R Instructions for the problem 1:

•Get familiar with data (means and variances in groups, useful graphs). What the graphs suggest about interaction? Which factor looks to be significant? Do variances in particular groups look similar? Do the means in particular groups look similar? Why is it important?

```
tapply(Crop$Yield,list(Crop$Irrigation,Crop$Fertilizer),mean)
tapply(Crop$Yield,list(Crop$Irrigation,Crop$Fertilizer),var)
```

!library(lattice)

```
xyplot(Yield~Irrigation:Fertilizer,data=Crop)
```

interaction.plot(Crop\$Irrigation,Crop\$Fertilizer,Crop\$Yield)
interaction.plot(Crop\$Fertilizer,Crop\$Irrigation,Crop\$Yield)

!library(sciplot) whiskers represent standard errors for means

lineplot.CI(Fertilizer, Yield, group=Irrigation, data=Crop)

lineplot.CI(Irrigation, Yield, group=Fertilizer, data=Crop)

lineplot.CI(Irrigation, Yield, data=Crop)

lineplot.CI(Fertilizer, Yield, data=Crop)

R Instructions for the problem 2:

•Perform a Shapiro-Wilks normality test for each group.

tapply(Crop\$Yield, list(Crop\$Irrigation, Crop\$Fertilizer), shapiro.test)[1:6]

R Instructions for the problem 3:

•Perform the Levene's Test for Homogeneity of Variances.

!library(DescTools)

LeveneTest(Yield~Irrigation*Fertilizer, data=Crop,center=mean)

R Instructions for the problem 4:

•Create an object in R bearing all essential results of two-way ANOVA.

AResults1<-lm(Yield~Irrigation+Fertilizer+Irrigation:Fertilizer,Crop)

(It is vital to remember which level of Irrigation and Fertilizer is set to be reference level.

In R it is the first one. Check it by

 ${\tt levels(Crop\$Irrigation)}, \ {\rm the \ first \ in \ order \ is \ the \ reference \ one.}$

levels(Crop\$Fertilizer)

R Instructions for the problem 5:

•Create an ANOVA table. Which factors/interactions are significant? anova(AResults1)

Analysis of Variance Table

Response: Yield

```
Df Sum Sq Mean Sq F value Pr(>F)

Irrigation 1 4.3 4.27 0.3582 0.552

Fertilizer 2 4994.1 2497.07 209.6418 < 2.2e-16 ***

Irrigation:Fertilizer 2 810.1 405.07 34.0075 2.764e-10 ***

Residuals 54 643.2 11.91
```

(Irrigation is insignifficant, however interaction of Irrigation with Fertilizer as well as Fertilizer itself are significant. See the p-values in a last column.)

R Instructions for the problem 6:

•Predict values in particular groups.

tapply(predict(AResults1), list(Crop\$Irrigator, Crop\$Fertilizer), mean)

```
littleF mediumF plentyF
littleI 17.8 30.2 46.4
plentyI 21.4 37.8 36.8
```

(Remark: results are the same like descriptive means. It is because we had balanced data.)

R Instructions for the problem 7:

•Create a linear model with treatment parametrization and interpret coefficients. (Tricky

Estimate Std. Error t value Pr(>|t|)

= 37.8

- for voluntiers!) Symbols of α and β have different meaning then in lecture!!!!! summary (AResults1)

Coefficients:

