(a). $A=[6,5,8,7,3,4,2,1]$.

P4

6	5	8	7	3	4	2	1

6 5 8 7   3 4 2 1

6 5   8 7   3 4   2 1

4compare 5 6   7 8   3 4   1 2

4compare 5 6 7 8   1 2 3 4

4compare 1 2 3 4 5 6 7 8

So, we have 12 comparsion in total.

(5-6, 7-8, 3-4, 1-2)

(5-7, 6-7, 1-3, 2-3)

(1-5, 2-5, 3-5, 4-5).

(b). when an array has even number of elements, it will be divided into two equal length subarrays, and each of them will have minimum comparsion $M(n/2)$, and the elements of two subarrays will be compared between themselves.

when they will compare between themselves, the minimum comparsion will happen when all the elements of one array is either less or greater than each element of the other array, the number of comparsions between themselves will be $n/2$, and because they are $n/2$ length each.

so, the total number of comparsions using Merge Sort will be $M(n) = 2M(n/2) + n/2$.

(c). We can try to prove it by induction.

First, we assume $M(n) = \frac{n \log n}{2}$,

we can take for $n = c$, c is a constant.

so we have $M(c) = \frac{c \cdot \log c}{2}$.

We can also let $n = 2c$,

so, we have

$$M(2d) = 2 \cdot M(d) + d$$
$$= 2 \cdot \frac{d \cdot \log d}{2} + d$$
$$= d \cdot \log d + d$$
$$= 2d\left(\frac{\log d + 1}{2}\right)$$
$$= 2d \cdot \frac{(\log d + \log_2 2)}{2}$$
$$= 2d \cdot \frac{\log_2 2d}{2}$$
$$= 2d \cdot \frac{\log 2d}{2} = \frac{2d \cdot \log 2d}{2}$$

then, we can find $M(n) = \frac{n \log n}{2}$, takes for $n = 2d$.

so, we can conclude that

$M(n) = \frac{n \log n}{2}$ is proved.

(d). From the question, we know total numbers of input $= 8$.

$P(\text{exactly } (2 \text{ comparsions})) = 1$.

Probability of getting single comparsion $= \frac{1}{2}$

so, we can apply for binomial distribution:

$$P(X=x) = n \cdot C_x p^x q^{n-x}$$

$$= 12 \cdot C_8 \left(\frac{1}{2}\right)^8 \cdot \left(\frac{1}{2}\right)^4$$

$$= 12 C_8 \left(\frac{1}{2}\right)^{12}$$

$$= 495 \cdot \frac{1}{4096}$$

$$= 0.1208$$

so, the probability ~~proof~~ of getting exactly 12 comparsions in merge sort will be 0.1208.