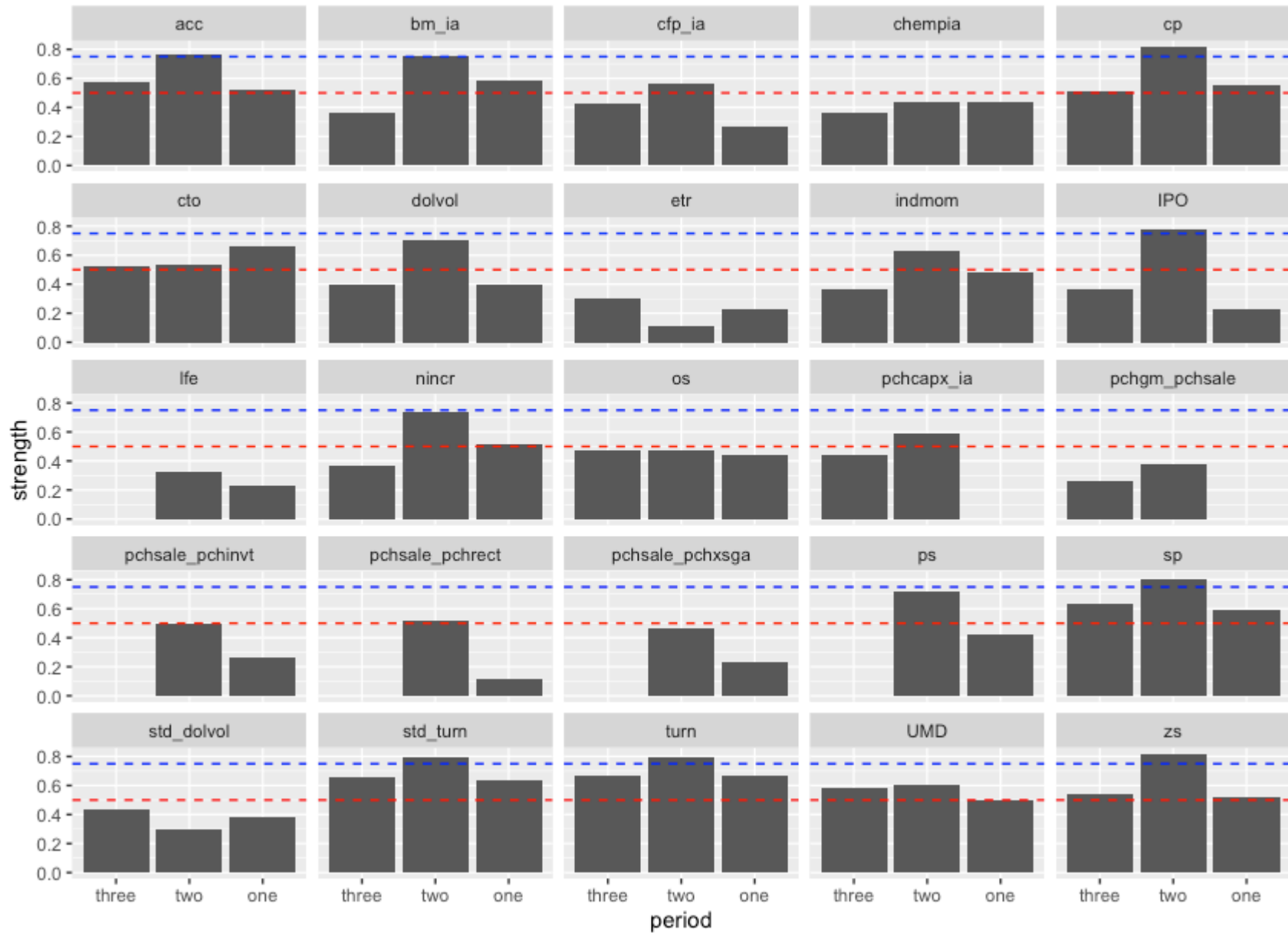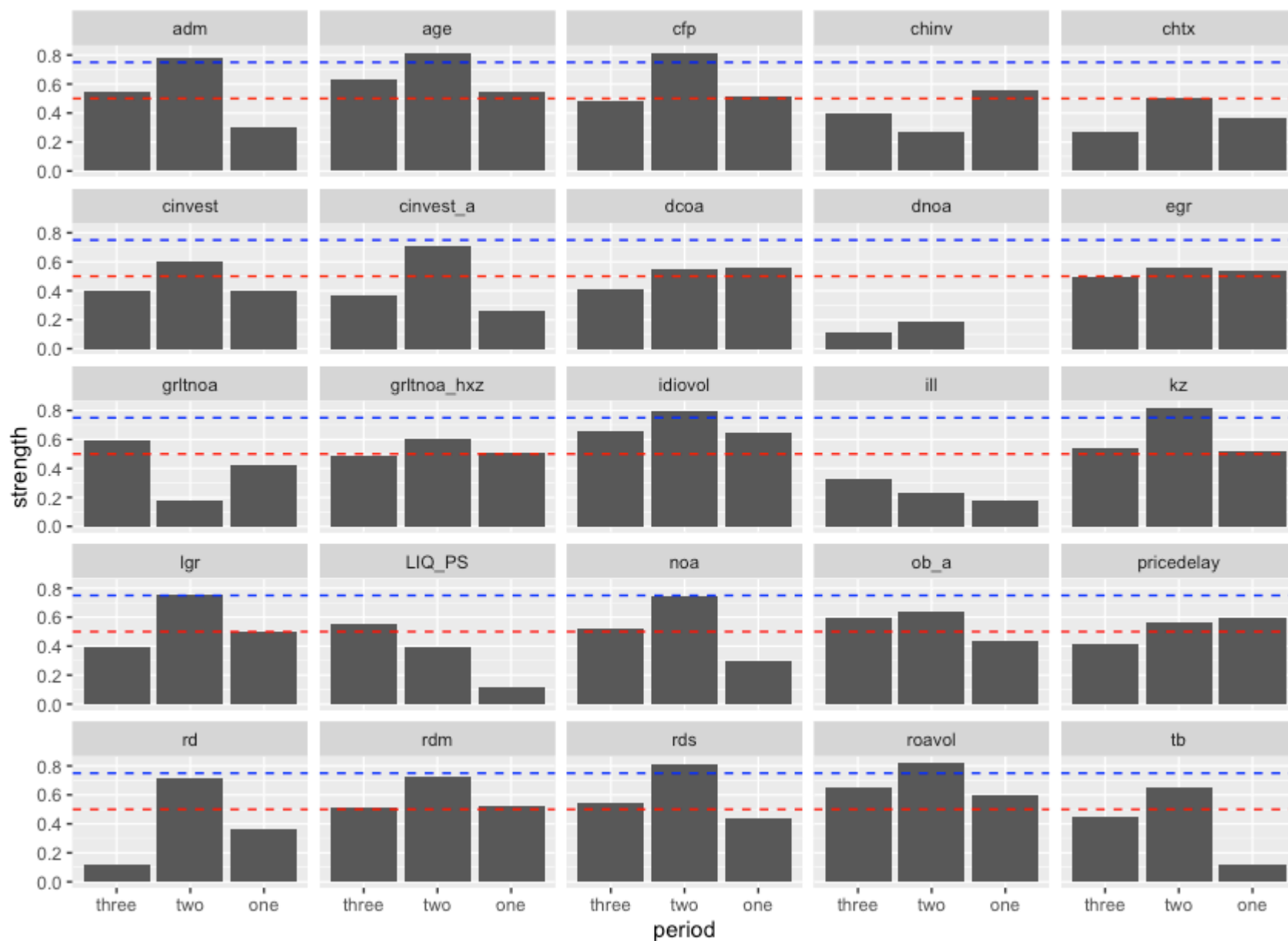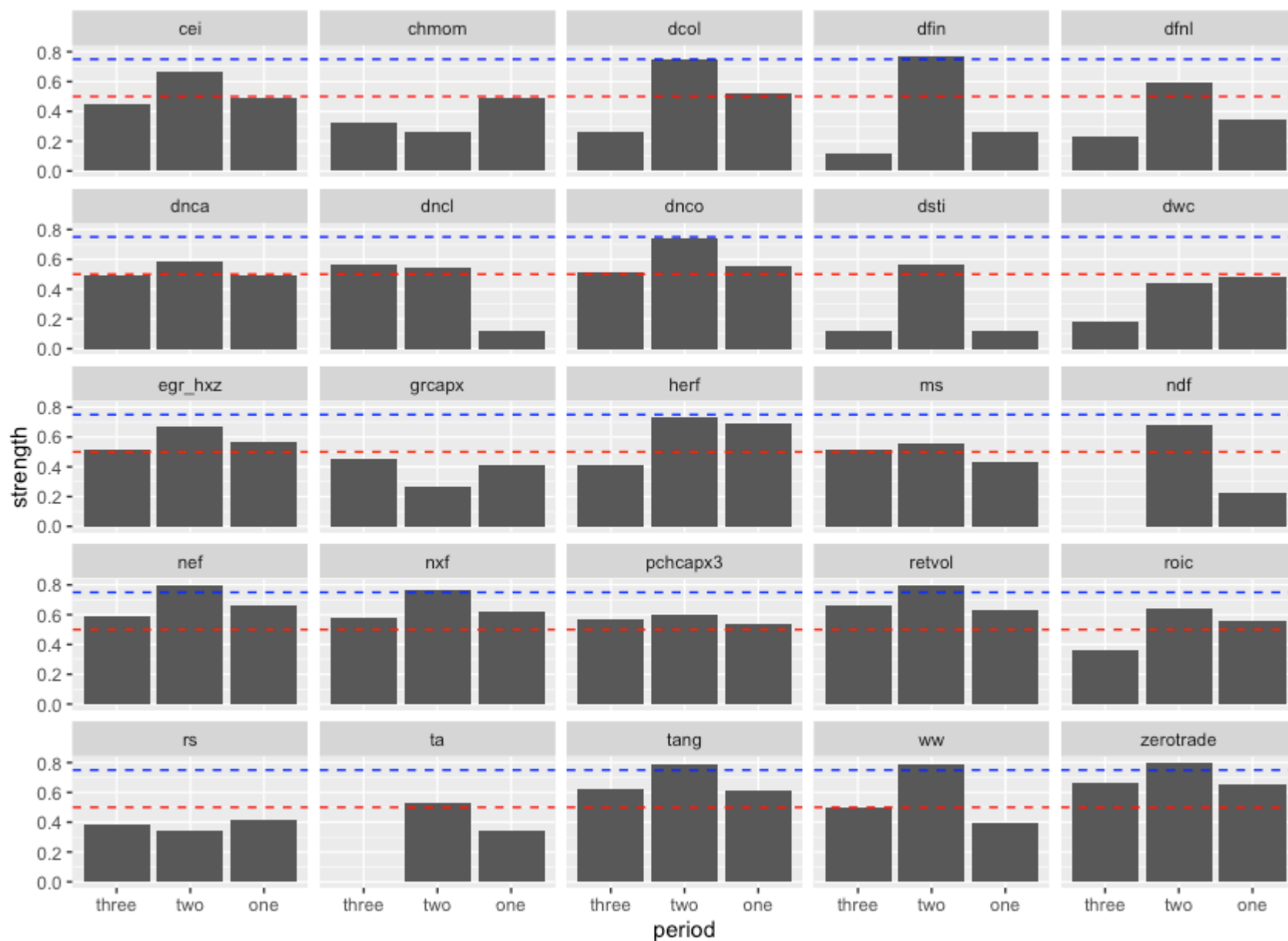
Notes: The figure compare the strength of factor using subsample from the thirty year data.. The x-axis indicates the subsample data set: three is third decade (January 2007 to December 2017), two is second decade (January 1997 to December 2007), and one is the first decade (January 1987 to December 1997). The red dash line and blue dash line represent 0.5 and 0.75 threshold value respectively.