Q1

20 Points

After finding out you are secretly part of the royal family of Genovia, you inherit a 16th-century
Genovian castle with an elaborate plumbing system that has accumulated pipes, junctions, and
clogs over four centuries. Instead of a diagram, you are given a list of pipes and their capacities
leading from the water source to your bathroom

Pipes	S,2	S,4	2,3	2,5	4,2	4,5	3,5	3,6	5,6	5,7	6,7	6,T	7, T
Capacity	4	7	6	2	4	3	6	2	5	4	1	9	4

Q1.1

10 Points

Draw the flow graph of your new castle and list:
i. The shortest augmenting path (and it's bottleneck)
ii. The highest capacity augmenting path (and it's bottleneck)

Please see the attached file for my answer.

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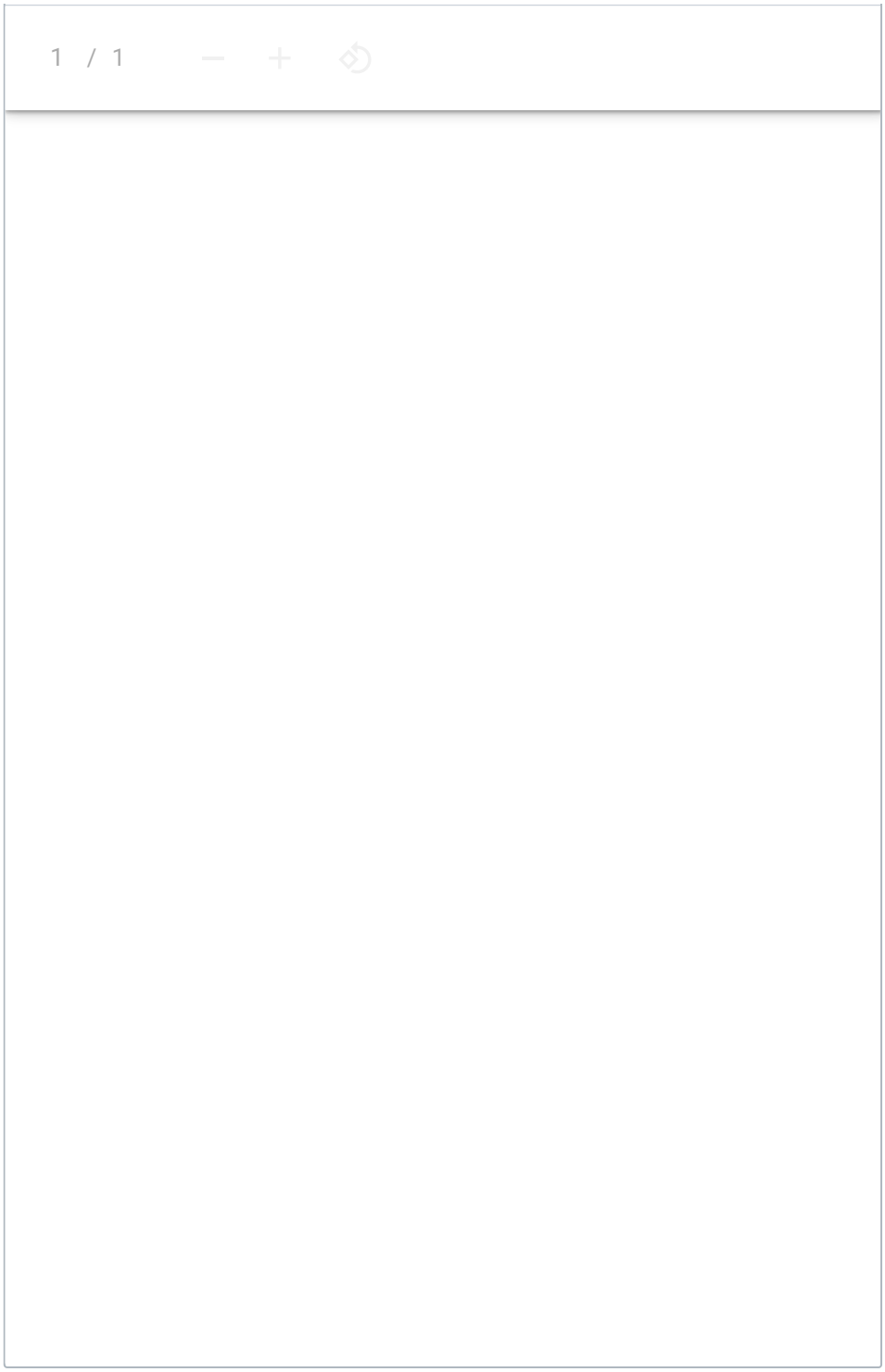

Q1.2
10 Points

You'd like to know if it is safe to install a modern shower, or if this will eventually overflow
the historic bathtub. Use the Ford Fulkerson algorithm to determine the max flow of this
flow network/graph. Please draw your final residual graph and write the calculated max
flow.

Please see the attached file for my answer.

