

From the question, we can know that we have a graph $G(V,E)$ and also have a integer k . We can convert this question to determine if there is a graph include an independent set of vertices of size $\geq k$.

Since NP-complete can have NP and NP-hard, so we can just to consider two parts of this proof.

1. The problem is in NP class 2. All other problems in NP class can be polynomial-time reducible to that. We can denote $B \leq_p C$, which B is polynomial-time reducible to C.

We can have a pseudocode to prove Independent Set is NP.

Initially, we set $flag = true$

for every pair $\{u, v\}$ in the subset V' :

 we check that these two don't

 have an edge between them

 if there is an edge,

 set $flag$ to false and break

if $flag$ is true:

 Solution is correct

else:

 solution is incorrect

2. Independent Set is NP-hard

First, we suppose that the graph G has a clique of size k_1 and the presence of clique implies that there are k_1 vertices in graph G . Because those edges are contained in graph G , so they will not appear in G' . So, those k_1 vertices are not adjacent to each other in G' and thus forming an independent set of size k_1 .

Second, we suppose the complementary G' has a set of independent vertices of size k_1' and they don't share with others. Then, those k_1' vertices will share an edge and so be adjacent to each other. So, the graph G will have a clique of size k_1' and thus the independent set is NP-hard.

Finally, we can prove independent set $\leq P$ and because the independent set problem is both NP and NP hard.

For the proof of statement, we need to check if a vertex cover of size at most integer k_2 exists in graph G .

Because we know NP Complete which consists of NP and NP hard. So, we can just from those two parts and then we can prove this problem.

1. Proof that vertex cover is in NP

I will have the following pseudocode for explanation:

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cnt is an integer
set cnt to 0
for each vertex v in V'
    remove all edges adjacent to v from set E
    increment cnt by 1
    if cnt = k2 and E is empty
    then
        given solution is correct
    else
        given solution is not correct
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The solution for a vertex cover problem is a subset V' of V that includes the vertices cover. Then, we will be able to check if the set V' is a vertex cover of size k_2 using above pseudocode in a graph $G(V,E)$.

So, we can easily get that this can be done in a polynomial time. Then, the vertex cover must be a NP.

2. Proof that vertex cover is NP hard

I will convert this problem to finding out if there is a clique of size k_2 in the graph $G(V,E)$, so we just need to check the existence of a clique of size k_2 in given graph G .

We suppose there is a clique of size k_2 in graph G and let the set of vertices be V' , which means $|V'| = k_2$ and in the complement graph G' , we can choose any edge (u,v) and then one of u or v must be in set $V-V'$ at least. As we can suppose if both u and v are from set V' , the edge (u,v) will belong to V' . So, all edges in G' are covered by vertices in the set of $V-V'$.

So, we can know that there is a clique of size k_2 in graph G if and only if there's a vertex cover of size $|V| - k_2$ in graph G' and also any instance of clique problem will be reduced to an instance of the vertex cover problem. So, it is a NP hard.

Finally, we can conclude that vertex cover must be NP-complete. Because vertex cover is both NP and NP hard.