

vedejaši podmnožina \$S_A\$: \$r = \text{počet skupin (úrovní faktorů)}\$
 $n = n_1 + \dots + n_r$

Musí platit $L = \sum_{i=1}^r n_i \cdot (M_i - M) = 0$

každá skupina

a) pro \$r=2\$

$$\begin{aligned} L &= n_1 \cdot (M_1 - M) + n_2 \cdot (M_2 - M) = n_1 \cdot \left[\frac{1}{n_1} \cdot \sum_{j=1}^{n_1} x_{1j} - \frac{1}{n_1+n_2} \cdot \left(\sum_{j=1}^{n_1} x_j + \sum_{j=1}^{n_2} y_j \right) \right] + \\ &\quad + n_2 \cdot \left[\frac{1}{n_2} \cdot \sum_{j=1}^{n_2} y_j - \frac{1}{n_1+n_2} \cdot \left(\sum_{j=1}^{n_1} x_j + \sum_{j=1}^{n_2} y_j \right) \right] = \\ &= \sum x_j - \frac{n_1}{n_1+n_2} (\sum x_j + \sum y_j) + \sum y_j - \frac{n_2}{n_1+n_2} (\sum x_j + \sum y_j) = \\ &= (\sum x_j + \sum y_j) \cdot \left[1 - \frac{n_1}{n_1+n_2} - \frac{n_2}{n_1+n_2} \right] = (\sum x_j + \sum y_j) \cdot \frac{n_1+n_2 - n_1 - n_2}{n_1+n_2} = 0 \\ &= \underline{\underline{0}} \end{aligned}$$

b) pro \$r \geq 3\$

$$\begin{aligned} \sum_{i=1}^r n_i (M_i - M) &= \sum_{i=1}^r n_i \left[\frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij} - \frac{1}{n} \sum_{i=1}^r \sum_{j=1}^{n_i} x_{ij} \right] = \\ &= \sum_{i=1}^r \left[\sum_{j=1}^{n_i} x_{ij} - \frac{n_i}{n} \sum_{i=1}^r \sum_{j=1}^{n_i} x_{ij} \right] = \sum_{i=1}^r \sum_{j=1}^{n_i} x_{ij} - \frac{n_1}{n} \sum_{i=1}^r \sum_{j=1}^{n_i} x_{ij} + \\ &\quad + \sum_{i=2}^r \sum_{j=1}^{n_i} x_{ij} - \frac{n_2}{n} \sum_{i=1}^r \sum_{j=1}^{n_i} x_{ij} + \dots + \sum_{i=r}^r \sum_{j=1}^{n_i} x_{ij} - \frac{n_r}{n} \sum_{i=1}^r \sum_{j=1}^{n_i} x_{ij} = \\ &= \sum \sum x_{ij} - \frac{n_1}{n} \sum \sum x_{ij} - \dots - \frac{n_r}{n} \sum \sum x_{ij} = \\ &= \sum \sum x_{ij} \cdot \left[1 - \frac{n_1}{n} - \frac{n_2}{n} - \dots - \frac{n_r}{n} \right] = \sum \sum x_{ij} \cdot \frac{n_1 + \dots + n_r - n_1 - \dots - n_r}{n} = 0 \end{aligned}$$