PS10-Q1.2.pdf Download

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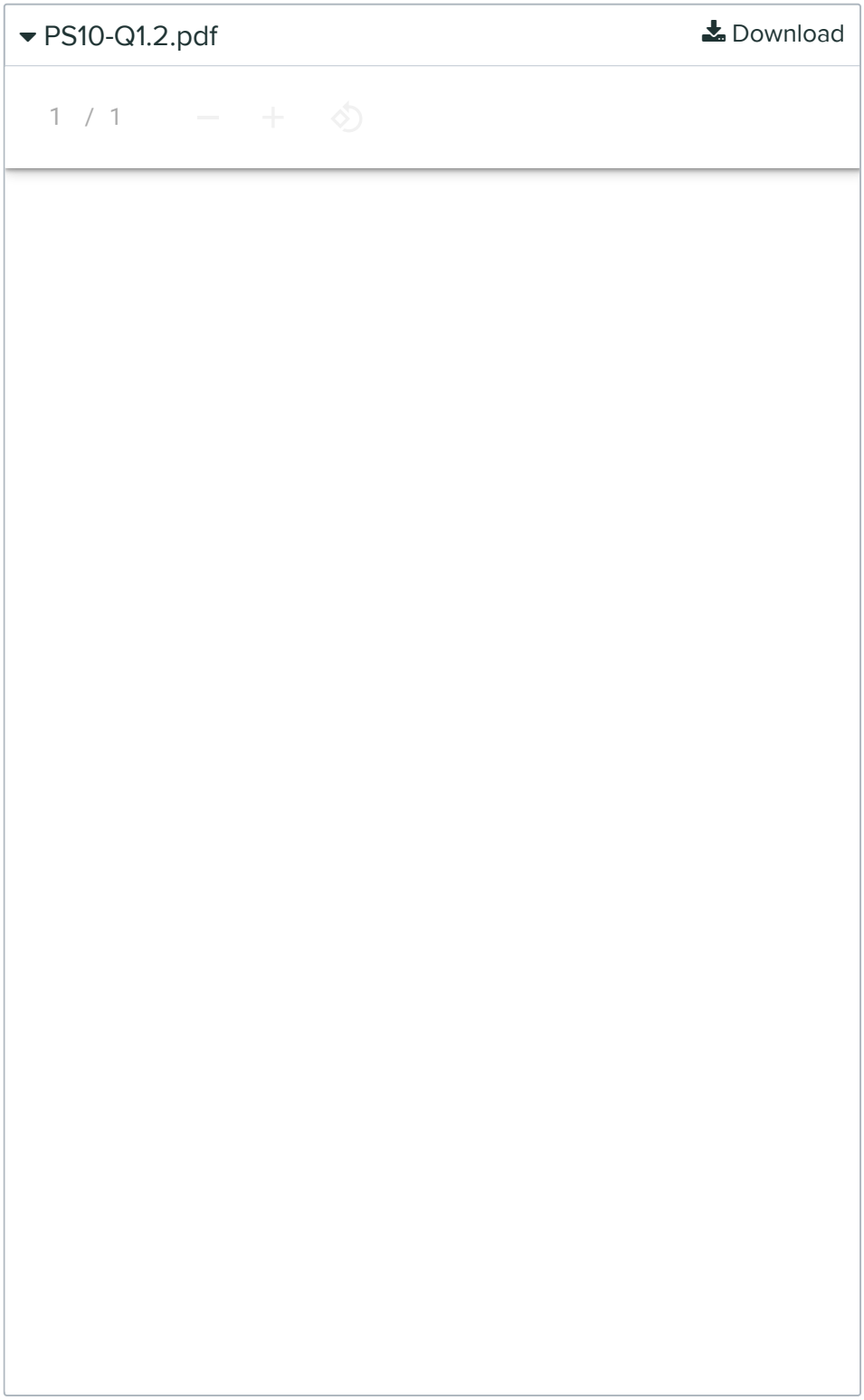

Q2

5 Points

Not all augmenting paths are equal, and starting with different paths leads to different

residual graphs, although all selections produce the same max-flow result. Determine a
process for selecting your augmenting paths. Justify your answer.
Hint: most implementations of Ford-Fulkerson take a greedy approach.

For the Ford-Fulkerson, we can have the following algorithm:
FORD–FULKERSON(G)
  FOREACH edge e ∈ E : f (e) ← 0
  Gf ← residual network of G with respect to flow f.
  WHILE (there exists an s↝t path P in Gf) [P is the augmenting path]
    f ← AUGMENT(f, c, P).
    Update Gf.
  Return f.

We know the fact that every edge in the graph contains a value, called the residual capacity, which is equal to the current capacity.
So, first, we can have augmenting path in residual graph through DFS/BFS.
then, we just update the residual graph as the following steps:
1. For each edge in augmenting path, we have a value of minimum capacity in the path that is subtracted from all edges of that path we have traversed.
2. For each successive node in the augmented path, we have an equal number of edges are added to the reverse direction.
3. Repeatedly find the augmenting path through the residual graph until there is no augmentation path.
Finally, we can get the overall flow is calculated.

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Problem Set 10

GRADED

STUDENT
Kejian Tong

TOTAL POINTS
25 / 25 pts

QUESTION 1

(no title) **20** / 20 pts

1.1 (no title) **10** / 10 pts

1.2 (no title) **10** / 10 pts

QUESTION 2

(no title) **5** / 5 pts