P1:

A: $3^{1500} \bmod 11$

$1500_{(10)} = 10\,1110\,1100_{(2)}$

$= 1\times2^1 + 1\times2^3 + 1\times2^4 + 1\times2^6 + 1\times2^7 + 1\times2^8 + 1\times2^{10}$

$= 4 + 8 + 16 + 64 + 128 + 256 + 1024$

So,

$3^{1500} = 3^4 \times 3^8 \times 3^{16} \times 3^{64} \times 3^{128} \times 3^{256} \times 3^{1024}$

$3^1 \equiv 3 \bmod 11$

$3^2 \equiv 9 \bmod 11$

$3^4 \equiv 9^2 \equiv 4 \bmod 11$

$3^8 \equiv 4^2 \equiv 5 \bmod 11$

$3^{16} \equiv 5^2 \equiv 3 \bmod 11$

$3^{32} \equiv 3^2 \equiv 9 \bmod 11$

$3^{64} \equiv 9^2 \equiv 4 \bmod 11$

...

$3^{1024} = 9^2 \equiv 4 \bmod 11$

So,

$3^{1500} \equiv 4\times5\times3\times4\times5\times3\times4 \bmod 11$

$\equiv 14400 \bmod 11$

$\equiv 1$

B: $5^{4858} \bmod 10$

Likewise, we use the same idea as the first one.

$4358_{10} = 1000100000110_{(2)}$

$= 1\times2^1 + 1\times2^2 + 1\times2^8 + 1\times2^{12}$

$= 2+4+256+4096$

So, $5^{4358} = 5^2 \times 5^4 \times 5^{256} \times 5^{4096}$

$5^1 \equiv 5 \bmod 10$

$5^2 \equiv 5 \bmod 10$

$5^4 \equiv 5 \bmod 10$

$5^8 \equiv 5 \bmod 10$

...

$5^{256} \equiv 5 \bmod 10$

$5^{4096} \equiv 5 \bmod 10$

So, $4358 = 4096 + 256 + 4 + 2$

$5^{4358} \bmod 10 \equiv 5\times5\times5\times5 \bmod 10$

$\equiv 5$

C: $6^{22345} \bmod 7$

$22345_{10} = 101011010001001_{(2)}$

$= 2+8+64+256+512+1024+4096+16384$

So,

$6^{22345} = 6^1 \cdot 6^0 \cdot 6^4 \cdot 6^{13} \cdot 6^{12} \cdot 6^{11} \cdot 6^{4096} \cdot 6^{1250}$

$6^1 \equiv 6 \mod 7$

$6^2 \equiv 1 \mod 7$

$6^4 \equiv 1 \mod 7$

$6^8 \equiv 1 \mod 7$

$6^{16} \equiv 1 \mod 7$

$6^{32} \equiv 1 \mod 7$

...

$6^{4096} \equiv 1 \mod 7$

So, $6^{22345} \equiv (1 \times 1 \times 1 \cdots \times 6) \mod 7$

$\equiv 6$

P2:

a. $GCD(648, 124) = GCD(124, 28)$

$= GCD(28, 12)$

$= GCD(12, 4)$

$= GCD(4, 0)$

$= 4$

b: $GCD(123456789, 123456788)$

$= GCD(123456788, 1)$

$= GCD(1, 0)$

$= 1$

c: $GCD(2^{300} \cdot 3^{200}, 2^{200})$

$= GCD(2^{100} \cdot 2^{200} \cdot 3^{200}, 2^{200})$

$= GCD(2^{200}, 0)$

$= 2^{200}$

P3:

lets say: we have modulus p and the base g, Alice and Bob want to send their own secret number a, b to each others they will perform a function like

$A = g^a \bmod p$,

p is a prime number, at least 2048 bits long,

g is a small number,

lets take an example to clarify that.

Bob and Alice has secure number $a=3, b=6$,

$p=17$, $g=4$.

then Alice:

$A = g^a \bmod p$

$A = 64 \bmod 17$

$A = 13$.

same perform for Bob:

$B = 4^6 \bmod 17$

$B = 4096 \bmod 17$

$B = 16$

Alice the send her result A to Bob, while Bob sends his result B to Alice. Alice the calculates the shared secret-s using the number she received from Bob and her number-a using the formula:

$s = B^a \bmod p$

$s = 16^3 \bmod 17$

$s = 16$

the do same job for Bob:

$s = A^b \bmod p$

$s = 13^b \bmod p$

$s = 4826809 \bmod 17$

$s = 16$

Another example:
we can define a decode function like encode = $s \cdot 3$ if Bob wants to send b=9. the encode_number $= s \cdot 3 = 16 \times 3 = 48$; Bob will send 48 to Alice; then Alice will decode it usin same encode function.

then, s is a shared key that is only known for Bob and Alice, and they can setup for symmetric encryption, allowing them to safely send info between themselves in a way that only they can access it.

P4:

```
    public static int criticalEvents(int[] arr, int t){
        int cnt = 0;
        for(int i = 0; i < arr.length; i++){
            for(int j = 0; j < arr.length; j++){
                if(arr[i] > arr[j] * t){
                    cnt++;
                }
            }
        }
        return cnt;
    }
}
```