Q1

5 Points

Let G be a graph, where each edge has a weight.

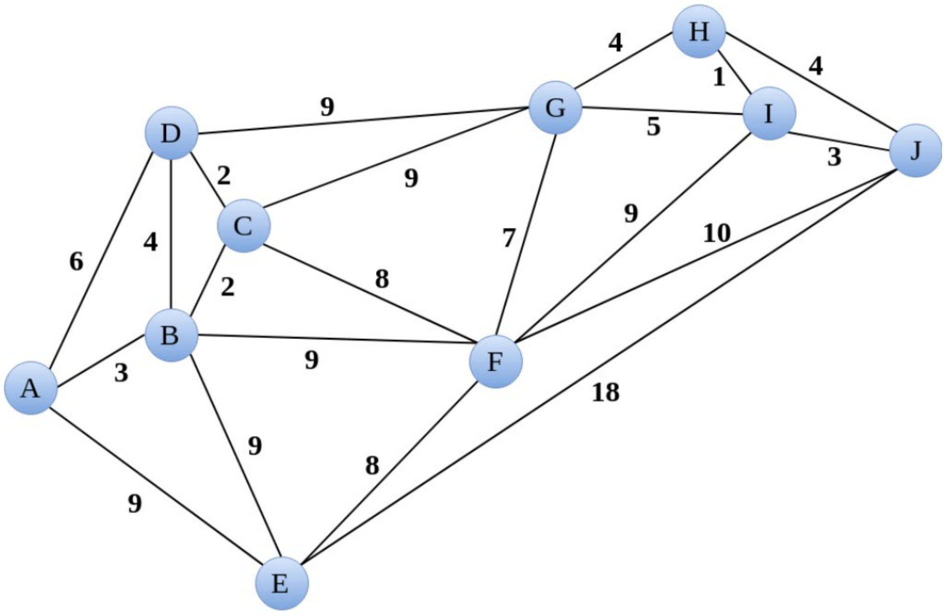
A spanning tree is a set of edges that connects all the vertices together, so that there exists a path between any pair of vertices in the graph.

A minimum-weight spanning tree is a spanning tree whose sum of edge weights is as small as possible.

Last week we saw how Kruskal's Algorithm can be applied to any graph to generate a minimum-weight spanning tree.

In this question, you will apply Prim's Algorithm on the graph below.

You must start with vertex A.



There are nine edges in the spanning tree produced by Prim's Algorithm, including AB, BC, and IJ.

Determine the exact order in which these nine edges are added to form the minimum-weight spanning tree.

AB(3)>BC(2)>CD(2)>CF(8)>FG(7)>GH(4)>HI(1)>IJ(3)>FE(8)

Q2

5 Points

Let G be a graph with V vertices and E edges. One can implement Kruskal's Algorithm to run in $O(E \log V)$ time, and Prim's Algorithm to run in $O(E + V \log V)$ time.

If G is a $dense$ graph with an extremely large number of vertices, determine which algorithm would output the minimum-weight spanning tree more quickly. Clearly justify your answer.

When we get a very dense graph with far more edges than vertices, So, Prim's algorithm is much faster in the dense graph than Kruskal's Algorithm in the dense graph.
For example:
We should use Kruskal when the graph is sparse, i.e.small number of edges,like E=O(V),when the edges are already sorted or if we can sort them in linear time. We should use Prim when the graph is dense, i.e number of edges is high ,like E=O(V^2).

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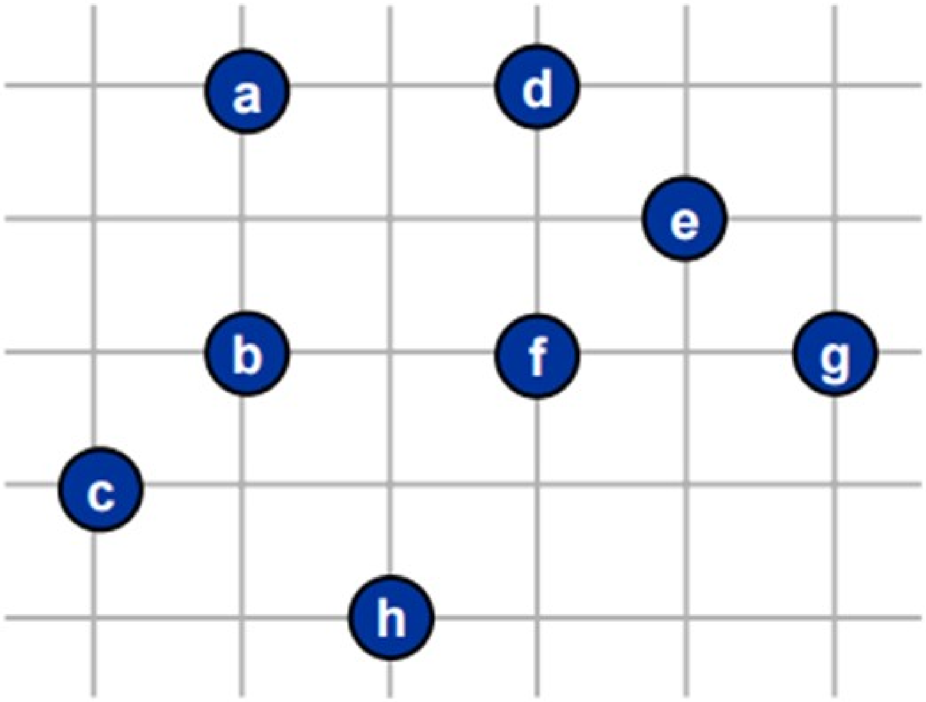
Q3

