I Chci dokázat $S_T = S_{AB} + S_E$

v modelu je $y_{irs} = m_{..} + [ab]_{rs} + e_{irs}$

$$y_{irs} - m_{..} = [ab]_{rs} + e_{irs}$$

proto $S_T = \sum_{r=1}^{R}\sum_{s=1}^{S}\sum_{i=1}^{n}(y_{irs}-m_{..})^2 = \sum_r\sum_s\sum_i \left([ab]_{rs} + e_{irs}\right)^2$

$$= \sum_r\sum_s\sum_i \left([ab]_{rs}^2 + 2[ab]_{rs}e_{irs} + e_{irs}^2\right) =$$

$$= \sum_r\sum_s n.[ab]_{rs}^2 + 2\sum_r\sum_s [ab]_{rs}\cdot\overbrace{\sum_i e_{irs}}^{0} + \sum_r\sum_s\sum_i e_{iss}^2 =$$

$$= \sum_r\sum_s n[ab]_{rs}^2 + \sum_r\sum_s\sum_T e_{irs}^2 =$$

$$= S_{AB} + S_E$$

$$\left(\sum_{i=1}^{n} e_{isr} = \sum_i (y_{irs} - m_{rs}) = n\cdot\frac{1}{n}\sum_i y_{irs} - \sum_i m_{rs} = n.m_{rs} - n\,m_{rs} = 0\right)$$

$$S_T = S_{AB} + S_E$$

II Chci dokázat, že $S_{AB} = S_A + S_B + S_{A\times B}$

v modelu je $[ab]_{rs} = (ab)_{rs} + a_r + b_s$ efekt buněk = ef. interakcí + hlavní efekty

$$S_{AB} = \sum_r\sum_s n.[ab]_{rs}^2 = \sum\sum n.\left((ab)_{rs} + a_r + b_s\right)^2 = n\sum_r\sum_s\left[(ab)_{rs}^2 + a_r^2 + b_s^2 + 2(ab)_{rs}a_r + 2(ab)_{rs}b_s + 2a_rb_s\right]$$

$$= n\sum\sum(ab)_{rs}^2 + n.\sum_r a_r^2\cdot S + n\sum_s b_s^2.R + 2n\sum_r a_r\cdot\overbrace{\sum_s(ab)_{rs}}^{0} + 2n\sum_s b_s\overbrace{\sum_r(ab)_{rs}}^{0} + 2n\sum_r a_r\overbrace{\sum_s b_s}^{0}$$

$$= S_{A\times B} + S_A + S_B$$

$$S_{AB} = S_{A\times B} + S_A + S_B$$

Vedlejší podmínky: $\sum_{s=1}^{S} b_s = 0$; $\sum_{s=1}^{S}(ab)_{rs} = 0$; $\sum_{r=1}^{R} a_r = 0$; $\sum_{s=1}^{S}(ab)_{rs} = 0$.

z I a II plyne $\boxed{S_T = S_{A\times B} + S_A + S_B + S_E}$

Pozn. ad II: odvození pro vyvážené třídění!