

## Q1

10 Points

In Modules 10 and 11 we discussed techniques to solve maximum flow and minimum cut problems. In this quiz, we shall go deeper in the exploration.

### Q1.1

5 Points

Describe an efficient algorithm that finds a new maximum flow if the capacity of a particular edge increases by one unit.

From the question, we can know that if the capacity of a particular edge increases by one unit, the min-cut will also increase by at most 1 and max flow can increase by at most 1. So, we try to find if there is an augmenting path  $p$  from  $s$  to  $t$ . If we can find such path  $p$ , then we will just increase the max flow by augmenting the path we found, otherwise, the current one is still the maximum flow.

### Q1.2

5 Points

Describe an efficient algorithm that finds a new maximum flow if the capacity of a particular edge decreases by one unit.

From the question, we can suppose that  $c(e') = f(e')$  and we also decrease the capacity by 1 unit. Then, we just need to remove one unit of flow from  $s$  to  $t$ , which will go through edge  $e'$ . Let's say  $e' = (u, v)$ .

First for the removing flow:

We run two BFS, one starting from  $s$  to  $u$  going through edges that has flow greater than 0, the other from  $v$  to  $t$  equally from edges that has flow greater than 0.

We then choose a path from  $s$  to  $u$  and remove 1 unit of flow from each edge and do the same with a path from  $v$  to  $t$ .

Second for the recover max flow:

We still have a valid flow network but not the network with the max flow.

We then run on iteration augmenting path with the capacity of edge  $e'$  reduced, which has the  $O(|V| + |E|)$  time complexity for one iteration of a DFS.

Finally, we can get the total time complexity would be  $O(|V|+|E|)$  that is linear which is corresponding to the number of nodes and edges.

## Q2

5 Points

Suppose you are organizing a conference where researchers present articles they have written. Researchers who want to present an article send a paper to the conference organizers. The conference organizers have access to a committee of reviewers who are each willing to read up articles each.

Each paper submission gets reviewed by up to reviewers. Moreover, each submission has a particular topic and each reviewer has a specialization for a set of topics, so papers on a given topic only get reviewed by those reviewers who are experts on that topic.

The conference organizers need to decide which reviewers will review each article (or equivalently, which articles will be reviewed by which reviewers). Explain how they could use a flow network to solve this problem.

1. From the question, we can have 2 sets. A set A of articles and a set B of reviewers.
2. We can add directed edge(a, b) to the graph to see if any article m in set A is reviewed by any reviewer n in set B.
3. We can set the capacity of that edge to be 1
4. We just add a vertex source node (S) and a terminal vertex node (T). For each a in A, add an edge (S, a) of capacity  $m_A$ . For each b in B, add an edge (b, T), of capacity  $m_B$ .
5. Now we run the Ford-Fulkerson algorithm to get the maximum flow and find the corresponding cut (A, B).

Finally, this gives us the set of edges that correspond to the article assignment of articles to reviewers.

Quiz 8 - MF, MC

● GRADED

STUDENT

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TOTAL POINTS

15 / 15 pts

QUESTION 1

(no title)

10 / 10 pts

1.1 (no title)

5 / 5 pts

1.2 (no title)

5 / 5 pts

QUESTION 2

(no title)

5 / 5 pts