10 Points

Consider eight points on the Cartesian two-dimensional x-y plane.



For each pair of vertices u and v, the weight of edge uv is the Euclidean (Pythagorean) distance between those two points. For example, $dist(a, h) = \sqrt{4^2 + 1^2} = \sqrt{17}$ and $dist(a, b) = \sqrt{2^2 + 0^2} = 2$.

Q3.1
5 Points

Using the algorithm of your choice, determine one possible minimum-weight spanning tree and compute its total distance, rounding your answer to one decimal place. Clearly show your steps.

From the question given, we can get the following distance.
dis(c,b)=√2, dist(b,h)= √5, dist(h,f)=√5, dist(f,e)√2, dist(e,g)=√2, dist(e,d)=√2, dist(a,f)=√2, dist(a,d)=2;
So, the total distance of MST is:
c-b-h-a-f-e-d-g
√2+√5+2+2+√2+√2+√2=11.9

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Q3.2
5 Points

Because many pairs of points have identical distances (e.g. $dist(h, c) = dist(h, b) = dist(h, f) = \sqrt{5}$), the above diagram has more than one minimum-weight spanning tree.

Determine the total number of minimum-weight spanning trees that exist in the above diagram. Clearly justify your answer.

Given to the above graph, we will get the following MST:
1: c-b-h-a-f-e-d-g
√2+√5+2+2+√2+√2+√2=11.9
2: c-b-f-h-a-e-d-g
(c,b)+(a,b)+(b,f)+(f,h)+(f,e)+(e,d)+(e,g) =
√2+2+2+√5+√2+√2+√2=11.9
3. (c,b)+(b,h)+(b,f)+(f,e)+(e,d)+(d,a)+(e,g)
√2+√5+2+3√2+2=11.9
4. (c,b)+(b,f)+(f,h)+(f,e)+(e,d)+(d,a)+(e,g)
√2+2+√5+3√2+2=11.9
Similarly, we also have another five MST which also have the same total number of value as others.
So, the total number of MST is 9.
Please see my attached for the 9 MST graph.

▾ Ps7-Q3.2.pdf Download

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Q4
5 Points

Let A, B, C, D be the vertices of a square with side length 100.

If we want to create a minimum-weight spanning tree to connect these four vertices, clearly this spanning tree would have total weight 300 (e.g. we can connect AB, BC, and CD).

But what if we are able to add extra vertices inside the square, and use these additional vertices in constructing our spanning tree?

Would the minimum-weight spanning tree have total weight less than 300? And if so, where should these additional vertices be placed to minimize the total weight?

Let G be a graph with the vertices A, B, C, D, and possibly one or more additional vertices that can be placed anywhere you want on the (two-dimensional) plane containing the four vertices of the square.

Determine the smallest total weight for the minimum-weight spanning tree of G. Round your answer to the nearest integer.

Yes, the minimum-weight spanning tree have total weight would be less than 300 if we are able to add additional vertex in the center of the square.

We can try to add a new vertex in the center of the square, mark vertex E, then we can get the smallest total weight for the minimum-weight spanning tree of G is:
given that the question ,we know that each of the edge is 100.
We can have two vertices E and F, which is in the middle line splitting 2 half of the ABCD graph, then we apply for the Pythagorean Theorem, and can get
AE=BE=√(50^2+30^2)=√3400,
Similarly, we get the same for CF and FE which equal to √3400;

Finally, the smallest total weight of the MST is 4 * √3400 + 40 = 273

Problem Set 7

GRADED

STUDENT

Kejian Tong

TOTAL POINTS

25 / 25 pts

QUESTION 1

(no title) **5** / 5 pts

QUESTION 2

(no title) **5** / 5 pts

QUESTION 3

(no title) **10** / 10 pts

3.1 (no title) **5** / 5 pts

3.2 (no title) **5** / 5 pts

QUESTION 4

(no title) **5** / 5 pts