

Q.1.1

We can use Merge sort

① sort the left half of array (recursively)

② sort the right half of array (recursively)

③ Merge the solutions.

Because we use recursive idea to solve this question,

$$\text{we can } T(n) = aT\left(\frac{n}{b}\right) + cn^k$$

we can use Master Theorem solves this.

$$\text{So we have } T(n) = 2T\left(\frac{n}{2}\right) + c \cdot n$$

if we apply master Theorem, we'll see that our case is one where  $a=b^k$ ,  $2=2^1$ ,

Our complexity is  $O(n \log n)$ , and it has been proven that an array can't be sorted faster than  $O(n \log n)$ ,

So, it's lower bound  $\Omega(n \log n)$ .

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1 ▾ public static void mergeSort(int[] array, int left, int right) {
2     if (right <= left) return;
3     int mid = (left+right)/2;
4     mergeSort(array, left, mid);
5     mergeSort(array, mid+1, right);
6     merge(array, left, mid, right);
7 }
8

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9 ▾ void merge(int[] array, int left, int mid, int right) {
10     // calculating lengths
11     int lengthLeft = mid - left + 1;
12     int lengthRight = right - mid;
13     int leftArray[] = new int [lengthLeft];
14     int rightArray[] = new int [lengthRight];
15     for (int i = 0; i < lengthLeft; i++)
16         leftArray[i] = array[left+i];
17     for (int i = 0; i < lengthRight; i++)
18         rightArray[i] = array[mid+i+1];
19     int leftIndex = 0;
20     int rightIndex = 0;
21 ▾ for (int i = left; i < right + 1; i++) {
22 ▾     if (leftIndex < lengthLeft && rightIndex < lengthRight) {
23 ▾         if (leftArray[leftIndex] < rightArray[rightIndex]) {
24             array[i] = leftArray[leftIndex];
25             leftIndex++;
26         }
27 ▾     else {
28         array[i] = rightArray[rightIndex];
29         rightIndex++;
30     }
31 }
32 ▾ else if (leftIndex < lengthLeft) {
33     array[i] = leftArray[leftIndex];
34     leftIndex++;
35 }
36 ▾ else if (rightIndex < lengthRight) {
37     array[i] = rightArray[rightIndex];
38     rightIndex++;
39 }
40 }
41 }
42

```