<pre>(Intercept) IrrigationplentyI FertilizermediumF FertilizerplentyF IrrigationplentyI:FertilizermediumF IrrigationplentyI:FertilizerplentyF</pre>					1	7.800 3.600 2.400 8.600 4.000 3.200		1.091 1.543 1.543 1.543 2.183 2.183	18.530 1.833	0.0234 8.70e-11 < 2e-16	* *** ***	
		α	+	$\beta_1.I_{pl}$	+	$\beta_2.F_{med}$	+	$\beta_3.F_{pl}$	$+ \beta_4$	$I_{pl}.F_{med}$	+	$\beta_5.I_{pl}.F_{pl}$
I:little I:little I:little I:plenty I:plenty I:plenty	F:little F:medium F:plenty F:little F:medium F:plenty	$\begin{array}{c c} & \alpha \\ & \alpha \end{array}$	+ + + + + +	$\beta_{1}.0$ $\beta_{1}.0$ $\beta_{1}.0$ $\beta_{1}.1$ $\beta_{1}.1$	+ + + + +	$ \beta_{2}.0 $ $ \beta_{2}.1 $ $ \beta_{2}.0 $ $ \beta_{2}.0 $ $ \beta_{2}.0 $ $ \beta_{2}.0 $	+ + + + +	$ \beta_{3}.0 $ $ \beta_{3}.0 $ $ \beta_{3}.1 $ $ \beta_{3}.0 $ $ \beta_{3}.1 $	+ + + + + +	$\beta_4.0$ $\beta_4.0$ $\beta_4.0$ $\beta_4.0$ $\beta_4.0$ $\beta_4.1$ $\beta_4.0$	+ + + + + +	$ \beta_{5}.0 $ $ \beta_{5}.1 $
I:little I:little I:little I:plenty	F:little F:medium F:plenty F:little	$\alpha \\ \alpha + \beta \\ \alpha + \beta \\ \alpha + \beta $	β_3		= = =	17.8 + 17.8 +	28.6			= 17.8 = 30.2 = 46.4 = 21.4		

R Instructions for the problem 8:

•Asses whether assumptions of homogeneity of variances and normality of residuals are met.

 $\alpha + \beta_1 + \beta_3 + \beta_5 = 17.8 + 3.6 + 28.6 - 13.2 = 36.8$

I:plenty F:medium $\alpha + \beta_1 + \beta_2 + \beta_4 = 17.8 + 3.6 + 12.4 + 4$

```
par(mfrow=(c(1:2)))
plot(AResults1,which=1:2)
```

I:plenty F:plenty

The first graph depicts dependence of residuals on predicted values (notice that residuals look to have homogeneous variance); the second suggests small problems with normality of residuals.

hist(residuals(AResults1))

R Instructions for the problem 9:

•Find out the confidence intervals for population means in particular groups. To process

that, first we have to perform two-way ANOVA in textbook parametrization. The easiest way to get this parametrization is to create new factor with 6 levels corresponding to 6 combinations of previous two factors. This will lead to "one way" ANOVA form without intercept. The new factor denote both

```
both<-paste(Crop$Irrigation,Crop$Fertilizer)
both<-factor(both)
Crop$both<-both
AResults2<-lm(Yield~both-1,Crop) "-1" is a symbol that the model is without intercept
anova(AResults2)
summary(AResults2)</pre>
```

Coefficients:

		Estimate	Std.	Error	t value	Pr(> t)	
bothlittleI	littleF	17.800		1.091	16.31	<2e-16	***
bothlittleI	${\tt medium} F$	30.200		1.091	27.67	<2e-16	***
bothlittleI	plentyF	46.400		1.091	42.52	<2e-16	***
bothplentyI	${\tt littleF}$	21.400		1.091	19.61	<2e-16	***
bothplentyI	${\tt medium} F$	37.800		1.091	34.63	<2e-16	***
bothplentyI	plentyF	36.800		1.091	33.72	<2e-16	***

In the column "Estimate" there are estimated population means in particular group and the p-value in a last column has clear interpretation: Is the population mean significantly different from zero? The 95% confidence interval for any population mean can be obtained by

confint(AResults2)

```
2.5 % 97.5 % bothlittleI littleF 15.61191 19.98809 bothlittleI mediumF 28.01191 32.38809 bothlittleI plentyF 44.21191 48.58809 bothplentyI littleF 19.21191 23.58809 bothplentyI mediumF 35.61191 39.98809 bothplentyI plentyF 34.61191 38.98809
```

Homework:

File Fuel_Consumption.RData contains data about the effects of two types of gas (factor A) and three different additives (factor B) on overall fuel consumption. The experiment is carried in a balanced design (equal number of observations for all factor combinations) and is repeated twice for each combination of factors. Consider a model with interaction and repeat the same exercises as before.