Q1 Reductions

3 Points

Suppose X and Y are decision problems for which $X \leq_P Y$, i.e., X is `polynomial-time reducible` to Y . If X is NP-complete and Y is in NP, explain why Y must also be NP-complete.

We can just follow the theorem to explain.
If Y is a problem in NP that X <= p Y, so Y is NP-complete.
Let's say that Z can be any problem in NP, so we have Z <= p and X <= p,
then we can get Z <= p Y.
So, Y is NP-complete.

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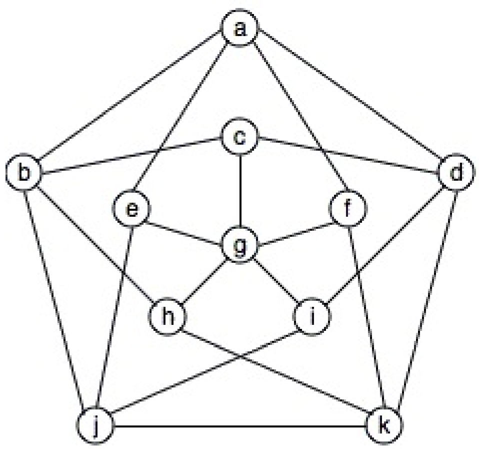
Q2 Vertex Cover

7 Points

Let G be a graph. We say that a set of vertices C form a vertex cover if every edge of G is incident to at least one vertex in C. We say that a set of vertices I form an independent set if no edge in G connects two vertices from I.



For example, if G is the graph above, $C = [b, d, e, f, g, h, j]$ is a vertex cover since each of the 20 edges in the graph has at least one endpoint in C, and $I = [a, c, i, k]$ is an independent set because none of these edges appear in the graph: ac, ai, ak, ci, ck, ik.

Q2.1
2 Points

In the example above, notice that each vertex belongs to the vertex cover C or the independent set I. Do you think that this is a coincidence?

Because we know for the theorem: we have vertex cover ≡ p independent set. We just let S is an independent set if and only if V - S is a vertex cover. We also consider an arbitrary edge(u, v) to explain this question. We let I is independent set, then we have u does not belong to I or v does not belong to I. Next, we can get u belongs to V - I or v belong to V - I.
Hence, we get for each of the vertex in the given graph, belonging to the vertex cover C or the independent set.

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Q2.2
5 Points

In the above graph, clearly explain why the maximum size of an independent set is 5. In other
words, carefully explain why there does not exist an independent set with 6 or more vertices.

From the given graph, we know we have 11 vertices in total. We know for any vertices C, construct a vertex cover if each edge of G is incident to at least one vertex in C, so the minimum size of the vertex cover in G can be C={a,b,d,g,k,g} = 6. Meanwhile, for every vertex that belongs to the vertex cover C or the independent set I. So, we can get the max size of the independent set is 5.

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Q3
5 Points

Let G be a graph with n vertices. If the maximum size of an independent set in G is k, clearly explain why the minimum size of a vertex cover in G is $n - k$.

From the question, we can apply to the theorem, then we have vertex cover ≡ p independent set. Because we know S can be an independent set if and only if V - S is a vertex cover. Furthermore, if the max size of an independent set in G is k and we also have G with n vertices. So, we can know the minimum size of a vertex cover in G should be n - k.

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Q4

5 Points

INDEPENDENT SET has the following `Decision Problem`: given a graph G and an integer k_1, does G have an independent set of size at least k_1?

VERTEX COVER has the following `Decision Problem`: given a graph G and an integer k_2, does G have a vertex cover of size at most k_2?

It is known that **INDEPENDENT SET** is `NP-complete`. Prove that **INDEPENDENT SET** $\leq_P$ **VERTEX COVER** (i.e that independent set is polynomial-time reducible to vertex cover). Use this to conclude that **VERTEX COVER** must also be NP-complete.

Please see my attached file.

PS11-Q4.pdf

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Q5 TSP

5 Points

In every instance (i.e., example) of the TSP, we are given n cities, where each pair of cities is connected by a weighted edge that measures the cost of traveling between those two cities. Our goal is to find the optimal TSP tour, minimizing the total cost of a Hamiltonian cycle in G.

Although it is NP-complete to solve the TSP, there is a simple 2-approximation achieved by first generating a minimum-weight spanning tree of G and using this output to determine our TSP tour.

Prove that our output is guaranteed to be a 2-approximation, provided the Triangle Inequality
holds. In other words, if OPT is the total cost of the optimal solution, and APP is the total cost of our approximate solution, clearly explain why $APP \leq \; 2 * OPT$.

Please see my attached file.

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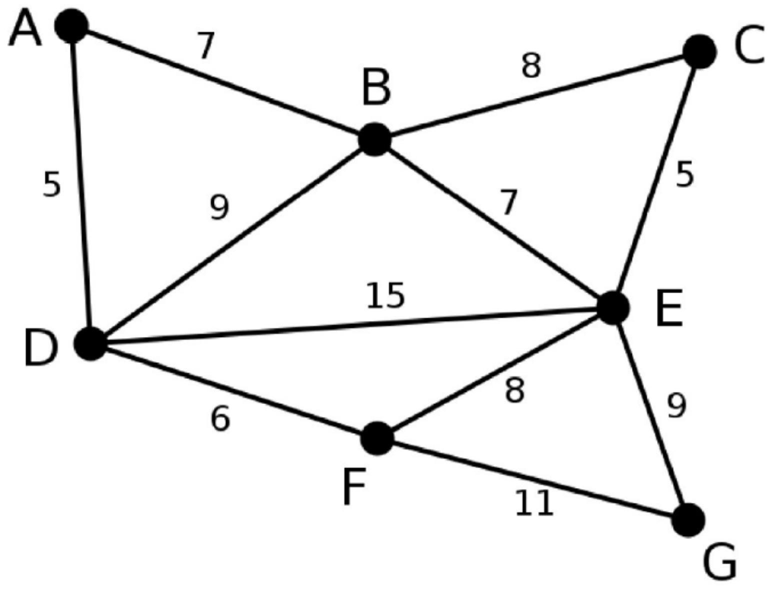
1 / 1

Q6 TSP - II

10 Points

Let G be this weighted undirected graph, containing 7 vertices and 11 edges.



For each of the 10 edges that do not appear $(AC, AE, AF, AG, BF, BG, CD, CF, CG, DG)$, assign a weight of 1000. It is easy to see that the optimal TSP tour has total cost 51.

Generate an approximate TSP tour using the algorithm from **Question 5**, and calculate the total cost of your solution. Explain why your solution is not a 2-approximation.

From the given graph, I just have a MST see my attached file. In the MST graph, I will using DFS to traverse each edge for two times and then we can get the total distance cost is 2*(AB+BC_CE+EG+AD+DF) = 80.
However, we know that the the optimal TSP tour has the cost is 51.
The optimal TSP tour cost is: AB+BC+CE+EG+GF+FD=51
So, because the total distance cost is 80 which is not equal to the optimal TSP that is 51.
Then, we can conclude that solution is not a 2-approximation.

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1 / 1

Problem Set 11

GRADED

STUDENT

Kejian Tong

TOTAL POINTS

28 / 35 pts

QUESTION 1

Reductions **3** / 3 pts

QUESTION 2

Vertex Cover **7** / 7 pts

2.1 (no title) **2** / 2 pts

2.2 (no title) **5** / 5 pts

QUESTION 3

(no title) **5** / 5 pts

QUESTION 4

(no title) **5** / 5 pts

QUESTION 5

TSP **5** / 5 pts

QUESTION 6

TSP - II **3** / 10 pts